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CONCEPT OF A NON-PROPORTIONALITY PARAMETER IN A COMPLEX FATIGUE LOAD STATE

Dariusz Skibicki
Janusz Sempruch

Department of Mechanical Engineering, University of Technology and Agriculture, Bydgoszcz
e-mail: darski@atr.bydgoszcz.pl; semjan@atr.bydgoszcz.pl

The article describes a concept of a stress parameter of the fatigue load non-proportionality. The parameter enables quantitative estimation of the load non-proportionality state. A criterion for the fatigue strength has been proposed on the basis of the parameter. The criterion verification results are presented in the article.

Key words: multiaxial fatigue, non-proportional loading

1. Introduction

Changing loads affecting structural elements may be complex due to their course, component multiaxiality or non-proportionality.

In most of practical solutions, a sinusoidally changing load, the simplest case of loading, is the basis for determination of the fatigue qualities of materials and structural elements. The only parameters used to describe the fatigue criterion quantities are those specifying the cycle geometry, like amplitude, average value or the cycle asymmetry coefficient. Parameters connected with time, like for example frequency, hardly ever appear in the descriptions.

Fatigue spectra depict courses that do not suit the simplified sinusoidal model, for instance random courses. Methods of creating fatigue spectra are based upon distributions of peak values, average values and amplitudes. In such cases the time parameter is not taken into account either.

A model presentation of disproportional loads is a biaxial load with components out of phase by angle ϕ (Fig. 1). A characteristic feature of this kind of loads is the rotation of the principal axis of stresses or strains. The phenomenon, when compared to proportional loads, causes a decrease in the fatigue life, decrease in fatigue limit, additional cyclical material strengthening (Socie, 1987), etc. Therefore, not taking into account the time parameter, e.g. phase angle, may lead to incorrectly assigning criterion values or incorrectly predicting the life.

$$\sigma_x = \frac{M_b}{W_y} \qquad \sigma_x = \sigma_x^a \sin(\omega t) \qquad\qquad \lambda = \frac{\sigma_x^a}{\tau_{xy}^a}$$
$$\tau_{xy} = \frac{M_t}{W_0} \qquad \tau_{xy} = \tau_{xy}^a \sin(\omega t - \phi)$$

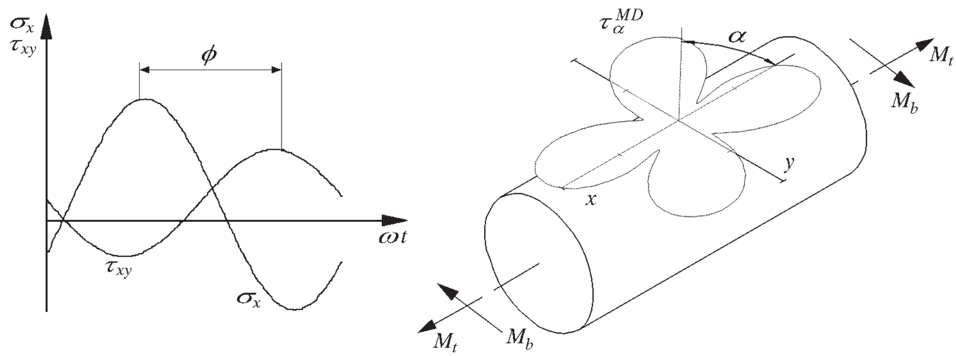


Fig. 1. Stress state components

Calculating the fatigue limit value according to McDiarmid's (1974) criterion can function as an example. The computational value is based only on the proportional criterion and does not take into account the phase shift ϕ; therefore, within the range of disproportional loads it appears to be underestimated when compared to the actual value (Fig. 2). The underestimation increases with the increase of the phase shift angle. This happens for different kinds of materials characterised by the fatigue limit quotient t/b, where t denotes the fatigue limit in torsion and b – fatigue limit in bending, and for different stress states described with the coefficient $\lambda = \sigma_x^a/\tau_{xy}^a$, where σ_x^a and τ_{xy}^a are the amplitudes of sinusoidally variable nominal stresses.

The easiest solution is that the phase shift ϕ should be introduced into the criterion notation of disproportional stresses. This is the case of Lee (1985, 1991) criteria, where the non-proportionality elements are suggested as functions of the phase angle ϕ.

More advanced criteria do not treat the phase angle as measure of load non-proportionality. They are functions of specially defined non-proportionality parameters, see (Duprat, 1997).

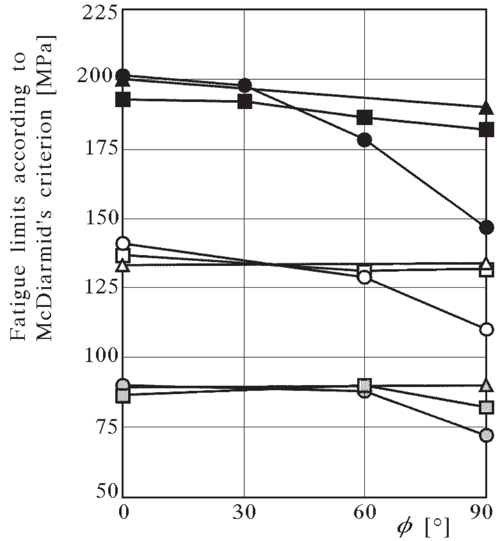


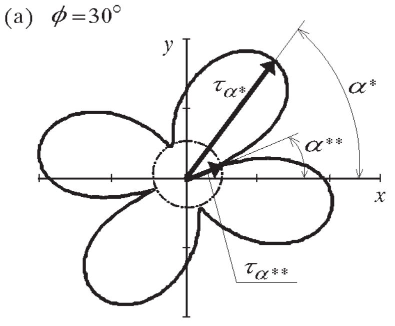
Fig. 2. Computational fatigue limits in function of the phase angle according to McDiarmid (1974). Denotations: square $\lambda = 1.21$, circle $\lambda = 0.5$, triangle $\lambda = 0.21$, white $t/b = 0.58$, black $t/b = 0.62$, grey $t/b = 0.98$

2. Load non-proportionality parameter

The above mentioned load non-proportionality parameter has become the basis for the fatigue strength criterion formulated in the paper by Skibicki and Sempruch (1999). The formulation was been based upon observations of stress hodographs for different load non-proportionality degrees. Hodographs are envelopes led upon cycle maximum vectors of normal, shear or reduced stresses acquired in all directions of the rotary transformation of the stress tensor (Fig. 1 and Fig. 3).

In order to find the form of the load non-proportionality parameter, the stress hodographs have been analysed for different degrees of load non-proportionality. In the analysed case, the hodographs have been obtained by calculating (in each direction α) the maximum value of reduced stress using McDiarmid's (1974) formula. In Fig. 3 two hodographs are presented, appropriately for the phase angles $\phi = 30°$ and $90°$, where the phase shift appears between the normal and shear stress of a biaxial load by bending and torsion.

The quantity τ_α^* (Fig. 3) is a reduced stress on the plane of the maximum shear stress. This plane is considered the critical one in the criterion. The quantity τ_α^* equals McDiarmid's (1974) computational fatigue limit.



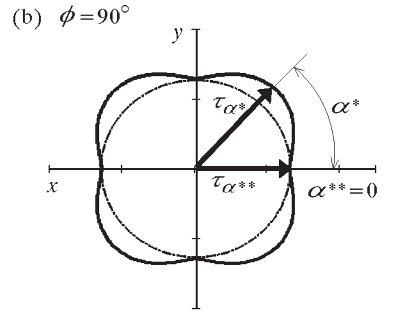


Fig. 3. Reduced stress envelopes according to McDiarmid for two phase angles

On the basis of both examples it is seen that increase in the angle ϕ from 30° to 90° leads to decrease in the stress value τ_α^*. At the same time, stress amplitudes on the remaining planes grow in relation to τ_α^*. Therefore, greater non-proportionality of the load entalis bigger participation of stresses from the outside of the critical plane in the whole fatigue process.

Consequently, the bigger value of such stresses, the more modified is the process of the fatigue damage accumulation that leads to destruction, and for which the principal stresses connected with the critical plane are responsible. The visible proof of the influence of the stresses connected with the critical plane may be observed on the level of dislocation interactions as changes within the dislocation structure picture.

Stress hodographs carry information about the load history during a fatigue cycle. Only after their analysis, and not by examining variability of nominal loads (i.e. analysis of the amplitude or average values of particular loads), the complete data about the effort of the material exposed to the load action can be obtained.

There are many ways to describe the changes of the load hodographs in Fig. 3. On the basis of further analyses it has been assumed that the changes can be represented by a vector absolute value to be described by the radius of a circle written into the hodograph. The quantity, described as τ_α^{**}, has been named the stress measure of load non-proportionality.

Functioning of the parameter τ_α^{**} has been tested for a wide range of stress state conditions and material properties (Fig. 4). In each analysed case the underestimation of the criterion value based on the proportional criterion, can be compensated with the growing value of the load non-proportionality parameter τ_α^{**}, Fig. 4.

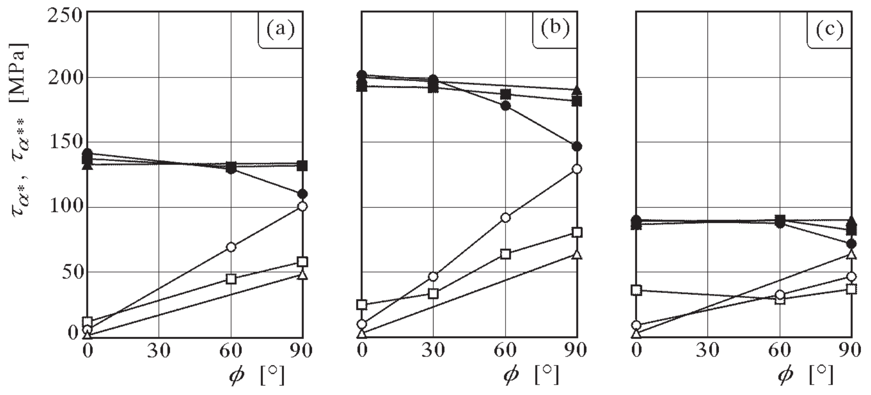


Fig. 4. Decrements τ_α^* (black markers) and increments τ_α^{**} (white markers) in function of the phase angle for $t/b = 0.58$ (a), $t/b = 0.62$ (b), $t/b = 0.94$ (c). Denotations: square $\lambda = 1.21$, circle $\lambda = 0.5$, triangle $\lambda = 0.21$

3. Load non-proportionality function

Further analysis of the diagrams in Fig. 4 proves that sensitiveness to load non-proportionality also depends also on the quotient of λ and the material properties coefficient t/b.

The influence of λ can be explained on the basis of observation of reduced stress hodographs for $\phi = 90°$ (Fig. 5). For the coefficient $\lambda = 0.5$ all planes are loaded with a similar value of the reduced stress. The course of the stress hodographs is approximately circular. The relation between the maximum and minimum values for this value of λ is small. In this case, the filling factor, which is the relation between the circle described on the envelope and the envelope-limited field, is the biggest and equals 90%. For $\lambda = 0.21$ it is 73% and for $\lambda = 1.21 - 79\%$. As one can see, λ affects the envelope course. The bigger the filling factor, the bigger stress values represented by the parameter τ_α^{**} in relation to τ_α^*. The coefficient λ nearer to 0.5 means more significant influence of the stresses not related with the critical plane.

An important conclusion of this part of analysis is that the suggested parameter τ_α^{**} is not sufficient to describe the changes in values of the stresses which take place outside the critical plane. The parameter does not take into account the character of the changes and the stress course, which additionally needs to be defined by the quantity λ. At this stage of research, the application of two quantities for describing the character of the stress envelope has been accepted. Remarks presented in this section, however, clearly define the

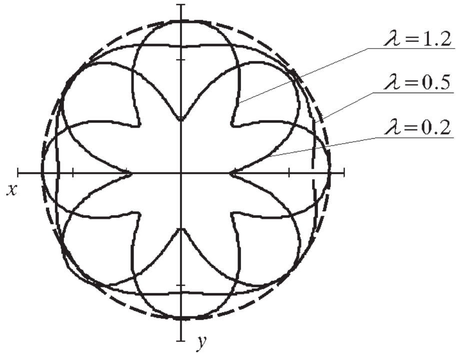


Fig. 5. Reduced stress envelopes τ_α for the angle $\phi = 90°$ and different λ

direction of further research into the function of load non-proportionality.

The influence of load non-proportionality on fatigue phenomena is not only connected with the stress state. Sensitiveness of the material to load non-proportionality is also very significant. It is directly connected with the relation of the fatigue limits t/b.

It has been assumed that the influence of all above mentioned parameters can be simultaneously presented as a load non-proportionality function

$$\tau^{**} = \tau^{**}\Big(\tau_{\alpha^{**}}, \lambda, \frac{t}{b}\Big) \tag{3.1}$$

4. Non-proportionality criterion

On the basis of the load non-proportionality function, a criterion for high-cycle fatigue strength for biaxial load with a phase shift of the elements has been formulated in the paper by Skibicki and Sempruch (1999). That load state is a model presentation of loads called disproportional.

As a basis for the formulated criterion, there is a physical model of the disproportional fatigue phenomenon, which is schematically presented in Fig. 6. The model assumes the possibility of fatigue description in conditions of the disproportional load by means of the correspondingly modified critical plane model that is known from the proportional load range (Fig. 6a). It is assumed that the plane also exists in conditions of principal axis rotation (Fig. 6b) and the connected with it quantities, like stresses or strains, are most significant for the course of fatigue damage accumulation. The process is, however, changed because of the load non-proportionality. The rotating vector of the maximum shear stress causes complex dislocation movement in many activated slip systems and, consequently, additional interactions between the dislocations. The phenomena modify the fatigue damage accumulation on the critical plane. Most frequently, the influence causes acceleration of the fatigue damage accumulation process.

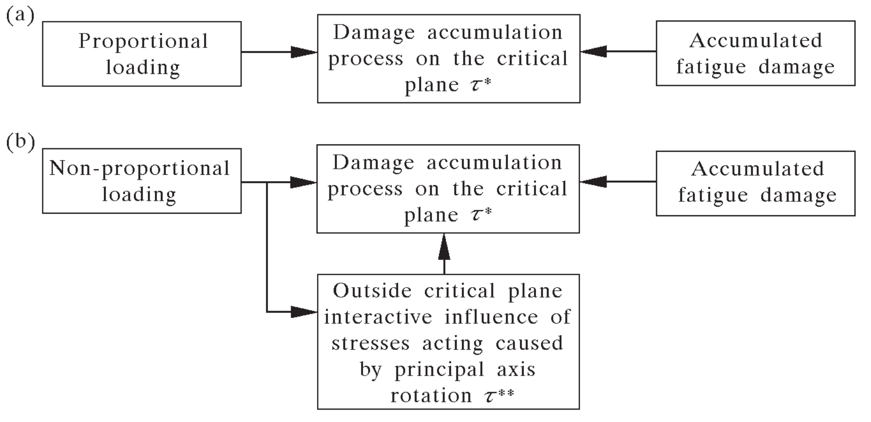


Fig. 6. A model of proportional and non-proportional fatigue

On the basis of the above mentioned remarks, the following general form of the criterion has been suggested

$$\tau^{red} = \tau^* + \tau^{**} = t \tag{4.1}$$

The first part is an arbitrary, known from the proportional load range, criterion based on the idea of the critical plane. The other part takes into account the load non-proportionality and is the earlier defined function of the load non-proportionality.

It has been assumed that the product form τ_α^{**} is possible

$$\tau^{**} = c \cdot h\big(\tau_{\alpha^{**}}\big) \cdot f(\lambda) \cdot g\Big(\frac{t}{b}\Big) \tag{4.2}$$

Eventually, the general form of the criterion is the following

$$\tau^{red} = \tau_{\alpha^*} + c \cdot h\big(\tau_{\alpha^{**}}\big) \cdot f(\lambda) \cdot g\Big(\frac{t}{b}\Big) \leqslant t \tag{4.3}$$

5. Detailed form of the load non-proportionality function

In order to alter the general form of the criterion into a detailed one, the forms of the three functions $h(\tau_{\alpha^{**}})$, $f(\lambda)$, $g(t/b)$, appearing in the formula (4) has to be found.

The process takes place in three stages, presented in Table 1. At first, functions defining the groups of literature data with invariable material features $t/b =$ const and uniform coefficients λ are found. The functions describe only variability of the criterion value for the range of phase angle changes. At the second stage, by means of a multinomial function, the notations from stage 1 are generalised into the range of the coefficient λ changes. At the last stage, a function describing also the changes in the material properties is found.

The detailed criterion form was obtained for the base McDiarmid's (1974) and Gough's (1950) criteria. In the first case it is as follows

$$\tau^{red} = \tau^{*} + \tau^{**} = \tau_{\alpha^{*}} + 2.7 \cdot 10^{5} \Big(\frac{t}{b}\Big)^{2} f(\lambda) \big(\tau_{\alpha^{**}}\big)^{3} = t \tag{5.1}$$

where: $f(\lambda) = -7.2\lambda^{2} + 11.5\lambda - 2.1$.

In the latter case, the obtained detailed form differes only by the coefficient values.

Table 1. Tabular presentation of the methodology of assigning the detailed form of the criterion

<table>
<tr><td colspan="3" rowspan="2">Data notations</td><td colspan="3">Non-proportionality element forms
at particular stages of criterion generalisation</td></tr>
<tr><td>Stage 1</td><td>Stage 2</td><td>Stage 3</td></tr>
<tr><td>i</td><td>j</td><td>k</td><td>$h_{ij}\big(\tau_{\alpha^{**}}^{MD}\big)$</td><td>$f_i(\lambda)h\big(\tau_{\alpha^{**}}^{MD}\big)$</td><td>$g(t/b)f_i(\lambda)h\big(\tau_{\alpha^{**}}^{MD}\big)$</td></tr>
<tr><td rowspan="5">t/b</td><td rowspan="3">λ_1</td><td>ϕ_1</td><td rowspan="3">$c_{ij}\big(\tau_{\alpha^{**}}^{MD}\big)^{3}$</td><td rowspan="5">$(a_i\lambda^{2} + b_i\lambda + c_i)\cdot$
$\cdot\big(\tau_{\alpha^{**}}^{MD}\big)^{3}$</td><td rowspan="8">$(t/b)^{2}\cdot$
$\cdot(-7.2\lambda^{2} + 11.5\lambda - 2.1)\cdot$
$\cdot\big(\tau_{\alpha^{**}}^{MD}\big)^{3}$</td></tr>
<tr><td>ϕ_2</td></tr>
<tr><td>ϕ_3</td></tr>
<tr><td rowspan="2">λ_2</td><td>ϕ_1</td><td rowspan="2">$c_{ij}\big(\tau_{\alpha^{**}}^{MD}\big)^{3}$</td></tr>
<tr><td>ϕ_2</td></tr>
<tr><td rowspan="3">t/b</td><td rowspan="3">λ_1</td><td>ϕ_1</td><td rowspan="3">$c_{ij}\big(\tau_{\alpha^{**}}^{MD}\big)^{3}$</td><td rowspan="3">$(a_i\lambda^{2} + b_i\lambda + c_i)\cdot$
$\cdot\big(\tau_{\alpha^{**}}^{MD}\big)^{3}$</td></tr>
<tr><td>ϕ_2</td></tr>
<tr><td>ϕ_3</td></tr>
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6. Verification

The verification was based on results gathered form literature (Nisihara and Kawamoto, 1945; Neugebauer, 1986; Lemmp, 1997; Sonsino, 1983). Those data include fatigue limits for different values of the parameters λ, ϕ and for a wide range of materials. The ratio t/b changes from 0.583 (for "mild steel" in Nisihara and Kawamoto (1945) to 0.949 (for "cast iron" also in Nisihara and Kawamoto (1945).

As a part of the verification, statistical parameters, like average values and standard deviations of the ratio – fatigue limit predicted experimentally to the obtained have been calculated separately for both origin criteria by Gough (Fig. 7a(1) and McDiarmid (Fig. 7.b(1)), and for both proposed modifications (see Fig. 7a(2) and Fig. 7b(2)), respectively. Then the results have been compared in Fig. 7.

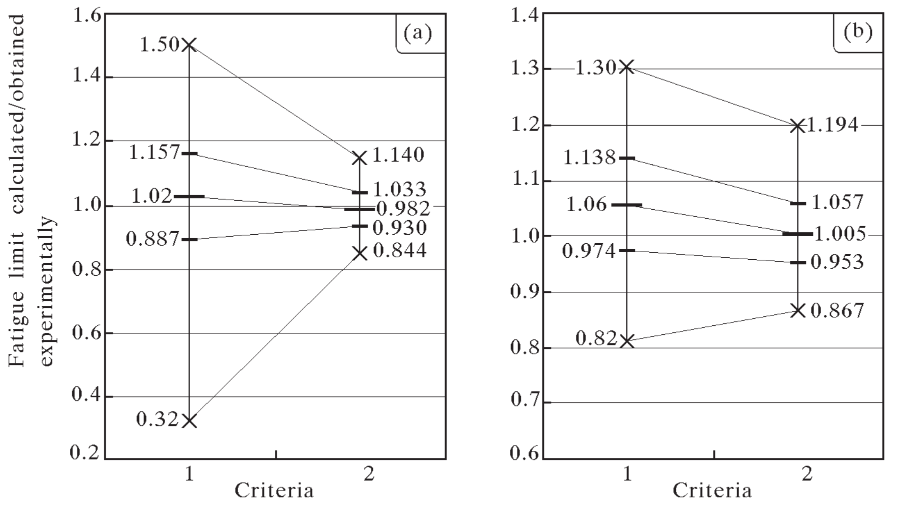

Fig. 7. Average values (long line), standard deviations (short line), and the error range (cross) for original criterion (1) and modified one (2), in the case of McDiarmid's criterion (a) and Gough's one (b)

The analysis shows that in the case of the suggested criterion, the average value moves towards one or values smaller than one (see the long line in Fig. 7). In practice, this entails safer results because the computational value of the limit is always smaller or, at the outmost, equals the experimental limit. What is more, it considerably decreases the span of the whole population of results (see Fig. 7 – the short line). Also, when taking into account single results, the error range is much smaller (Fig. 7 – the cross).

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Koncepcja parametru nieproporcjonalności w złożonym stanie obciążenia

Streszczenie

W artykule opisano koncepcję naprężeniowego parametru nieproporcjonalności obciążenia. Parametr umożliwia ilościową ocenę nieproporcjonalności obciążenia. Na podstawie parametru zostało zaproponowane kryterium wytrzymałości zmęczeniowej. W artykule zamieszczono wyniki weryfikacji kryterium.

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