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DECOMPOSITION-BASED EVOLUTIONARY COMPUTING IN MULTICRITERIA OPTIMIZATION ENVIRONMENT

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The paper presents strategies for implementing decomposition based genetic algorithms in multicriteria design optimization. The decomposition approach requires that the system design problem be partitioned into smaller sized subsystems, and the system solution obtained as a combination of the solutions from the subsystems. The absence of gradient information in a genetic algorithm based search strategy requires alternative methods for communicating the design information in different subsystems. Two newly developed methods referred to as experiential inheritance and interspecies migration were used to coordinate the solutions of subsystems in the decomposition based approach. Both the weighted sum and weighted minimax methods were explored in the solution to the multicriteria design problem. The proposed strategies were validated through implementation in representative algebraic and structural design problems.

Key words: multicriterion optimization, decomposition-based design

1. Introduction

Many practical design optimization problems require that multiple criteria be considered simultaneously, a task often requiring compromise between conflicting goals. Among the efforts to help in this decision-making process, developing search methods to find Pareto solutions have been central. Pareto optimality (Pareto, 1906), often referred to as "non-dominance" or "non-inferiority" is a mathematical concept that can be defined as a characteristic of multicrieria optimization solutions that cannot yield an improvement in one criterion without adversely affecting another criterion. When using GA's to locate a Pareto optimal solution, it is interesting to note that multiple nondominated solutions can be identified in a single run due to the population-based nature of the search process. A number of interesting multicriteria genetic algorithms have been introduced in recent years (Horn *et al.*, 1994; Srinivas and Deb, 1994, Fonseca and Fleming, 1995; Zitzler and Thiele, 1999; Deb *et al.*, 2002). Most of these approaches do indeed exploit the population based GA or evolutionary algorithm to identify the complete Pareto front in multicriteria optimization problems.

Among the many techniques developed to identify Pareto solutions is the well-known weighted sum method, wherein a sum of weighted criteria values are minimized or maximized. Another well-established method is the weighted minimax method. In minimization problems for example, the weighted minimax method minimizes the criterion that has the maximum value among the weighted multiple criteria. The present paper describes the adaptation of these methods in a decomposition based design optimization strategy for multicriteria problems, with genetic algorithms (GA's) used as the search technique.

Decomposition based design methods have been proposed as a solution to large-scale coupled problems, wherein the original problem is decomposed into a number of smaller, more tractable subproblems (Sobieszczański-Sobieski and Haftka, 1997). The ability to create smaller subproblems that represent the full complexity of the original problem, may allow for parallel processing of solutions and contribute to a better understanding of the problem domain. In order that the optimal solution to the original design problem is obtained through solutions of several smaller sized subproblems, solution coordination is necessary to account for any interactions among the decomposed subproblems. When using traditional gradient-based optimization methods, such interactions are typically considered on the basis of sensitivity information. Many problems, specifically those involving discrete and integer design variables are not amenable to such an approach due to the lack of gradient information; other problems may suffer from the existence of multiple relative optima and require that global search algorithms be used instead. The GA has emerged as a leading global search method that increases the probability of identifying the global optimum in such generically difficult optimization problems. The issue of solution coordination, however, requires special attention in the context of genetic search.

Two strategies, referred to as the experiential inheritance strategy and interspecies migration method (Ryoo and Hajela, 2002), have been proposed to facilitate the coordination in GA driven decomposition based design. As in a traditional decomposition approach, each subproblem is assigned a subset of design variables and constraints. In each subproblem, GA based searches are conducted in parallel (co-evolution) and the aforementioned strategies are used to communicate changes in a subproblem to other subproblems. Experiential inheritance compares the shared design variable values from two different subproblems and modifies the survival chances of each on the basis of its ability to generate a globally coordinated compatible design. The interspecies migration method directly carries information from one subproblem to another where the information is needed for evaluation of objectives and/or constraints. In the present paper, the experiential inheritance and interspecies migration methods have been extended to coordinate a global solution in multicriteria design optimization problems.

2. Communication based on experiential inheritance and interspecies migration

The following mathematical representation describes a coupled design problem where the design variable vector $\boldsymbol{X} = \{X_S, X_A, X_B\}$ can be categorized into three subsets of variables; subsets X_A and X_B contain design variables encountered only in constraints g_A and g_B, respectively

$$\begin{array}{rl} \text{minimize} & f = f(X_S, X_A, X_B) \\ \text{with respect to} & g_A = g_A(X_S, X_A) \leqslant 0 \\ & g_B = g_B(X_S, X_B) \leqslant 0 \end{array} \tag{2.1}$$

The subset X_S represents shared variables common to both of these constraints. In this representation, all of design variables may be used in the objective. The optimization problem can be decomposed into two subproblems as follows

$$\begin{array}{rl} \text{minimize} & f_A = f_A(X_S, X_A, X_B^*) \\ \text{with respect to} & g_A = g_A(X_S, X_A) \leqslant 0 \\ & g_B = g_B^* \leqslant 0 \\ \\ \text{minimize} & f_B = f_B(X_S, X_A^*, X_B) \\ \text{with respect to} & g_A = g_A^* \leqslant 0 \\ & g_B = g_B(X_S, X_B) \leqslant 0 \end{array} \tag{2.2}$$

In GA-based search, these two subproblems can be assigned their respective subpopulations and co-evolved in parallel. Note that the variables with asterisk are referred to as migration variables, and are not included in the chromosomal representation of the design for a particular subproblem. Therefore, these variables do not participate in the crossover and mutation operations. The constraints with asterisk are referred to as migration constraints, and are not calculated within the subproblem but are rather carried from other subproblems. These two subproblems cannot be solved independently as a unique value for X_S is required. There is a need, therefore, for coordination between the solutions to the two subproblems. To coordinate a global solution, both experiential inheritance and interspecies migration are used to communicate information between the subproblems.

In experiential inheritance approach, the fitness of an individual design in a particular subproblem is evaluated in terms of the objective and constraint functions of that particular subproblem. The shared variables of this individual design are compared to those from a different subproblem, and this comparison used to modify the objective values of the former.

In addition to the experiential inheritance method, another strategy referred to as interspecies migration method was used to carry information from one subproblem to another. In Eq. (2.2), objectives using variables with asterisk should be calculated using information from other subproblems; similarly, constraint values with asterisk should be carried from other subproblems. In the context of co-evolution, when the evaluation of an individual design in subproblem A requires information from subproblem B, the interspecies migration method uses a binary tournament selection method to choose an individual from this subpopulation B and uses the selected individual's information for the evaluation of the individual in subpopulation A.

The co-evolution process is schematically shown in Fig. 1. As can be seen from this figure, a population pool with modified objective values is created in both subpopulations and referred to as the experiential inheritance population. This pool serves as the selection source for all experiential inheritance and interspecies migration operations during the co-evolution process.

For the first generation of evolution, no comparison of shared variables is performed and the modified objective has the same value as the original objective. In subsequent cycles, the designs and their associated modified objective values comprise an experiential inheritance population and preserved for use in the next generation of evolution. For example, for evaluation of every individual in one population, the interspecies migration method selects information from the experiential inheritance population of another subproblem. Similarly, for comparison of every individual in one population, experiential inheritance operation chooses a comparison mate from the experiential inheritance po-

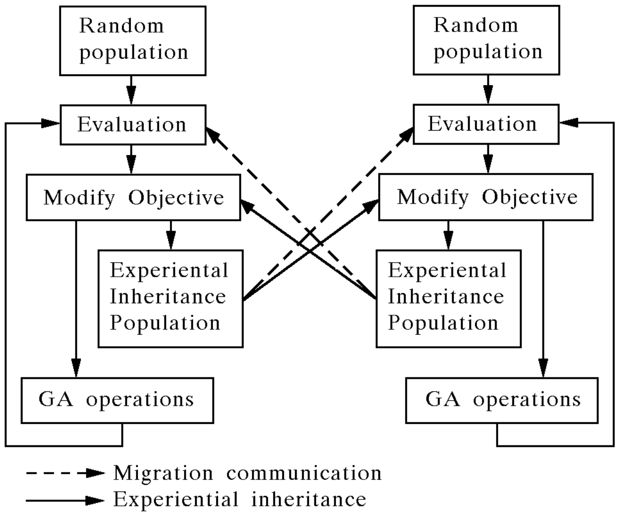

Fig. 1. Schematic illustration of co-evolution

pulation of another subproblem. This is repeated for each individual in all subproblems. Since the experiential inheritance population includes results of both local evaluation and those obtained through comparison, the information in this population is indicative of the global performance of the individual.

Three distinct issues are pertinent in the experiential inheritance approach. These include the selection of the two individuals for comparison, the metric for comparing these individuals, and the algorithm to modify their original objective values.

The binary tournament selection was used to choose an individual to participate in the comparison process. From an experiential inheritance population, two individuals were selected and the better of these used for comparison.

The difference of real values of the variables was used as a metric for comparison of shared design variables; this metric, referred to as the difference measure, was defined as follows

$$DM = \sum_{i=1}^{n} \alpha_i |x^A{}_{Si} - x^B{}_{Si}| \tag{2.3}$$

where x_{Si} indicates shared design variables, n is the number of shared design variables, x^A indicates that the value is from current subproblem A and x^B indicates that the value is from the experiential inheritance population of subproblem B. The coefficients α_i are weighting factors. For example, if x_1 varies from 0.0001 to 0.0002 while x_2 varies from 2 to 3, simple addition of differences may de-emphasize the difference in x_1. Proper coefficient values α_i would balance the significance of x_1 and x_2.

Following a comparison of individuals, the objective values were modified and resulted in changing the survival chance of the individual. In this study, the difference between two designs (DM) was appended as a penalty to the objective function; in a function minimization problem, decreasing the penalty would ensure greater compatibility between the shared design variables of the two subproblems. The modified objective function is as follows

$$\text{modified objective} = \text{objective} + \gamma\, DM \tag{2.4}$$

where the coefficient γ should be increased as DM decreases to improve the matching continually. Note that the objective term, as defined in Eq. (2.4), may include any penalty term associated with the violation of the design constraints.

3. Elitist communication and consistency of selection

Before the GA operations (crossover and mutation) of the local population, the best performer is identified and preserved for the next generation; the best performer in an experiential inheritance population is also similarly identified. These individuals are then matched against each other for experiential inheritance and migration communication exclusively.

In this study, only one tournament selection was performed for the experiential inheritance and interspecies migration. Therefore, the shared design variable values for comparison, the migration design variables for objective function, and the constraint values are from one individual in an experiential inheritance population. In this way, combining favorable factors from various designs is avoided; such an approach may otherwise generate unrealistic designs.

Figure 2 shows how the combination of favorable factors occurs if independent selections were allowed for the experiential inheritance and interspecies migration.

Experiential inheritance takes place between two designs which have $x_1 = 0$ and interspecies migration takes place with a design which has $x_2 = 0$ in experiential inheritance population B. Although this combination gives a compatible design in terms of shared design variable values and also gives a low objective function value in subproblem A, the combination of $x_1 = 0$ and

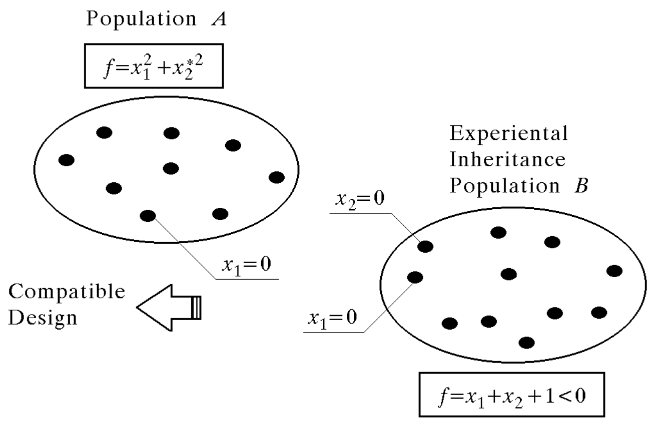

Fig. 2. Combination of favorable factors

$x_2 = 0$ in subproblem B violates its local constraint. The consistency of selection in the comparison process can help eliminate this unrealistic combination.

A stepwise description of the process used in this study is outlined below.

Step 1. Generate populations randomly for each subproblem.

Step 2. For each individual in subproblem populations, find a comparison mate by performing binary tournament selection from experiential inheritance populations (of another subproblem) defined in Step 5; the tournament selection is based on the modified objective value. The preserved elitist individuals in Step 6 are specifically matched against each other instead of using the tournament selection. For the first generation of co-evolution, perform random selection.

Step 3. Evaluate the objective values of each population (note that the objective includes an appended measure of constraint violation). When the evaluation needs a migration design variable value and/or constraint value, use the values associated with the comparison mate identified as described in Step 2.

Step 4. Use the experiential inheritance approach to obtain modified objective values.

Step 5. Associate design variables and constraint values with the modified objective value computed in Step 4 to form experiential inheritance populations. These experiential inheritance populations are used in Step 2 where comparison mates are identified.

Step 6. Perform ordinary genetic algorithm operations with modified objective values and go to Step 2. Repeat until a prescribed termination criterion has been satisfied.

4. Multi-criteria design optimization in decomposition based environment

A multi-criteria design problem can be represented as below

$$\begin{array}{rll} \text{minimize} & f_i = f_i(X_S, X_A, X_B) & i = 1, \dots, k \\ \text{with respect to} & g_A = g_A(X_S, X_A) \leqslant 0 & \\ & g_B = g_B(X_S, X_B) \leqslant 0 & \end{array} \tag{4.1}$$

Both the weighted sum and the weighted minimax approaches were implemented in the solution to this problem in a decomposition-based design environment. Using the weighted sum method, Eq. (4.1) can be decomposed into two subproblems as follows

$$\begin{array}{rl} \text{minimize} & F_A = \displaystyle\sum_{i=1}^{k} w_i f_{A_i}(X_S, X_A, X_B^*) \\ \text{with respect to} & g_A = g_A(X_S, X_A) \leqslant 0 \\ & g_B = g_B^* \leqslant 0 \\ \text{minimize} & F_B = \displaystyle\sum_{i=1}^{k} w_i f_{B_i}(X_S, X_A^*, X_B) \\ \text{with respect to} & g_A = g_A^* \leqslant 0 \\ & g_B = g_B(X_S, X_B) \leqslant 0 \end{array} \tag{4.2}$$

where F_A and F_B represent the weighted sums of the multi-criteria in each subproblem. Note that the weights associated with the same criterion in both subproblems are identical. Experiential inheritance and interspecies migration methods were applied to communicate between subproblems decomposed as in Eqs. (4.2).

When the weighted minimax method is applied to a decomposition problem with multiple criteria, the problem can be represented as follows

$$\begin{array}{rl} \text{minimize} & \displaystyle\max_{i=1,k} w_i f_{A_i}(X_S, X_A, X_B^*) \\ \text{with respect to} & g_A = g_A(X_S, X_A) \leqslant 0 \\ & g_B = g_B^* \leqslant 0 \\ \text{minimize} & \displaystyle\max_{i=1,k} w_i f_{B_i}(X_S, X_A^*, X_B) \\ \text{with respect to} & g_A = g_A^* \leqslant 0 \\ & g_B = g_B(X_S, X_B) \leqslant 0 \end{array} \tag{4.3}$$

Experiential inheritance and interspecies migration methods were applied to this problem structure as defined earlier.

5. Numerical examples

The proposed strategies have been implemented in three algebraic problems and a truss design problem.

A simple algebraic problem (Problem 1) with one shared variable is stated as follows

$$\begin{array}{rll} \text{minimize} & f_1 = x_0 + 3\mathrm{e}^{x_1} & f_2 = x_0 + 2x_2^2 \\ \text{with respect to} & g_1 = x_0 - 2x_1 - 2 \leqslant 0 & \\ & g_2 = 1 - x_0 + x_2 \leqslant 0 & \\ \text{with} & 0 \leqslant x_0 \leqslant 1 & -1.0 \leqslant x_1 \leqslant -0.5 \\ & -1.0 \leqslant x_2 \leqslant 0 & \end{array} \tag{5.1}$$

This problem can be decomposed into two subsystems as follows

$$\begin{array}{rll} \text{minimize} & f_{A1} = x_0 + 3\mathrm{e}^{x_1} & f_{A2} = x_0 + 2x_2^{*2} \\ \text{with respect to} & g_1 = x_0 - 2x_1 - 2 \leqslant 0 & g_2 = g_2^* \leqslant 0 \\ \text{with} & 0 \leqslant x_0 \leqslant 1 & -1.0 \leqslant x_1 \leqslant -0.5 \\ \text{minimize} & f_{B1} = x_0 + 3\mathrm{e}^{x_1^*} & f_{B2} = x_0 + 2x_2^2 \\ \text{with respect to} & g_1 = g_1^* \leqslant 0 & g_2 = 1 - x_0 + x_2 \leqslant 0 \\ \text{with} & 0 \leqslant x_0 \leqslant 1 & -1.0 \leqslant x_2 \leqslant 0 \end{array} \tag{5.2}$$

The optimal solution to this problem results in a convex Pareto front, and both of the proposed solution strategies are expected to yield the correct solutions. The experiential inheritance and interspecies migration strategies were implemented to obtain a coordinated solution from the decomposed subsystems.

Another algebraic problem (Problem 2) as described below was also tested to validate the proposed approach; the optimal solution to this problem results in a non-convex Pareto front

$$\begin{array}{rll} \text{minimize} & f_1 = x_0 & f_2 = x_1 + x_2 \\ \text{with respect to} & g_1 = 1 - (x_0 - 1)^2 - (x_1 - 1)^2 \leqslant 0(10) & \\ & g_2 = 1 - (x_0 - 2)^2 - (x_2 - 2)^2 \leqslant 0 & \\ \text{with} & 2 \leqslant x_0 \leqslant 3 & 1 \leqslant x_1 \leqslant 2 \\ & 2 \leqslant x_2 \leqslant 3 & \end{array} \tag{5.3}$$

The problem was decomposed into two subproblems as follows

$$
\begin{array}{rll}
\text{minimize} & f_{A1} = x_0 & f_{B2} = x_1 + x_2^* \\
\text{with respect to} & g_1 = 1 - (x_0 - 1)^2 - (x_1 - 1)^2 \leqslant 0 & \\
 & g_2 = g_2^* \leqslant 0 & \\
\text{with} & 2 \leqslant x_0 \leqslant 3 & 1 \leqslant x_1 \leqslant 2 \\
 & & \\
\text{minimize} & f_{A1} = x_0 & f_{B2} = x_1^* + x_2 \\
\text{with respect to} & g_1 = g_1^* \leqslant 0 & \\
 & g_2 = 1 - (x_0 - 2)^2 - (x_0 - 2)^2 \leqslant 0 & \\
\text{with} & 2 \leqslant x_0 \leqslant 3 & 2 \leqslant x_2 \leqslant 3
\end{array}
\tag{5.4}
$$

As in the previous example, both the weighted sum and the minimax approaches were used in the proposed decomposition environment.

An algebraic problem with three subsystems (Problem 3) was used as below in order to test the approach in the multi-subsystem environment

$$
\begin{array}{rll}
\text{minimize} & f_1 = \dfrac{1}{16(x_0 + x_1 + x_2 + x_3)^2} & \\
 & f_2 = \dfrac{4}{x_0 + x_1 + x_2 + x_3} & \\
\text{with respect to} & g_1 = x_0 + 3x_1 - 4 \geqslant 0 & \\
 & g_2 = x_0 + x_2 - 2 \geqslant 0 & \\
 & g_3 = 3x_0 + x_3 - 3 \geqslant 0 & \\
\text{with} & 0 \leqslant x_0 \leqslant 2 & 0 \leqslant x_1 \leqslant 2 \\
 & 0 \leqslant x_2 \leqslant 2 & 0 \leqslant x_3 \leqslant 2
\end{array}
\tag{5.5}
$$

The problem was decomposed into three subproblems as follows

$$
\begin{array}{rll}
\text{minimize} & f_1 = \dfrac{1}{16(x_0 + x_1 + x_2^* + x_3^*)^2} & \\
 & f_2 = \dfrac{4}{x_0 + x_1 + x_2^* + x_3^*} & \\
\text{with respect to} & g_1 = x_0 + 3x_1 - 4 \geqslant 0 & \\
 & g_2 = g_2^* \geqslant 0 & g_3 = g_3^* \geqslant 0 \\
\text{with} & 0 \leqslant x_0 \leqslant 2 & 0 \leqslant x_1 \leqslant 2
\end{array}
$$

$$
\begin{aligned}
\text{minimize} \quad & f_1 = \frac{1}{16(x_0 + x_1^* + x_2 + x_3^*)^2} \\
& f_2 = \frac{4}{x_0 + x_1^* + x_2 + x_3^*} \\
\text{with respect to} \quad & g_1 = g_1^* \geqslant 0 \qquad g_2 = x_0 + x_2 - 2 \geqslant 0 \\
& g_3 = g_3^* \geqslant 0 \\
\text{with} \quad & 0 \leqslant x_0 \leqslant 2 \qquad 0 \leqslant x_2 \leqslant 2
\end{aligned}
\tag{5.6}
$$

$$
\begin{aligned}
\text{minimize} \quad & f_1 = \frac{1}{16(x_0 + x_1^* + x_2^* + x_3)^2} \\
& f_2 = \frac{4}{x_0 + x_1^* + x_2^* + x_3} \\
\text{with respect to} \quad & g_1 = g_1^* \geqslant 0 \qquad g_2 = g_2^* \geqslant 0 \\
& g_3 = 3x_0 + x_3 - 3 \geqslant 0 \\
\text{with} \quad & 0 \leqslant x_0 \leqslant 2 \qquad 0 \leqslant x_3 \leqslant 2
\end{aligned}
$$

As in the previous examples, both the weighted sum and the minimax approaches were used in the proposed decomposition environment.

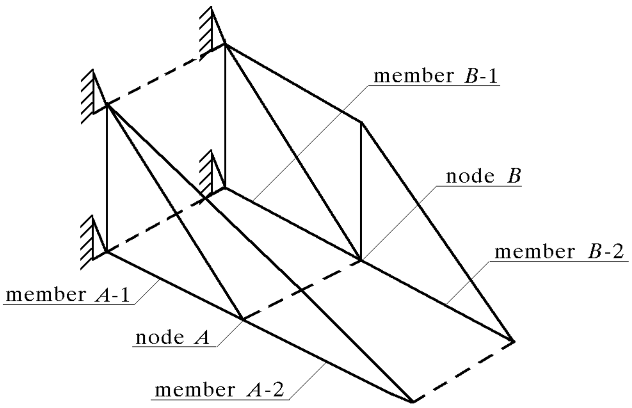

Fig. 3. Global structure of truss problem

A truss system (see Figure 3) with multiple design criteria (Problem 4) was considered as the third test problem. The weight of the truss structure and the nodal displacements were minimized, and constraints on stress levels in the bar elements were imposed in the design problem.

The structure can be decomposed into two substructures. One substructure has a 7 bar truss as shown in Fig. 4a and the other has a 4 bar truss as shown in Fig. 4b. Two substructures are under independent boundary conditions and loads. However, the cross-sectional area of truss members, $A-1$ and $B-1$, and of $A-2$ and $B-2$ should be set equal for manufacturing convenience. In

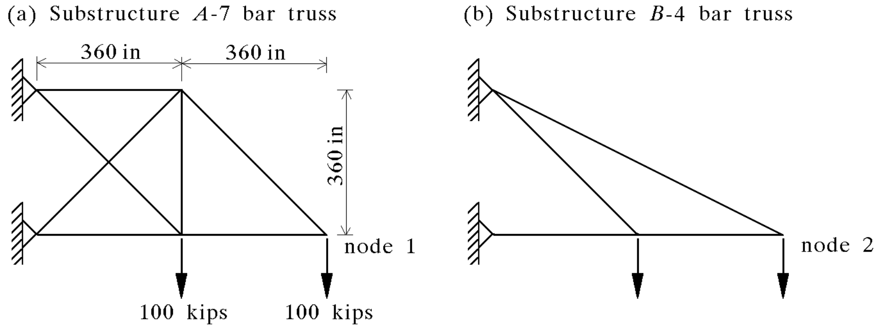

Fig. 4. Sub structures of truss problem

addition, horizontal locations of node A and node B, which can move freely horizontally, are to be the same; these are considered as shared design variables for the problem. Stresses in the truss members were restricted to $172 \cdot 10^3$ Pa. Young's modulus was given as $6.9 \cdot 10^8$ Pa and material density was set to $2768\,\mathrm{kg/m^3}$. In this problem, the weight of the structure and the average of the nodal displacement of node 1 and node 2 were minimized.

6. Results and discussions

For each of the test problems, multiple simulations were conducted to account for the random nature of the search process. Figure 5 shows the simulation results for Problem 1 using the weighted sum approach in the proposed decomposition environment. The solid line represents the criteria values of designs with the active constraints and Pareto front.

The results obtained using the weighted minimax methods in the decomposition based environment are shown in Fig. 6. In each case, simulations with three different initial populations were conducted and the results from different initial populations are marked with different symbols. The weight factors used for f_1 in the weighted sum method varied from 0 to 1 in increments of 0.1, and the sum of two weights was set to unity. The weight factors for f_1 in the weighted minimax method varied from 0.3 to 0.7 in increments of 0.05, and the sum of weights was set to unity. For each simulation, the number of function evaluations was restricted to 100000. Table 1 shows the objective values of a test simulation of Problem 1 from an initial population. The search process was able to converge to the known Pareto optimal front of solutions.

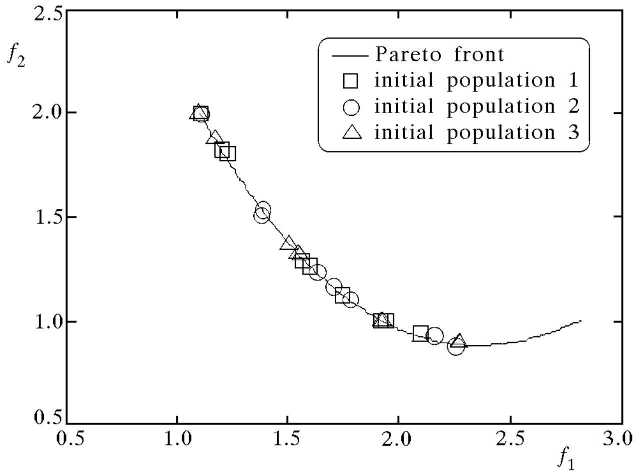

Fig. 5. Simulation results of Problem 1 using weighted sum approach

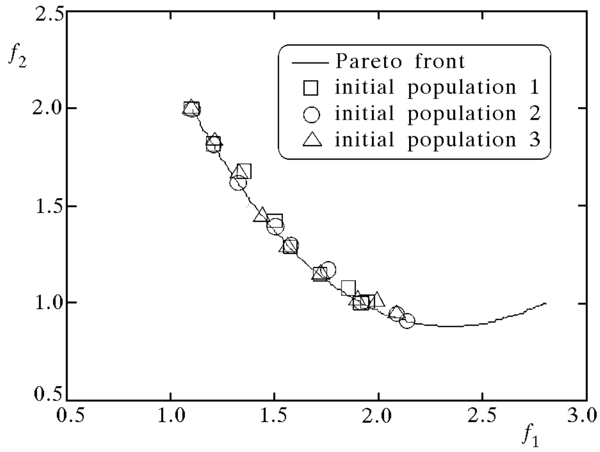

Fig. 6. Simulation results of Problem 1 using weighted minimax approach

The shaded area in Fig. 7 represents the feasible region for Problem 2. The optimal front for this problem is non-convex and was correctly identified in simulation using the minimax method. Simulation results using the weighted sum method for this problem were only able to find two Pareto optimal solutions for various combinations of weights. These were designs with $f_1 = 3$, $f_2 = 2$, and $f_1 = 2$, $f_2 = 4$ located at the extremities of the Pareto front, demonstrating the inappropriateness of the weighted sum method in such situations. The weighted minimax method was used with the weight factors for f_1 varying between 0.5 and 0.7 in increments of 0.01, and the sum of we-

Table 1. Simulation results of Problem 1

Weighted sum			Weighted minimax		
w_1	f_1	f_2	w_1	f_1	f_2
0.0	1.916	0.999	0.3	2.112	0.932
0.1	1.942	0.992	0.35	1.917	1.006
0.2	2.099	0.933	0.4	1.775	1.182
0.3	1.749	1.120	0.45	1.576	1.287
0.4	1.566	1.289	0.5	1.500	1.390
0.5	1.598	1.263	0.55	1.322	1.666
0.6	1.228	1.799	0.6	1.183	1.912
0.7	1.207	1.824	0.65	1.152	1.964
0.8	1.104	2.0	0.7	1.104	2.0
0.9	1.104	2.0			
1.0	1.104	2.0			

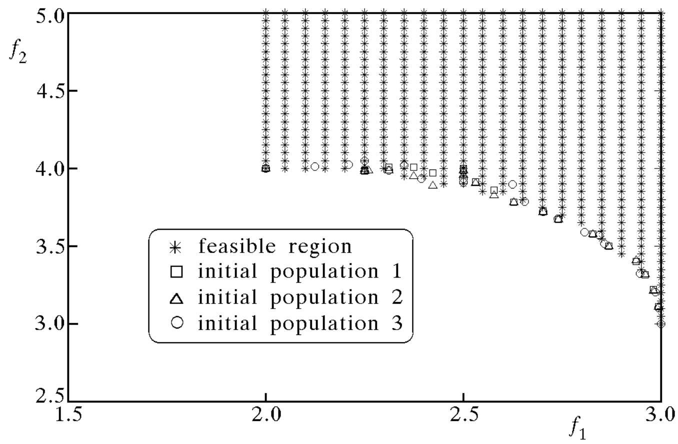

Fig. 7. Simulation results of Problem 2 using weighted minimax method

ights set to unity. Three different initial populations were used and marked respectively. The weighted minimax method produced a smooth and complete Pareto front. A total of 100000 function evaluations were allowed in the search process. Objective values of Problem 2 from an initial population are given in Table 2.

Table 2. Simulation results of Problem 2

Weighted minimax					
w_1	f_1	f_2	w_1	f_1	f_2
0.5	3.0	3.0	0.61	2.531	3.912
0.51	2.995	3.111	0.62	2.5	4.001
0.52	2.981	3.221	0.63	2.424	4.094
0.53	2.961	3.317	0.64	2.25	3.983
0.54	2.938	3.403	0.65	2.5	3.985
0.55	2.868	3.501	0.66	2.312	4.006
0.56	2.828	3.576	0.67	2.375	4.006
0.57	2.741	3.672	0.68	2.25	3.982
0.58	2.703	3.72	0.69	2.25	4.248
0.59	2.629	3.782	0.70	2.0	4.0
0.6	2.578	3.862			

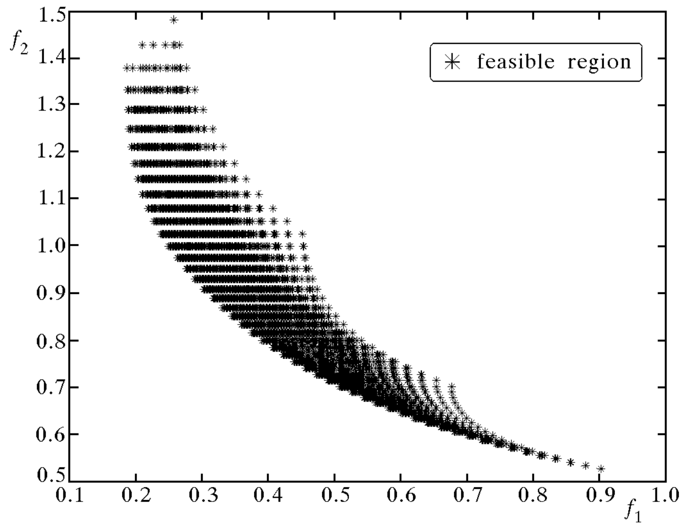

Fig. 8. Feasible region of Problem 3

The feasible region for Problem 3 was shaded with asterisk in Fig. 8. The optimal front was identified in simulation using both of the weighted sum and weighted minimax methods and was represented in Fig. 9 and Fig. 10, respectively. The weight factors of f_1 for both approaches varied between 0.0 and 1.0 in increments of 0.1. Three different initial populations were used and marked respectively. A total of 100000 function evaluations were allowed in the search process. Objective values of Problem 3 from an initial population are given in Table 3.

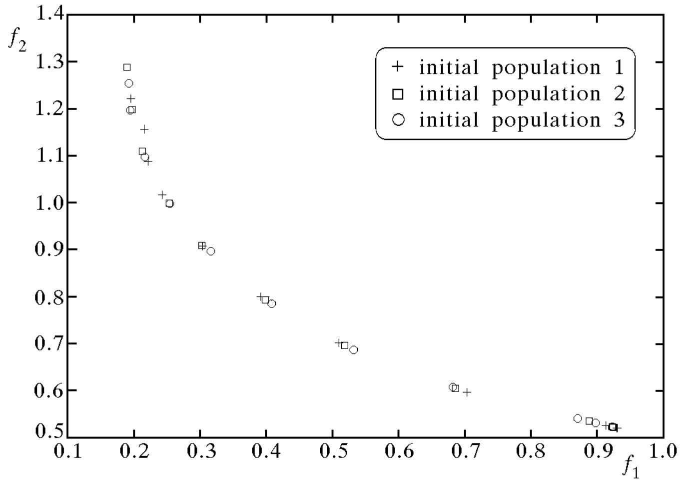

Fig. 9. Simulation results of Problem 3 using weighted sum method

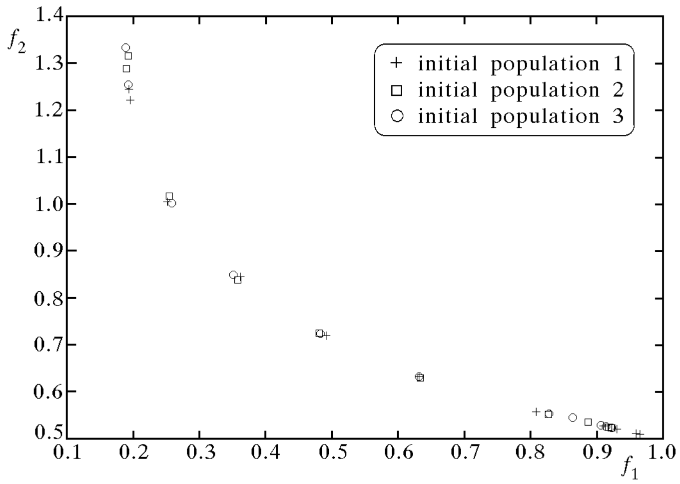

Fig. 10. Simulation results of Problem 3 using weighted minimax method

Table 3. Simulation results of Problem 3

w_1	Weighted sum		Weighted minimax	
	f_1	f_2	f_1	f_2
0.0	0.9308	0.5194	0.9308	0.5194
0.1	0.9132	0.5251	0.9600	0.5107
0.2	0.9299	0.5196	0.9654	0.5091
0.3	0.7036	0.5961	0.9108	0.5260
0.4	0.5099	0.7008	0.8091	0.5563
0.5	0.3913	0.7995	0.6319	0.6319
0.6	0.3039	0.9074	0.4918	0.7184
0.7	0.2427	1.0168	0.3619	0.8446
0.8	0.2211	1.0869	0.2511	1.0043
0.9	0.2153	1.1554	0.1931	1.2445
1.0	0.1956	1.2206	0.1956	1.2206

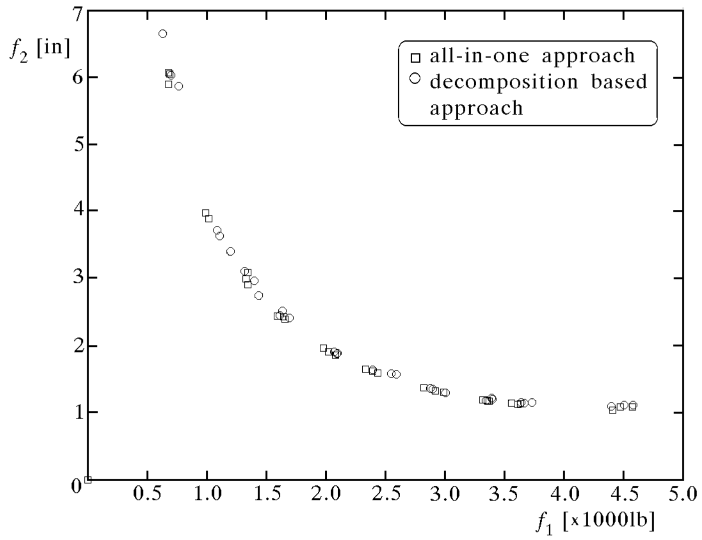

Fig. 11. Simulation results of Problem 4 using weighted sum method

Figure 11 shows the results from the all-in-one solution and decomposition-based approach for the truss problem (Problem 4) using the weighed sum method. Similarly, Fig. 12 shows the results of simulations using the weighted minimax method. Three simulations with different initial populations were used and the weights for f_1 were varied from 0 to 1 in increments of 0.1 for all cases. A total of 20000 function evaluations were allowed in the search process. The search results were comparable for both the all-in-one and decomposition-based methods; both the weighted sum and the weighted minimax approaches identified almost identical solutions. Table 4 shows objective values of Problem 4 for each case.

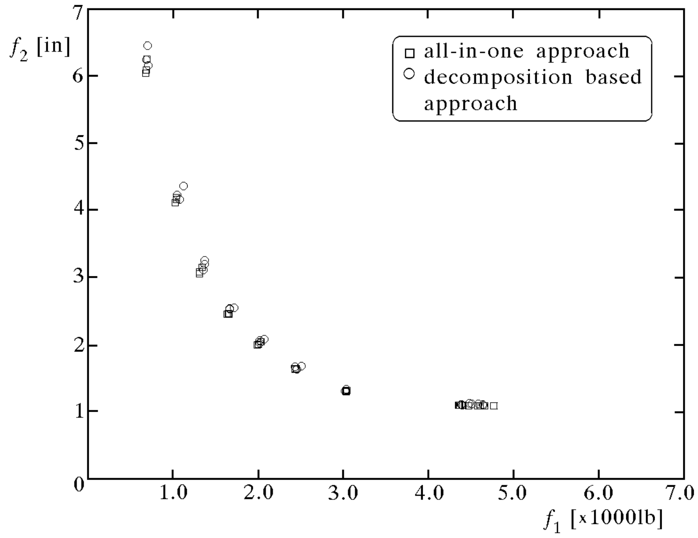

Fig. 12. Simulation results of Problem 4 using weighted minimax method

Table 4. Simulation results of Problem 4

w_1	All-in-one approach				Decomposition-based approach			
	Weighted sum		Weighted minimax		Weighted sum		Weighted minimax	
	f_1 [lb]	f_2 [in]	f_1 [lb]	f_2 [in]	f_1 [lb]	f_2 [in]	f_1 [lb]	f_2 [in]
0.0	4580	1.09	4580	1.09	4510	1.11	4511	1.11
0.1	3562	1.14	4771	1.09	3732	1.15	4637	1.10
0.2	3371	1.17	4303	1.09	3397	1.21	4394	1.11
0.3	2825	1.37	3032	1.30	2881	1.36	3021	1.30
0.4	2439	1.59	2438	1.64	2395	1.64	2457	1.63
0.5	2026	1.91	2039	2.04	2099	1.89	2074	2.07
0.6	1596	2.44	1636	2.45	1695	2.41	1664	2.54
0.7	1345	3.09	1315	3.08	1439	2.75	1371	3.20
0.8	994	3.98	1039	4.16	1110	3.63	1053	4.22
0.9	683	5.90	685	6.04	699	6.04	711	6.16
1.0	339	13.22	334	13.42	313	14.51	313	14.51

7. Closing remarks

The paper examines new strategies for adapting a GA search in a decomposition-based multicriteria design environment. To identify Pareto solutions in a multicriteria design problem, two well-established methods, namely the weighted sum and weighted minimax methods, were implemented in a decomposition-based approach. The approach requires communication between the decomposed subproblems so as to direct the search towards a globally compatible solution. These communication strategies are based on the previously developed mechanism of experiential inheritance and interspecies migration, developed specifically for GA implementations in the decomposition-based design. The decomposition based approach is successfully extended to incorporate multicriteria design problems; the proposed strategies are validated through numerical experiments conducted with algebraic and structural design problems. The numerical examples considered include problems involving both convex and non-convex Pareto fronts.

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Rachunek ewolucyjny w obszarze wielokryterialnej optymalizacji oparty na zagadnieniu dekompozycji

Streszczenie

W pracy zaprezentowano metodę zastosowania genetycznych algorytmów opartych na zagadnieniu dekompozycji w zadaniu wielokryterialnej optymalizacji obiektu. Zagadnienie dekompozycji wymaga rozbicia danego zadania na mniejsze podproblemy i znalezienia cząstkowych rozwiązań, by w efekcie otrzymać rozwiązanie ogólne na podstawie wcześniej wyznaczonych cząstkowych. Brak gradientowego charakteru informacji w metodzie poszukiwania rozwiązania opartej na algorytmie genetycznym skłania do zastosowania alternatywnej metody przekazu informacji pomiędzy obszarami rozbitych grup problemowych. W zagadnieniu dekompozycji użyto dwie nowosformułowane metody określone mianem dziedziczenia eksperymentalnego i migracji międzygatunkowej. W poszukiwaniu rozwiązania zadania wielokryterialnej optymalizacji obiektu wykorzystano metody sumy ważonej i wartości min-max. Zaproponowane strategie postępowania zweryfikowano na reprezentatywnych modelach algebraicznych i projektowych.

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