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SENSITIVITY OF CONTI-CASTING PROCESS WITH RESPECT TO COOLING CONDITIONS

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In the first part of this paper mathematical description of heat transfer processes proceeding in the domain of continuous casting is presented. The approach called the second generation modelling is taken into account. In particular, the capacity of a source function in the energy equation is determined by the nucleation and growth laws and the Johnson-Mehl-Avrami-Kolmogorow theory. Next, on the basis of the methods of sensitivity analysis the influence of cooling conditions of the cast slab surface on the course of solidification process is analyzed. At the stage of numerical computations the boundary element method has been used. In the final part of the paper an example of computations is presented.

Key words: continuous casting, solidification process, sensitivity analysis, boundary element method

1. Introduction

From the mathematical point of view the solidification and cooling processes in the domain of continuous casting belong to a group of moving boundary-initial problems and they are described by the Fourier-Kirchhoff equation and adequate boundary-initial conditions. It should be pointed out that taking into account the complicated form of the above mathematical description the application of typical analytical methods is rather impossible, and only the numerical methods allow to find the effective solution to the problem discussed. In the case of a continuous casting process a big number of numerical algorithms can be used, in particular the finite difference method (Saatdjian, 2000; Kapusta and Mochnacki, 1988; Mochnacki, 1993b; Mochnacki and Suchy, 1994), finite element method (Mochnacki and Suchy, 1994; Lewis et al., 1996), boundary element method (Majchrzak, 1993; Mochnacki, 1993a, 1996; Mochnacki and Majchrzak, 1995; Majchrzak and Mochnacki, 1996; Mochnacki and Suchy, 1997a,b; Majchrzak et al., 1999), collocation method (Majchrzak and Mochnacki, 1988; Mochnacki and Suchy, 1995) and also the certain non-typical numerical algorithms (e.g. Kapusta and Wawrzynek, 1992; Majchrzak and Wawrzynek, 1992).

A numerical solution as a rule concerns the analysis of a course of the solidification process for different technological parameters (grade of the metal, shape of the cast strand, pouring temperature, pulling rate, cooling conditions etc.) but one can find the more complex numerical models. It is possible to analyse the effects of macroscopic segregation in the domain of casting volume (Mochnacki et al., 1999; Majchrzak et al., 1998), the mechanical aspects of the process (e.g. thermal stresses) and the course of crystallization process (the second generation models (Stefanescu, 19993; Fraś et al., 1993; Majchrzak and Longa, 1996)).

The very interesting problem consists in the analysis of mutual connections between the cooling conditions of the cast strand surface and the kinetics of solidification in the domain considered. Here, the methods of the sensitivity analysis constitute a very effective tool for such a kind of investigations. In this paper a direct approach will be used. In particular, the influence of the heat transfer coefficient determining the heat exchange between the casting surface and the cooling system on the course of the solidification process will be discussed.

2. Governing equations

The construction of a numerical algorithm simulating the solidification and cooling processes in a cast strand volume requires assuming a certain mathematical model in the form of an adequate partial differential equation as well as geometrical, physical, initial and boundary conditions. We consider a rectangular aluminium casting produced by a vertical continuous casting machine. In a general case, the heat transfer processes proceeding in the casting domain Ω are described by the energy equation of the form

$$c(T)\Big(\frac{\partial T}{\partial t} + \boldsymbol{w} \cdot \operatorname{grad} T\Big) = \operatorname{div}[\lambda(T) \operatorname{grad} T] + Q \tag{2.1}$$

where $c(T)$, $\lambda(T)$ are the thermophysical parameters (volumetric specific heat and thermal conductivity), $\boldsymbol{w}$ is a velocity field, Q is the capacity of internal heat sources whereas T, t denote the temperature and time, respectively.

The distribution of the velocity field $\boldsymbol{w}$ results from the technological conditions and, as a rule, this element of the energy equation is treated as a pulling rate w (e.g. for the vertical cast strand $\boldsymbol{w} = [0, 0, w]$) though the convective effects in the liquid state sub-domain $\Omega_1(t)$ associated with the interaction of the molten metal stream and (or) the action of the rotational magnetic field should be in a certain way taken into account. The exact analysis of these very complex phenomena is rather difficult both from the theoretical and numerical points of view, but recently there have appeared commercial codes using the FEM on the basis of which such problems can be simulated (Parkitny and Sowa, 2000). In numerous mathematical models of the continuous casting process it is assumed that the convective heat transfer in the adequate sub-domain $\Omega_0(t)$ is substituted with an *unrealistic* heat conduction, which means that in the place of the real thermal conductivity $\lambda(T)$ the so-called effective thermal conductivity for molten metal is introduced (Skladostev, 1974). The component Q in equation (2.1) determines the evolution of the latent heat L_V [J/m^3], and this source function can be expressed as

$$Q = L_V \frac{\partial f_S}{\partial t} \tag{2.2}$$

where f_S is the local volumetric fraction of the solid metal in the neighbourhood of the point considered. The capacity of the source function Q results from the analysis of the crystallization process (e.g. Stefanescu, 1993; Fraś et al., 1993; Majchrzak and Longa, 1996).

In the Cartesian co-ordinate system $\{x, y, z\}$ (cf. Figure 1) energy equation (2.1) is as follows

$$\begin{aligned} c(T)\Big[\frac{\partial T}{\partial t} + w\frac{\partial T}{\partial z}\Big] &= \\ = \frac{\partial}{\partial x}\Big[\lambda(T)\frac{\partial T}{\partial x}\Big] + \frac{\partial}{\partial y}\Big[\lambda(T)\frac{\partial T}{\partial y}\Big] &+ \frac{\partial}{\partial z}\Big[\lambda(T)\frac{\partial T}{\partial z}\Big] + L_V\frac{\partial f_S}{\partial t} \end{aligned} \tag{2.3}$$

The basic energy equation can be considered in a simplified form. The heat conduction process in the domain of continuous casting proceeds, first

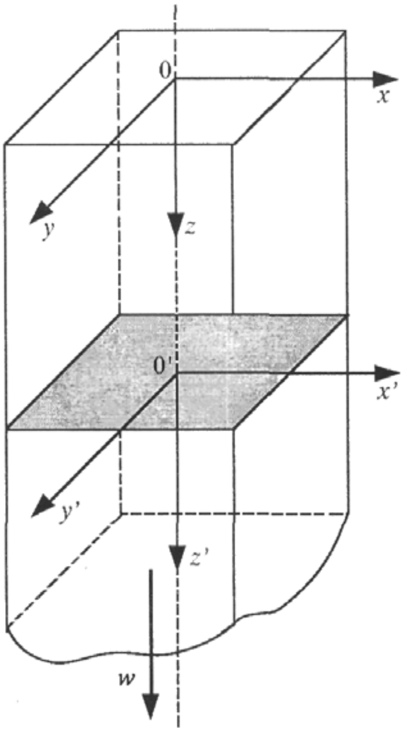

Fig. 1. Rectangular cast slab

of all, from the axis to the lateral surface and its vertical component can be neglected. Thus we have

$$c(T)\left[\frac{\partial T}{\partial t}+w\frac{\partial T}{\partial z}\right]=\frac{\partial}{\partial x}\left[\lambda(T)\frac{\partial T}{\partial x}\right]+\frac{\partial}{\partial y}\left[\lambda(T)\frac{\partial T}{\partial y}\right]+L_V\frac{\partial f_S}{\partial t} \tag{2.4}$$

Let we rewrite equation (2.4) in a coordinate system *tied* to a certain section of the shifting casting, namely $x'=x$, $y'=y$, $z'=z-wt$. We assume, as previously, that the heat conduction in the z direction can be neglected. It is easy to check up that in the energy equation we *lose* the component $\partial T/\partial z$

$$c(T)\frac{\partial T}{\partial t}=\frac{\partial}{\partial x'}\left[\lambda(T)\frac{\partial T}{\partial x'}\right]+\frac{\partial}{\partial y'}\left[\lambda(T)\frac{\partial T}{\partial y'}\right]+L_V\frac{\partial f_S}{\partial t} \tag{2.5}$$

The above presented idea constitutes a basis of a certain numerical algorithm called a *wandering cross section method* (WCSM) (Majchrzak, 1993; Mochnacki and Suchy, 1993). The WCSM corresponds to the assumption that a certain casting section, in which the temperature field is described by equation (2.4), is formally fixed and the conditions given on its boundary change with time. In this way the variable conditions simulate the displacement of the section considered through the installation. In spite of the fact that 2D solutions can be found, in the reality we obtain a 3D temperature field. In order to simplify the form of the next formulas we denote again $x' = x$ and $y' = y$.

Typical boundary conditions describing the heat exchange on the lateral surface of the continuous casting are of the form

$$-\lambda(T)\frac{\partial T}{\partial n} = q = \alpha(T - T_w) \tag{2.6}$$

where $\partial T/\partial n$ is the normal derivative at the boundary point, α is the heat transfer coefficient, T_w is the cooling water temperature. For the upper surface of the casting it can be assumed that $T = T_0$, where T_0 is the pouring temperature and this value constitutes the initial condition of the problem considered. It should be pointed out that the wandering cross section method applied here does not require the formulation of boundary conditions for the upper and lower surfaces limiting the casting domain.

Now, the problem of the source function will be discussed. A temporary value of the solid fraction f_S of the metal at the considered point is given by the Johnson-Mehl-Avrami-Kolmogorov equation (Kolmogorov, 1937; Fraś, 1992; Fraś et al., 1993)

$$f_S = 1 - \exp(-\omega) \tag{2.7}$$

where

$$\omega = \frac{4}{3}\pi\nu \int\limits_{o}^{1} \frac{\partial N}{\partial t}\Big[\int\limits_{t'}^{t} u\; d\tau\Big]^3\; dt \tag{2.8}$$

In equation (2.8) N is the number of nuclei (more precisely: density [nuclei/m^3]), ν is the coefficient equal to 1 for spherical grains and $\nu < 1$ for dendritic growth, u is the rate of the solid phase growth, t' is the initial instant of the crystallization process. If we assume a constant number of the nuclei, then

$$\omega = \frac{4}{3}\pi\nu N\Big[\int\limits_{t'}^{t} u\; d\tau\Big]^3 \tag{2.9}$$

The solid phase growth (equiaxial grains) is determined by equation

$$u = \frac{\partial R}{\partial t} = \mu \Delta T^p \tag{2.10}$$

where R is the grain radius, μ is a growth coefficient, p is the exponent from within the interval $p \in [1,2]$, and

$$\Delta T = T_{cr} - T \tag{2.11}$$

is the undercooling below the solidification point T_{cr}. Additionally, we assume that for $T > T_{cr}$: $\Delta T = 0$, in other words $u = 0$, and then the lower limit in integrals (2.8) and (2.9) equals $t' = 0$. Hence

$$\omega = \frac{4}{3}\pi\nu N\Big[\int\limits_0^t \mu\Delta T^p\, d\tau\Big]^3 \tag{2.12}$$

Introducing (2.7) to equation (2.5) one obtains

$$c(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\Big[\lambda(T)\frac{\partial T}{\partial x}\Big] + \frac{\partial}{\partial y}\Big[\lambda(T)\frac{\partial T}{\partial y}\Big] + L_V \exp(-\omega)\frac{\partial \omega}{\partial t} \tag{2.13}$$

In order to simplify the further consideration we assume constant values of the thermophysical parameters c and λ (the crystallization process proceeds in a rather small interval of temperature and this assumption does not introduce essential errors). Thus

$$c\frac{\partial T}{\partial t} = \lambda\nabla^2 T + Q \tag{2.14}$$

where

$$Q = 4\pi\nu N L_V \mu\Delta T^p \Big(\int\limits_0^t \mu\Delta T^p\, d\tau\Big)^2 \exp\Big[-\frac{4}{3}\pi\nu N\Big(\int\limits_0^t \mu\Delta T^p\, d\tau\Big)^3\Big] \tag{2.15}$$

This equation supplemented with Robin's boundary conditions (2.6) and the initial one determining the pouring temperature of the molten metal constitutes a base for numerical computations of the temperature field in the continuous casting domain

$$\begin{cases} P \in \Omega: & c\dfrac{\partial T}{\partial t} = \lambda\nabla^2 T + Q \\ P \in \Gamma: & -\lambda\dfrac{\partial T}{\partial n} = \alpha(T - T_w) \\ t = 0: & T = T_0 \end{cases} \tag{2.16}$$

3. Sensitivity analysis

In this section a sensitivity analysis of the solidification process with respect to the heat transfer coefficient on the lateral surface of the casting is presented, at the same time the direct method is applied (Dems, 1986; Dems and Rousselet, 1999).

Differentiation of the equations forming the mathematical model discussed with respect to α leads to an additional boundary initial problem

$$\begin{cases} P \in \Omega: & c\dfrac{\partial}{\partial t}\Big(\dfrac{\partial T}{\partial \alpha}\Big) = \lambda \nabla^2 \Big(\dfrac{\partial T}{\partial \alpha}\Big) + \dfrac{\partial Q}{\partial \alpha} \\ P \in \Gamma: & -\lambda \dfrac{\partial}{\partial n}\Big(\dfrac{\partial T}{\partial \alpha}\Big) = \alpha \dfrac{\partial T}{\partial \alpha} + T - T_w \\ t = 0: & \dfrac{\partial T_0}{\partial \alpha} = 0 \end{cases} \tag{3.1}$$

where $P \in \Omega$ is the point from the interior of the casting domain, $P \in \Gamma$ is the boundary one.

We denote $U = \partial T/\partial \alpha$ and additionally

$$r_S = \int\limits_0^t \mu \Delta T^p \, d\tau \qquad\qquad r_U = \int\limits_0^t \mu \Delta T^{p-1} U \, d\tau \tag{3.2}$$

Then

$$\begin{aligned} Q_u &= \frac{\partial Q}{\partial \alpha} = 4p\pi\nu N L_V \exp\Big(-\frac{4}{3}\pi\nu N r_S^3\Big) \cdot \\ &\Big[4\pi\nu N \mu \Delta T^p \rho_U r_S^4 - 2\mu \Delta T^p \rho_U r_S - \mu U r_S^2 \Delta T^{p-1}\Big] \end{aligned} \tag{3.3}$$

At the stage of numerical computations the value $p = 1$ has been assumed. So, boundary initial problem (3.1) can be written in the form

$$\begin{cases} P \in \Omega: & c\dfrac{\partial U}{\partial t} = \lambda \nabla^2 U + Q_u \\ P \in \Gamma: & -\lambda \dfrac{\partial U}{\partial n} = \alpha (U - U_w) \\ t = 0: & U_0 = 0 \end{cases} \tag{3.4}$$

where

$$U_w = \frac{T_w - T}{\alpha} \tag{3.5}$$

One can notice that sensitivity model (3.4) from the mathematical point of view is the same as the basic one (2.16). In other words, we can use the same numerical algorithm in order to solve both the basic and additional boundary initial problems (the different form of the source functions must be taken into account, of course). It should be pointed out that the sensitivity model cannot be analysed separately because it is coupled with the solidification model by the components Q_u and U_w. The problem discussed has been solved using the boundary element method for parabolic equations.

4. Boundary element method

In order to solve the basic and sensitivity problems the 1st scheme of the boundary element method has been applied. We consider the following Fourier equation

$$(x,y) \in \Omega : \qquad c\frac{\partial F(x,y,t)}{\partial t} = \lambda \nabla^2 F(x,y,t) + S(x,y,t) \tag{4.1}$$

where F denotes the temperature or function U, while $S(x,t)$ is the source function.

We introduce a time grid with the step $\Delta t = t^f - t^{f-1}$, and then, for the transition $t^{f-1} \to t^f$ we consider the following boundary integral equation (Mochnacki and Majchrzak, 1995; Majchrzak and Mochnacki, 1996; Majchrzak, 2001; Brebbia et al., 1984)

$$\begin{aligned}
&B(\xi,\eta)F(\xi,\eta,t^f) + \frac{1}{c}\int\limits_{t^{f-1}}^{t^f}\int\limits_{\Gamma} F^*(\xi,\eta,x,y,t^f,t)J(x,y,t)\,d\Gamma dt = \\
&= \frac{1}{c}\int\limits_{t^{f-1}}^{t^f}\int\limits_{\Gamma} J^*(\xi,\eta,x,y,t^f,t)F(x,y,t)\,d\Gamma dt + \\
&+ \frac{1}{c}\iint\limits_{\Omega} F^*(\xi,\eta,x,y,t^f,t^{f-1})F(x,y,t^{f-1})\,d\Omega + \\
&+ \frac{1}{c}\int\limits_{t^{f-1}}^{t^f}\iint\limits_{\Omega} S(x,y,t)F^*(\xi,\eta,x,y,t^f,t)\,d\Omega dt
\end{aligned} \tag{4.2}$$

In equation (4.2) F^* is the fundamental solution (Mochnacki and Majchrzak, 1995; Majchrzak and Mochnacki, 1996; Majchrzak, 2001; Brebbia et al., 1984)

$$F^*(\xi,\eta,x,y,t^f,t) = \frac{1}{4\pi a(t^f-t)}\exp\Big[-\frac{r^2}{4a(t^f-t)}\Big] \tag{4.3}$$

where r denotes the distance between the observation point (ξ, η) and the point under considerations (x, y), whereas

$$J^*(\xi, \eta, x, y, t^f, t) = -\lambda \frac{\partial F^*(\xi, \eta, x, y, t^f, t)}{\partial n} \tag{4.4}$$

and

$$J(x, y, t) = -\lambda \frac{\partial F(x, y, t)}{\partial n} \tag{4.5}$$

The coefficient $B(\xi)$ is from within the interval $(0, 1)$.

For the elements constant with respect to time (Majchrzak, 2001; Brebbia et al., 1984) one obtains

$$\begin{aligned} & B(\xi, \eta)F(\xi, \eta, t^f) + \int_\Gamma J(x, y, t^f) g(\xi, \eta, x, y)\, d\Gamma = \\ & = \int_\Gamma F(x, y, t^f) h(\xi, \eta, x, y)\, d\Gamma + \iint_\Omega S(x, y, t^{f-1}) g(\xi, \eta, x, y)\, d\Omega + \\ & + \iint_\Omega J^*(\xi, \eta, x, y, t^f, t^{f-1}) F(x, y, t^{f-1})\, d\Omega \end{aligned} \tag{4.6}$$

where

$$\begin{aligned} h(\xi, \eta, x, y) &= \frac{1}{c} \int_{t^{f-1}}^{t^f} J^*(\xi, \eta, x, y, t^f, t)\, dt \\ g(\xi, \eta, x, y) &= \frac{1}{c} \int_{t^{f-1}}^{t^f} F^*(\xi, \eta, x, y, t^f, t)\, dt \end{aligned} \tag{4.7}$$

Discretisation of the boundary and the interior of the considered domain using the constant internal and boundary elements leads to the following system of equations $(i = 1, 2, ..., N)$

$$\sum_{j=1}^N G_{ij} J_j^f = \sum_{j=1}^N H_{ij} F_j^f + \sum_{l=1}^L P_{il} F_l^{f-1} + \sum_{l=1}^L Z_{il} S_l^{f-1} \tag{4.8}$$

where

$$G_{ij} = \int_{\Gamma_j} g(\xi^i, \eta^i, x, y)\, d\Gamma_j$$

$$H_{ij} = \begin{cases} \int\limits_{\Gamma_j} h(\xi^i, \eta^i, x, y)\, d\Gamma_j & i \neq j \\ -\dfrac{1}{2} & i = j \end{cases} \tag{4.9}$$

$$P_{il} = \iint\limits_{\Omega_l} F^*(\xi^i, \eta^i, x, y, t^f, t^{f-1})\, d\Omega_l$$

$$Z_{il} = \iint\limits_{\Omega_l} g(\xi^i, \eta^i, x, y)\, d\Omega_l$$

Introducing the Robin boundary condition

$$J_j^f = \alpha(F_j^f - F_w) \tag{4.10}$$

to equations (4.8) we have

$$\sum_{j=1}^{N} (\alpha G_{ij} - H_{ij}) F_j^f = \sum_{j=1}^{N} \alpha G_{ij} F_w + \sum_{l=1}^{L} P_{il} F_l^{f-1} + \sum_{l=1}^{L} Z_{il} S_l^{f-1} \tag{4.11}$$

The system of equations (4.11) allows one to determine the boundary values of F, and then, the fluxes J, see equation (4.10). In the second step the internal values of the function F can be found using the formula ($i = N+1, ..., N+L$)

$$F_i^f = \sum_{j=1}^{N} H_{ij} F_j^f - \sum_{j=1}^{N} G_{ij} J_j^f + \sum_{l=1}^{L} P_{il} F_l^{f-1} + \sum_{l=1}^{L} Z_{il} S_l^{f-1} \tag{4.12}$$

5. Example of computations

The aluminium cast slab of 0.15×0.15 m size is considered. The pulling rate is assumed to be $w = 0.02$ m/s, while the pouring temperature $T_0 = 700°$C. The following parameters of the metal are introduced: $\lambda = 150$ W/mK, $c = 3 \cdot 10^6$ J/(m^{3}K), $L_V = 9.75 \cdot 10^8$ J/m^3, $T_{cr} = 660°$C, $N = 10^{11}$ nuclei/m^3, $\mu = 3 \cdot 10^6$ m/(sK), $\nu = 1$, $p = 1$. The heat transfer coefficient $\alpha = 1200$ W/(m^{2}K), cooling water temperature $T_w = 30°$C.

The quarter of the casting section is divided into 256 constant internal cells, along which the boundary 64 constant boundary elements are distinguished, and the time step is $\Delta t = 0.05$ s.

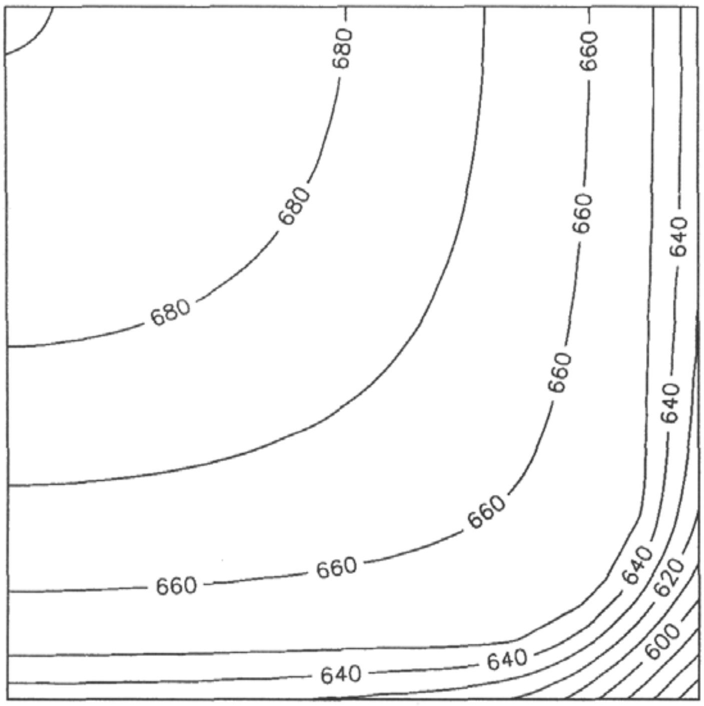

Fig. 2. Temperaure field ($z = 0.3$, $\alpha = 1200$)

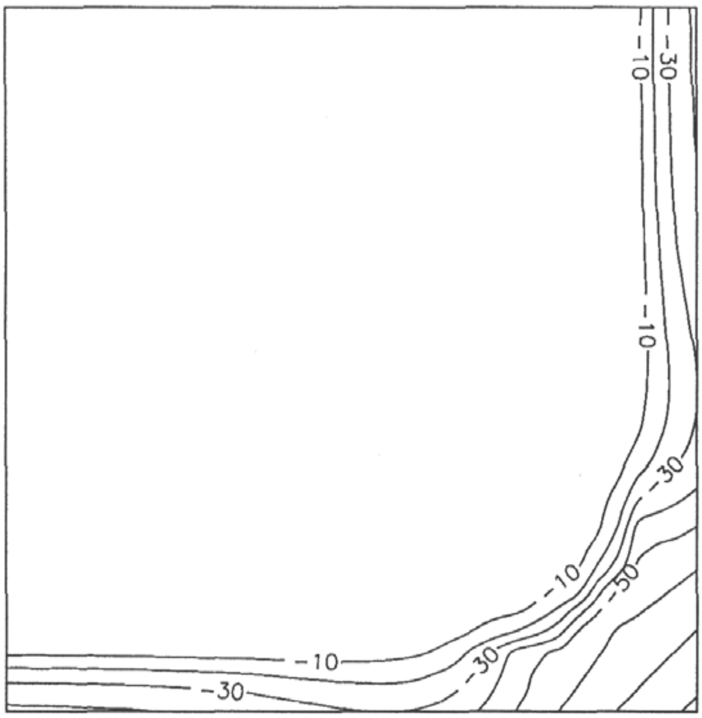

Fig. 3. Sensitivity field ($z = 0.3$, $\alpha = 1200$)

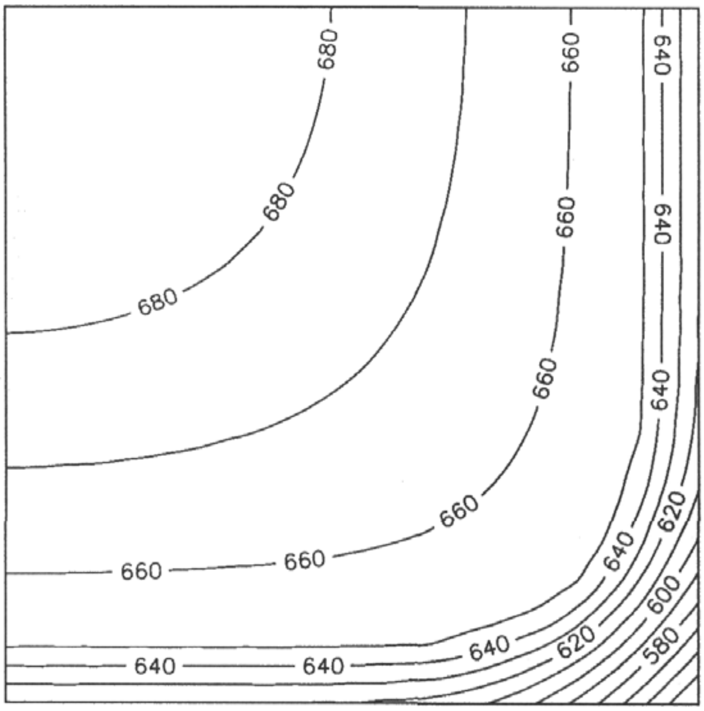

Fig. 4. Direct solution ($z = 0.3$, $\alpha = 1400$)

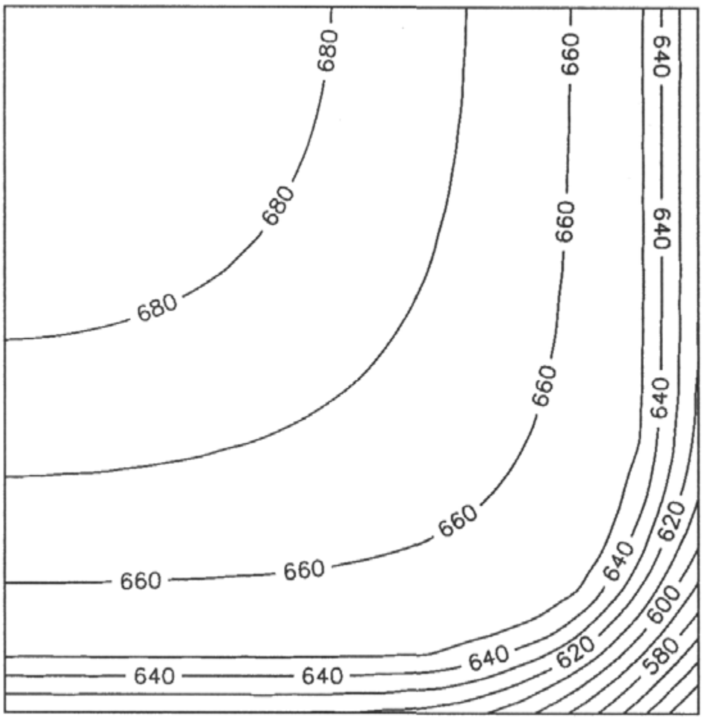

Fig. 5. Indirect solution ($z = 0.3$, $\alpha = 1400$)

In Fig. 2 and Fig. 3 the temperature and sensitivity fields after 15 s are shown (they correspond to the distance $z = 0.3$ m from the upper surface of the casting). Fig. 4 and Fig. 5 illustrate the temperature field for more intensive cooling conditions ($\alpha = 1400\,\mathrm{W/(m^2K)}$). In Fig. 4 the temperature field is found directly, while in Fig. 5 the temperature field is calculated on the basis of the sensitivity field. In order to rebuilt the basic solution on the solution corresponding to the new heat transfer coefficient the Taylor formula should be used

$$T(\alpha + \Delta\alpha) = T(\alpha) + \frac{\partial T}{\partial \alpha}\Delta\alpha = T(\alpha) + U\Delta\alpha \tag{5.1}$$

One can notice that both results are practically the same. Figures 6, 7, 8, 9 show the results corresponding to the distance $z = 0.6$ m. The possibility of transformation of the basic numerical solution to the solution for other cooling conditions is one of the essential practical applications of the sensitivity analysis.

The presented results of computations show that even in the case of non-steady and non-linear complex problems the application of the sensitivity analysis leads to quite exact solutions. Propably, in the case of bigger values of $\Delta\alpha$ second order sensitivity coefficients should be introduced.

The cooling curves shown in Fig. 10 correspond to basic solution (1) and the indirect solutions for $\Delta\alpha = \pm 200\,\mathrm{W/(m^2K)}$ (2, 3). It is clearly visible that the second generation models give the information concerning the recalescence effect and the results are closer to the real course of the process. The propelling force of crystallization is the undercooling below the temperature T_{cr}. The beginning of activity of the internal heat sources takes place when the local value of temperature T drops below T_{cr} (Figure 12: 1 – basic, 2, 3 – indirect). Initially, the capacity of the internal sources is small and the temperature decreases as before. The progressive growth of Q causes an increase in the local temperature. At the final stage, the capacity of Q decreases ($Q \to 0$) and the typical cooling process takes place.

The solidification model based on the well known Stefan boundary condition or the model, in which the artificial mushy zone is introduced (the fixed domain method) (Crank, 1984; Idelsohn et al., 1994), do not allow one to take into account the processes proceeding on the micro scale. On the other hand, however, such approaches (the I generation models) are also very useful in the thermal theory of foundry processes.

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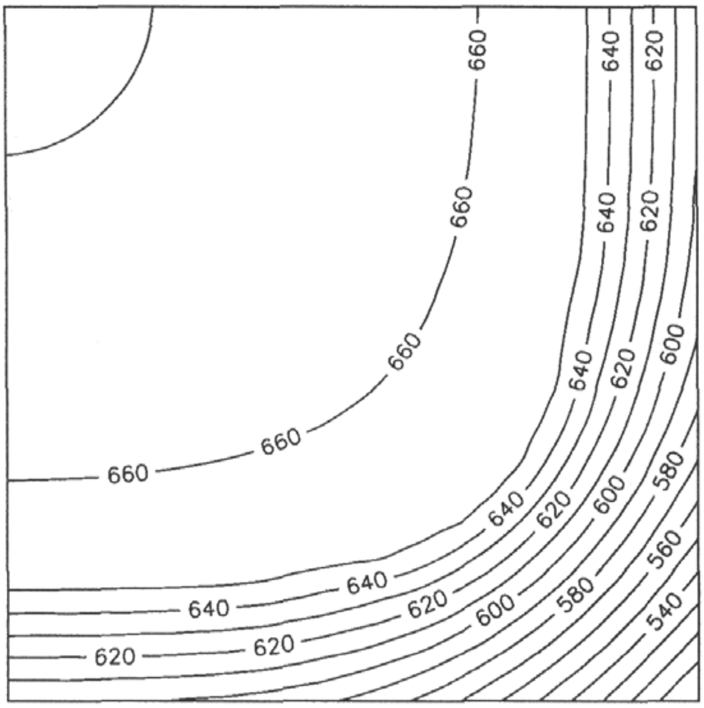

Fig. 6. Temperaure field ($z = 0.6$, $\alpha = 1200$)

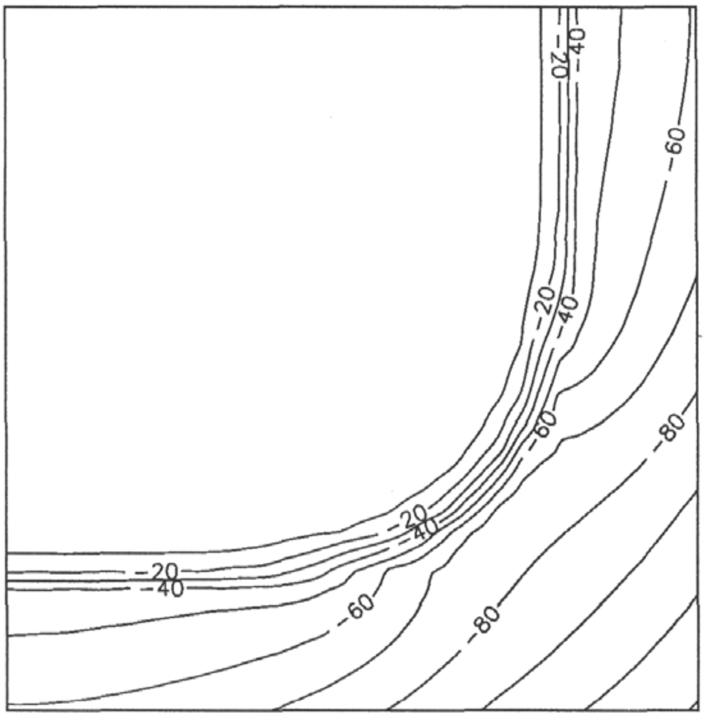

Fig. 7. Sensitivity field ($z = 0.6$, $\alpha = 1200$)

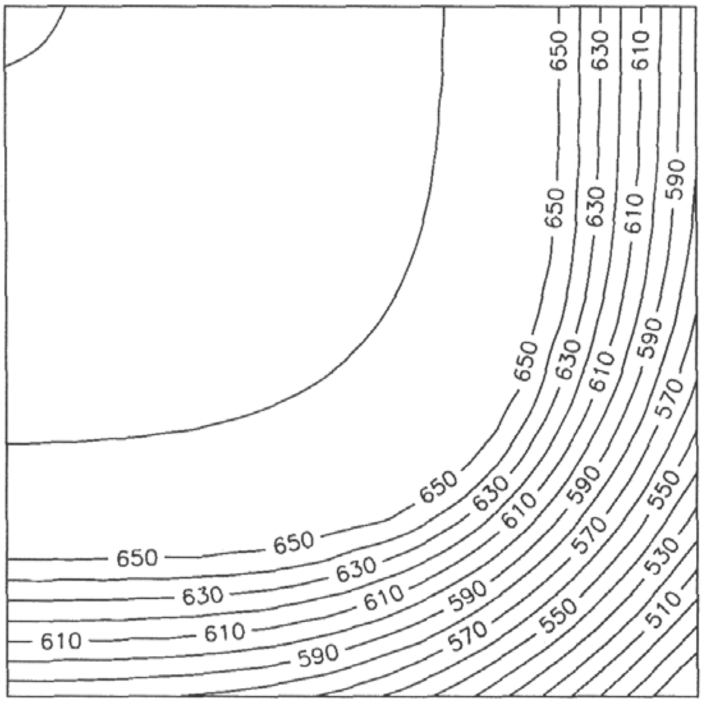

Fig. 8. Direct solution ($z = 0.6$, $\alpha = 1200$)

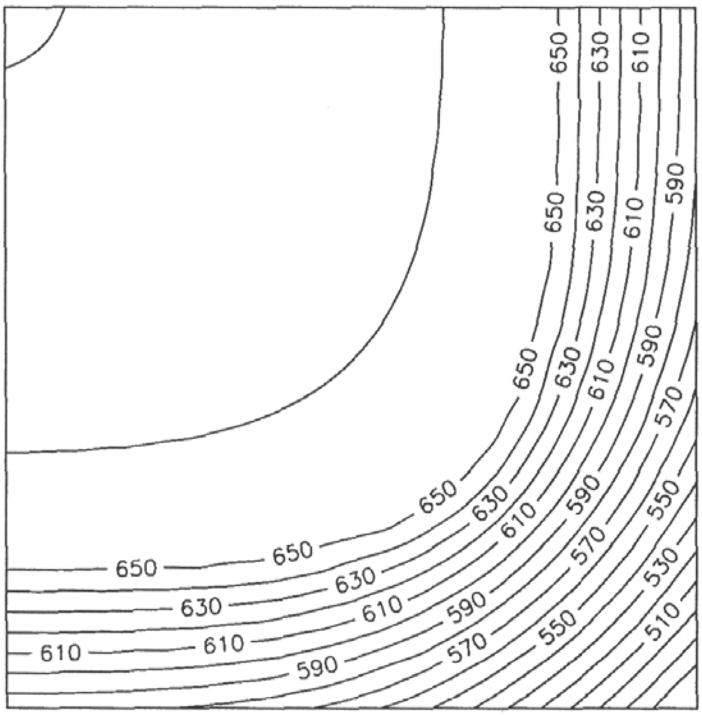

Fig. 9. Indirect solution ($z = 0.6$, $\alpha = 1400$)

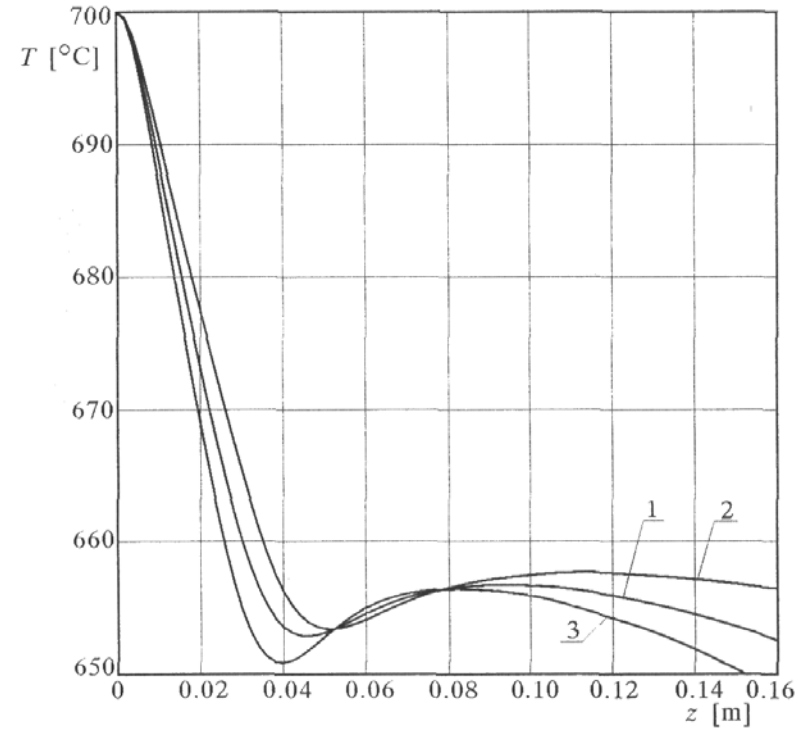

Fig. 10. Cooling curves along the z axis near the casting corner

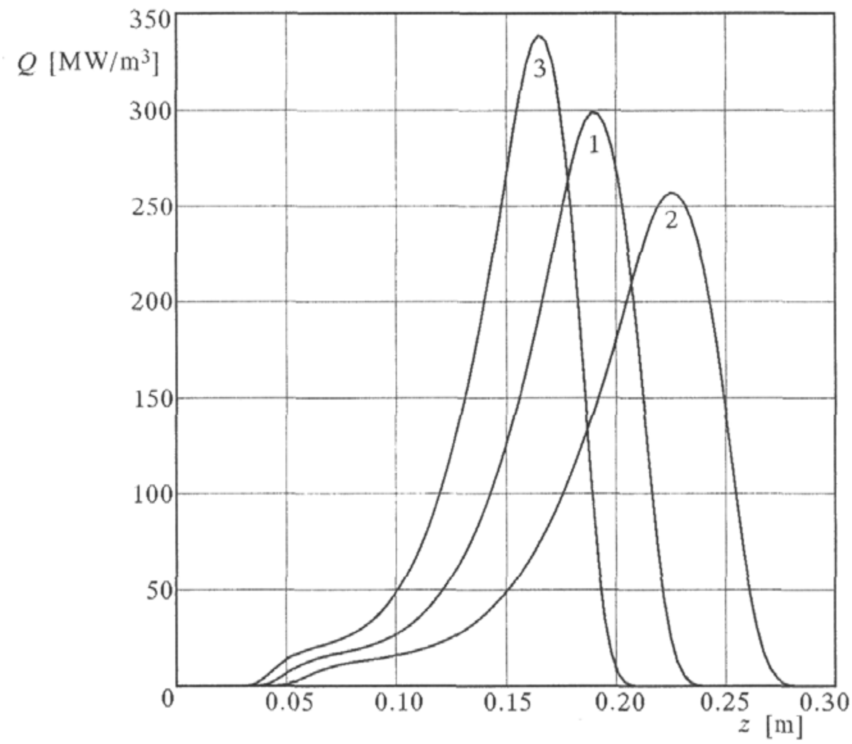

Fig. 11. Capacity of internal heat sources

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Analiza wrażliwości procesu ciągłego odlewania na warunki chłodzenia wlewka

W pierwszej części pracy przedstawiono opis matematyczny krzepnięcia wlewka ciągłego wytwarzanego z czystego metalu. Do opisu wykorzystano podejście nazywane modelem drugiej generacji. W szczególności, wydajność wewnętrznych źródeł ciepła będąca składnikiem równania energii jest wyznaczana na podstawie praw zarodkowania i wzrostu oraz teorii Johnsona-Mehla-Avrami-Kołmogorowa. Następnie, wykorzystując metodę bezpośrednią analizy wrażliwości, badano wpływ warunków chłodzenia powierzchni wlewka na proces jego krzepnięcia. Na etapie obliczeń numerycznych zastosowano metodę elementów brzegowych. W końcowej części pracy pokazano przykład obliczeń numerycznych.

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