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MODELLING AND OPTIMIZATION OF TRANSMISSION SYSTEMS WITH AN ASYNCHRONOUS MOTOR

Arkadiusz Mężyk

Department of Applied Mechanics, Silesian University of Technology, Gliwice
e-mail: mezyk@polsl.gliwice.pl

Mathematical models of an electromechanical system with an induction motor are dealt with in the paper. An electric motor model is described in the equivalent axial coordinate system (x, y). The mathematical model of the gear train has been formulated by means of a hybrid method of rigid and deformable finite elements. Special attention has been paid to problems of modelling of planetary gears. The sensitivity analysis and optimization have been carried out using an objective function describing the maximal forces in kinematic pairs. The applied algorithm of direct differentiation makes it possible to examine the influence of parameters of the electromechanical model on the dynamic phenomena occurring in the system. From the performed investigations it appears that, when modelling the mechanisms under consideration, it is necessary to take electromechanical couplings into account.

Key words: mathematical model, gear train, electromechanical system, sensitivity analysis, optimization

1. Introduction

Because of high requirements to be met by modern driving systems of working machines, the necessity arises to reduce masses of the systems and to assure transmission of high powers at the same time. It also happens that excitation frequencies are situated in resonance zones of the systems. The effect of the design parameters on the dynamic phenomena in the system can be directly determined in the simplest cases only. In addition, the constructed models become more complicated, when systems of complex physical nature, e.g. electromechanical systems, are described. A great number of such systems composed of an electric motor and of a multi-stage gear train or planetary gear are incorporated in drives of working machines. In such cases it turns out that the use of simulation results for modifying the dynamic characteristics is a complicated and labour-consuming procedure. Methods of the sensitivity analysis and optimization can serve for aiding the investigations performed, because this approach to the problem proves to be effective as well as professional software packages and the computer technology are being developed (Haug *et al.*, 1986; Haftka *et al.*, 1990; Haug and Aurora, 1979; Kleiber *et al.*, 1995). Initially, the optimization procedures found applications for dealing with static problems, such as e.g. minimization of costs, mass, overall dimensions or maximization of transmitted power, etc. At present, the sensitivity analysis and optimization are more and more often used to solve complicated problems of dynamic machinery systems and to aid the process of designing (see Haug and Aurora, 1979; Mężyk, 1994; Zeman and Hlaváč, 2001). This approach is very efficient, especially in cases, in which it is necessary to assure high operational parameters, high durability and reliability and to comply with a number of limitations ensuing from technical conditions and safety requirements (Sobieszczański-Sobieski and Haftka, 1997). Dynamic properties of a system are one of the factors determining its durability and reliability. The use of the optimization method allows these properties to be effectively selected (Mężyk, 1994; Mężyk and Świtoński, 2001; Zeman and Hlaváč, 2001). A function that describes eigenvalues of a system serves as an objective function being used most often during optimization-oriented investigations. The problem is usually involved in the optimization of the objective function in the frequency domain. The optimization can be carried out for objective functions being described both in the frequency and time domain. In the latter case, difficulties crop up because it is necessary to analyse the time-varying objective function. It entails the necessity of numerical solving of differential equations of motion of the system. However, the thus formulated problem makes it possible to make the calculations when considering the phenomena of energy dissipation, the state of the external load in the system as well as the effect of coupling between the electrical drive and the mechanical subsystem.

This paper presents an algorithm of calculations that constitutes an effective tool for aiding the process of designing high-power electromechanical driving systems with an asynchronous motor. The application of the modelling, sensitivity analysis and optimization for selecting the design parameters allows the dynamic characteristics of complex machinery systems, adequate from the point of view of minimization of the reactions in kinematic pairs and of decreasing of the vibration level, to be obtained.

2. Problem formulation

The problem of excessive vibration level occurs in many technical applications. When this is a case, the high durability and reliability of a system cannot be assured during operation under a complex dynamic state of load, even though the most precise design methods have been employed. It is possible to solve this problem by modelling and analysing the dynamic characteristics, which depend on the structure of the system and on its design parameters. The optimization by using objective functions described in the time domain is employed in the paper for minimization of the dynamic forces of electromechanical driving systems in unsteady states (start-up, sudden change of the load). The purpose of the investigations is realised through minimization of the peak values of time courses of the dynamic reactions in selected kinematic pairs, e.g. by means of the following optimization problem

$$\mathrm{Min}\,\psi = P_{max}^2(\boldsymbol{b},t) = [k_i \varDelta q_i(\boldsymbol{b},t)]^2 \tag{2.1}$$

where P is the calculated value of the dynamic force in the selected kinematic pair of the system, k_i is the stiffness coefficient in ith kinematic pair, t is time, $\boldsymbol{b}$ is the vector of design variables, $\varDelta q_i$ is the relative generalized displacement between two nodes of the model.

The optimization procedure uses basic parameters of a structural state space model as design variables (e.g. inductances, resitances, moments of inertia, stiffness coefficients, damping coefficients). Minimization of the vibration level is obtained by a proper selection of the design features of the system under consideration. The procedure enables the optimal selection of an active control procedure as well, but this case has not been considered during the numerical simulations. The block diagram of the applied algorithm of calculation is presented in Fig. 1 and described in the paper.

3. Modelling of an electromechanical driving system

For carrying out the investigations of dynamic phenomena of complex electromechanical systems an assumption of a physical model of the real object under consideration is needed. Both mechanical and electrical parts of the drive are dynamic systems coupled one with another. When analysing the dynamic phenomena, especially in unsteady states, it is necessary to use a model, which enables the realisation of an electromechanical coupling. The model of

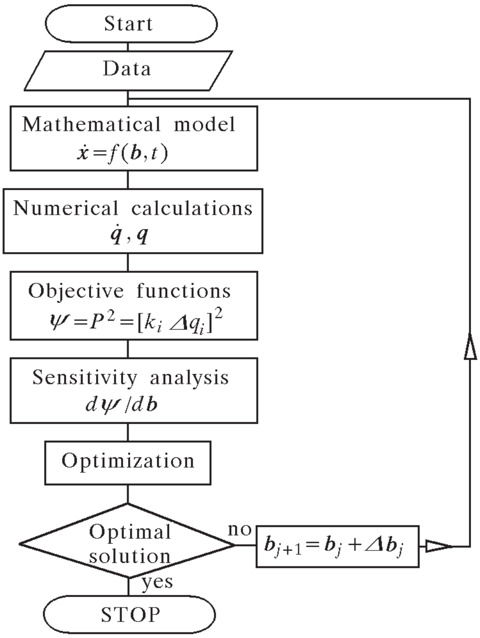

Fig. 1. Block diagram of the algorithm of optimization of the dynamic properties; $\boldsymbol{x}$ is the vector of state variables, $\boldsymbol{q}$ is the vector of generalized coordinates

the electromechanical driving system with an induction motor is described by a system of differential equations whose matrix form is as follows

$$
\begin{aligned}
&\mathbf{M}\ddot{\boldsymbol{q}} + \mathbf{C}_v\dot{\boldsymbol{q}} + \mathbf{K}\boldsymbol{q} = \boldsymbol{Q} \\
&\frac{d}{dt}\mathbf{L}\mathbf{i} + \mathbf{R}\mathbf{i} = \mathbf{U} \\
&M_{el} = \frac{1}{2}\mathbf{i}^\top \frac{\partial}{\partial\varphi_1}\mathbf{L}\mathbf{i}
\end{aligned}
\tag{3.1}
$$

where: $\mathbf{M}$, $\mathbf{C}_v$, $\mathbf{K}$ are the matrices of inertia, damping and stiffness, respectively, $\boldsymbol{q}$ is the vector of generalized coordinates, $\boldsymbol{Q}$ is the vector of generalized forces, $\mathbf{L}$, $\mathbf{R}$, $\mathbf{i}$, $\mathbf{U}$ are the matrices of inductance, resistance, currents and supply voltages, respectively, M_{el} is the torque of the motor, φ_1 is the angular displacement of the rotor.

The electromechanical coupling is effected via the joint solution to equations of motion (3.1) of the model, coupled with the angular displacement of the rotor φ_1, determined from the model of the mechanical subsystem, and via the electromagnetic moment M_{el}, calculated from the model of the electric motor.

When describing the physical model by means of the state coordinates, the system of differential equations is as follows

$$
\begin{aligned}
\dot{\boldsymbol{x}}(t) &= \mathbf{A}\boldsymbol{x}(t) + \mathbf{B}\boldsymbol{u}(t)\\
\boldsymbol{y}(t) &= \mathbf{C}\boldsymbol{x}(t) + \mathbf{D}\boldsymbol{u}(t)
\end{aligned}
\tag{3.2}
$$

where $\mathbf{A}$ is the matrix of the system, $\mathbf{B}$ is the matrix of inputs, $\mathbf{C}$ is the matrix of outputs, $\mathbf{D}$ is the matrix of direct effects of the input vector $\boldsymbol{u}(t)$ on the output vector $\boldsymbol{y}(t)$.

The matrices $\mathbf{A}$ and $\mathbf{B}$ of the model of the mechanical system, described by equations (3.2), are determined in the following way

$$
\mathbf{A} = -\mathbf{S}_1^{-1}\mathbf{S}_2 \qquad\qquad \mathbf{B} = \mathbf{S}_1^{-1} \tag{3.3}
$$

where $\mathbf{S}_1$, $\mathbf{S}_2$ are auxiliary block matrices in the form

$$
\mathbf{S}_1 = \begin{bmatrix} \mathbf{0} & \mathbf{M} \\ \mathbf{M} & \mathbf{C}_V \end{bmatrix} \qquad\qquad \mathbf{S}_2 = \begin{bmatrix} -\mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix} \tag{3.4}
$$

Such a form of equations of the model is suitable for a uniform description of the considered class of electromechanical systems and allows the control of the system to be taken into account.

4. Model of an induction motor

The dynamic forces in kinematic pairs depend, to a considerable degree, on the characteristics and power of the driving motor. The dynamic properties of the electric motor are usually described by means of circuit models with a number of circuits being dependent on the assumed accuracy of the representation of a real object (Kopylov, 1984; Vas, 1992). A proper selection of coordinate systems can produce further simplification. One of the biaxial coordinate systems used to analyse the induction machines is a system of the axial coordinates (x, y) rotating round a stator at an angular velocity ω_x.

When constructing the model of an induction motor, a number of simplifying assumptions is made. The following are the most important ones:

- the dependences between the magnetic induction in the gap and the specific electric loading are linear,

- the system under consideration is a holonomic system,
- the influence of higher harmonics of the motor magnetic field is so little that can be ignored,
- the magnetic induction field in the gap is distributed sinusoidally along the perimeter of the motor gap,
- the motor is supplied with a sinusoidal voltage,
- the generalized forces are the voltages supplying the stator and the loading moment of the motor,
- energy dissipation is equal to losses in the resistances of electric circuits of the machine and to possible internal friction losses,
- friction occurring in the system is of a viscous character.

The equations of the mathematical model, described in the biaxial coordinate system (x, y), take the following form in a general matrix notation

$$\begin{bmatrix} \mathbf{U}_s \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{SS} & \mathbf{R}_{SR} \\ \mathbf{R}_{RS} & \mathbf{R}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{i}_S \\ \mathbf{i}_R \end{bmatrix} + \begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{SR}^{\top} & \mathbf{L}_{RR} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \mathbf{i}_S \\ \mathbf{i}_R \end{bmatrix} \tag{4.1}$$

The electromagnetic moment is determined by the expression

$$M_{el} = pL_m(i_{mx}i_{sy} - i_{my}i_{sx}) \tag{4.2}$$

A model with two substitute circuits in the rotor is sufficient for an asynchronous squirrel-cage motor (see Fig. 2).

In such a case particular quantities and matrices existing in the equations (4.1) and (4.2) are determined as follows:

— magnetizing current

$$i_{mx} = i_{sx} + i_{rx} + i_{px} \qquad\qquad i_{my} = i_{sy} + i_{ry} + i_{py} \tag{4.3}$$

— vector of voltages supplying the stator

$$\mathbf{U}_S = [U_{sx}, U_{sy}]^{\top} \qquad\qquad \mathbf{0} = [0, 0, 0, 0]^{\top} \tag{4.4}$$

— current vectors of the stator and the rotor

$$\mathbf{i}_S = [i_{sx}, i_{sy}]^{\top} \qquad\qquad \mathbf{i}_R = [i_{rx}, i_{ry}, i_{px}, i_{py}]^{\top} \tag{4.5}$$

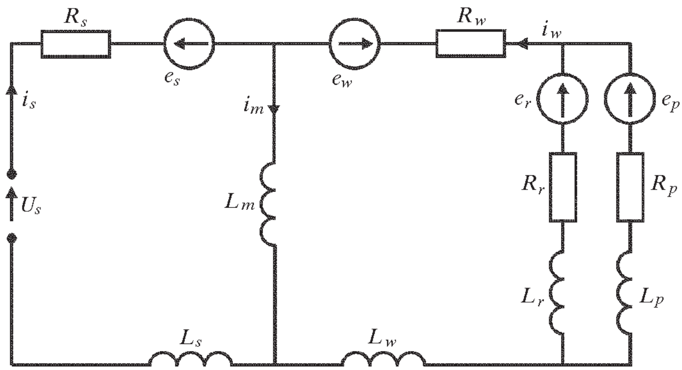

Fig. 2. Circuit model of an inductance motor with two substitute circuits in the rotor; e_s, e_w, e_r, e_p are rotation voltages in circuits of the stator and rotor

— matrices containing the resistances and inductances

$$\mathbf{L}_{SS} = \begin{bmatrix} L_m + L_s & 0 \\ 0 & L_m + L_s \end{bmatrix} \qquad \mathbf{L}_{SR} = \begin{bmatrix} L_m & 0 & L_m & 0 \\ 0 & L_m & 0 & L_m \end{bmatrix}$$

$$\mathbf{L}_{RR} = \begin{bmatrix} L_r + L_w + L_m & 0 & L_w + L_m & 0 \\ 0 & L_r + L_w + L_m & 0 & L_w + L_m \\ L_w + L_m & 0 & L_p + L_w + L_m & 0 \\ 0 & L_w + L_m & 0 & L_p + L_w + L_m \end{bmatrix}$$

$$\mathbf{R}_r = \begin{bmatrix} R_r + R_w & 0 & R_w & 0 \\ 0 & R_r + R_w & 0 & R_w \\ R_w & 0 & R_p + R_w & 0 \\ 0 & R_w & 0 & R_p + R_w \end{bmatrix}$$

$$\mathbf{R}_s = \begin{bmatrix} R_s & 0 \\ 0 & R_s \end{bmatrix} \qquad \mathbf{\Omega}_s = \begin{bmatrix} 0 & -\omega_x \\ \omega_x & 0 \end{bmatrix} \tag{4.6}$$

$$\mathbf{\Omega}_r = \begin{bmatrix} 0 & -(\omega_x - \omega_1) & 0 & 0 \\ \omega_x - \omega_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(\omega_x - \omega_1) \\ 0 & 0 & \omega_x - \omega_1 & 0 \end{bmatrix}$$

$$\mathbf{R}_{SS} = \mathbf{R}_s + \mathbf{\Omega}_s \mathbf{L}_{SS} \qquad \mathbf{R}_{SR} = \mathbf{\Omega}_s \mathbf{L}_{SR}$$

$$\mathbf{R}_{RR} = \mathbf{R}_r + \mathbf{\Omega}_r \mathbf{L}_{RR} \qquad \mathbf{R}_{RS} = \mathbf{\Omega}_r \mathbf{L}_{SR}^{\top}$$

where L_m, L_s are the inductances of windings connected with the main magnetic flux and with the leakage flux of the stator, L_w, L_r, L_p are the inductances of windings connected with the leakage fluxes, which are common for the two squirrel-cages, a starting cage and an operating cage in stator terms, p is the number of pole pairs, R_s is the resistance of the stator, R_r, R_p are the resistances of the starting cage and of the operating one in stator terms, U_s is the supply voltage, ω_0 is the synchronous velocity, ω_1 is the angular velocity of the rotor.

The major problems, when modeling the induction motor, refer to the determination of parameters of the assumed model because it is impossible to determine these parameters without full access to the technical documentation of the motor.

5. Model of a mechanical system

In general, the mechanical part of a system is formed by gear trains, which are modelled in a discrete or discrete-continuous form, with characteristics of the meshing stiffness, pitch plays, flexibility of bearings, rigidity of shafts and bodies, gyroscopic forces, etc. being taken into account. When beginning the process of modelling, it is necessary to have regard to the fact that the major problem we face with consists in the determination of parameters of the assumed model. Thus, only the properties of the real object, which have the decisive effect on the investigated dynamic phenomena, should be considered. The testing of transverse torsional vibrations needs developing of expanded spacial dynamic models of the system (Zeman and Nemecek, 1995). A spacial model is composed of subsystems. Shafts with gear wheels are usually these subsystems, the interaction of which is realised via interteeth forces or reactive forces of bearings. Such models are usually constructed by a hybrid method, which is a combination of the finite element method with the rigid finite elements (Fig. 3).

When this is a case, the shaft is divided into deformable beam-type finite elements, whereas the gear wheel, the stiffness of which is by several orders higher than the shaft stiffness, is taken as a stiff solid. In the case of elements modelled by the finite element method there will be band inertia and stiffness matrices of the beam elements (see Kruszewski *et al.*, 1984). The rigid finite element method uses non-deformable, rigid finite elements and massless, deformable elastic and damping elements. The matrices of inertia of the rigid finite elements are determined with respect to its principal axes and they are diagonal ones

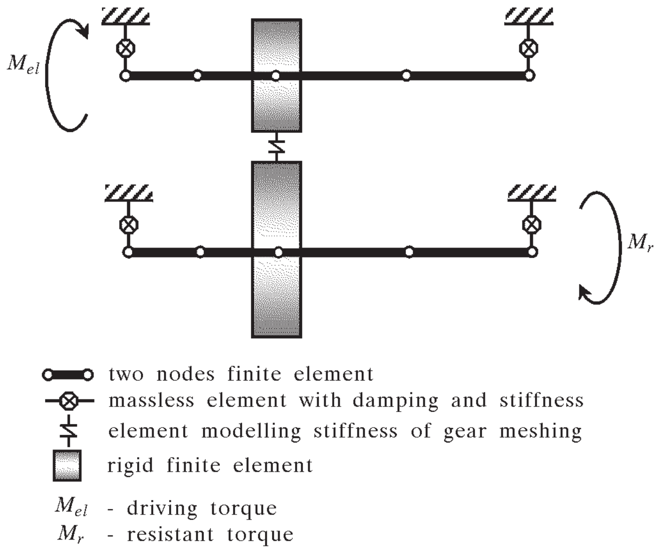

Fig. 3. Physical model of a spur gear pair modelled by the hybrid method

$$\mathbf{M}_i = diag(m_i, m_i, m_i, I_{ix}, I_{iy}, I_{iz}) \tag{5.1}$$

where m_i, I_{ix}, I_{iy}, I_{iz} are the mass and mass moments of inertia of the ith rigid finite element with respect to a system of principal axes.

The blocks of the stiffness matrix of the system which describe the stiffness of elastic and damping elements are obtained in the following way

$$\begin{aligned} \mathbf{K}_{rrk} &= \mathbf{S}_{rk}^{\top}\mathbf{\Theta}_{rk}^{\top}\mathbf{K}_k\mathbf{\Theta}_{rk}\mathbf{S}_{rk} \\ \mathbf{K}_{rpk} &= -\mathbf{S}_{rk}^{\top}\mathbf{\Theta}_{rk}^{\top}\mathbf{K}_k\mathbf{\Theta}_{pk}\mathbf{S}_{pk} \end{aligned} \tag{5.2}$$

where $\mathbf{K}_k$ is the diagonal matrix of stiffness of the kth elastic and damping element with respect to its principal axes, $\mathbf{S}_{rk}$, $\mathbf{S}_{pk}$ are the blocks of fastening of the elastic and damping element to the rigid finite element marked with the numbers r and p, $\mathbf{\Theta}_{rk}$, $\mathbf{\Theta}_{pk}$ are the blocks of the direction factors of the angles between the system of principal axes of the elastic and damping elements and the system of principal axes of the inertia of rigid finite elements marked with the numbers r and p,

The displacement parallel to the system of principal axes of the rigid finite element towards the point of fastening of the elastic and damping element determines the matrix $\mathbf{S}_{rk}$. The transformation matrix $\mathbf{\Theta}_{rk}$ contains cosines of angles between the axes of the coordinate systems.

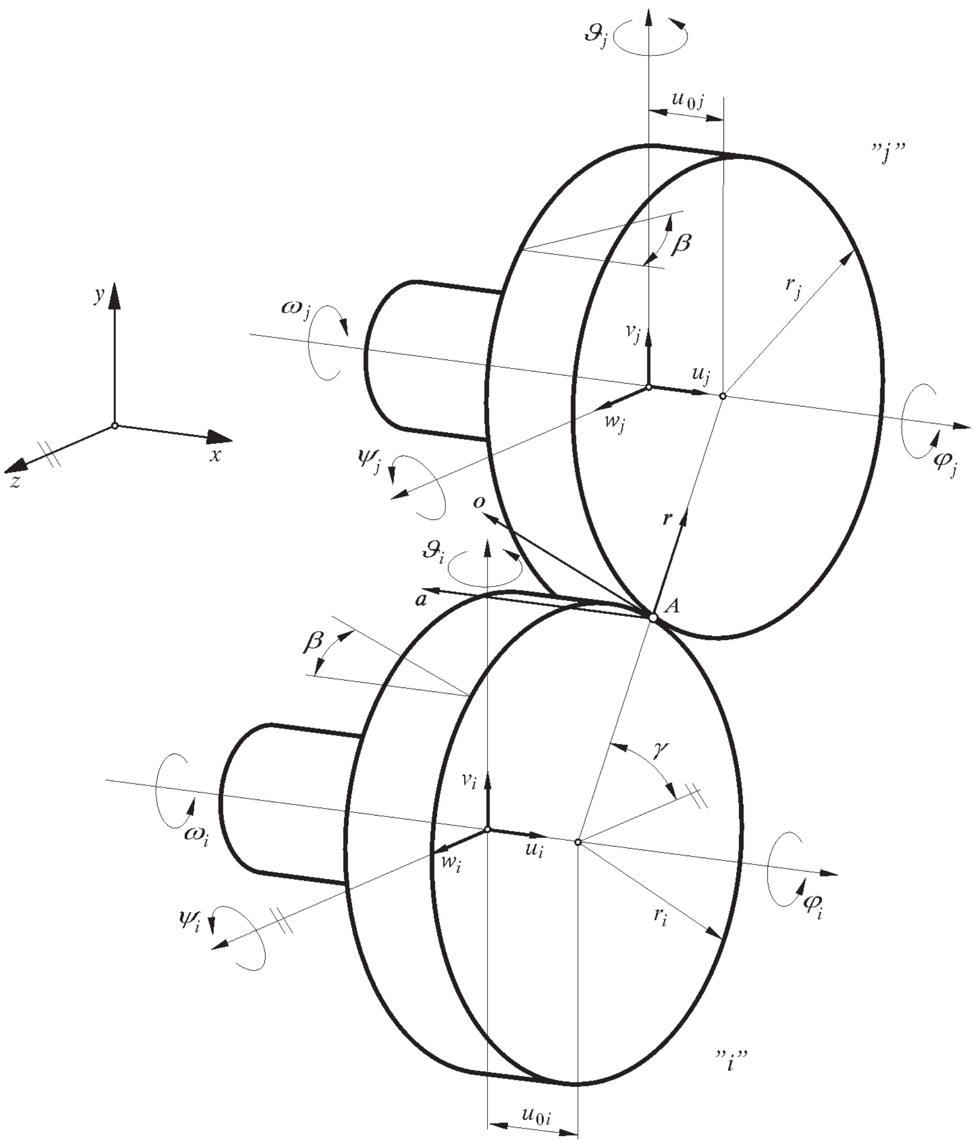

Fig. 4. Single-stage spur gear

In Fig. 4 the following denote: u_i, v_i, w_i, φ_i, ν_i, ψ_i – linear and angular displacement in relation to the coordinate system of the ith node.

The application of the rigid finite element method proves to be very effective for the modelling of gear wheels and shaft bearings.

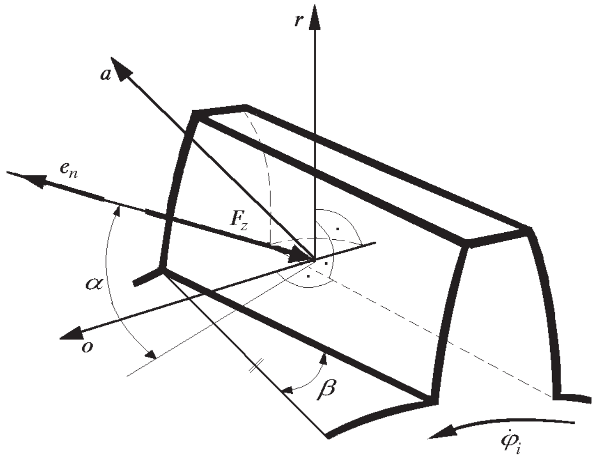

Fig. 5. Forces acting on the tooth of the gear wheel reduced to the contact point

The interaction between the subsystems (Fig. 5) is obtained by the normal teeth forces occurring in the meshings of the mating gear wheels

$$F_z = k_z(\boldsymbol{d}_i - \boldsymbol{d}_j)\boldsymbol{e_n} \tag{5.3}$$

where $\boldsymbol{d}_i$, $\boldsymbol{d}_j$ are the vectors of displacements of the points of contact in the pole of meshing, $\boldsymbol{e_n}$ is the versor of the axis perpendicular to the contact plane in the point of meshing, k_z is the stiffness coefficient of the meshing.

As the model of the system is described by means of generalized coordinates, the interteeth force is to be expressed as a function of these coordinates. To do this, we determine coordinates of the vectors $\boldsymbol{d}_i$, $\boldsymbol{d}_j$ in the cartesian coordinate system ora the origin of which is in the pole of the meshing

$$\begin{aligned}
d_{oi} &= v_i\cos\gamma + w_i\sin\gamma + r_i\varphi_i - u_{oi}(\vartheta_i\sin\gamma - \psi_i\cos\gamma)\\
d_{ri} &= v_i\sin\gamma - w_i\cos\gamma + u_{oi}(\vartheta_i\cos\gamma + \psi_i\sin\gamma)\\
d_{ai} &= -u_i + r_i\psi_i\sin\gamma + r_i\vartheta_i\cos\gamma\\
d_{oj} &= v_j\cos\gamma + w_j\sin\gamma - r_j\varphi_j - u_{oj}(\vartheta_j\sin\gamma - \psi_j\cos\gamma)\\
d_{rj} &= v_i\sin\gamma - w_i\cos\gamma + u_{oj}(\vartheta_j\cos\gamma + \psi_j\sin\gamma)\\
d_{aj} &= -u_j - r_j\psi_j\sin\gamma - r_j\vartheta_j\cos\gamma
\end{aligned} \tag{5.4}$$

where α is the pressure angle (for left turning of the pinion $\alpha = \alpha_n$, for right turning $\alpha = \pi - \alpha_n$), α_n is the nominal presure angle, β is the helix angle, γ is the angle defining the configuration of the system, u_{oi}, u_{oj} are the distances of the meshing pole from nodes of the model referring to the centres of gravity of the first and second gear wheel, r_i, r_j are the radii of the pitch circle of the first and second gear wheel.

The unit vectors expressed in the same coordinate system have the following components

$$\begin{aligned} e_{no} &= \cos\alpha\cos\beta \\ e_{nr} &= \sin\alpha \\ e_{na} &= \cos\alpha\sin\beta \end{aligned} \tag{5.5}$$

When inserting expressions (5.4) and (5.5) into relationship (5.3), the normal force in the meshing is determined in the following way

$$F_z = k_z(\boldsymbol{\delta}_i^\top \boldsymbol{q}_i - \boldsymbol{\delta}_j^\top \boldsymbol{q}_j) \tag{5.6}$$

where $\boldsymbol{\delta}_i$, $\boldsymbol{\delta}_j$ are the vectors of geometric parameters of the gear wheels.

The vectors of geometric parameters of the gear wheels are determined when considering the relationships between the vectors of displacements of the contact points in the pole of meshing as well as between the versor of the axis perpendicular to the meshing plane and generalized coordinates of the nodes in which the gear wheels are situated

$$\begin{aligned} \boldsymbol{\delta}_i &= \begin{bmatrix} -\cos\alpha\sin\beta \\ \sin\alpha\sin\gamma + \cos\alpha\cos\beta\cos\gamma \\ r_i\cos\alpha\sin\beta\sin\gamma + u_{oi}(\sin\alpha\sin\gamma + \cos\alpha\cos\beta\cos\gamma) \\ -\sin\alpha\cos\gamma + \cos\alpha\cos\beta\sin\gamma \\ r_i\cos\alpha\sin\beta\cos\gamma + u_{oi}(\sin\alpha\cos\gamma - \cos\alpha\cos\beta\sin\gamma) \\ r_i\cos\alpha\cos\beta \end{bmatrix} \\ \boldsymbol{\delta}_j &= \begin{bmatrix} -\cos\alpha\sin\beta \\ \sin\alpha\sin\gamma + \cos\alpha\cos\beta\cos\gamma \\ -r_j\cos\alpha\sin\beta\sin\gamma + u_{oj}(\sin\alpha\sin\gamma + \cos\alpha\cos\beta\cos\gamma) \\ -\sin\alpha\cos\gamma + \cos\alpha\cos\beta\sin\gamma \\ -r_j\cos\alpha\sin\beta\cos\gamma + u_{oj}(\sin\alpha\cos\gamma - \cos\alpha\cos\beta\sin\gamma) \\ -r_j\cos\alpha\cos\beta \end{bmatrix} \end{aligned} \tag{5.7}$$

The components of vectors of the generalized coordinates for particular elements of the gear train are taken in the form $\boldsymbol{q} = [u, v, \psi, w, \vartheta, \varphi]^\top$. The derived relationships can be used to determine the matrices of meshing stiffness both for the spure and helical gear as well as for any arrangement of the shafts and gear wheels. The following blocks form a matrix of the meshing stiffness

$$
\begin{aligned}
&\mathbf{K}_{zii} = k_z \boldsymbol{\delta}_i \boldsymbol{\delta}_i^\top \qquad && \mathbf{K}_{zij} = -k_z \boldsymbol{\delta}_i \boldsymbol{\delta}_j^\top \\
&\mathbf{K}_{zji} = -k_z \boldsymbol{\delta}_j \boldsymbol{\delta}_i^\top \qquad && \mathbf{K}_{zjj} = k_z \boldsymbol{\delta}_j \boldsymbol{\delta}_j^\top
\end{aligned}
\tag{5.8}
$$

$$
\mathbf{K}_z = \begin{array}{c}
\left[\begin{array}{c|c|c|c|c}
 & & & & \\ \hline
 & \mathbf{K}_{zii} & & \mathbf{K}_{zij} & \\ \hline
 & & & & \\ \hline
 & \mathbf{K}_{zji} & & \mathbf{K}_{zjj} & \\ \hline
 & & & &
\end{array}\right]
\end{array}
\begin{array}{l}
\\ \}\,\boldsymbol{q_i} \\ \\ \}\,\boldsymbol{q_j} \\ \\
\end{array}
$$

The blocks of matrices of the meshing stiffness are added to the blocks of the global matrix of stiffness of the system in places corresponding to the generalized coordinates of the nodes of the gear wheels.

In the case of gear wheels with moving axles (e.g. planetary gears), the angle γ defining the configuration of the system (Fig. 6) is a time function. When taking a design form of the planetary gear into consideration, a modified dynamic model has been constructed, which enables one to analyse the vibrations of elements that are in rotary motion and in plane motion. The matrix of constraints stiffness of the planetary gear $\mathbf{K}_{pp}$ has been determined in the following way

$$
\mathbf{K}_{pp} = \left[\begin{array}{c|c|c}
\mathbf{K}_c & & \\ \hline
 & & \\ \hline
 & & \mathbf{K}_j
\end{array}\right] + \mathbf{K}_v
\tag{5.9}
$$

where $\mathbf{K}_c$, $\mathbf{K}_j$ are the matrices of stiffness of the bearing of the central gear and of the planet carrier (see Eq. (5.2)), $\mathbf{K}_v$ is the matrix of the meshing stiffness and stiffness of the bearing of the planet wheel.

In the case of meshing of the central gear with a planet wheel it is possible to make use of the relationships that have been deduced for a single-stage gear $(5.7)_1$

$$
\boldsymbol{\delta}_c = \left[\begin{array}{c}
-\cos\alpha\sin\beta \\
\sin\alpha\sin\gamma + \cos\alpha\cos\beta\cos\gamma \\
r_c\cos\alpha\sin\beta\sin\gamma + u_{oc}(\sin\alpha\sin\gamma - \cos\alpha\cos\beta\cos\gamma) \\
-\sin\alpha\cos\gamma + \cos\alpha\cos\beta\sin\gamma \\
r_c\cos\alpha\sin\beta\cos\gamma + u_{oc}(\sin\alpha\cos\gamma + \cos\alpha\cos\beta\sin\gamma) \\
r_c\cos\alpha\cos\beta
\end{array}\right]
\tag{5.10}
$$

$$
\boldsymbol{\delta}_o = \begin{bmatrix} -\cos\alpha\sin\beta \\ \sin\alpha \\ -r_o\cos\alpha\sin\beta + u_{0o}\sin\alpha \\ \cos\alpha\cos\beta \\ -u_{0o}\cos\alpha\cos\beta \\ -r_o\cos\alpha\cos\beta \end{bmatrix}
$$

where r_c, r_o are the radii of the central gear and of the planet wheel.

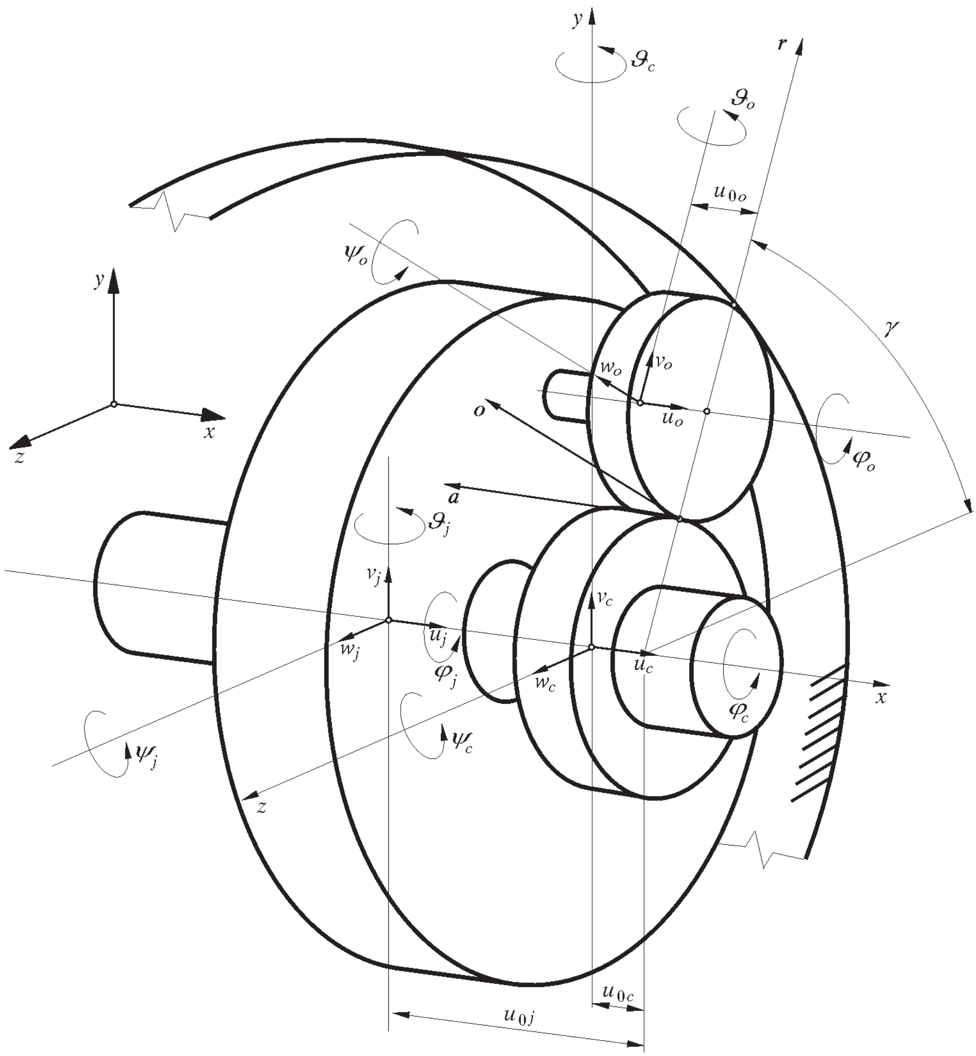

Fig. 6. Model of a planetary gear

The stiffness matrix of the meshing $\mathbf{K}_{vco}$ can be determined when applying Eqs (5.8) and (5.11).

Assuming that the toothed outer ring gear with internal teeth is undeformable, we obtain the following expression for the force in the meshing "planet gear-outer ring gear"

$$
\begin{aligned}
F_{ow} &= k_{ow}\boldsymbol{\delta}_{ow}^{\top}\boldsymbol{q}_o \\
\boldsymbol{\delta}_{ow} &= \begin{bmatrix} \cos\alpha\sin\beta \\ \sin\alpha \\ -r_o\cos\alpha\sin\beta + u_{0o}\sin\alpha \\ -\cos\alpha\cos\beta \\ u_{0o}\cos\alpha\cos\beta \\ -r_o\cos\alpha\cos\beta \end{bmatrix}
\end{aligned}
\tag{5.11}
$$

The matrix of stiffness of this meshing is described by the following relationship

$$
\mathbf{K}_{vow} = k_{ow}\boldsymbol{\delta}_{ow}\boldsymbol{\delta}_{ow}^{\top} \tag{5.12}
$$

In order to determine the matrix of bearing stiffness of the planet wheel $\mathbf{K}_{voj}$, elastic and damping elements are introduced, and they connect the planet wheels with their axles mounted in the planet carrier. The matrices of stiffness of the elastic and damping elements are determined on the basis of relationships (5.2).

Proceeding in this way with a greater number of planet wheels, we determine the stiffness matrix of the constraints existing between the elements of the model of the whole planetary gear

$$
\mathbf{K}_v = \sum_{i=1}^{n}(\mathbf{K}_{vco_i} + \mathbf{K}_{vow_i} + \mathbf{K}_{voj_i}) \tag{5.13}
$$

where n is the number of planet wheels in the gear.

The blocks of matrices (5.9) are inserted into these blocks of stiffness matrix of the whole system which correspond to relevant nodes of the model.

Because of a complicated nature of energy dissipation phenomena it is necessary to make a number of simplifying assumptions when describing these phenomena. Constructing the damping matrix as a linear combination of the matrices of inertia and stiffness is a common practice in the process of modelling of the systems under consideration (see Ginsberg, 2001)

$$\mathsf{C}_v = \alpha_v \mathsf{M} + \beta_v \mathsf{K} \tag{5.14}$$

where α_v, β_v are scalar factors.

The method is very simple to apply and considerably simplifies the analysis of vibrations of systems with a vibration damper. It is often assumed that $\alpha_v = 0$, and β_v is determined on the basis of material constants. Thus, the damping matrix is considered to be proportional to the stiffness matrix.

6. Sensitivity analysis

A finite differences method provides the simplest way to calculate the derivatives of the objective function. This method uses a very simple algorithm. Nevertheless, the results obtained when employing this method can be burdened with numerical errors. The accuracy of this method is sufficient for solving of problems which are not too complicated. The semianalytical methods are more accurate and quicker, but they are more labour-consuming at the stage of data preparation. These methods cover direct and adjoint methods, which are to be mentioned (Haug *et al.*, 1986). The sensitivity analysis of time courses needs solving of a mathematical model of the system. Such a model, described in state coordinates, is conditioned by the selected design variables as well. The equations of motion can be written as follows

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{b}) \qquad\qquad \boldsymbol{x}(t_0) = \boldsymbol{h}(\boldsymbol{b}) \tag{6.1}$$

where $\boldsymbol{x}(t)$ is the vector of state variables, $\boldsymbol{b}$ is the vector of design variables, t_0 is the initial time.

The time t_k defining the moment of occurrence of the tested system state can also be a design variable. When a general case is considered, it is described by the following function

$$F(t_k, \boldsymbol{x}(t_k), \boldsymbol{b}) = 0 \tag{6.2}$$

The objective function can incorporate an integral term related to the description in a certain time interval and a term describing the state of the system at a definite moment of time t_k

$$\psi = g(t_k, \boldsymbol{x}(t_k), \boldsymbol{b}) + \int\limits_{t_0}^{t_k} T(t, \boldsymbol{x}, \boldsymbol{b})\; dt \tag{6.3}$$

When differentiating functional (6.3) with respect to $\boldsymbol{b}$, the following expression is obtained after transformations

$$\frac{d\psi}{d\boldsymbol{b}} = \boldsymbol{G}^\top(t_k,\boldsymbol{x}(t_k),\boldsymbol{b})\frac{\partial \boldsymbol{x}(t_k)}{\partial \boldsymbol{b}} + \frac{\partial g}{\partial \boldsymbol{b}} - $$

$$-\frac{1}{\dot{F}(t_k)}\Big[\frac{\partial g}{\partial t_k} + \frac{\partial g}{\partial \boldsymbol{x}}\boldsymbol{f}(t_k)\Big]\frac{\partial F}{\partial \boldsymbol{b}} + \int\limits_{t_0}^{t_k}\Big(\frac{\partial T}{\partial \boldsymbol{x}}\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{b}} + \frac{\partial T}{\partial \boldsymbol{b}}\Big)\,dt \tag{6.4}$$

$$\boldsymbol{G}^\top(t_k,\boldsymbol{x}(t_k),\boldsymbol{b}) = \Big[\frac{\partial g(t_k)}{\partial \boldsymbol{x}}\Big]^\top - \frac{1}{\dot{F}(t_k)}\Big[\frac{\partial g}{\partial t_k} + \frac{\partial g}{\partial \boldsymbol{x}}\boldsymbol{f}(t_k) + T(t_k)\Big]\Big[\frac{\partial F}{\partial \boldsymbol{x}}\Big]^\top$$

The partial derivatives in expression $(6.4)_1$ can be determined analytically, except for the term $\partial\boldsymbol{x}(t_k)/\partial\boldsymbol{b}$, because $\boldsymbol{x}(t)$ is determined by numerical integration of the equations of motion. A direct differentiation method or an adjoint variable method should be employed to determine the lacking expression.

The direct differentiation method consists in the formulation of an additional initial problem, obtained in consequence of the differentiation of model equations (6.1) with respect to $\boldsymbol{b}$

$$\frac{\partial \dot{\boldsymbol{x}}}{\partial \boldsymbol{b}} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\frac{\partial \boldsymbol{x}}{\partial \boldsymbol{b}} + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{b}} \qquad\qquad \frac{\partial \boldsymbol{x}(t_0)}{\partial \boldsymbol{b}} = \frac{\partial \boldsymbol{h}}{\partial \boldsymbol{b}} \tag{6.5}$$

By solving initial value problem (6.5) we determine values of the derivatives $\partial\boldsymbol{x}(t)/\partial\boldsymbol{b}$ being sought. A method for formulating the equations of the direct differentiation method for an electromechanical model will be presented thereinafter. With a view to the clarity of the description, the vector $\boldsymbol{x}$ is written as composed of two vectors containing the coordinates of the electrical part $\boldsymbol{x}_e$ and of the mechanical one $\boldsymbol{x}_m$, respectively

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_e \\ \boldsymbol{x}_m \end{bmatrix} \tag{6.6}$$

where the subscripts e, m refer to the quantities which describe the models of the electrical system and the mechanical one, respectively.

Thus, equations of motion of the induction motor model (4.1) take the form

$$\dot{\boldsymbol{x}}_e = \underbrace{(\mathbf{L}^*)^{-1}\mathbf{U}_z - (\mathbf{L}^*)^{-1}\mathbf{R}^*\boldsymbol{x}_e}_{\boldsymbol{f}_e(\boldsymbol{x},\boldsymbol{b})}$$

$$\mathbf{L}^* = \begin{bmatrix} \mathbf{L}_{SS} & \mathbf{L}_{SR} \\ \mathbf{L}_{RS} & \mathbf{L}_{RR} \end{bmatrix} \qquad\qquad \mathbf{R}^* = \begin{bmatrix} \mathbf{R}_{SS} & \mathbf{R}_{SR} \\ \mathbf{R}_{RS} & \mathbf{R}_{RR} \end{bmatrix} \tag{6.7}$$

When differentiating system of equations $(6.7)_1$ with respect to the design variables, we obtain the following relationship

$$\frac{\partial \dot{\boldsymbol{x}}_e}{\partial \boldsymbol{b}} = \frac{\partial \boldsymbol{f}_e}{\partial \boldsymbol{b}} + \frac{\partial \boldsymbol{f}_e}{\partial \boldsymbol{x}_e}\frac{\partial \boldsymbol{x}_e}{\partial \boldsymbol{b}} \tag{6.8}$$

where particular matrices of the partial derivatives are determined as

$$\begin{aligned} \frac{\partial \boldsymbol{f}_e}{\partial \boldsymbol{x}_e} &= -(\mathbf{L}^*)^{-1}\Big(\frac{\partial \mathbf{R}^*}{\partial \boldsymbol{x}_e}\boldsymbol{x}_e + \mathbf{R}^*\frac{\partial \boldsymbol{x}_e}{\partial \boldsymbol{x}_e}\Big) \\ \frac{\partial \boldsymbol{f}_e}{\partial \boldsymbol{b}} &= -(\mathbf{L}^*)^{-1}\Big[\frac{\partial \mathbf{L}^*}{\partial \boldsymbol{b}}(\mathbf{L}^*)^{-1}(\mathbf{U}_z - \mathbf{R}^*\boldsymbol{x}_e) + \frac{\partial \mathbf{R}^*}{\partial \boldsymbol{b}}\boldsymbol{x}_e\Big] \end{aligned} \tag{6.9}$$

The form of the derivatives of matrices of the inductance and resistance as well as of the electromagnetic moment of the motor will depend on the simplifications assumed when constructing the model and on the number of the assumed substitute circuits. Similarly, equations of the mathematical model (3.2) are formulated, and the derivatives of coordinates of the mechanical part of the system are determined

$$\begin{aligned} \dot{\boldsymbol{x}}_m &= \underbrace{\mathbf{A}_m\boldsymbol{x}_m + \mathbf{B}_m\boldsymbol{u}_m}_{\boldsymbol{f}_m(\boldsymbol{x},\boldsymbol{b})} \\ \frac{\partial \dot{\boldsymbol{x}}_m}{\partial \boldsymbol{b}} &= \frac{\partial \boldsymbol{f}_m}{\partial \boldsymbol{b}} + \frac{\partial \boldsymbol{f}_m}{\partial \boldsymbol{x}_m}\frac{\partial \boldsymbol{x}_m}{\partial \boldsymbol{b}} \\ \frac{\partial \boldsymbol{f}_m}{\partial \boldsymbol{x}_m} &= \mathbf{A}_m + \mathbf{B}_m\frac{\partial \boldsymbol{u}_m}{\partial \boldsymbol{x}_m} \\ \frac{\partial \boldsymbol{f}_m}{\partial \boldsymbol{b}} &= \frac{\partial \mathbf{A}_m}{\partial \boldsymbol{b}}\boldsymbol{x}_m + \frac{\partial \mathbf{B}_m}{\partial \boldsymbol{b}}\boldsymbol{u}_m + \mathbf{B}_m\frac{\partial \boldsymbol{u}_m}{\partial \boldsymbol{b}} \end{aligned} \tag{6.10}$$

Particular partial derivatives of the matrix of the system are defined by the relationships

$$\begin{aligned} \frac{\partial \mathbf{A}_m}{\partial \boldsymbol{b}} &= -\mathbf{S}_1^{-1}\Big(\frac{\partial \mathbf{S}_1}{\partial \boldsymbol{b}}\mathbf{A}_m + \frac{\partial \mathbf{S}_2}{\partial \boldsymbol{b}}\Big) \\ \frac{\partial \mathbf{B}_m}{\partial \boldsymbol{b}} &= -\mathbf{B}_m\frac{\partial \mathbf{S}_1}{\partial \boldsymbol{b}}\mathbf{B}_m \\ \frac{\partial \mathbf{S}_1}{\partial \boldsymbol{b}} &= \begin{bmatrix} \mathbf{0} & \dfrac{\partial \mathbf{M}}{\partial \boldsymbol{b}} \\ \dfrac{\partial \mathbf{M}}{\partial \boldsymbol{b}} & \dfrac{\partial \mathbf{C}_V}{\partial \boldsymbol{b}} \end{bmatrix} \qquad \frac{\partial \mathbf{S}_2}{\partial \boldsymbol{b}} = \begin{bmatrix} -\dfrac{\partial \mathbf{M}}{\partial \boldsymbol{b}} & \mathbf{0} \\ \mathbf{0} & \dfrac{\partial \mathbf{K}}{\partial \boldsymbol{b}} \end{bmatrix} \end{aligned} \tag{6.11}$$

Because of the couplings existing between the models of the mechanical and electrical parts it is necessary to take the derivatives of conjugate quantities, i.e. the angular velocity of the rotor and the electromagnetic torque of the driving motor into account. The couplings are considered when the following equations have been introduced

$$\begin{aligned}\frac{\partial \boldsymbol{f}_m}{\partial \boldsymbol{x}_e} &= \frac{\partial \mathbf{B}_m}{\partial \boldsymbol{x}_e}\boldsymbol{u}_m + \mathbf{B}_m\frac{\partial \boldsymbol{u}_m}{\partial \boldsymbol{x}_e}\\ \frac{\partial \boldsymbol{f}_e}{\partial \boldsymbol{x}_m} &= -(\mathbf{L}^*)^{-1}\frac{\partial \mathbf{R}^*}{\partial \boldsymbol{x}_m}\boldsymbol{x}_e\end{aligned} \tag{6.12}$$

Having regard to the above considerations, an additional system of differential equations of the direct differentiation method for the electromechanical model with an asynchronous motor can have the following form

$$\begin{aligned}\begin{bmatrix}\dfrac{\partial \dot{\boldsymbol{x}}_e}{\partial \boldsymbol{b}}\\ \dfrac{\partial \dot{\boldsymbol{x}}_m}{\partial \boldsymbol{b}}\end{bmatrix} &= \begin{bmatrix}\dfrac{\partial \boldsymbol{f}_e}{\partial \boldsymbol{x}_e} & \dfrac{\partial \boldsymbol{f}_e}{\partial \boldsymbol{x}_m}\\ \dfrac{\partial \boldsymbol{f}_m}{\partial \boldsymbol{x}_e} & \dfrac{\partial \boldsymbol{f}_m}{\partial \boldsymbol{x}_m}\end{bmatrix}\begin{bmatrix}\dfrac{\partial \boldsymbol{x}_e}{\partial \boldsymbol{b}}\\ \dfrac{\partial \boldsymbol{x}_m}{\partial \boldsymbol{b}}\end{bmatrix} + \begin{bmatrix}\dfrac{\partial \boldsymbol{f}_e}{\partial \boldsymbol{b}}\\ \frac{\partial \boldsymbol{f}_m}{\partial \boldsymbol{b}}\end{bmatrix}\\ \begin{bmatrix}\dfrac{\partial \boldsymbol{x}_e(t_0)}{\partial \boldsymbol{b}}\\ \dfrac{\partial \boldsymbol{x}_m(t_0)}{\partial \boldsymbol{b}}\end{bmatrix} &= \begin{bmatrix}\dfrac{\partial \boldsymbol{h}_e}{\partial \boldsymbol{b}}\\ \dfrac{\partial \boldsymbol{h}_m}{\partial \boldsymbol{b}}\end{bmatrix}\end{aligned} \tag{6.13}$$

The advantage of the direct differentiation method consists in that the algorithm is relatively simple and it is possible to calculate the derivative $\partial \boldsymbol{x}(t)/\partial \boldsymbol{b}$ and $\boldsymbol{x}(t)$ at the same time. However the necessity of solving a great number of additional differentiation equations is a disadvantage.

7. Optimization

The discussed problem of optimization is a problem of a MinMax type, in which maximal values of the objective function will be minimized. Hence, the optimization problem can be formulated in the form

$$\mathrm{Min}\,\mathrm{Max}\,\psi(t) \tag{7.1}$$

When beginning the calculations, we do not know at which step of the calculations the maximal values occur. First of all, it is also necessary to find the maximal value in each iteration and then to state the values of design variables, which minimize the maximal values. One of the methods used to solve the optimization problem of the MinMax type consists in its transformation in the problem of the Min type. In this instance, the variable ψ_{max} is taken as an additional unknown subjected to additional limitations. The most important limitation is that the value of the objective function should not exceed ψ_{max} at each step of calculations. In this way the optimization problem becomes transformed into a sequence of linear optimization tasks. With a very little increase in this parameter it is possible to assume a linear increase in the objective function in the neighbourhood $\boldsymbol{b}$. Optimization problem (2.1) can take a form

$$\begin{aligned} &\mathrm{Min}\,\psi_{max} \\ &\psi_j + \nabla\psi_j \varDelta\boldsymbol{b} - \psi_{max} \leqslant 0 \qquad\qquad j = 1,...,k \end{aligned} \tag{7.2}$$

where k is the number of peak values of the objective function under analysis, ψ_j is the value of the objective function for jth peak value, $\nabla\psi_j$ is a gradient of ψ_j with respect to the design variable, $\varDelta\boldsymbol{b}$ is an increase in the value $\boldsymbol{b}$.

8. Numerical calculations

The model has been constructed on the basis of the technical documentation of a prototype driving system of a real working machine. The system consists of an asynchronous squirrel-cage electric motor without any soft-start system, a multistage gear transmission and a planetary gear. Power transmitted by the system is about 300 kW. The selection of a number of degrees of freedom and the division into finite elements has been carried out when analysing the design form and the predominant forms of vibrations (see Fig. 7).

In Fig. 7 the following designations are: 1,...,31 – numbers of division nodes into finite elements, E_1-E_4 – elastic and damping elements modelling fragments of the shafts, S_1-S_{19} – rigid finite elements.

Simulation and optimization-oriented investigations were carried out in MATLAB environment. The forms of matrices of stiffness and inertia of the model are presented in Fig. 8 and Fig. 9. Because of the character of the matrices of stiffness and inertia which have a large number of nonzero terms, the sparse matrix algebra has been employed. This allowed the required operational memory to be reduced and the numerical calculations to be accelerated.

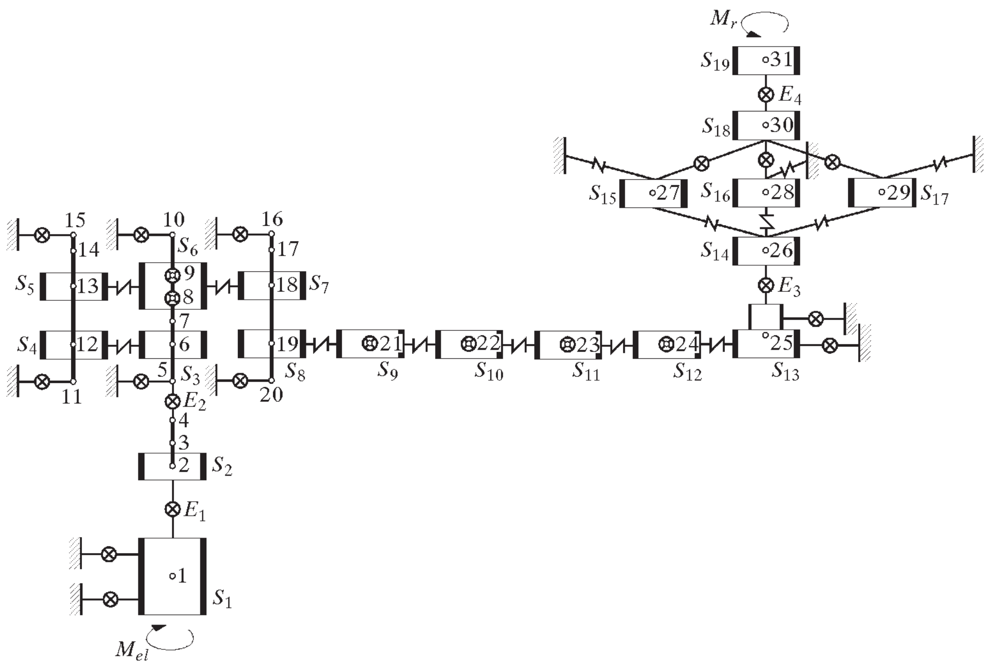

Fig. 7. Dynamical model of transverse torsional vibrations of the transmission system

Further savings are possible by decreasing the number of degrees of freedom. That is why the models with fewer degrees of freedom were utilized at the further stage of the optimization-oriented investigations, whereas the complete model served for verifying the effect the parameters obtained in the process of optimization had on the dynamic characteristics of the system. Lower accuracy in the frequency domain is obtained in the case of neglecting these degrees of freedom, which are of no importance for the objective of the investigations being performed. But, this method makes it possible to maintain a physical sense of the generalized coordinates of the reduced model and of its parameters, which is of vital significance when selecting the objective function in the time domain.

The objective function describing the maximal values in kinematic pairs of the system was applied for the purpose of investigations. The dynamic torque in the main shaft during the start up of the machine was utilized as an objective function selected for the optimization-oriented investigations (see Fig. 11). Three peaks of the torque caused by a change in the electromagnetic torque of the motor are the subject of consideration. Such an objective function

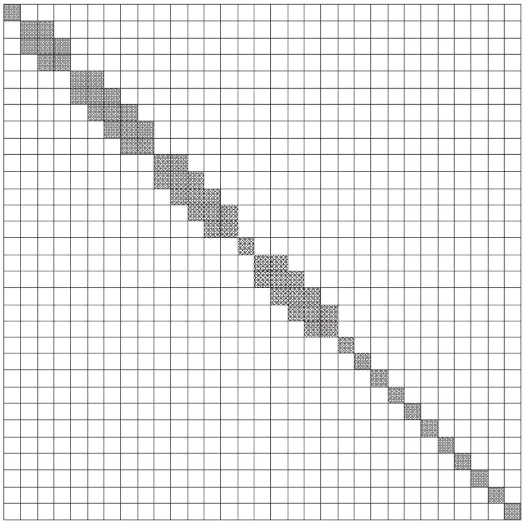

Fig. 8. Nonzero blocks of the inertia matrix referring to particular nodes of the model

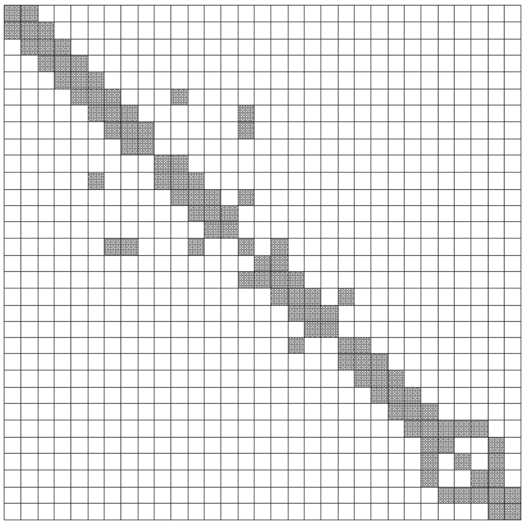

Fig. 9. Nonzero blocks of the stiffness matrix referring to particular nodes of the model

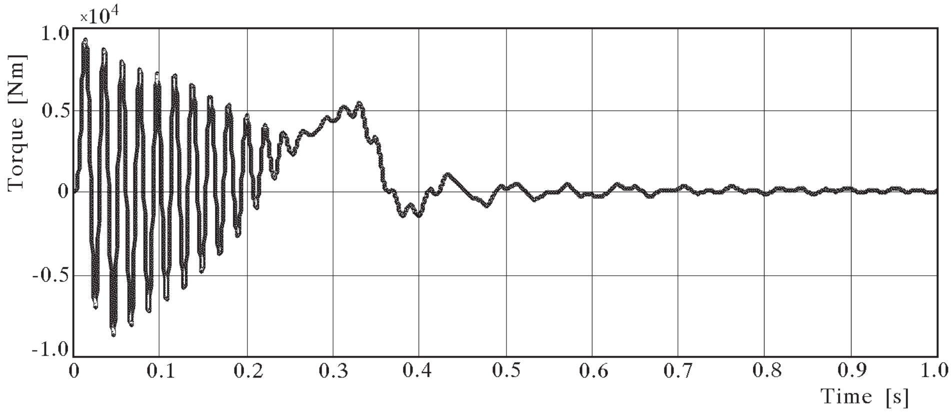

Fig. 10. Electromagnetic torque of the driving motor

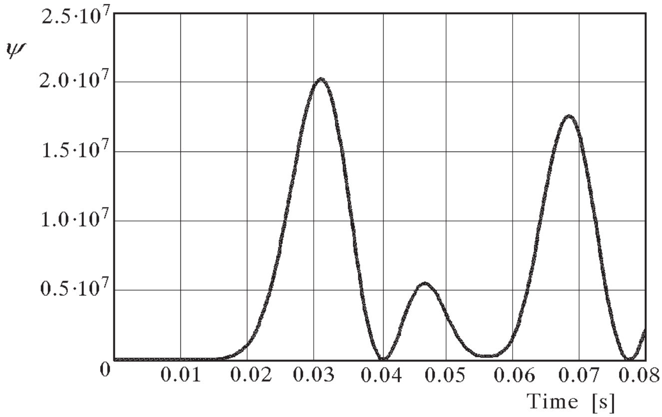

Fig. 11. Objective function $\psi = M_{15}^2(t)$

enables one to minimize the response of the system to a varying external driving force (see Fig. 10). The results of investigations have proved that such an objective function provides a minimization of the dynamic response both during the start up of the system and the steady state (see Fig. 10, Fig. 13 and Fig. 14).

The torsional stiffness of the safety shaft, installed in the driving motor (between nodes 2-4, see Fig. 7), has been selected as a design variable in the process of the optimization of dynamic characteristics of the system. When analysing the sensitivity of the objective functions, which describe dynamic forces in 15 selected kinematic pairs, to the value of the coefficient of stiffness of the shaft, it has been stated that this parameter has a similar effect on most of the peak values (Fig. 12).

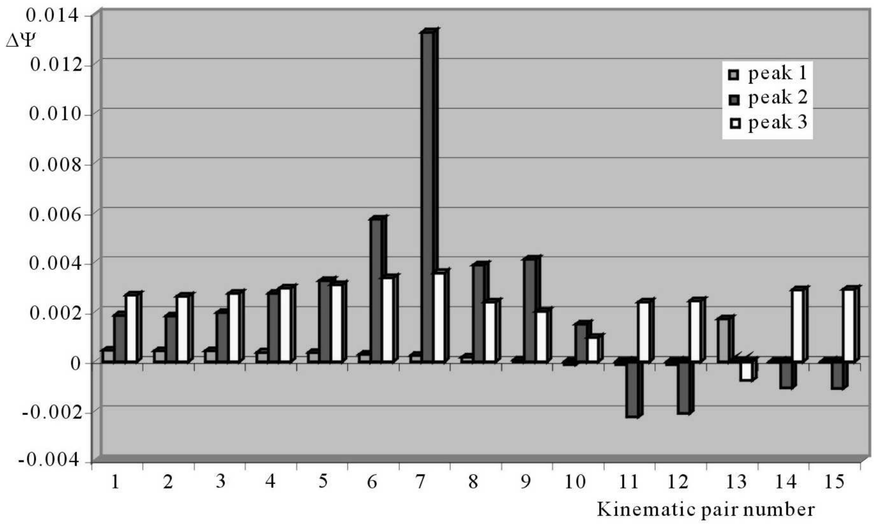

Fig. 12. Influence of the stiffness coefficient of the safety shaft on the peaks of dynamic forces in particular kinematic pairs

Using a sequential linear programming method, the optimization process has been performed. The obtained optimum value of the stiffness coefficient was used for further numerical simulations. The aim of the calculations was to compare the dynamic reactions taking place in the system before the optimization with those occurring after the optimization.

The results of the calculations made for the selected kinematic pair are presented in Fig. 13 and Fig. 14. From these results it is evident that the proper selection of stiffness of the shaft under consideration brings about a decrease both in the torsional moments (Fig. 13) in kinematic pairs of the system and in the reactions in the points of support of the shafts (Fig. 14). This statement has been proved true in the course of experimental investigations consisting in the measurement of vibration accelerations in the gear housing.

The system under investigation is driven by an asynchronous, squirrel-cage motor that has been modelled by means of the equations of motion formulated in the coordinate system (x, y). The influence of major parameters of the motor (such as L_m, L_s – inductances of the windings connected with the main magnetic flux and with the leakage flux of the stator L_r – leakage inductance of the rotor, R_s – resistance of the stator phase and R_r - resistance of the rotor cage) on values of the dynamic reactions in particular kinematic pairs

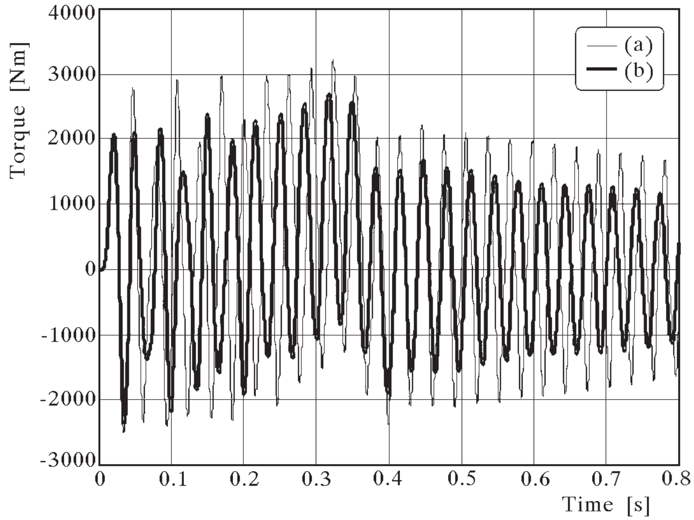

Fig. 13. Dynamic torsional torque in the 1st kinematic pair with the safety shaft (a) before and (b) after optimization

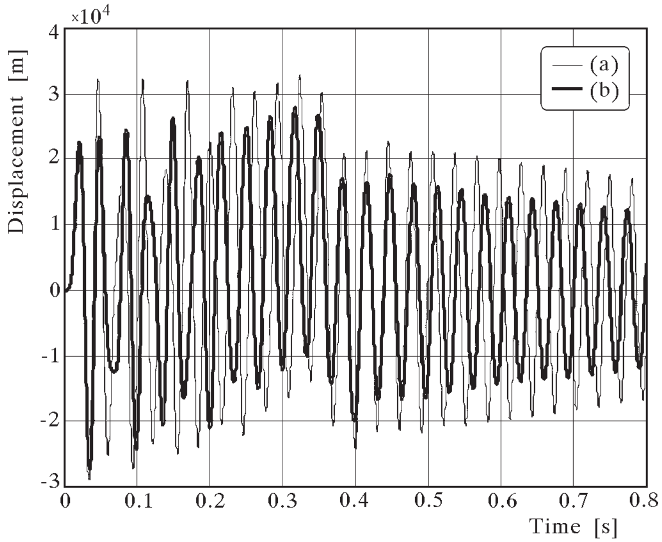

Fig. 14. Displacement of one of the shaft bearings in the direction of the x-axis (a) before optimization and (b) after optimization

of the mechanical system has been determined. The results have been utilized both for the optimization and for mutual "tuning" of the subsystems as well as for increasing of the accuracy of the dynamic model.

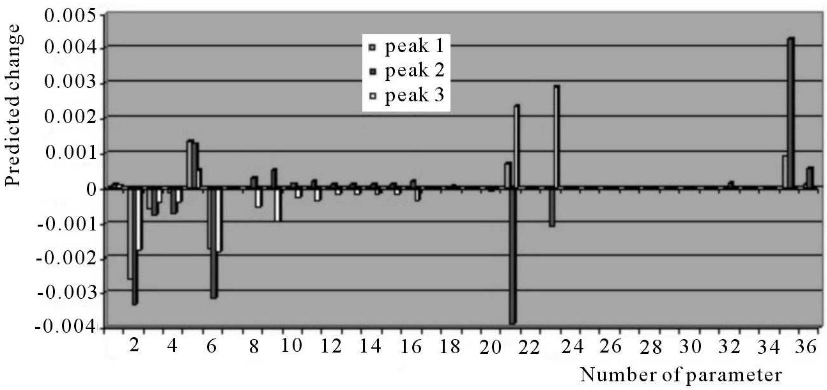

Fig. 15. Relative changes of the peak values of the objective function describing the torque on the output shaft due to an increase in some parameters of the electric (1-5) and mechanical (6-36) parts of the system during the start-up, where: 1 – L_m, 2 – L_s, 3 – L_r, 4 – R_s, 5 – R_r, 6-21 – selected mass moments of inertia and masses, 22-36 – selected stiffness coefficients of the shafts, meshes and bearings

From the analysis of the influence of particular parameters on the examined dynamic phenomena it appears that the electric parameters have a considerable effect on the peaks of forces in the shafts of the transmission gear during the start-up (Fig. 15) and in the course of operation under a load. The further optimization process was performed by using the selected parameters of the electric motor (L_m, R_r) and torsional stiffness of the safety shaft. The presented results of numerical calculations (Fig. 16) indicate that the most effective approach to the problem of optimization of dynamic properties of electromechanical driving systems consists in mutual tuning of the electromagnetic part with the mechanical part through a proper selection of their design parameters. The simultaneous application of mechanical and electrical parameters makes that the least peak values of dynamic reactions in kinematic pairs of the system are obtained. High sensitivity of the force peaks in the mechanical part to electromagnetic parameters indicates that the negligence of dynamic phenomena in the driving motor can lead to considerable errors in the results obtained from computer simulations, especially in unsteady state conditions.

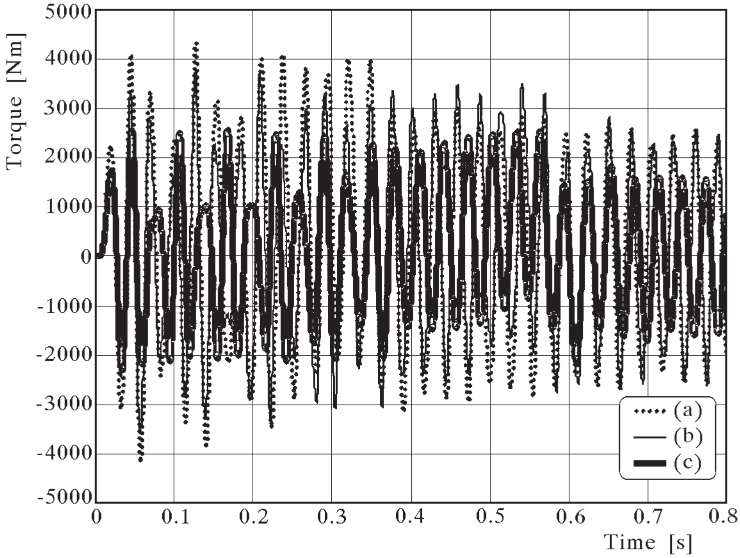

Fig. 16. Torque on the safety shaft (a) before optimization, (b) after optimization of selected parameters of the electric motor (L_m, R_r), (c) after optimization of the parameters L_m, R_r of the electric motor and stiffness of the shaft

9. Conclusions

The results of carried out investigations have proved that the application of the sensitivity analysis and the optimization method enables an effective shaping of dynamic characteristics of electromechanical driving systems. Considerable decreasing of the maximal amplitudes of the forces in kinematic pairs is a result. Such investigations can be successfully used to aid the process of designing and constructing of prototypes of machines and their equipment. The developed algorithms and computer programs are of a general character and can be sucessfully utilized for determination of design parameters of electromechanical systems, the design form of which is similar to the example being considered.

The proposed mathematical model of electromechanical driving systems takes the major phenomena that are determined by dynamic properties of the systems into account. From the results of the sensitivity analysis of an electromechanical system it is evident that parameters of the driving motor have considerable influence on maximal values and on courses of the dynamic forces in kinematic pairs, which indicates that the electromechanical couplings must be taken into consideration in formulated models, especially when unsteady states are analysed.

The assumptions made in the course of the modelling of the system have proved to be true and the formulated algorithms of calculations related to the qualitative description of dynamic phenomena proved to be correct during the experimental measurements. The results of the investigations, which have been performed hitherto, indicate that the mathematical modelling of the dynamic phenomena, methods of the sensitivity analysis and optimization are very effective.

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Modelowanie i optymalizacja układów napędowych z silnikami asynchronicznymi

Streszczenie

W pracy omówiono sposób modelowania elektromechanicznych układów napędowych z silnikami indukcyjnymi. Model silnika elektrycznego opisano w zastępczym układzie współrzędnych (x, y). Model matematyczny przekładni zębatej opracowano z wykorzystaniem hybrydowej metody sztywnych i odkształcalnych elementów skończonych. Szczególną uwagę zwrócono na modelowanie przekładni planetarnej. Analizę wrażliwości i optymalizację przeprowadzono dla funkcji celu opisującej wartości maksymalne sił dynamicznych. Zaprezentowana metoda bezpośredniego różniczkowania umożliwia określenie wpływu parametrów silnika napędowego na zjawiska dynamiczne w części mechanicznej.

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