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EVOLUTIONARY ALGORITHMS AND BOUNDARY ELEMENT METHOD IN GENERALIZED SHAPE OPTIMIZATION

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The coupling of modern, alternative optimization methods such as evolutionary algorithms with the effective tool for analysis of mechanical structures – BEM, gives a new optimization method, which allows one to perform the *generalized shape optimization* (simultaneous shape and topology optimization) for elastic mechanical structures. This new evolutionary method is free from typical limitations connected with classical optimization methods. In the paper, results of researches on the application of evolutionary methods in the domain of mechanics are presented. Numerical examples for some optimization problems are presented, too.

Key words: evolutionary algorithms, genetic algorithms, generalized shape optimization, topology optimization

1. Introduction

For the last twenty years the optimization of mechanical structures has been divided into four main directions:

- material property optimization,
- size optimization,
- shape optimization,
- topology optimization.

When the optimal solutions for two dimensional problems are searched for, mainly the shape optimization and topology optimization is applied. The topology optimization reaches wider and wider applications due to its advantages.

The topology optimization concerns problems, in which the topology of a structure is changed. Within the last ten years three directions of development of the topology optimization can be observed. The first steps in this field concerned optimization problems of truss structures where the optimal layout was looked for. Most of the first papers presenting the topology optimization concerned just the optimization of truss structures e.g.: Achtziger (1995), Dems (1996), Kirsch (1994, 1995, 1996), Mróz and Piekarski (1995, 1996), Rozvany (1995, 1996), Rozvany and Birker (1995), Rozvany and Károlyi (1997), Rozvany *et al.* (1993).

Design variables in such optimization are parameters describing member positions in the structure, cross section (zero value is equivalent to removing the member) and the position of trusses joints.

The second type of *topology optimization* concerns mechanical structures, mainly surfaces, where material properties and layout are taken as design variables. The leading researches in this field are: Allaire (1996a,b), Bendsoe (1995), Bendsoe and Kikuchi (1988), Olhoff (1989, 1993), Pedersen (1995).

In recent years the third direction of the development of the topology optimization has appeared. It is based on generating a new void inside a domain on the basis on special criteria and next on conducting simultaneous shape and topology optimization for outside and inner structure boundaries. The researches on this field are conducted by Eschenauer *et al.* (1994), Eschenauer and Schumacher (1995), Schumacher (1995), Sokołowski and Żochowski (1999), Burczyński and Kokot (1997, 1998). From the mathematical point of view those types of optimization relay on replacing a homogenous domain by a non-homogenous domain.

One of the main goals of this type of the topology optimization of continuous structures, where a material continuum fills an area, is to place the material inside the domain in such a way, that the optimization criteria with constraints are satisfied. The placing of the materials inside the domain can be equivalent to the change of the material density, which eventually leads to generating subregions with zero density. This special case, particularly desirable from the structural point of view, is equivalent with generating interior boundaries in the form of different shape voids. Further transformations of generated voids and outer boundary require applying the classical shape optimization, which leads to connecting topology optimization and shape optimization.

Both problems – topology optimization and shape optimization – connected with each other, create *a generalized shape optimization problem*, which has crucial importance in the optimal design of structures.

Regardless of the type of the optimization, the formulated optimization problem must always be solved. To do this, one of well-known optimization methods can be used (e.g. the optimality criteria approach or mathematical programming methods).

Most of them, despite their advantage, have some limitations, namely:

- the objective function must be continuous,
- the hessian of the objective function should be positive definite,
- there is a substantial probability of getting a local optimum,
- calculations start from a single point, narrowing the search domain,
- the choice of the starting point influences the method convergence.

The limitations mentioned above cause that for some optimization problems the optimal solution is either very difficult or quite impossible to obtain. It leads to significant problems in getting the optimal solution in some cases. Therefore, new optimization methods, free from the limitations mentioned above, are still being looked for.

Relatively not long ago, the research on simulating processes occurring in nature has been undertaken (Goldberg, 1989; Holland, 1975; Michalewicz, 1992). Resulting from those experiments various algorithms for searching optimal solutions were created. Those algorithms are known as GAs, evolutionary programming, evolutionary strategies, neural networks, classifier systems and simulated annealing. Many of them turn out to be alternative methods of optimization for classical methods such as e.g. well-known gradient methods. They have been widely applied in solving the search problems and optimization problems in many disciplines for many years, but their application to solve optimization problems in mechanics started relatively not long ago.

Particularly, the GAs are often used in solving optimization problems (Michalewicz, 1992). In GAs, the search of the optimal solution is based only on the values of the objective function (the fitness function) and does not require meeting the limitations listed for classical optimization methods. It gives a free hand in defining optimization problems.

On the basis of articles presented on various conferences and symposia one may suppose that evolutionary methods will give also good results when they are applied to mechanical engineering problems. The first attempts of applying GAs to the optimization of mechanical structures was undertaken in the early 1990s (Nagendra *et al.*, 1993; Ponslet *et al.*, 1993; Sakamoto and Oda, 1993). The existing papers in this field concern: truss structures (the optimal trusses selection, their layouts or looking for proper cross sections) (Oshaki, 1995; Ponslet *et al.*, 1993; Rajeev and Krishnamoorthy, 1997; Sakamoto and Oda, 1993), the optimization of laminate (composite) structures (Nagendra *et al.*, 1993; Okumura *et al.*, 1995; Punch *et al.*, 1994); optimization of two dimensional structures (Annicchiarico and Cerrolaza, 1998; Burczyński and Kokot, 1998; Chapman and Jakiela, 1994; Fernandes *et al.*, 1998; Kallassy and Marcelin, 1997; Kita and Tanie, 1997), the optimizaton of vibrating structures (Burczyński *et al.*, 1999b). The genetic algorithms have been also applied in crack and void identification (Burczyński *et al.*, 1999a). The main goal of this paper is to present the results of research on the application of a new alternative optimization approach – *the evolutionary method* based on GAs and BEM, to solve problems in the field of the generalized shape optimization.

2. The generalized shape optimization problem

Consider the following class of optimization problems

$$\min_{\boldsymbol{x}} : \; J_0(\boldsymbol{x}) \tag{2.1}$$

with imposed constraints

$$\begin{aligned} &J_\alpha(\boldsymbol{x}) = 0 && \alpha = 1,2,\ldots,n \\ &J^*_\beta(\boldsymbol{x}) \geqslant 0 && \beta = 1,2,\ldots,m \\ &x_{i_{\max}} \geqslant x_i \geqslant x_{i_{\min}} && i = 1,2,\ldots,k \end{aligned} \tag{2.2}$$

where $\boldsymbol{x} = (x_i)$ is a vector of design variables.

The functionals J_0, J_α and J^*_β can have the following forms

$$J(\boldsymbol{x}) = \int\limits_{\Omega} \Psi(\sigma,\varepsilon,u)\; d\Omega + \int\limits_{\Gamma} \Phi(\boldsymbol{p},\boldsymbol{u})\; d\Gamma \qquad \text{or} \qquad J(\boldsymbol{x}) = \int\limits_{\Omega} C\; d\Omega \tag{2.3}$$

where Ψ is an arbitrary continuous function of stresses σ, strains ε and displacements $\boldsymbol{u}$ in the domain Ω of the structures and Φ is an arbitrary function of the displacements $\boldsymbol{u}$ and tractions $\boldsymbol{p}$ on the boundary Γ.

Functionals (2.1) and $(2.2)_{1,2}$ can represent the objectives or constraints described by the stresses or displacements e.g. in the form of the complementary energy, von Mises stresses, or the cost of the structure. Constraints $(2.2)_3$ are imposed on the design variables, simply the geometry constrains.

In the next paragraphs it will be shown how to solve the optimization problem presented above. The proposed method consists of a few steps: geometry modelling and choosing the design variables, applying the numerical method for evaluation of the fitness function, creating the internal voids (if necessary) and applying the evolutionary process. All steps are described below.

3. Geometry modelling

The choice of the geometry modelling method and the design variables has great influence on the final solution of the optimization process. There is a lot of methods for geometry modelling. In the proposed approach NURBS or Bsplines (Piegl and Tiller, 1995) are used to the modelling of the geometry of the structures. The Bsplines curves are defined as follows (Piegl and Tiller, 1995)

$$C(u) = \sum_{i=0}^{n} N_{i,p}(u)\boldsymbol{P}_i \qquad a \leqslant u \leqslant b \tag{3.1}$$

where $\{N_{i,p}(u)\}$ are the pth-degree Bspline basis function, $\{P_i\}$ are the control points.

The NURBS curves are defined as (Piegl and Tiller, 1995)

$$C(u) = \frac{\sum\limits_{i=0}^{n} N_{i,p}(u)w_i\boldsymbol{P}_i}{\sum\limits_{i=0}^{n} N_{i,p}(u)w_i} \qquad a \leqslant u \leqslant b \tag{3.2}$$

where $\{N_{i,p}(u)\}$ are the pth-degree Bspline basis functions defined on a nonperiodic (and nonuniform) knot vector, $\{P_i\}$ are the control points, $\{w_i\}$ are the weigts.

The possibility of an easy control of the shape by only a few control points and the local changes of shapes without affecting on the rest of the structure are their main advantages (Fig. 1), which are very helpful in the optimization process.

In the shape optimization process the co-ordinates of the control points become design variables, which gives a small number of the design variables and simplicity of data preparation in comparison with other methods (e.g. when the coordinates of boundary nodes (in BEM) or mesh nodes (in FEM) are taken as the design variables).

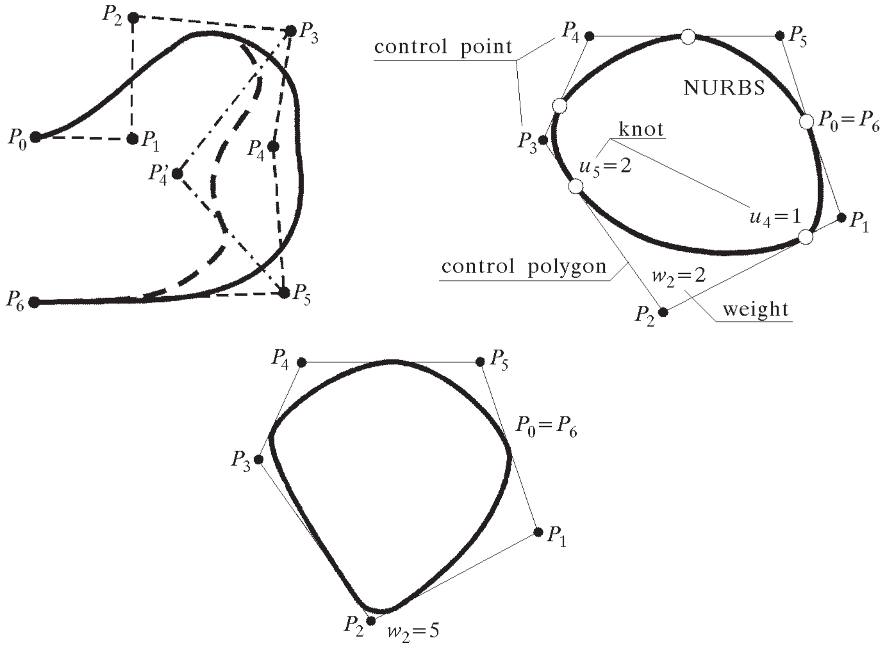

Fig. 1. Bspline and NURBS

4. Boundary element method in evaluation of fitness function

In the GAs, a fitness function with constraints plays the role of the environment (Michalewicz, 1992). In the cases of the optimization problems of mechanical structures the fitness function, called the objective function, depends on stresses, strains or displacements. In order to determine those quantities the computational methods of mechanics such as *the finite element method* (FEM) or *the boundary element method* (BEM) are used in practical engineering applications.

BEM is selected as the method of structure analysis. Particularly, BEM fits to the sensitivity analysis and the optimization of elastic structures for the sake of its specific features. The main advantages of BEM are the easy way of discretization (only the boundary of a structure is discretized) and the computation precision of boundary values. In a iteratively solved shape optimization problem the easy way of discretization is very important, because the rediscretization of the domain Ω any time it changes is very inconvenient (it occurs using FEM).

In the shape optimization the boundary of the structure is subjected to variation, and the goal of the optimization is to shape the boundary in such a way, that the objective function gets the extremum (satisfying the constraints). BEM has turned out to be a convenient and natural numerical technique in the classical shape optimization, because it enables precise describing of the design variables on the varying boundary and fast meshing of the structure during the iterative process of boundary shape evolution (only the boundary is discretizing).

The list of papers in this field can be found by Burczyński (1993).

In BEM, the boundary value problem is described by the following vector boundary integral equations (Kleiber, 1998)

$$\begin{aligned} \boldsymbol{c}(\boldsymbol{x})\boldsymbol{u}(\boldsymbol{x}) + \int\limits_{\Gamma} \boldsymbol{U}^*(\boldsymbol{x},\boldsymbol{y})\boldsymbol{p}(\boldsymbol{y})\, d\Gamma(\boldsymbol{y}) &= \\ = \int\limits_{\Gamma} \boldsymbol{P}^*(\boldsymbol{x},\boldsymbol{y})\boldsymbol{u}(\boldsymbol{y})\, d\Gamma(\boldsymbol{y}) &+ \int\limits_{\Omega} \boldsymbol{U}^*(\boldsymbol{x},\boldsymbol{y})\boldsymbol{b}(\boldsymbol{y})\, d\Omega(\boldsymbol{y}) \end{aligned} \tag{4.1}$$

where $\boldsymbol{c}(\boldsymbol{x})$ is a coefficient matrix, $\boldsymbol{u}(\boldsymbol{y})$ and $\boldsymbol{p}(\boldsymbol{y})$ are vectors of displacements and tractions, respectively, on the boundary Γ, $\boldsymbol{b}(\boldsymbol{y})$ is a vector of the body forces in the domain Ω, $\boldsymbol{U}^*(\boldsymbol{x},\boldsymbol{y})$ and $\boldsymbol{P}^*(\boldsymbol{x},\boldsymbol{y})$ are fundamental solutions of elastostatics.

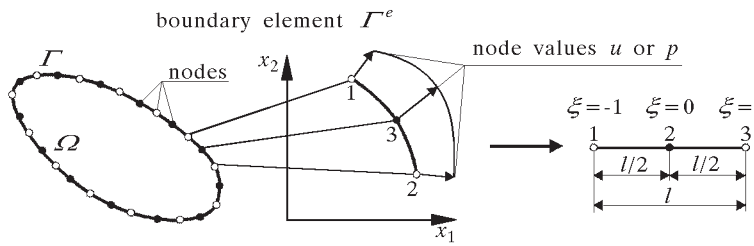

Fig. 2. Discretization of the boundary using quadratic boundary elements

Discretizing the boundary Γ by means of the boundary elements Γ^e, $e = 1, ..., E$

$$\boldsymbol{x}(\xi) = N_e^n(\xi)(\boldsymbol{x})_e^n \tag{4.2}$$

and approximating the fields of displacements and tractions on each boundary element Γ^e, in terms of the nodal values u_e^n, p_e^n and the shape functions N_e^n

$$\boldsymbol{u}(\boldsymbol{y}) = N_e^n(\xi)u_e^n \qquad\qquad \boldsymbol{p}(\boldsymbol{y}) = N_e^n(\xi)p_e^n \tag{4.3}$$

one obtains a discrete form of boundary integral equation (4.1)

$$\begin{aligned}\boldsymbol{c}(\boldsymbol{x})\boldsymbol{u}(\boldsymbol{x}) &= \sum_{e=1}^{E}\sum_{n=1}^{N_e}(\boldsymbol{u})_e^n \int\limits_{\Gamma^e} \boldsymbol{P}^*[\boldsymbol{x},\boldsymbol{y},\xi] N_e^n(\xi) J(\xi)\, d\Gamma(\xi) - \\ &- \sum_{e=1}^{E}\sum_{n=1}^{N_e}(\boldsymbol{p})_e^n \int\limits_{\Gamma^e} \boldsymbol{U}^*[\boldsymbol{x},\boldsymbol{y},\xi] N_e^n(\xi) J(\xi)\, d\Gamma(\xi) + \int\limits_{\Omega} \boldsymbol{U}^*(\boldsymbol{x},\boldsymbol{y})\boldsymbol{b}(\boldsymbol{y})\, d\Omega(\boldsymbol{y})\end{aligned} \tag{4.4}$$

where $(\boldsymbol{u})_e^n$, $(\boldsymbol{p})_e^n$ are the nodal values of the displacements and tractions fields, $J(\xi)$ – jacobian, $\boldsymbol{P}^*[\boldsymbol{x},\boldsymbol{y},\xi]$, $\boldsymbol{U}^*[\boldsymbol{x},\boldsymbol{y},\xi]$ – fundamental solutions in the local coordinate system.

Finally, equation (4.4) can be transformed into a system of linear algebraic equations

$$\mathbf{AX} = \mathbf{F} \tag{4.5}$$

where the unknown values of the boundary displacements and tractions are placed in the column matrix $\mathbf{X}$.

Solving equation (4.5), allows one to obtain all the unknown boundary values of the displacements and tractions. Knowing all boundary displacements and tractions on the boundary Γ, the components of the stress tensor $\boldsymbol{\sigma} = (\sigma_{ij})$ can be calculated in selected internal points $\boldsymbol{x} \in \Omega$ using the following equation

$$\sigma(\boldsymbol{x}) = \int\limits_{\Gamma} D(\boldsymbol{x},\boldsymbol{y})\boldsymbol{p}(\boldsymbol{y})\, d\Gamma(\boldsymbol{y}) - \int\limits_{\Gamma} S(\boldsymbol{x},\boldsymbol{y})\boldsymbol{u}(\boldsymbol{y})\, d\Gamma(\boldsymbol{y}) + \int\limits_{\Omega} D(\boldsymbol{x},\boldsymbol{y})\boldsymbol{b}(\boldsymbol{y})\, d\Omega(\boldsymbol{y}) \tag{4.6}$$

where $S(\boldsymbol{x},\boldsymbol{y})$ and $D(\boldsymbol{x},\boldsymbol{y})$ are the third-order fundamental solution tensors obtained from suitable differentiation of $\boldsymbol{U}^*(\boldsymbol{x},\boldsymbol{y})$ and $\boldsymbol{P}^*(\boldsymbol{x},\boldsymbol{y})$ with respect to the source point $\boldsymbol{x}$ and application of Hooke's law.

The application of BEM to the optimization shows the great advantage over other computational methods for the analysis of mechanical structures, because the discretization is to be done only along the boundary of the structure, which reduces the problem size in the analysis process.

5. Generating internal voids – the bubble method

Majority of methods of solving optimization problems are based rather on finite element procedures. BEM has been applied to shape optimization problems but till now it has not been used to the topology optimization. In the classical shape optimization one optimizes the shape of the existing boundaries. BEM is a exceptionally natural and convenient numerical analysis technique for such optimization process. It appears, however, that it is impossible to insert any changes inside the domain. As the discretization is made only on the boundary, there is no possibility to insert the void during the optimization of the structure.

In order to eliminate this drawback, the idea of using the so-called *topology derivative*, proposed by Sokołowski and Żochowski (1999) has been applied. The *topology derivative* is defined as follows

$$\Im(\boldsymbol{x}) = \lim_{\rho \to 0} \frac{J\Big(\Omega \setminus \overline{B}_\rho(\boldsymbol{x})\Big) - J(\Omega)}{|\overline{B}_\rho(\boldsymbol{x})|} \qquad \boldsymbol{x} \in \Omega \tag{5.1}$$

or by an equivalent formula

$$\widetilde{\Im}(\boldsymbol{x}) = \lim_{\rho \to 0} \frac{J\Big(\Omega \setminus \overline{B}_\rho(\boldsymbol{x})\Big) - J(\Omega)}{\rho^N} \qquad \boldsymbol{x} \in \Omega \tag{5.2}$$

where J is an arbitrary shape functional, $\Omega \subset R^N$ is a domain in the R^N space, $B_\rho(\boldsymbol{x})$ is a bubble with the radius $\rho > 0$ such that $B_\rho(\boldsymbol{x}) = \{\boldsymbol{y} \in R^N : \|\boldsymbol{y} - \boldsymbol{x}\| < \rho\}$, $\overline{B}_\rho(\boldsymbol{x})$ is the bubble surroundings $B_\rho(\boldsymbol{x})$. The function $\Im(\boldsymbol{x})$, called *the topology derivative* of the functional $J(\Omega)$, gives information about the infinitesimal variation of the shape functional J if a small void is inserted into the domain Ω. Applying the *topology derivative* approach, it is possible to determine the position of the bubble with any shape in the domain.

When the functional describes the complementary energy, the *topology derivative* problem can be reduced to a method presented by Eschenauer and Schumacher (1994, 1995), called *"the bubble method"*. It is assumed that inserting an infinitesimally small void into the domain will produce only local stress concentration in the vicinity of the bubble, and the global stress field remains unchanged. Looking for the best position of such a bubble, an optimization process is carried out, and for some shapes of the bubble (i.e. a circle, triangle, ellipse) the so-called *characteristic functions* (Eschenauer and Schumacher, 1995) can be determined:

- for a circular-shaped void

$$H(\sigma_1, \sigma_2) = \frac{1}{2E}\Big[(\sigma_1 + \sigma_2)^2 + 2(\sigma_1 + \sigma_2)^2\Big] \tag{5.3}$$

- for a ellipse-shaped void

$$H = \frac{1}{4\pi E}\Big[5.65(\sigma_1+\sigma_2)^2 + 5.52(\sigma_1-\sigma_2)^2 - \\ -0.22(\sigma_1^2-\sigma_2^2)\cos 2\alpha + 2.34(\sigma_1-\sigma_2)^2\cos 2\alpha\Big] \tag{5.4}$$

- for a triangle-shape void

$$H = \frac{1}{2E}\Big[7.0(\sigma_1+\sigma_2)^2 + 14.5(\sigma_1-\sigma_2)^2\Big] \tag{5.5}$$

The coordinates, where the characteristic function gets the minimum, point out the centre of the newly generated void. As the characteristic functions depend on stresses, it is easy to generate internal voids in the topology optimization process using BEM. The coordinates which are the centre of a new "bubble" inserted into the domain are generated on the basis of the objective function. In the problem considered above the objective function is expressed by the complementary energy which is a measure of the mean compliance of the structure. The point where the complementary energy gets the minimum is the point of the centre of the new "bubble".

6. Evolutionary optimization

6.1. Genetic algorithms

In general, the GAs simulate a natural evolutionary process. The GAs are able to find the optimal solution satisfying the constraints without the calculation of derivatives. Many papers or books contain a lot of benchmark tests for optimization problems, including ones which are very difficult to solve by means of the classical methods (Michalewicz, 1992; Michalewicz *et al.*, 1994, 1996).

The GAs map an evolutionary process of nature over a span of time, in order to adapt an individual to conditions of life as fit as possible. It is just nothing more than the main goal of optimization. Those algorithms are procedures for searching the space of solutions. They take advantage of the mechanisms of natural selection and genetic inheritance, using the neo-Darwinian principle of reproduction and the survival of the fittest.

The GAs start with a population of randomly generated candidates from the feasible solution domain (Fig. 3). These candidates, called chromosomes,

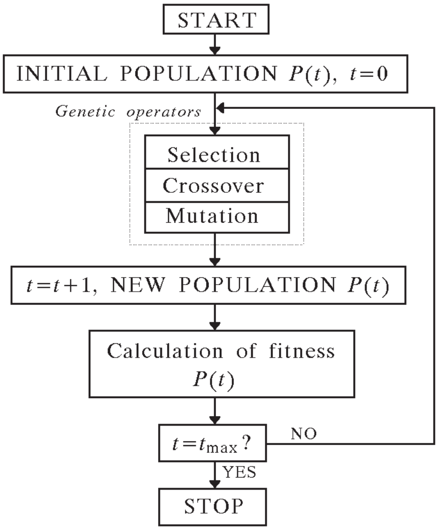

Fig. 3. Flow chart of a genetic algorithm

evolve towards better solutions by applying genetic operators such as a mutation and crossover, simulated on the basis of heredity principles (genetic) and a selection simulated on the basis of the natural selection (theory of evolution).

Ordinarily, after applying genetic operators the new population has a better fitness. The population of individuals (chromosomes) undergoes the evolution. The objective function with imposed constraints plays the role of the environment to distinguish between good and bad solutions (Michalewicz, 1992).

6.2. Chromosomal representation and genetic operators

The first step in a GA is to create chromosomes that describe possible solutions. There are a few possibilities of chromosomal representation (Michalewicz, 1992). The binary coding, the Gray coding, the logarithmic coding and the floating point coding are the best known ones. The selection of the kind of coding depends on the optimization problem. The floating point coding is the best representation of the problem described in this paper.

GAs with population of chromosomes coded by floating point representation use the modified operators (Michalewicz, 1992; Michalewicz *et al.*, 1994, 1996) which are applied to get good convergence, "jumping" far from a local optimum, stability, good fitness, better local tuning.

The chromosome is represented by the design vector $\boldsymbol{c}$

$$\boldsymbol{c} = (c_1, c_2, \ldots, c_i, \ldots, c_N) \qquad c_i^{\min} \leqslant c_i \leqslant c_i^{\max} \tag{6.1}$$

where c_i, $i = 1, ..., N$ are genes.

The population consists of a randomly generated set of feasible chromosomes. The population evolves towards better individuals by applying genetic operators to find a better fitness. The next "evolved" population is created in the following steps:

1. ***Selection***

   This operator allows the selection of parents from the population. The rest of genetic operators produce offspring by acting on the parents. The roulette wheel method (Michalewicz, 1992) is the best known selection. In this method parents are selected proportionally to the magnitudes of their fitness.

   Having selected chromosomes from the initial population the genetic operators "work on them" in order to create a new population. The six genetic operators for the floating point representation of chromosomes, briefly shown below, have been completely described by Michalewicz (1992), Michalewicz *et al.* (1994, 1996).

2. ***Mutations***

   2.1. *Uniform mutation*

   This operator produces a single offspring $\boldsymbol{c}'$ from a single parent $\boldsymbol{c}$. One gene taken from the parent is randomly selected and changed to a new one whose value is randomly selected from the design space. For example, when the gene ci is selected

$$\boldsymbol{c} = (c_1, c_2, \ldots, c_i, \ldots, c_N) \qquad \boldsymbol{c}' = (c_1, c_2, \ldots, c_i', \ldots, c_N) \tag{6.2}$$

   This operator is very important in the early phases of the evolution process, because children are allowed to move freely within the feasible domain. It is helpful in the case when the local optimum exists, because a chromosome can "escape" from it.

   2.2. *Boundary mutation*

   This operator produces a single offspring $\boldsymbol{c}'$ from a single parent $\boldsymbol{c}$, too, but the changed genes of a chromosome can take only boundary values of the design space $c_i^{\min}$ or $c_i^{\max}$. The boundary mutation works very well when the solution lies either on or near the boundary of the feasible search space.

2.3. *Non-uniform mutation*

This operator produces a single offspring $\boldsymbol{c}'$ from a single parent $\boldsymbol{c}$. If the gene ci has been chosen, the new mutated gene will have the following value

$$c_i' = \begin{cases} c_i + \Delta[t, right(c_i) - c_i] & \text{if a random digit is 0} \\ c_i - \Delta[t, c_i - left(c_i)] & \text{if a random digit is 1} \end{cases} \tag{6.3}$$

where t is the generation number. The function $\Delta(t, y)$ returns a value from the range $[0, y]$. The probability of $\Delta(t, y)$ increases when t increases. This operator causes that initially the design space is searched uniformly, but with the increase of t the space is searched very locally, which gives fine tuning of the algorithm.

3. ***Crossovers***

The crossover operation swaps some chromosomes of the selected parents in order to create an offspring.

3.1. *Simple crossover*

This operator needs two parents and produces two children. For a randomly generated crossing parameter k ($k = 1, ...,$ number of parameters) it works as follows (e.g. $k = 2$)

$$\begin{aligned} \boldsymbol{c}_1 &= (c_1, c_2, |c_3) & \qquad \boldsymbol{c}_2 &= (s_1, s_2, |s_3) \\ \boldsymbol{c}_1' &= (c_1, c_2, |s_3) & \qquad \boldsymbol{c}_2' &= (s_1, s_2, |c_3) \end{aligned} \tag{6.4}$$

A simple crossover may produce a child outside the design space. To avoid this, a parameter $\alpha \in [0, 1]$ is applied. New chromosomes have the following forms ($\boldsymbol{c}_1$, $\boldsymbol{c}_2$ are parents)

$$\begin{aligned} \boldsymbol{c}_1 &= (c_1, ..., c_q) \qquad \boldsymbol{c}_2 = (s_1, ..., s_q) \\ \boldsymbol{c}_1' &= [c_1, ..., c_k, ..., s_{k+1}\alpha + c_{k+1}(1-\alpha), ..., s_q\alpha + c_q(1-\alpha)] \\ \boldsymbol{c}_2' &= [s_1, ..., s_k, ..., c_{k+1}\alpha + s_{k+1}(1-\alpha), ..., c_q\alpha + s_q(1-\alpha)] \end{aligned} \tag{6.5}$$

and they always lie within the feasible design space.

3.2. *Arithmetical crossover*

This operator produces two children that are a linear combination of two parents

$$\boldsymbol{c}_1' = \alpha\boldsymbol{c}_1 + (1-\alpha)\boldsymbol{c}_2 \qquad\qquad \boldsymbol{c}_2' = \alpha\boldsymbol{c}_2 + (1-\alpha)\boldsymbol{c}_1 \tag{6.6}$$

where α is a random parameter from the range $[0,1]$. Applying this parameter guarantees that the children are in the feasible domain.

3.3. *Heuristic crossover*

This operator produces a single offspring from two parents

$$\boldsymbol{c}_3' = r(\boldsymbol{c}_2 - \boldsymbol{c}_1) + \boldsymbol{c}_2 \tag{6.7}$$

where r is a random value from the range $[0,1]$, and the parent $\boldsymbol{c}_2$ is not worse than $\boldsymbol{c}_1$ (i.e. the fitness for $\boldsymbol{c}_2$ is better than for $\boldsymbol{c}_1$ for a maximization problem). This operator has a few features: it uses values of the objective function to determine the direction of the search, secures fine local tuning and the searching in the promising direction.

When a new population is created, the fitness is calculated and the whole evolutionary process is repeated to produce next populations with better fitness. The process is stopped after n cycles or when the solution can not attain better solution after k cycles.

6.3. Evolutionary algorithm

A flow chart of the proposed approach of the evolutionary optimization is presented in Fig. 4.

The first step in an optimization process is to find the optimal shape of the external boundary. It means that the typical shape optimization is carried out. The genetic algorithm is applied as an optimization module. In order to obtain the information about the fitness function for each individual in the population, BEM is used. When the main goal of the optimization is to find only the optimal shape without any changes inside the domain then the optimization process is finished and the obtained solution is the optimal one.

When it is possible to make any change inside the domain, the second step of the optimization – *the topology optimization* – is carried out. A new void, which changes the topology class, is inserted into the domain. In order to generate a new void inside the domain the *"bubble method"* is used. It secures the optimal position for the inserted void. The coordinates, where the characteristic functions get the minimum, are the co-ordinates of the centre of

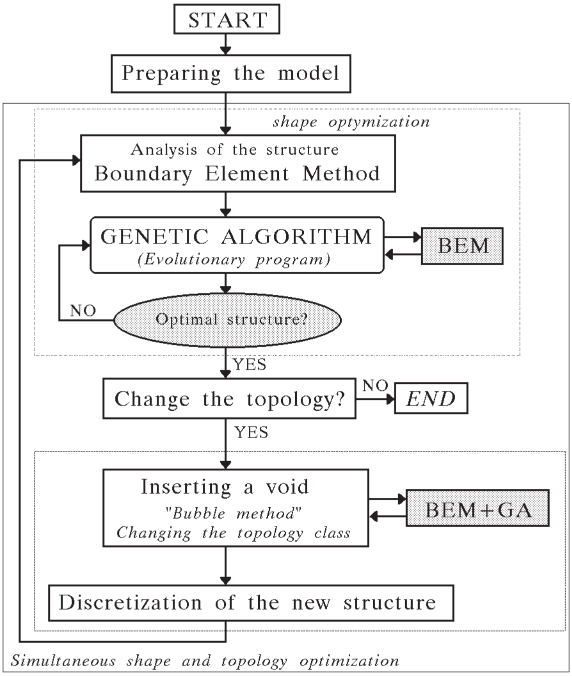

Fig. 4. Evolutionary optimization

the new void. The characteristic functions depend on stresses, so the minimum values are calculated using BEM and GAs. After inserting the void, during the optimization process the best place for the inserted void, the optimal shape of the external boundary and the internal boundary are searched for. It means that the shape optimization and the topology optimization are being carried out simultaneously.

7. Numerical examples

7.1. Example 1

A cantilever beam subjected to a point force is considered (Fig. 5a). The objective function is to minimize the complementary energy with the constraint imposed on the volume of the structure ($V_{end} = 0.5V_{start}$). All boundaries of the structure are modelled by NURBS. The final shape of the external boundary after the shape optimization is presented in Fig. 5b. The final form of the structure after the simultaneous shape and topology optimization is presented in Fig. 5c and Fig. 5d. Figure 5e shows fitness function values versus the number of iterations. Curve 1 presents the evolution process for the structure in Fig. 5b. The final solution is found very fast in the early stage of the evolution process. After inserting the first void in the initial iterations the optimization process converge nearly to the final solution (curve 2). The next iterations are connected with good tuning to the final solution and possible checking if there is no other global optimum. Curve 3 represents the optimization process for the structure with two voids. It can be seen that finding the final solution for this structure requires much more iterations than for the initial structure.

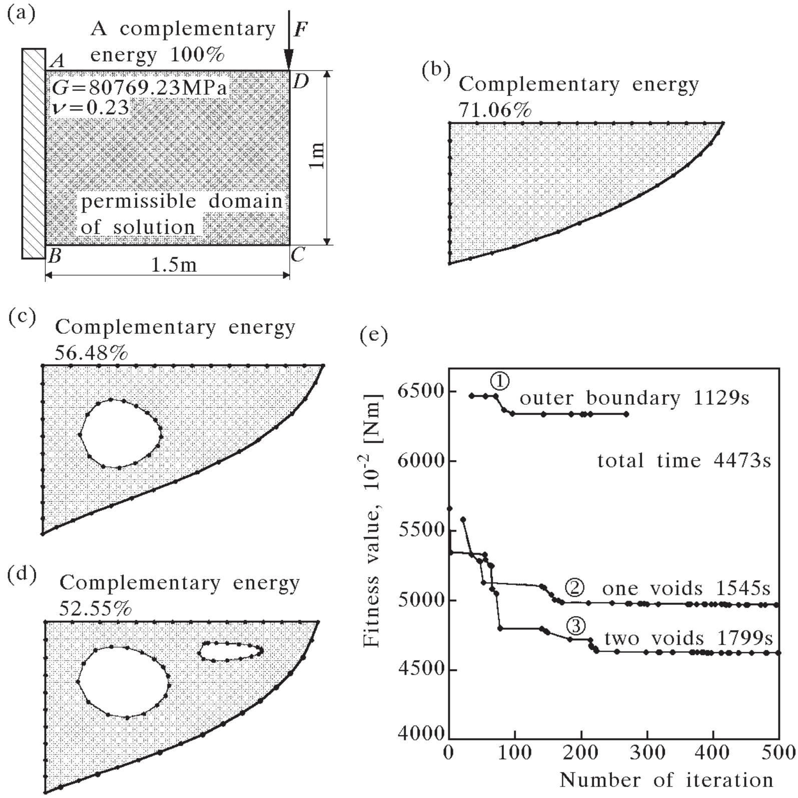

Fig. 5. Evolutionary optimization of a cantilever beam

The problem was solved on the 120 MHz PC Pentium, and the total time of the calculations was 75 min.

In all presented examples the following genetic algorithm parameters have been used:

GA parameters	Value
population size	70
number of generations	300
selective pressure	0.057*
nonuniform mutation	0.028*
crossover	0.143*

* – number of chromosomes undergo the operators to the population size.

7.2. Example 2

The example shows the results of the optimization process of the support presented in Fig. 6a. The optimization goal is to find the optimal shape and topology of the structure for the minimum area of the structure with a constraint for displacements of loaded nodes $(u \leqslant 1.3u_{start})$. All boundaries of the structure are modelled by NURBS. The final shape of the external boundary is presented in Fig. 6b. The form of the structure after the simultaneous shape and topology optimization is shown in Fig. 6c.

Figure 6d shows the objective function values versus the number of iterations. Curve 1 presents the evolution process for the initial structure. The final solution is found very fast in the early stage of the evolution process. After inserting a void in the initial iterations the optimization process converge very fast to the final solution (curve 2). The next iterations are connected with good tuning to the final solution and possible checking if there is no other global optimum. The problem was solved on the 120 MHz PC Pentium and the total time of the calculations was 118 min.

7.3. Example 3

The example shows the results of the optimization process of the rectangular plate presented in Fig. 7a. The objective function is to minimize the complementary energy with a constraint condition in the form of a volume constraint $(V_{end} = V_{start})$. It is assumed that the inserted voids changing the structure topology are circular and the outer boundary is modelled by NURBS. The final shape of the external boundary is presented in Fig. 7b. The form of the structure after the simultaneous shape and topology optimization is shown in Fig. 7c and Fig. 7d. Figure 7e shows the objective function values versus the number of iterations. Curve 1 shows the optimization process for

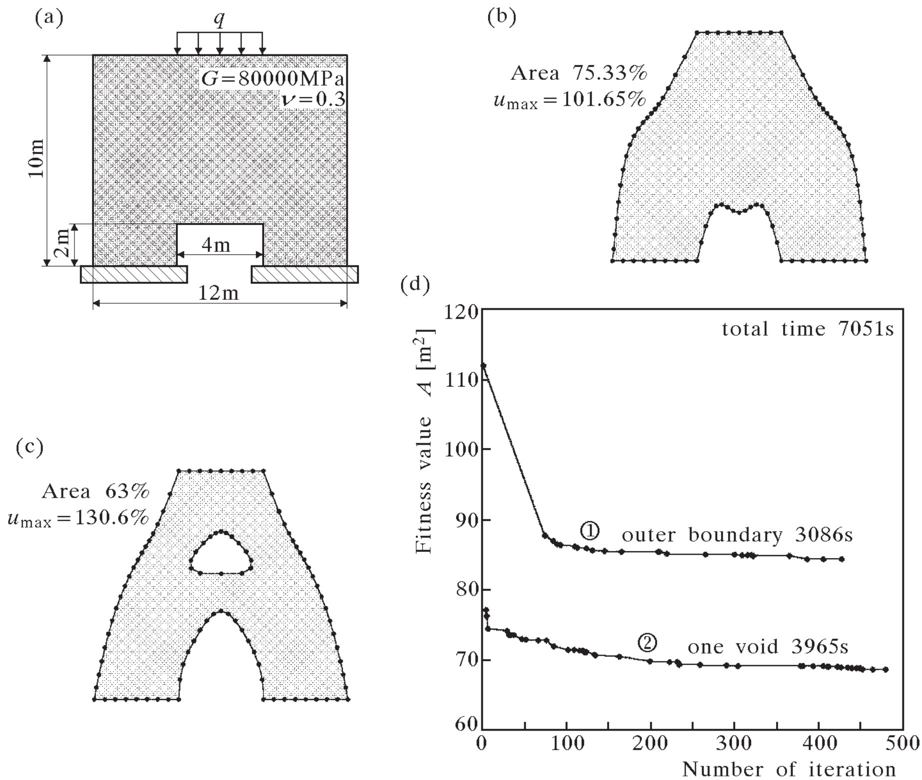

Fig. 6. Evolutionary optimization of a support

the structure without voids. The final solution is found in the first steps of opimization process. Curve 2 presents the optimization process for the structure with two voids. In this case the evolution goes slower. Finally, curve 3 presents the evolution process for the structure with four voids. In the initial iterations the final solution is found and the next iterations give no improvement. Those iterations are connected only with good tuning to the final solution and possible checking if there is no other global optimum. The problem was solved on the 120 MHz PC Pentium and the total time of the calculations was 160 min.

8. Conclusions

Numerical tests prove that the proposed approach of *the evolutionary opitmization* can be applied in order to solve a large class of *generalized shape*

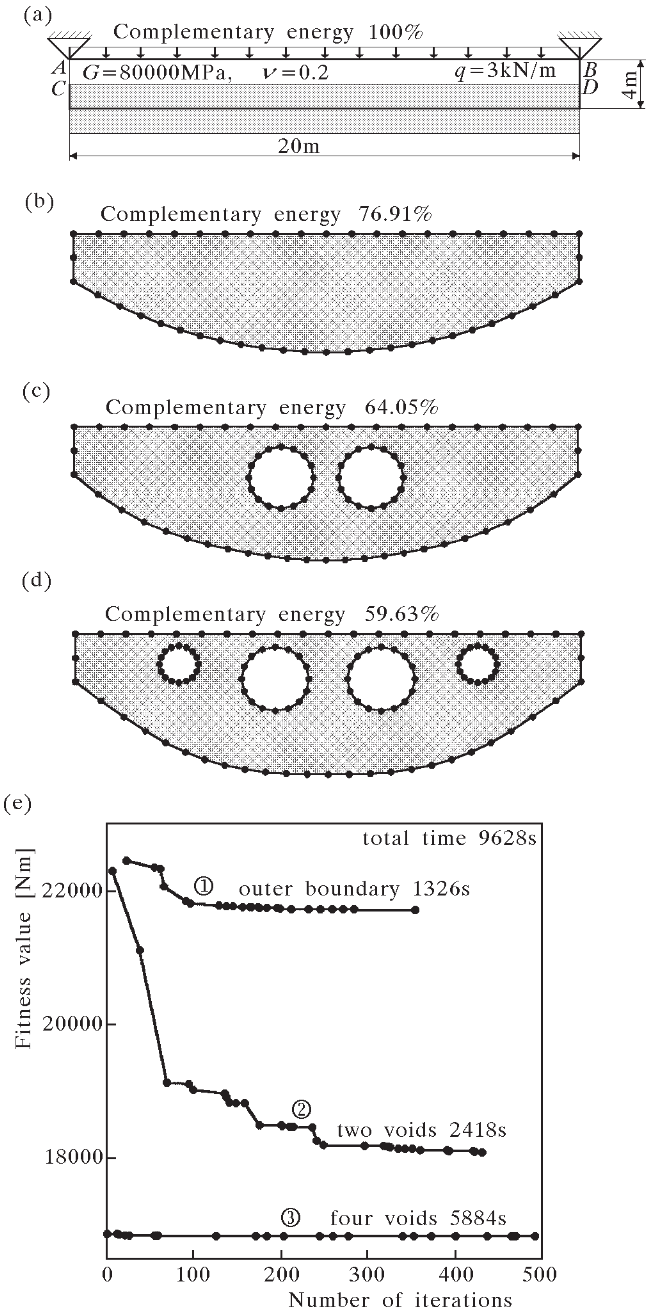

Fig. 7. Evolutionary optimization of a rectangular plate

optimization problems of mechanical structures (the simultaneous shape and topology optimization). The application of GAs as the optimization module makes this method free from limitations typical for the classical optimization methods. The coupling of the boundary element method with the genetic algorithm gives an effective and efficient alternative optimization tool.

The application of BEM in the optimization shows the great advantage over other computational methods used in the analysis of mechanical structures, because only the boundary of the structure is discretized. Also the simplicity of data preparation and the small number of data should be taken into consideration.

The proposed approach of the evolutionary optimization allows the performance of either a complex optimization process in the form of the *generalized shape optimization* or a partial process in the form of *the shape optimization* without the change in the topology class, or *the topology optimization* not changing the shape of the external boundary of the structure.

Now, the main direction of the research is to develop this method and apply it to different fields of mechanics such as identification, dynamic problems, thermo-elastic problems, etc.

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Algorytmy ewolucyjne i metoda elementów brzegowych w uogólnionej optymalizacji kształtu

Streszczenie

Połączenie nowoczesnych algorytmów optymalizacji, jakimi są algorytmy ewolucyjne, z metodą elementów brzegowych pozwala opracować alternatywną metodę optymalizacji sprężystych układów mechanicznych w zakresie uogólnionej optymalizacji kształtu (połączenie optymalizacji kształtu z optymalizacją topologiczną). Metoda ta jest pozbawiona wad związanych z typowymi klasycznymi metodami optymalizacji (ciągłość funkcji celu, wyznaczanie gradientu funkcji itp.), co znacznie rozszerza możliwości jej zastosowań. W artykule przedstawiono proponowaną metodę optymalizacji wraz z przykładami optymalizacji wybranych układów mechanicznych.

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