JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
41, 3, pp. 521-536, Warsaw 2003

STUDY OF THE THERMOSTRESSED STATE OF ELECTRICALLY CONDUCTIVE NONFERROMAGNETIC SHELLS

Oleksandr Hachkevych[1,2]
Zygmunt Kasperski[2]
Borys Chornyj[1]
Oleh Dzyubachyk[1]

[1] *Pidstryhach Institute for Applied Problems of Mechanics and Mathematics, NASU, Lviv, Ukraine; e-mail: dept13@iapmm.lviv.ua*

[2] *Technical University of Opole, Poland; e-mail: kaspersk@po.opole.pl*

A method of determination of parameters describing electromagnetic, temperature and mechanical fields in electrically conductive shells caused by external quasisteady electromagnetic fields under presence of the strong skin effect is considered in this paper.

Key words: electrically conductive shell, temperature fields, stresses, quasisteady electromagnetic field, resonance phenomena

1. Introduction

In recent years, treatment and production of structural elements as well as traditional and new materials widely incorporates the use of the electromagnetic filed (EMF). For determination of the rational parameters of such a treatment, development of the corresponding methodology of the mathematical modelling and effective ways of research of mechanical, thermal and electromagnetic processes that happen in a deformable material medium under conditions of various-typed EMF influence is necessary. Also it concerns the exploitation of elements and products that work under electromagnetic loading.

The method of determination of parameters describing electromagnetic, thermal and mechanical processes in electrically conductive shells, caused by the action of external quasisteady EMF under presence of the strong skin effect is discussed in the paper. The method is a development of the known one for electrically conductive bodies in the case of electrically conductive shells.

2. Problem formulation

Consider a thin electrically conductive shell with the thickness $2h$ with linear electric and magnetic material properties. The shell space D is free from strange charges and currents. The shell is placed in a dielectric medium D_0, close to vacuum with respect to electric and magnetic properties. It is subjected to the quasisteady EMF. The field is created by the system of given in D_0 solenoidal currents (inductor) of AM diapason (radiofrequency with amplitude modulation) with the density (Tamm, 1976; Gaczkiewicz and Kasperski, 1999)

$$\begin{aligned} &\boldsymbol{j}^{(0)}(\boldsymbol{r},t) = \boldsymbol{j}_0^*(\boldsymbol{r},t)\cos(\omega t+\psi_0) \equiv \operatorname{Re}\boldsymbol{j}_*^{(0)}(\boldsymbol{r},t) \\ &\operatorname{div}\boldsymbol{j}_0^*(\boldsymbol{r},t) = 0 \end{aligned} \tag{2.1}$$

where

$\boldsymbol{r}$	–	radius vector
ω	–	circular frequency
t	–	time
ψ_0	–	initial phase
$\boldsymbol{j}_*^{(0)}(\boldsymbol{r},t)$	–	complex vector of current density; $\boldsymbol{j}_*^{(0)}(\boldsymbol{r},t) = \boldsymbol{j}_0^*(\boldsymbol{r},t)\mathrm{e}^{\mathrm{i}(\omega t+\psi_0)} \equiv \boldsymbol{j}^{(0)}(\boldsymbol{r},t)\mathrm{e}^{\mathrm{i}\omega t}$
$\boldsymbol{j}_0^*(\boldsymbol{r},t)$	–	modulated amplitude; $\boldsymbol{j}_0^*(\boldsymbol{r},t) = j(t)\boldsymbol{j}_0(\boldsymbol{r})$
$\boldsymbol{j}_0(\boldsymbol{r})$	–	amplitude of carrying signal
$\boldsymbol{j}^{(0)}(\boldsymbol{r},t)$	–	complex modulated amplitude of vector of current density; $\boldsymbol{j}^{(0)}(\boldsymbol{r},t) = \boldsymbol{j}_0^*(\boldsymbol{r},t)\mathrm{e}^{\mathrm{i}\psi_0}$
$j(t)$	–	function, describing the law of changing the signal in time, modulating the amplitude of electromagnetic oscillations (amplitude of carrying regime).

This function is slowly changing in the period $f_* = 2\pi/\omega$ (close to a constant), so that the condition

$$\Big|\frac{dj(t)}{dt}\Big| \ll \omega|j(t)| \qquad t > 0 \tag{2.2}$$

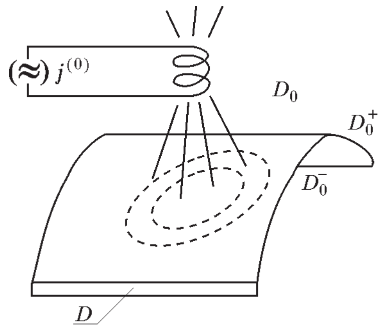

Fig. 1.

is satisfied (condition of the regime quasisteadiness (Tamm, 1976; Gaczkiewicz and Kasperski, 1999)).

We accept that the parameters of electromagnetic action $(\boldsymbol{j}_0(\boldsymbol{r}), j(t), \omega)$ are such that the action concerns "non-shocking" EMF under the value of magnetic field strength less than 10^6 A/m ($H_0 < 10^6$ A/m, where H_0 – the maximal value of the magnetic field strength in the body (Tamm, 1976; Wainberg, 1967). Under such parameters of the electromagnetic action we assume that displacements, deformations and their velocities are so small that the assumptions of the linear elasticity theory are satisfied, and the influence of movement on the characteristics of quasisteady EMF in the shell is negligible (Moon, 1978; Nowacki, 1986). We consider materials for which electromechanical and thermoelectric effects are small, and can be neglected. Thus, we treat the EMF as an external action to the shell. Its influence on the heat-conduction and deformation processes reduces to taking into account heats and ponderomotive factors (electromagnetic (ponderomotive) forces and turning moments). For materials, being linear with respect to electric and magnetic properties, vectors of strengths and inductions (displacements) of electric and magnetic fields are parallel (Rawa, 1994; Tamm, 1976), and electric and magnetic turning moments per unit volume are equal to zero. As a result, the connections between electromagnetic, temperature and mechanical fields in the considered case are taken into account through Joule's heat, ponderomotive forces, and also through the dependence between the deformation and temperature fields (thermoelastic dissipation of energy). In such an approach, accordingly to the known mathematical model of thermomechanics of electrically conductive shells under an action of a quasisteady EMF (Gaczkiewicz *et al.*, 1997), the initial problem of determination of parameters describing thermomechanical behavior of the shell, will be resolved through two stages.

At the first stage, from the equation of electrodynamics of shells (neglecting the influence of displacement currents in the shell region), we determine EMF parameters in it, and then, on their base – Joule's heat and ponderomotive forces (as functions of electrodynamic parameters). At the second stage, we find the temperature and mechanical fields. With it we set forth the equations of the coupled problem of thermomechanics under the known sources of heat and volume forces, which are correspondingly the Joule heats and ponderomotive forces, under given conditions of thermal exchange of the shell with the external medium and conditions of mechanical fixation.

Consider frequencies, for which the parameter $\delta = (2\mu\omega\sigma h^2)^{-1/2}$ (characterizing relative depth of the inductive current penetration into the shell (Podstrigach *et al.*, 1977; Tamm, 1976); μ – magnetic penetrance; σ – coefficient of electrical conductivity) is small in comparison with unit

$$\delta = \frac{1}{\sqrt{2\mu\omega\sigma h^2}} \ll 1 \tag{2.3}$$

(condition of presence of the strong skin effect (Rawa, 1994; Tamm, 1976)).

For determination of the parameters of examined fields we will use:

- method of solving singularly perturbed equations of electrodynamics in the form of asymptotic expansions on powers of the small parameter δ
- method of spectral expansions on the shell thickness for determination of the temperature
- known methods of solving the equations of thermoelasticity of thin shells for determination of mechanical fields.

In the region D of the shell and in the certain round area D_0 we introduce a mixed curvilinear coordinate system $(\alpha_1, \alpha_2, \gamma)$, in which α_j $(j = 1, 2)$ are the lines of the main curvatures k_j of the medium surface of the shell, and where the coordinate γ determines the place of the point on the line normal to this surface. In the further part of the paper, all metric characteristics are given with respect to the half of the shell thickness h.

3. Mathematical model

At the first stage, i.e. in the determination of the electromagnetic field parameters under the given distribution of external currents, we assume that the equations of the electrodynamics of shells are simplified due to negligibly small

values $k_j\gamma$ in comparison with the unit. We also assume that the parameters of EMF in the system "shell – external medium" approximately look like (quasisteady approximation) (Gaczkiewicz and Kasperski, 1999; Podstrigach *et al.*, 1977):
— vector of the current density

$$\boldsymbol{j}_* = j(t)\mathrm{Re}\Big\{\boldsymbol{j}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega t}\Big\}$$

— electric field strength

$$\boldsymbol{E}_* = j(t)\mathrm{Re}\Big\{\boldsymbol{E}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega t}\Big\} \qquad \boldsymbol{E}_*^{(0)} = j(t)\mathrm{Re}\Big\{\boldsymbol{E}^{(0)}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega t}\Big\}$$

— magnetic field strength

$$\boldsymbol{H}_* = j(t)\mathrm{Re}\Big\{\boldsymbol{H}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega t}\Big\} \qquad \boldsymbol{H}_*^{(0)} = j(t)\mathrm{Re}\Big\{\boldsymbol{H}^{(0)}(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\omega t}\Big\}$$

in the region of the shell D and the region of the external medium D_0. Then, the determination of this parameters reduces to solving of the boundary problem, governed by the equations (Gaczkiewicz *et al.*, 1997; Podstrigach *et al.*, 1977)

$$\Big[\delta^2\Big(\frac{\partial^2}{\partial\gamma^2} + \nabla^2 - k^2\Big) - \frac{\mathrm{i}}{2}\Big]\boldsymbol{E} = \boldsymbol{0} \qquad x \in D \tag{3.1}$$

– for the shell region and

$$(\Delta + k_{0*}^2)\boldsymbol{E}_{\pm}^{(0)} = \mathrm{i}\mu_0\omega h^2 \boldsymbol{j}_{\pm}^{(0)} \qquad x \in D_0^{\pm} \tag{3.2}$$

– for the subregions $D_0^{\pm}$ of the region D_0 of the external medium, external to the surface $\gamma = \pm 1$ of the shell. On the surfaces $\gamma = \pm 1$, dividing D and D_0, conditions of the ideal electromagnetic contact (Podstrigach *et al.*, 1977) are satisfied

$$\begin{aligned}
&E_j^{\pm} = E_j^{(0)\pm}\\
&\Big(\frac{\partial}{\partial\gamma} + k_j - k\Big)E_j^{\pm} - \frac{1}{A_j}\frac{\partial E_\gamma^{\pm}}{\partial\alpha_j} = \mu_*\Big[\Big(\frac{\partial}{\partial\gamma} + k_j\Big)E_j^{(0)\pm} - \frac{1}{A_j}\frac{\partial E_\gamma^{(0)\pm}}{\partial\alpha_j}\Big]\\
&E_\gamma^{\pm} = 2\mathrm{i}\mu_* k_{0*}^2\delta^2 E_\gamma^{(0)\pm}\\
&\Big(\frac{\partial}{\partial\gamma} + k\Big)E_\gamma^{\pm} = \Big(\frac{\partial}{\partial\gamma} + 2k\Big)E_\gamma^{(0)\pm}
\end{aligned} \tag{3.3}$$

In infinity, the radiation conditions (Gaczkiewicz and Kasperski, 1999; Gaczkiewicz *et al.*, 1997; Podstrigach *et al.*, 1977; Tamm, 1976) is

$$\lim_{\gamma\to\pm\infty}\gamma\Big(\frac{\partial}{\partial\gamma}\pm \mathrm{i}k_{0*}\Big)\boldsymbol{E}_{\pm}^{(0)}=\boldsymbol{0} \tag{3.4}$$

In (3.1)-(3.3) the repeated indices are not the summation ones; they, i.e. j, $(j=1,2)$ correspond to α_j coordinates. In the formulas above, the following denote:

$\boldsymbol{E}_{\pm}^{(0)},\boldsymbol{j}_{\pm}^{(0)}$	–	complex amplitudes of the electric field strength and density of current in the subregions $D_0^{\pm}$, external to the surface $\gamma=\pm1$ of the shell
ε_0,μ_0	–	dielectric and magnetic penetrances of vacuum
$A_j,\ (j=1,2)$	–	coefficients of the first quadratic form of the medial surface (Korn and Korn, 1968)

and

$$\boldsymbol{E}^{(0)+}=\boldsymbol{E}_{+}^{(0)}\Big|_{\gamma=1}\qquad \boldsymbol{E}^{(0)-}=\boldsymbol{E}_{-}^{(0)}\Big|_{\gamma=-1}$$
$$k_{0*}^2=\varepsilon_0\mu_0\omega^2h^2\qquad \mu_*=\mu\mu_0^{-1}\qquad 2k=k_1+k_2$$

The components of the vector $\nabla^2\boldsymbol{E}$ can be written down as (Gaczkiewicz *et al.*, 1997)

$$\begin{aligned}(\nabla^2\boldsymbol{E})_j&=L_j^2E_j+(-1)^lLE_l+L_j^{-1}E_\gamma \qquad l,j=1,2\quad (l\neq j)\\ (\nabla^2\boldsymbol{E})_\gamma&=L_\gamma^2E_\gamma-L_1^{+}E_1-L_2^{+}E_2\end{aligned} \tag{3.5}$$

where

$$\begin{aligned}
L_j^2&=\nabla^2-B^2-k_j^2 \qquad\qquad L_\gamma^2=\nabla^2-k_1^2-k_2^2\\
L&=\frac{B_1}{A_1}\Big(2\frac{\partial}{\partial\alpha_1}-\frac{1}{A_1}\frac{\partial A_1}{\partial\alpha_1}\Big)-\frac{B_2}{A_2}\Big(2\frac{\partial}{\partial\alpha_2}-\frac{1}{A_2}\frac{\partial A_2}{\partial\alpha_2}\Big)+\\
&\quad+\frac{1}{A_1A_2}\Big(\frac{1}{A_1}\frac{\partial^2A_1}{\partial\alpha_1\partial\alpha_2}-\frac{1}{A_2}\frac{\partial^2A_2}{\partial\alpha_1\partial\alpha_2}\Big)\\
L_j^{\pm}&=\frac{2k_j}{A_j}\frac{\partial}{\partial\alpha_j}+(k_j\pm k_l)B_l+\frac{1}{A_j}\frac{\partial k_j}{\partial\alpha_j} \qquad l,j=1,2\quad (l\neq j)\\
\nabla^2&=\frac{1}{A_1A_2}\Big[\frac{\partial}{\partial\alpha_1}\Big(\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}\Big)+\frac{\partial}{\partial\alpha_2}\Big(\frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\Big)\Big]\\
B^2&=B_1^2+B_2^2 \qquad\qquad B_j=\frac{1}{A_jA_l}\frac{\partial A_j}{\partial\alpha_l}
\end{aligned}$$

and the Laplace operator in equation (3.2) is

$$\Delta = \frac{\partial^2}{\partial\gamma^2} + 2k\frac{\partial}{\partial\gamma} + \nabla_*^2$$

$$\nabla_*^2 = \frac{1}{A_1A_2}\Big[\frac{\partial}{\partial\alpha_1}\Big(\frac{A_2}{A_1}\frac{\partial}{\partial\alpha_1}\Big) + \frac{\partial}{\partial\alpha_2}\Big(\frac{A_1}{A_2}\frac{\partial}{\partial\alpha_2}\Big)\Big]$$

In the case when one of the external subregions $D_0^\pm$ (for example D_0^-) is constrained, it is necessary to replace radiation condition (3.4) in this subregion with the condition of the finiteness of the function $\boldsymbol{E}_-^{(0)}$ to be determined.

While considering nonclosed shells, constrained by surfaces of the coordinate system $\alpha_j = \alpha_j^\pm$, we set conditions similar to (3.3) on these surfaces.

On the base of condition (2.3) equations (3.1), (3.2) for complex amplitudes of the electric field strength (of an elliptic type) are singularly perturbed. Therefore, to construct their solution we use the known method of asymptotic expansions (Gaczkiewicz *et al.*, 1997; Vishyk and Liusternik, 1960). In the environment of bases $\gamma = \pm 1$ of the shell (subregions with the thickness $\beta^\pm = \beta^\pm(\delta)$, $0 < \beta^\pm < 1$) we introduce the regularizing substitution $\xi^\pm = \rho^\pm/\delta$, where $\rho^\pm = 1 \pm \gamma$. Further, the solution of the initial problem in the subregions $D_0^\pm$ and $D^\pm$ is given in terms of series with respect to the small parameter δ. Each next approximation is searched in the form of a function of the boundary-layer type that corresponds to quick attenuation of EMF in the direction normal to the surface $\gamma = \pm 1$. As a result, the solutions in the chosen shell subregions (Gaczkiewicz and Kasperski, 1999) will be

$$\begin{aligned} E_j^\pm &= \delta \sum_{n=0}^{\infty} \delta^n \sum_{k=0}^{\left[\frac{n}{2}\right]} B_{j,k}^{\pm(n)}(\alpha_1,\alpha_2)(\xi^\pm)^k \mathrm{e}^{-\xi^\pm} \\ E_\gamma^\pm &= \delta^2 \sum_{n=0}^{\infty} \delta^n \sum_{k=0}^{\left[\frac{n+1}{2}\right]} B_{\gamma,k}^{\pm(n)}(\alpha_1,\alpha_2)(\xi^\pm)^k \mathrm{e}^{-\xi^\pm} \end{aligned} \tag{3.6}$$

The coefficients $B_{j,k}^{\pm(n)}$, $B_{\gamma,k}^{\pm(n)}$ are determined consecutively with the help of recurrent correlations through their previous values and solutions of corresponding boundary problems for the external medium region

$$\begin{aligned} &(\Delta + k_0^2)\boldsymbol{E}_0^{\pm(n)} = (\mathrm{i}\omega\mu_0 h^2 \boldsymbol{j}_0^\pm)\delta_{n0} \qquad n = 0,1,2,\ldots \\ &E_{0j}^{\pm(n)} = B_{j,0}^{\pm(n-1)} \\ &\Big(\frac{\partial}{\partial\gamma} + 2k\Big)E_{0\gamma}^{\pm(n)}\Big|_{\gamma=\pm1} = \pm\frac{\mathrm{i}+1}{2}B_{\gamma,0}^{\pm(n-1)} \mp B_{\gamma,1}^{\pm(n-1)} + kB_{\gamma,0}^{\pm(n-2)} \end{aligned} \tag{3.7}$$

under radiation conditions in infinity. There δ_{n0} denotes Kronecker's symbol, values with negative indices $(n-p)$ are identically equal to zero.

While solving the sequence of boundary problems (3.7) we use a representation in the form of a series with respect to the small parameter k_{0*}. The first approximation (that corresponds to the zero power) is the solution to problem (3.7) under quasisteady conditions, i.e. under neglecting the displacement currents in the region $D_0^{\pm}$ of the external medium.

The solution to the formulated problem for the shell region D is constructed by the method of prolongating solutions to (3.6) from the subregions $D^{\pm}$ on the whole region D with the help of the smoothing multipliers $\psi^{\pm}$ (Vishyk and Liusternik, 1960) in the appearance

$$\boldsymbol{E} = \psi^{+}\boldsymbol{E}^{+} + \psi^{-}\boldsymbol{E}^{-}$$

For such a solution it is possible to make the evaluation of the reminder in the metric L_2 (Gaczkiewicz *et al.*, 1997).

Notice, that the problem of determination of EMF parameters can be formulated in a similar way with respect to the magnetic field strength $\boldsymbol{H}$ as well.

At the second stage of solving, i.e. during determination of the thermostressed state parameters, the initial ones will be Joule's heats Q_* and ponderomotive forces $\boldsymbol{F}_*$, which in the considered case can be written down as

$$\begin{aligned} Q_* &= \boldsymbol{j}_{**}\cdot\boldsymbol{E}_{**} = \sigma\boldsymbol{E}_{**}^2 \equiv \frac{1}{\sigma}\operatorname{rot}\boldsymbol{H}_{**}^2 \\ \boldsymbol{F}_* &= \boldsymbol{F}_{*A} = \boldsymbol{j}_{**}\times\boldsymbol{B}_{**} = \mu\sigma\boldsymbol{E}_{**}\times\boldsymbol{H}_{**} = \mu\operatorname{rot}\boldsymbol{H}_{**}\times\boldsymbol{H}_{**} \end{aligned} \tag{3.8}$$

where $\boldsymbol{j}_{**}$, $\boldsymbol{E}_{**}$, $\boldsymbol{H}_{**}$ are real current density and sterengths of the electric and magnetic field, respectively. For considered quasisteady EMF that corresponds to real parts of complex vectors taking into account the dependence $\operatorname{Re}\boldsymbol{a} = (\boldsymbol{a}+\overline{\boldsymbol{a}})/2$ (where $\overline{\boldsymbol{a}}$ is a complex conjugate to $\boldsymbol{a}$), we obtain (Gaczkiewicz and Kasperski, 1999)

$$Q_* = Q_{(1)} + Q_{(2)} \qquad\qquad \boldsymbol{F}_* = \boldsymbol{F}_{(1)} + \boldsymbol{F}_{(2)} \tag{3.9}$$

In dependences (3.9)

$$
\begin{aligned}
Q_{(1)} &= \frac{\sigma}{2}\boldsymbol{E}_* \cdot \overline{\boldsymbol{E}}_* = \frac{\sigma}{2}\varphi(t)\boldsymbol{E}(\boldsymbol{r}) \cdot \overline{\boldsymbol{E}}(\boldsymbol{r}) \\
Q_{(2)} &= \frac{\sigma}{4}(\boldsymbol{E}_*^2 + \overline{\boldsymbol{E}}_*^2) = \frac{1}{4}\varphi(t)\Big(\boldsymbol{E}^2(\boldsymbol{r})\mathrm{e}^{2\mathrm{i}\omega t} + \overline{\boldsymbol{E}}^2(\boldsymbol{r})\mathrm{e}^{-2\mathrm{i}\omega t}\Big) \equiv \\
&\equiv \frac{1}{2}\varphi(t)\mathrm{Re}\left(\boldsymbol{E}^2\mathrm{e}^{2\mathrm{i}\omega t}\right) \\
\boldsymbol{F}_{(1)} &= \frac{1}{2}\sigma\mu\varphi(t)\mathrm{Re}\left(\boldsymbol{E}(\boldsymbol{r}) \times \overline{\boldsymbol{H}}(\boldsymbol{r})\right) \\
\boldsymbol{F}_{(2)} &= \frac{1}{2}\sigma\mu\varphi(t)\mathrm{Re}\left(\boldsymbol{E}(\boldsymbol{r}) \times \boldsymbol{H}(\boldsymbol{r})\mathrm{e}^{2\mathrm{i}\omega t}\right) \\
\varphi(t) &= j^2(t)
\end{aligned}
\tag{3.10}
$$

Taking into account (3.6) and writing down the Maxwell equation

$$
\boldsymbol{H} = -\frac{1}{\mathrm{i}\mu\omega}\,\mathrm{rot}\,\boldsymbol{E}
$$

for the shell case

$$
\begin{aligned}
H_j &= \frac{(-1)^{j+1}}{\mathrm{i}\mu\omega}\Big[\Big(\frac{\partial}{\partial\gamma} + k_l\Big)E_l - \frac{1}{A_l}\frac{\partial E_\gamma}{\partial\alpha_l}\Big] \qquad\qquad l,j = 1,2, \quad l \neq j \\
H_\gamma &= -\frac{1}{\mathrm{i}\mu\omega A_1 A_2}\Big[\frac{\partial}{\partial\alpha_1}(A_2E_2) - \frac{\partial}{\partial\alpha_2}(A_1E_1)\Big]
\end{aligned}
\tag{3.11}
$$

we express the functions $Q_{(j)}$ and $\boldsymbol{F}_{(j)}$ $(j = 1,2)$ in the form of asymptotic expansions on the powers of the parameter δ.

Correspondingly, due to the formal structure of $\boldsymbol{E}$ and $\boldsymbol{H}$ for quasisteady EMF representation (3.9) of heats and ponderomotive forces in function of $Q_{(1)}$, $\boldsymbol{F}_{(1)}$ and $Q_{(2)}$, $\boldsymbol{F}_{(2)}$ we search the temperature and components of the tensor of stresses as

$$
w = w^{(1)} + w^{(2)} \qquad\qquad w \equiv \{T, \widehat{\sigma}\} \tag{3.12}
$$

Slow-changing components $T^{(1)}$, $\widehat{\sigma}^{(1)}$ are searched in a quasistatic formulation neglecting the coupling between temperature and deformation fields (Kovalenko, 1975; Nowacki, 1986), i.e. on the base of correlations of the quasistatic thermomechanics of shells (Kovalenko, 1975; Nowacki, 1986; Podstrigach and Shvets, 1978). The components $T^{(2)}$, $\widehat{\sigma}^{(2)}$ are searched in a quasisteady representation

$$
w^{(2)} = 2\sum_{m=1}^{\infty}\mathrm{Re}\left(w_*^{(2m)}(\boldsymbol{r},t)\mathrm{e}^{2\mathrm{i}m\omega t}\right) \equiv 2\varphi(t)\sum_{m=1}^{\infty}\mathrm{Re}\left(w^{(2m)}(\boldsymbol{r})\mathrm{e}^{2\mathrm{i}m\omega t}\right) \tag{3.13}
$$

(there $w_*^{(2m)}(\boldsymbol{r},t) = \{T^{(2m)}(\boldsymbol{r},t), \widehat{\boldsymbol{\sigma}}^{(2m)}(\boldsymbol{r},t)\}$) – slow-changing in the period f_* functions) from the corresponding dynamic task of coupled thermoelasticity. Taking into account the known data (Gaczkiewicz and Kasperski, 1999) indicating negligible influence of quite periodical components of Joule's heat on the thermostressed state of electrically conductive bodies under the conditions of strong skin effect in comparison with the influence of ponderomotive forces, we assume that the components $T^{(2)}$, $\widehat{\boldsymbol{\sigma}}^{(2)}$ are caused by the quasisteady component $\boldsymbol{F}_{(2)}$ of the ponderomotive force. Since thermal perturbation in this case is caused by the deformation from a quick-changing dynamic action, the process of deformation is considered as adiabatic (Kovalenko, 1975; Nowacki, 1986), and the components $T^{(2)}$, $\widehat{\boldsymbol{\sigma}}^{(2)}$ in (3.12), (3.13) are determined from the correlations of the dynamical problem of thermoelasticity in an adiabatic quasisteady approximation. Thus, from the correlations for shells, we obtain an increment in the temperature $T^{(2)}$ equal to (Kovalenko, 1975; Nowacki, 1986)

$$T^{(2)} = -\frac{(3\lambda_* + 2\mu_*)\alpha_t a T_0 \varepsilon_{kk}}{\lambda} \tag{3.14}$$

where λ_* and μ_* are isothermal Lamé moduli; ε_{kk} – the first invariant of the deformation tensor. Then

$$T^{(2)} = -\frac{(3\lambda_* + 2\mu_*)\alpha_t T_0}{c_\varepsilon(3\lambda_s + 2\mu_*)}\sigma_{kk}^{(2)} \equiv -\frac{\alpha_t a T_0}{\Big(1 + 3\varepsilon_* \frac{1-\nu}{1+\nu}\Big)\lambda}\sigma_{kk}^{(2)} \tag{3.15}$$

In (3.15), (Kovalenko, 1975; Nowacki, 1986)

$$\lambda_s = \lambda_* + \frac{(3\lambda_* + 2\mu_*)^2\alpha_t^2 a T_0}{\lambda} \equiv \frac{\nu E}{(1+\nu)(1-2\nu)}\Big(1 + \varepsilon_*\frac{1-\nu}{\nu}\Big)$$

is the adiabatic Lamé module (Kovalenko, 1975), and

$$\varepsilon_* = \frac{(3\lambda_* + 2\mu_*)^2\alpha_t^2 T_0}{(\lambda_* + 2\mu_*)c_\varepsilon} \equiv \frac{(1+\nu)\alpha_t^2 a E T_0}{(1-\nu)(1-2\nu)\lambda}$$

denotes the parameter of conjugation of the deformation and temperature fields (Kovalenko, 1975; Nowacki, 1986), where

$$\sigma_{kk}^{(2)} = \sigma_{11}^{(2)} + \sigma_{22}^{(2)} + \sigma_{33}^{(2)} \qquad\qquad c_\varepsilon = \frac{\lambda}{a}$$

and

α_t	–	coefficient of linear thermal expansion
ν	–	Poisson's coefficient
T_0	–	initial temperature, [K]

λ – coefficient of heat-conduction
a – coefficient of heat diffusion.

Notice, that the slow-changing component $T^{(1)}$ on the assumption that the shell stays in conditions of convective heat transfer with the external medium, can be effectively determined from the simplified (due to the fact that the shell is thin-walled) equation of heat-conduction (Podstrigach and Shvets, 1978) and corresponding initial and boundary conditions making use of the spectral expansions on the thickness coordinate (the method of finite integral transformations) (Galitsin and Zhukovskii, 1976). The solution is searched in the form

$$T(\alpha_1, \alpha_2, \gamma, \tau) = \sum_{m=1}^{\infty} K_m(\gamma)\widetilde{T}_m(\alpha_1, \alpha_2, \tau) \tag{3.16}$$

The coefficients $\widetilde{T}_m$ are determined from the solution to the equation

$$\Big(\nabla^2 - \frac{\partial}{\partial\tau} - \beta_{0m}^2\Big)\widetilde{T}_m = -\widetilde{Q}_{*m} \qquad\qquad \widetilde{T}_m\Big|_{\tau=0} = 0 \tag{3.17}$$

under given conditions on the shell edges. In equation (3.17)

$$\widetilde{Q}_{*m} = \frac{h^2}{\lambda}\int\limits_{-1}^{1} Q_{(1)}K_m(\gamma)\,d\gamma \tag{3.18}$$

where

$K_m(\gamma)$ – kernel of the transformation; $K_m(\gamma) = \varPhi_m(\gamma)/\|\varPhi_m\|$
β_{0m} – characteristic values
$\varPhi_m(\gamma)$ – characteristic functions of the corresponding Sturm-Liouville task;
$\varPhi_m(\gamma) = (\mathrm{Bi}^- + k)\sin[\beta_m(1+\gamma)] + \beta_m\cos[\beta_m(1+\gamma)]$
$\|\varPhi_m\|$ – normalizing multiplier, $\|\varPhi_m\| = [\int_{-1}^{1}\varPhi_m^2(\gamma)d\gamma]^{1/2}$
β_m – positive roots of the equation

$$\Big[\beta_m^2 - (\mathrm{Bi}^+ - k)(\mathrm{Bi}^- + k)\Big]\tan^2\beta_m = (\mathrm{Bi}^+ + \mathrm{Bi}^-)\beta_m$$

$$\beta_m = \sqrt{\beta_{0m}^2 - k^2}$$

$\tau = at/h^2$ – Fourier criterium
$\mathrm{Bi}^{\pm} = hH^{*\pm}$ – Biot criterium
$H^{*\pm}$ – coefficients of convective heat exchange from the surfaces $\gamma = \pm 1$.

Under condition that EMF is harmonic ($j(t) = 1$ and the density of heats $Q^* = Q_{(1)}h^2/\lambda$ is a function of coordinates only), we can obtain an asymptotic solution to the three-dimensional problem of heat-conduction for the initial period of heating ($\tau \ll 1$). As a small parameter we take the value $1/s$, where s is the parameter of Laplace's transformation. Specifically, we will obtain such an expression for the temperature field (in the second approximation)

$$
\begin{aligned}
T \;=\; & \tau Q_* - \psi^+ \sqrt{(4\tau)^3} I^3 \mathrm{erfc}\Big(\frac{1+\gamma}{2\sqrt{\tau}}\Big)\Big[\frac{\partial}{\partial\tau}Q_* + \Big(\mathrm{Bi}^+ - k\Big)\Big]_{\gamma=1} + \\
+ \; & \psi^- \sqrt{(4\tau)^3} I^3 \mathrm{erfc}\Big(\frac{1+\gamma}{2\sqrt{\tau}}\Big)\Big[\frac{\partial}{\partial\tau}Q_* - \Big(\mathrm{Bi}^- + k\Big)\Big]_{\gamma=-1}
\end{aligned}
\tag{3.19}
$$

where (Lykov, 1967)

$$
I^p \mathrm{erfc}(x) = \int\limits_x^\infty I^{p-1}\mathrm{erfc}(\xi)\, d\xi \qquad\qquad I^0\mathrm{erfc}(x) = \mathrm{erfc}(x)
$$

In expression (3.19) components corresponding to the boundary layer take into account the influence of the heat exchange process on the temperature field during short heating times.

4. Calculations results

In Figures 2 and 3 the distributions of the temperature T and components of the stress tensor $\widehat{\boldsymbol{\sigma}}$ for a cylindric shell of radius $R = 0.40\,\mathrm{m}$ subject to induction heating by the external currents of a constant amplitude j_0 ($j(t) = 1$) applied coaxially to the shell surface of radius $R = 0.42\,\mathrm{m}$ in the direction tangential to the line of the cross-section are shown. The shell, which is made of the rustless steel (X18H9T) (Gaczkiewicz and Kasperski, 1999) is heat-insulated on the bases $\gamma = \pm 1$.

In Fig. 2 and Fig. 3 the solid lines depict functions of temperature and stress on the surface nearest to the inductor ($\gamma = 1$), the dash-dotted ones on the medium surface ($\gamma = 0$), and the dashed – on the interior surface ($\gamma = -1$) of the shell. Lines 1 correspond to $\delta = 0.1$, and lines 2 – to $\delta = 0.2$.

The value of the current j_0 was determined from the condition that at the instant τ_* the shell is heated up to the given temperature T_*.

In Fig. 2 the dependance of the temperature on the parameter τ/τ_* for $\tau_* = 1$ is given. One can see that the temperature level essentially decreases with a drop in γ. On the inductor side of the surface the temperature level is

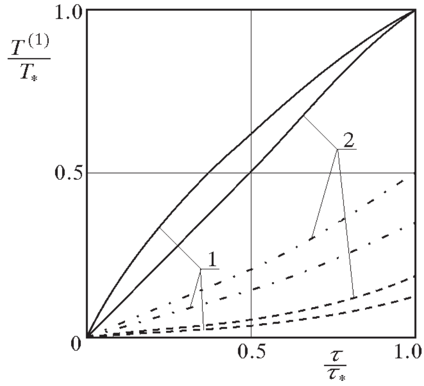

Fig. 2.

higher for lower δ, and on the medium and interior ones on the contrary — higher for greater values of the parameter δ.

In Fig. 3 the dependance of stresses $\sigma_0^* = \sigma_{\phi\phi} = \sigma_{xx}$ in time is given. The stresses on the surface $\gamma = 1$ are compressive, and on $\gamma = -1$ – tensile. The stresses for $\gamma = 1$ are growing quicker, and their absolute value exceeds the stresses on the interior surface. With decreasing δ the level of stresses on the exterior surface is growing, and on the interior – decreasing.

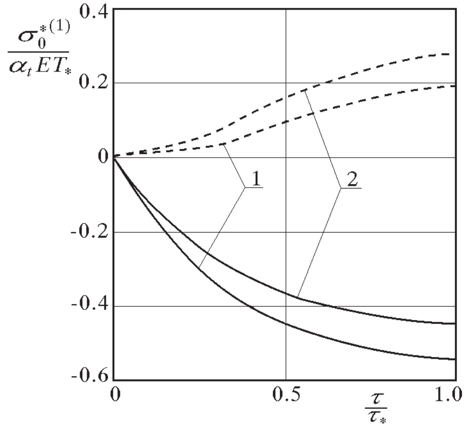

Fig. 3.

Notice that for the considered parameters δ the components of the temperature and stresses $T^{(2)}$, $\sigma_0^{*(2)}$ are negligible in comparison with $T^{(1)}$ and $\sigma_0^{*(1)}$

$$\sigma_0^{*(1)} = \frac{\alpha_t E}{1-\nu}(T_1 - T) \qquad\qquad T_1 = \frac{1}{2h}\int\limits_{-1}^{1} T\, d\gamma$$

The quasistatic stresses caused by the ponderomotive forces are also negligible.

Carried out investigations on the thermomechanical behavior of the shells in function of the penetration depth (frequencies of the external EMF) showed, that as well as for the bodies of a simple geometrical structure (Gaczkiewicz and Kasperski, 1999) in the neigbourhood of the EMF frequencies $\omega_n = 0.5\omega_n^*$, $n = 1, 2, ...$ (where ω_n^* – eigenfrequency of the thermoelastic shell oscillations) the levels of quite periodic components of the temperature and stresses in a non-polarized ferromagnetic significantly grow and become proportional (resonance phenomena take place). High levels of the quite periodic components of temperature are caused by the coupling between deformation and temperature fields. With the growth in the number n of resonance frequency the amplitudes of quite periodic components $T^{(2)}$, $\widehat{\boldsymbol{\sigma}}^{(2)}$ decrease.

With respect to the weakness of the coupling between the deformation and temperature field parameter ε_* ($\varepsilon_* \ll 1$) for steel shells (for which the phenomena of the strong skin effect take place), we notice that each eigenfrequency of the thermoelastic oscillations ω_* practically equals to the corresponding eigenfrequency of the elastic oscillations of the considered shell. Fig. 4 the solid lines illustrate the dependance of the first two resonance frequencies of EMF ω_n (curves 1 and 2 correspondingly) on the shell thickness (for a shell of radius $R = 0.40$ m made of the steel X18H9T). The dashed lines correspond to the dependance of the "resonance" parameter of the relative depth of the currents penetration on the thickness. For the given shell thickness lower δ corresponds to higher resonance frequencies. With the growth of the frequency the resonance frequencies decrease.

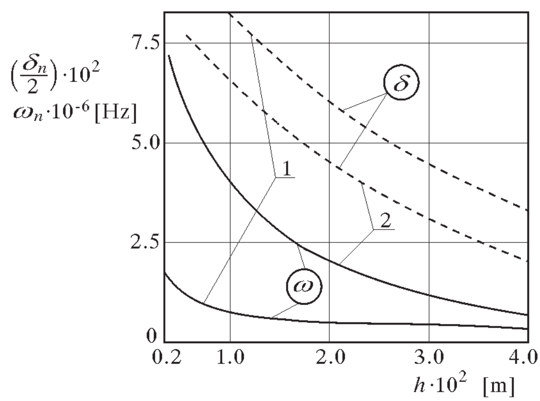

Fig. 4.

5. Conclusions

The area of the resonance frequency (value of the deviation $\Delta\omega_1$ of the EMF frequency ω from the first resonance ω_1, for which the maximal value of the stress $\widehat{\boldsymbol{\sigma}}^{(2)}$ constitutes not less than 10 % of the maximal value of $\widehat{\boldsymbol{\sigma}}^{(1)}$ in a steady regime) does not depend in linear materials on the EMF characteristics and is narrow ($\Delta\omega_1 \leqslant 10^{-4} \div 10^{-5}\omega_1$ – for non-magnetic materials and $\Delta\omega_1 \leqslant 10^{-4} \div 10^{-5}\mu_*^2\omega_1$ – for magnetic ones). In this area the maximal values of $T^{(2)}$ and $\widehat{\boldsymbol{\sigma}}^{(2)}$ are caused by the ponderomotive forces (the influence of the Joule heats is negligible) and not significantly depend on Biot's criterium. Outside of the resonance area the thermostressed state of the shell is determined by the slow-changing components $Q^{(1)}$ and $\boldsymbol{F}^{(1)}$ of the heats and ponderomotive forces (which coincide with the averaged ones over the period f_*, i.e.

$$M^* = \frac{2\pi}{\omega} \int\limits_t^{t+\frac{2\pi}{\omega}} M\ dt$$

where $M \equiv \{Q_*, \boldsymbol{F}_*\}$). For $\mu_* < 30$ it is possible to neglect outside of the area of resonance frequencies the effect of the ponderomotive force as well, i.e. to use the approach, which is usually used for solving problems of induction heating in shells with the skin effect condition (Gaczkiewicz and Kasperski, 1999; Podstrigach and Shvets, 1978).

The given research method can be used for study of thermomechanical behavior of shells undergoing an action of EMF due to quasisteady currents that are used in practice for inductive thermotreatment of longitudinal and cross welding seams in welded shells, especially cylindrical ones.

References

1. GACZKIEWICZ A., KASPERSKI Z., 1999, *Modele i metody matematyczne w zagadnieniach brzegovych termomechaniki ciał przewodzących*, OWPO, Opole

2. GACZKIEWICZ A., KASPERSKI Z., CZORNYJ B., 1997, Asymptotyczna metoda rozwiązywania zagadnień termomechaniki powłok przewodząncych przy powierzchniowym nagrzewaniu indukcyjnym, *Zeszyty Naukowe Politechniki Opolskiej*, **N230**, *Matematyka*, **14**, 21-29

3. GALITSIN A., ZHUKOVSKII A., 1976, *Integral Transforms and Special Functions in Heat-Conduction Problems* [in Russian], Naukova Dumka, Kiev

4. KORN G., KORN T., 1968, *Mathematical Handbook for Scientists and Engineers*, 2nd ed., Mc.Graw-Hill, New York
5. KOVALENKO A.D., 1975, *Thermoelasticity* [in Russian], Vishcha Shkola, Kiev
6. LYKOV A.V., 1967, *Heat-conduction Theory* [in Russian], Vysshaja Shkola, Moscow
7. MOON F.C., 1978, Problems in magneto-solid-mechanics, *Mechanics Today*, **4**, 307-390
8. NOWACKI W., 1983, *Efekty elektromagnetyczne w stałych ciałah odkształcalnyh*, PWN, Warszawa
9. NOWACKI W., 1986, *Thermoelasticity*, Pergamon Press, Oxford
10. PODSTRIGACH YA., BURAK YA., GACHKEVICH A., CHERNYAVSKAYA L., 1977, *Thermoelasticity of Electrically Conducting Bodies* [in Russian], Naukova Dumka, Kiev
11. PODSTRIGACH YA., SHVETS R.N., 1978, *Thermoelasticity of Thin Shells* [in Russian], Naukova Dumka, Kiev
12. RAWA H., 1994, *Electryczność i magnetyzm w technice*, PWN, Warszawa
13. TAMM N.E., 1976, *Foundations of the Theory of Electricity* [in Russian], Nauka, Moscow
14. VISHYK M.I., LIUSTERNIK L.A., 1960, Asymptotic Behavior of the Solutions of the Linear Differential Equations with Big or Quick-changing Coefficients and Boundary Conditions [in Russian], *Successes of Mathematical Sciences*, **15**, 4, 27-95
15. WAINBERG A.M., 1967, *Electronic Ovens. Educational Manual for High Schools, 2-nd ed.* [in Russian], Energija, Moscow

Badania termicznego stanu naprężenia nieferromagnetycznych powłok przewodzących prąd elektryczny

Streszczenie

W pracy zaprezentowano metodę wyznaczania parametrów elektromagnetycznych, temperaturowych oraz mechanicznych w powłokach przewodzących prąd elektryczny, znajdujących się pod wpływem pola elektromagnetycznego przy uwzględnieniu silnego efektu naskórkowości.

Manuscript received December 3, 2002; accepted for print March 5, 2003