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INTERACTION OF CYLINDRICAL SHELL AND SPHERICAL BODY IN IDEAL COMPRESSIBLE MEDIUM

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This paper presents analytical and numerical investigations of the interaction between a cylindrical elastic shell surrounded by an unbounded ideal compressible liquid and containing other compressible liquid with a vibrating spherical inclusion in it. Only small amplitudes of the vibrations are considered, therefore the linear theory of elastic shells is used, and the behavior of liquids is described by the Helmholtz equations. Approach to the solution of such a problem is based on the re-expansion of partial solutions to the Helmholtz equation written in cylindrical coordinates by spherical functions and *vice versa*. The results obtained may be used for researching processes of vibro-displacement and localization, decontamination of liquid media, airing and dispersion, in bioacoustics, defectoscopy, cardiovascular medicine and in technologies for reconstruction of oil production in corked wells.

Key words: thin elastic circular cylindrical shell, vibrating spherical body, ideal compressible liquid, cylindrical and spherical wave functions

1. Introduction

Development of different fields of contemporary technics, elaboration of intensive technological processes give rise to necessity of investigation of the interaction between rigid or elastic bodies (shells) and a liquid or elastic medium. The study on the interaction of bodies in a liquid or elastic medium appear in bioacoustics, defectoscopy, cardiovascular medicine and in technologies reconstructing oil production in corked wells.

A significant part of the general problem on the interaction between such bodies and media is formed by coupled problems. Among the well-known classical interaction problems one can choose the following: investigation of diffraction of electromagnetic waves (Ivanov, 1968), acoustic waves (Shenderov, 1972; Belov *et al.*, 1998) and elastic waves (Guz *et al.*, 1978) in multilinked bodies. A characteristic feature of most solved problems is the identical configuration of the boundary surfaces. Recently, some simulation work has been carried out, and peculiarities of the interaction process in a liquid (both incompressible and compressible) in the system of bodies with different geometry have been found (Olsson, 1993; Kubenko and Savin, 1995; Kubenko and Kruk, 1995; Kubenko and Dzyuba, 2000, 2001).

The aim of the paper is to develop mathematical methods and to investigate the dynamic interaction of bodies with different geometric form in an ideal compressible liquid under periodic dynamic action. The mathematical technique is expected to allows one to rewrite a general solution to the corresponding constitutive equations from one to an other coordinate system. It enables getting an exact analytical solution (as a Fourier series) to the interaction problem for a collection of rigid and elastic bodies, as well as gas bubbles; to research vibrations of cylindrical vesseles (elastic shells) filled with a liquid and containing spherical inclusions (particles, bubbles, etc.) and to study character of a stream in the space occupied by structurally or arbitrarily disposed bodies with spherical, cylindrical and other forms.

2. Problem formulation

We consider the following hydrodynamic system: an infinite thin elastic circular cylindrical shell with the thickness h is surrounded by an unbounded ideal compressible liquid with the parameter: c_2 – sound speed in the liquid, γ_2 – liquid density, and contains an other compressible liquid (c_1, γ_1) and a vibrating spherical inclusion in it. The spherical body is supposed to harmonically vibrate according to a given law along the shell axis. The spherical body and the cylindrical shell do not intersect. They are described by spherical and cylindrical coordinate systems, see Fig. 1. A steady-state vibration is considered, so the exponential factor expressing time dependency is neglected.

The boundary problem consists in searching solutions to the following Helmholtz equations relative to wave potentials

$$\nabla^2\varphi^{(l)} + \frac{\omega^2}{c_l^2}\varphi^{(l)} = 0 \qquad l = 1, 2 \tag{2.1}$$

The vector of the liquid speed and its pressure in an arbitrary point of the liquid volume are expressed through the wave potential as follows

$$\boldsymbol{U}_l = \operatorname{grad}\varphi^{(l)} \qquad p^{(l)} = \mathrm{i}\gamma_l\omega\varphi^{(l)} \qquad l = 1, 2 \tag{2.2}$$

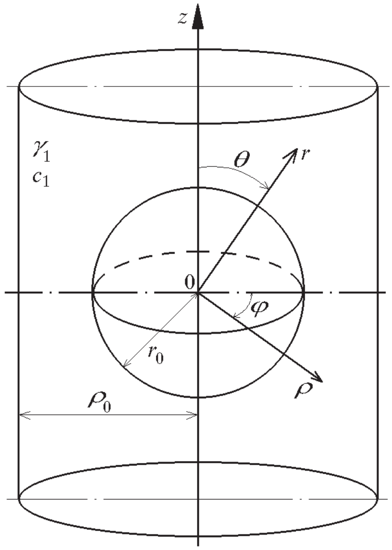

Fig. 1. Geometry of the system

Thus, it is necessary to find the solutions to Eqs (2.1), which whould satisfy the boundary conditions:
— on the sphere surface

$$\frac{\partial \varphi^{(1)}}{\partial r}\Big|_{r=r_0} = V(\theta) \qquad\qquad V(\theta) = \sum_{n=0}^{\infty} V_n P_n(\cos\theta) \tag{2.3}$$

— on the thin elastic shell surface

$$\frac{\partial \varphi^{(l)}}{\partial \rho}\Big|_{\rho=\rho_0} = -\frac{\partial w}{\partial t} = \mathrm{i}\omega w \qquad\qquad l = 1,2 \tag{2.4}$$

In equations (2.1)-(2.4) $\varphi^{(1)}$, $\varphi^{(2)}$ denote the wave potentials inside and outside the cylindrical volume, respectively; ω – sphere vibration frequency; w – the cylindrical shell deflection (the deflection w is assumed to be positive in the direction of the shell curvature center); $V(\theta)$ – function describing motion of the sphere surface which can be presented in the form of Legendre's polynomials series.

The following non-dimensional variables are introduced afterwards

$$\begin{array}{lll} \overline{r} = \dfrac{r}{\rho_0} & \overline{f} = \dfrac{\gamma_1}{\gamma_m} & \overline{\omega} = \dfrac{\omega\rho_0}{c_1} \\ \overline{U} = \dfrac{U}{c_1} & \overline{\varphi} = \dfrac{\varphi}{\rho_0 c_1} & \overline{p} = \dfrac{p}{\gamma_1 c_1^2} \end{array} \tag{2.5}$$

Further considerations only they will be used, and the overbars shall be omitted for convenience in all expressions.

The cylindrical shell undergoes an action of the hydrodynamic load

$$q\Big|_{\rho=\rho_0} = \Big(-p^{(1)} + p^{(2)}\Big)\Big|_{\rho=\rho_0} \tag{2.6}$$

which is symmetric relative to the shell axis. Consequently, the deformations of the shell middle surface do not depend on the angle of rotation around the Oz-axis, and the displacement of the middle surface along the arc is identically equal to zero.

As we consider a thin elastic cylindrical shell, its motion is discribed by equations of the linear shell theory based on the Khirgoff-Love hypotheses (Volmir, 1979). Let us write these equations in non-dimensional variables (2.5) in the case of axisymmetric deformation of the shell

$$\begin{aligned}
&\frac{\partial^2 u(z)}{\partial z^2} - \nu\frac{\partial w(z)}{\partial z} = -\omega^2\frac{c_1^2}{c_m^2}u(z)\\
&-\nu\frac{\partial u(z)}{\partial z} + \Big(1 + \frac{h^2}{12}\frac{\partial^4}{\partial z^4}\Big)w(z) = \frac{c_1^2}{c_m^2}\Big(\frac{f}{h}q(\rho_0, z) + \omega^2 w(z)\Big)
\end{aligned} \tag{2.7}$$

where u is the displacement of the shell middle surface in the axial direction; γ_m – density of the shell material; c_m – sound speed in the shell material, $c_m = \sqrt{E/[\gamma_m(1-\nu^2)]}$; E – elasticity modulus; ν – Poisson's ratio.

The problem statement will be complete if the shell deflection is expressed through the unknown wave potential. So, the Fourier transformation according to the z-coordinate is used in Eqs (2.7). As a result, the system of equations of shell motion in the image space is obtained

$$\begin{aligned}
&-\xi^2 u^F(\xi) - \mathrm{i}\nu\xi w^F(\xi) = -\omega^2\frac{c_1^2}{c_m^2}u^F(\xi)\\
&-\mathrm{i}\nu\xi u^F(\xi) + \Big(1 + \frac{h^2}{12}\xi^4\Big)w^F(\xi) = \frac{c_1^2}{c_m^2}\Big(\frac{f}{h}q^F(\rho_0, \xi) + \omega^2 w^F(\xi)\Big)
\end{aligned} \tag{2.8}$$

It is a system, which allows one to extract the correlation between the shell deflections and liquid speed potentials in the image space

$$\begin{aligned}
w^F(\xi) &= \frac{R(\xi, \omega)}{\mathrm{i}\omega}\Big[\varphi^{(1)F}(1, \xi) - \frac{\gamma_2}{\gamma_1}\varphi^{(2)F}(1, \xi)\Big]\\
R(\xi, \omega) &= \frac{\omega^2\frac{c_1^2}{c_m^2}\frac{f}{h}\Big(\omega^2\frac{c_1^2}{c_m^2} - \xi^2\Big)}{\nu^2\xi^2 + \Big(\omega^2\frac{c_1^2}{c_m^2} - \xi^2\Big)\Big(1 + \frac{h^2}{12}\xi^4 - \omega^2\frac{c_1^2}{c_m^2}\Big)}
\end{aligned} \tag{2.9}$$

3. Solution technique

The potential outside the cylindrical volume, which is the solution to the Helmholtz equation (2.1) in the cylindrical coordinates when $\rho \to \infty$, looks like

$$\varphi^{(2)}(\rho, z) = \int\limits_{-\infty}^{\infty} C(\xi) H_0\Big(\sqrt{\omega^2 \frac{c_1^2}{c_2^2} - \xi^2}\rho\Big) e^{i\xi z}\, d\xi \tag{3.1}$$

where $C(\xi)$ is the unknown function.

The liquid potential inside the shell is constructed in the form of a superposition

$$\varphi^{(1)} = \varphi_s^{(1)} + \varphi_c^{(1)} \tag{3.2}$$

of the potential caused by action due to the spherical body on the liquid and the potential defining the liquid disturbance carried into it through the cylindrical shell. The first function must decrease when $r \to \infty$, and the second one must be a limited function when $\rho \to 0$.

The component of the total potential caused by presence of the sphere and damped when the radial coordinate grows looks like

$$\varphi_s^{(1)}(r, \theta) = \sum_{n=0}^{\infty} x_n h_n(\omega r) P_n(\cos\theta) \tag{3.3}$$

where x_n is the unknown constants; h_n – spherical Hankel's function; P_n – Legendre's polynomials.

The potential which presents the solution to the Helmholtz equation in the cylindrical coordinates, limited when the radial coordinate tends to zero, looks like

$$\varphi_c^{(1)}(\rho, z) = \int\limits_{-\infty}^{\infty} B(\xi) J_0(\sqrt{\omega^2 - \xi^2}\,\rho) e^{i\xi z}\, d\xi \tag{3.4}$$

where $B(\xi)$ is the unknown function and J_0 a cylindrical Bessel's function.

The investigation is based on the possibility of representing the solution to the Helmholtz equation in form (3.2) both in the cylindrical and spherical coordinate systems. It is necessary for the boundary conditions to be satisfied on the surface of each body. In accordance with the developed solution technique the correlations that express the cylindrical wave function through spherical ones and vice versa are used (Yerofeyenko, 1972)

$$\begin{aligned}
&\mathrm{e}^{\mathrm{i}\xi z}J_0(\sqrt{\omega^2-\xi^2}\,\rho) = \sum_{n=0}^{\infty}\mathrm{i}^n(2n+1)P_n\Big(\frac{\xi}{\omega}\Big)j_n(\omega r)P_n(\cos\theta)\\
&h_n(\omega r)P_n(\cos\theta) = \frac{\mathrm{i}^{-n}}{2\omega}\int\limits_{-\infty}^{\infty} H_0(\sqrt{\omega^2-\xi^2}\,\rho)P_n\Big(\frac{\xi}{\omega}\Big)\mathrm{e}^{\mathrm{i}\xi z}\,d\xi
\end{aligned} \tag{3.5}$$

As a result, we obtain a representation of the total potential inside the shell in the spherical

$$\begin{aligned}
&\varphi^{(1)}(r,\theta) = \sum_{n=0}^{\infty}[x_nh_n(\omega r) + B_nj_n(\omega r)]P_n(\cos\theta)\\
&B_n = \mathrm{i}^n(2n+1)\int\limits_{-\infty}^{\infty} B(\xi)P_n\Big(\frac{\xi}{\omega}\Big)\,d\xi
\end{aligned} \tag{3.6}$$

and in the cylindrical coordinate systems

$$\begin{aligned}
&\varphi^{(1)}(\rho,z) = \int\limits_{-\infty}^{\infty}\Big[A(\xi)H_0(\sqrt{\omega^2-\xi^2}\,\rho) + B(\xi)J_0(\sqrt{\omega^2-\xi^2}\,\rho)\Big]\mathrm{e}^{\mathrm{i}\xi z}\,d\xi\\
&A(\xi) = \frac{1}{2\omega}\sum_{n=0}^{\infty}x_n\mathrm{i}^{-n}P_n\Big(\frac{\xi}{\omega}\Big)
\end{aligned} \tag{3.7}$$

Now, we can satisfy boundary conditions (2.3), (2.4).

At first it is necessary to write the boundary conditions on the shell surface (2.4) in the Fourier image space

$$\frac{\partial\varphi^{(l)F}(\rho,\xi)}{\partial\rho}\Big|_{\rho=\rho_0} = \mathrm{i}\omega w^F(\xi) \qquad l=1,2 \tag{3.8}$$

where, in accordance with expressions (3.1), (3.7)

$$\begin{aligned}
&\varphi^{(1)F}(\rho,\xi) = A(\xi)H_0(\sqrt{\omega^2-\xi^2}\,\rho) + B(\xi)J_0(\sqrt{\omega^2-\xi^2}\,\rho)\\
&\varphi^{(2)F}(\rho,\xi) = C(\xi)H_0\Big(\sqrt{\omega^2\frac{c_1^2}{c_2^2}-\xi^2}\rho\Big)
\end{aligned}$$

and the expression for $w^F(\xi)$ is determined by formula (2.9).

Satisfying boundary conditions (3.8) one can express the unknown functions $B(\xi)$, $C(\xi)$ through the coefficients x_n of the expansion of the "internal" fluid speed potential, caused by the sphere presence, into a Fourier series according to the Legendre polynomials

$$
\begin{aligned}
B(\xi) &= -\frac{A(\xi)}{\mathcal{D}(\xi)}\Big\{\sqrt{\omega^2-\xi^2}[1-\mathcal{M}(\xi)]H_1(\sqrt{\omega^2-\xi^2})+ \\
&+ R(\xi,\omega)H_0(\sqrt{\omega^2-\xi^2})\Big\} \\
C(\xi) &= \frac{A(\xi)}{\mathcal{D}(\xi)}\frac{R(\xi,\omega)\sqrt{\omega^2-\xi^2}}{\sqrt{\omega^2\frac{c_1^2}{c_2^2}-\xi^2}H_1\Big(\sqrt{\omega^2\frac{c_1^2}{c_2^2}-\xi^2}\Big)}\times \\
&\times\ \Big[J_0(\sqrt{\omega^2-\xi^2})H_1(\sqrt{\omega^2-\xi^2})-H_0(\sqrt{\omega^2-\xi^2})J_1(\sqrt{\omega^2-\xi^2})\Big]
\end{aligned}
\tag{3.9}
$$

The following designations are introduced here

$$
\begin{aligned}
\mathcal{D}(\xi) &= \sqrt{\omega^2-\xi^2}[1-\mathcal{M}(\xi)]J_1(\sqrt{\omega^2-\xi^2})+R(\xi,\omega)J_0(\sqrt{\omega^2-\xi^2}) \\
\mathcal{M}(\xi) &= \frac{\gamma_2}{\gamma_1}R(\xi,\omega)\frac{H_0\Big(\sqrt{\omega^2\frac{c_1^2}{c_2^2}-\xi^2}\Big)}{\sqrt{\omega^2\frac{c_1^2}{c_2^2}-\xi^2}\ H_1\Big(\sqrt{\omega^2\frac{c_1^2}{c_2^2}-\xi^2}\Big)}
\end{aligned}
$$

From the boundary condition on sphere surface (2.3) and by virtue of the orthogonality of the Legendre polynomials, we obtain the following relation for any n

$$
x_n h_n'(\omega r_0)+B_n j_n'(\omega r_0)=\frac{V_n}{\omega}
$$

Substitution of relations (3.6), $(3.9)_1$ into the last expression leads to an infinite system of linear algebraic equations

$$
x_n-\frac{1}{2\omega}(2n+1)\frac{j_n'(\omega r_0)}{h_n'(\omega r_0)}\sum_{m=0}^{\infty}\mathrm{i}^{n-m}q_{mn}x_m=\frac{V_n}{\omega h_n'(\omega r_0)}
\tag{3.10}
$$

for $n=0,1,2,...$, which enables to define coefficients of the expansion of the "internal" fluid speed potential into a Fourier series according to the Legendre polynomials.

The coefficients q_{mn} are determined as follows

$$
q_{mn} = \begin{cases} 2\int\limits_0^\infty \Big\{ \sqrt{\omega^2-\xi^2}[1-\mathcal{M}(\xi)]H_1(\sqrt{\omega^2-\xi^2}) + \\ +R(\xi,\omega)H_0(\sqrt{\omega^2-\xi^2})\Big\} \dfrac{P_n\Big(\frac{\xi}{\omega}\Big)P_m\Big(\frac{\xi}{\omega}\Big)}{\mathcal{D}(\xi)}\, d\xi & (n+m) - \text{even} \\ 0 & (n+m) - \text{odd} \end{cases} \tag{3.11}
$$

4. Numerical results

The complex and coupled problem has been reduced to the investigation and solution of an infinite system of linear algebraic equations (3.10). The system was solved by a truncation technique. The truncation order of this system was defined by a test in such a way that the sufficient accuracy of satisfying boundary conditions was reached.

The coefficients q_{mn} were determined by formula (3.11). The integration interval was divided into three segments: $0 \leqslant \xi < \omega c_1/c_2$, $\omega c_1/c_2 < \xi < \omega$ and $\omega < \xi < \infty$. The upper infinite limit was replaced with a finite one which guaranteed stability of the obtained results at least in the third decimal digid. In integration within the limits of the second and third of the above intervals the integrands were expressed through the modified Bessels functions. It should be mentioned that the integrands have singularities at points where their denominators are equal to zero. The investigation of the behaviour of the integrands in the ε-neighbourhood of the singular points showed that they tended to the same absolute value and opposite sign if calculated from the right and from the left of these points. During computation these points were isolated by a small ε-neighborhood.

All calculations were executed in the nondimensional variables. The following parameters of the internal and external liquids and shell material were considered:

- Internal medium: $c_1 = 1500\,\mathrm{m/s}$, $\gamma_1 = 1000\,\mathrm{kg/m^3}$
- External medium: $c_2 = 3000\,\mathrm{m/s}$, $\gamma_2 = 3000\,\mathrm{kg/m^3}$
- Shell material: $f = 1/8$, $\nu = 0.3$, $E = 2 \cdot 10^{11}\,\mathrm{N/m^2}$.

The internal fluid patameters were considered as normalization multipliers.

The spherical body surface was assumed to pulsate with the non-dimensional amplitude equal to one, according to the law

$$V(\theta) = 1 \tag{4.1}$$

or to oscillate in accordance with the relation

$$V(\theta) = \cos\theta \tag{4.2}$$

Note once more that the harmonic time-dependence is omitted.

The hydrodynamic and elastic characteristics of the concerned system "spherical inclusion-internal compressible liquid-elastic cylindrical shell-external compressible liquid" were investigated. At the same time we studied the influence of geometric proportions of the considered bodies and sphere vibration frequency on these characteristics. Comparisons were made with the cases of the sphere vibrations along the axis of the thin elastic cylindrical shell loaded only by the internal compressible liquid (without accounting for the external one).

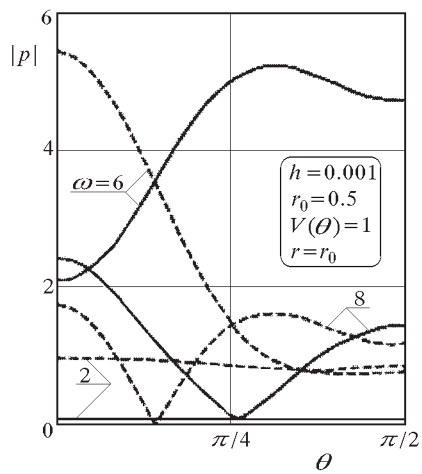

Fig. 2. Pressure distribution along the surface of the pulsating sphere for different frequencies

The influence of the frequency of sphere vibrations on the distribution of absolute values of the fluid pressure and shell flexures along the surfaces of considered bodies is shown in Fig. 2 - Fig. 5. A sphere with the radius equal to half of the cylinder one is considered. Here the law describing the sphere

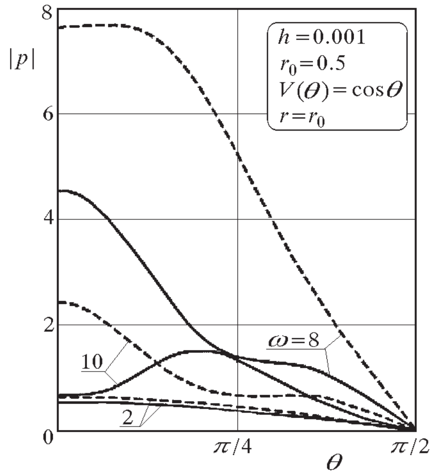

Fig. 3. Pressure distribution along the surface of the oscillating sphere for different frequencies

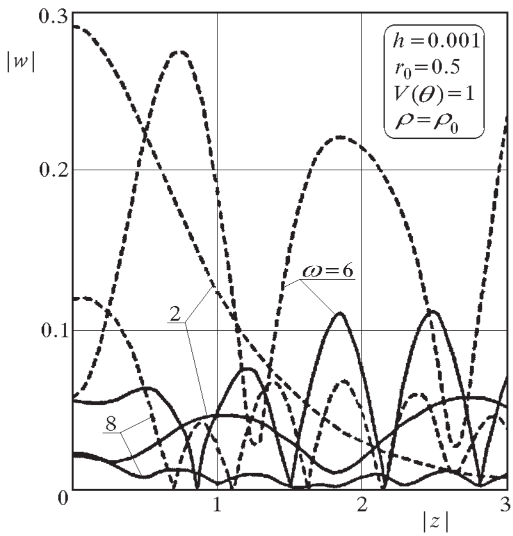

Fig. 4. Distribution of the deflections of the shell along its surface for different frequencies of sphere pulsations

surface vibrations was defined by relation (4.1), see Fig. 2 and Fig. 4, and by relation (4.2), see Fig. 3 and Fig. 5. Figures 2 and 3 illustrate the pressure distribution along the sphere surface in the region $0 \leqslant \theta \leqslant \pi/2$; figures 4 and 5 show the distribution of the shell deflections along its generatrix in the region $0 \leqslant |z| \leqslant 3$. The firm lines correspond to the characteristics calculated with taking into consideration the external medium; the dotted lines correspond to the characteristics calculated disregarding its influence.

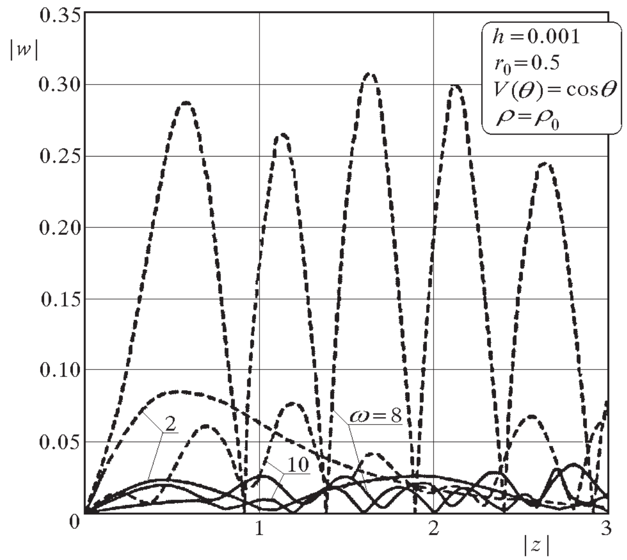

Fig. 5. Distribution of the deflections of the shell along its surface for different frequencies of sphere oscillations

Graphic dependences of absolute values of the pressure in the external liquid and its particles speed from the distance r between the spherical body center and a point being considered in shell external region ($\theta = \pi/2$; $\rho_0 \leqslant r \leqslant 3\rho_0$) for different frequencies of the sphere surface pulsations are given in Fig. 6 and Fig. 7.

The figures show, for the excitation frequency $\omega = 6$ for the sphere pulsations and $\omega = 8$ for the sphere oscillations, the absolute values of the pressure, shell deflections and liquid speed, both in the external and in the internal shell region, have essentially (by a few times) increasing amplitude. This circumstance apparently witnesses that there are families of cutoffs of waveguide modes in the cylindrical domain, which presents a great challenge in further investigations of such a hydrodynamic system.

5. Conclusions

An exact analytical solution (as a Fourier series to the problem) of interaction of an infinite thin elastic circular cylindrical shell, surrounded by an

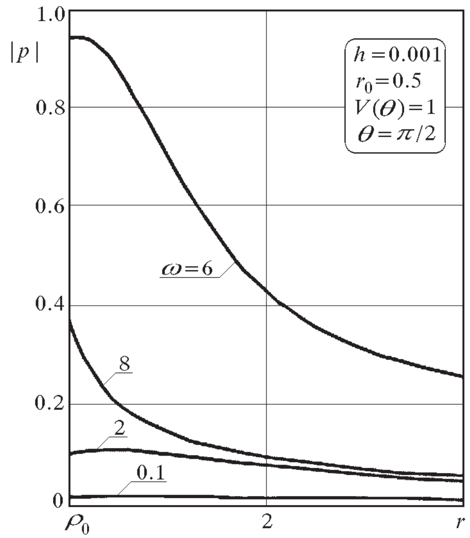

Fig. 6. Distribution of the pressure in the shell external region on the plane $z = 0$

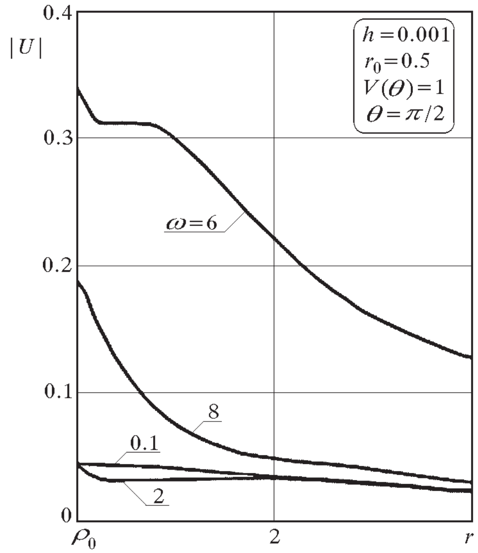

Fig. 7. Distribution of the external fluid speed on the plane $z = 0$

unbounded ideal compressible liquid and containing an other compressible liquid with a vibrating spherical inclusion in it has been obtained in the paper. The suggested approach has been based on the re-expansion of a particular solution to the Helmholtz equation, written in the spherical coordinates, by a system of cylindrical harmonic functions and *vice versa*. Such an approach enables one to meet boundary conditions on both the spherical and cylindrical surfaces. The developed method of construction of the exact liquid speed potential allows:

- investigation of the fields of speeds and pressures of the compressible liquids and the deformation state of the cylindrical shell also with the predetermined precision;
- study on some applied and technological processes (for example, vibrodisplacement and localization, purification and decontamination of liquid media, airing and dispersion, technologies of reconstructing oil production in corked wells) on the basis of more exact input data.

The theory has been tested numerically on a steel shell immersed into granite, filled with water and containing vibrating spherical inclusion on its axis. The obtained results can find and found real application to model and to investigate the problem of reconstruction of oil production in corked wells (for Sumy oil-and-gas production department).

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Oddziaływanie cylindrycznej powłoki z ciałem kulistym zanurzonym w idealnym ośrodku ściśliwym

Streszczenie

W pracy przedstawiono rezultaty badań analitycznych i numerycznych dotyczących problemu interakcji pomiędzy sprężystą powłoką cylindryczną w otoczeniu idealnego ośrodka ściśliwego, która wewnątrz zawiera inny ośrodek ściśliwy, a w nim drgający obiekt o kształcie kulistym. Analizowano małe drgania układu w ramach liniowej teorii sprężystości, a dynamikę ośrodków ściśliwych opisano równaniami Helmholtza. Rozwiązanie problemu otrzymano w drodze rozwinięcia rozwiązań cząstkowych równań Helmholtza wyrażonych we współrzędnych walcowych za pomocą funkcji sferycznych i na odwrót. Otrzymane wyniki mogą być przydatne w badaniach zagadnień transportu i pozycjonowania wibracyjnego, oczyszczania płynów, osuszania i rozpraszania, w bioakustyce, defektoskopii, medycynie układu krążenia, jak również w technologiach rekonstrukcji wydobycia ropy naftowej z zasypanych odwiertów.

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