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PRESSURE-INDUCED RADIAL VIBRATION OF A CYLINDRICAL SHELL MADE OF A PIEZOELECTRIC FIBER COMPOSITE

PIOTR M. PRZYBYŁOWICZ

Institute of Machine Design Fundamentals, Warsaw University of Technology
e-mail: piotrp@ipbm.simr.pw.edu.pl

The paper is concerned with the problem of active damping of transverse (radial) vibration of a cylindrical shell made of a piezoelectric fiber composite (PFC). Smart PFC laminates allow for the access to new possibilities in the field of active vibration damping in various structural elements. In the work, the vibration induced by the pressure pulsating inside the shell is analysed. A simplified model is introduced to the studies. On account of the character of the excitation, the couplings between radial, circumferential and axial vibration modes are neglected in the model. The examinations concentrate on the efficiency of the active control in damping the transverse, i.e. radial vibration of the shell. Numerical simulations prove that resonance characteristics are very sensitive to the lamination angle, which gives a designer the possibility of adjusting the shell structure to the a priori known operating conditions. The main emphasis, however, is put on the effect of active control on the profile of amplitude-frequency characteristics. The simulations carried out clearly show that the application of a simple control strategy based on velocity feedback yields a significant reduction of the vibration level, which is found to be promising and encourages developing more sophisticated models.

Key words: shell, piezoelectric fiber, lamination, active damping, resonance characteristics

1. Introduction

Recent development observed in the field of material mechanics indicates an appearance of interesting solutions for improving the quality of active damping methods based on application of piezoelectric materials. The idea of combining mechanical structures with piezoceramic elements has already been adopted to trusses in which the extensional capability of piezoelements was harnessed to damp bending vibration of those systems. For instance, such a concept was elaborated by dell'Isola and Vidoli (1998) who incorporated lead zirconate titanate (PZT) actuators to truss modular beams coupled with fourth-order electric transmission lines. In their examinations the actuators work as mechanical truss elements on the one hand, and as capacitors, i.e. purely electrical components on the other. Dell'Isola and Vidoli proved the feasibility of electric damping of mechanical vibration via the distributed PZT control. What is especially promising, they confirmed that the optimal choice of control parameters for any vibration frequency is possible.

The integration of piezoceramic (PZT) fibers within composite materials represents a new type of material evolution. Tiny PZT fibers of $30\,\mu$m in diameter can be aligned in an array, electrodized with interdigitated electrodes and then integrated into planar architectures. Such architectures are embedded within the glass or graphite fiber-reinforced polymers and become piezoelectric after being poled (Sporn and Schoencker, 1993). The idea of combining piezoceramics with polymers occurred in the 1980s (Newnham et al., 1980) and several years later it evolved towards smart composite materials. Piezoelectric Fiber Composites (PFCs) have a large potential for controlling. Matrix and ceramic combinations, volume fractions, and ply angles contribute to the tailorability of PFCs (Bent et al., 1995), which make them applicable to structures requiring highly distributed actuation and sensing. Manufacturing technologies of PFCs have been adopted from graphite/epoxy manufacturing methods. Today, PFCs are being equipped with an interdigitated electrode pattern (the so-called IDEPFCs, see Bent and Hagood, 1997). Regardless of the electrode arrangement, the piezoelectric composites create a class of active materials that can cover entire structures – the actuators that are conformable to curved elements such as tubes or shells.

In the present work a cylindrical shell made of piezoelectric fiber composite is examined. The laminate is symmetric, angle-ply. There is little reported in the literature on active vibration control of shells. Saravann et al. (2000) discussed the problem of active damping in composite cylindrical shells with polyvinylidene fluoride (PVDF) elements. The authors thoroughly studied the effect of skew angle and axial location for the patch of collocated sensors/actuators on the active damping ration. This paper concentrates on the efficiency of making use of PZT fiber composites to suppress resonant vibration induced by pulsating pressure inside the shell. The work aims at determining the amplitude-frequency characteristics drawn for disabled and enabled control system. The effect of ply angle is to be discussed as well. Governing equations are taken from the first order shear deformation theory supplemented with constitutive equations of the piezoelectricity. As the study has an introductory character, a possibly simple model is formulated with many approximations involved. The most far-reaching one neglects the couplings between vibration modes corresponding to different principal directions of the shell. Only the transverse modes (radial direction) are taken into account as the shell undergoes a harmonic excitation from the internal medium, which does not flow but changes its pressure sinusoidally.

2. Considered model and governing equations

The system under consideration consists of a cylindrical shell made of an angle-ply symmetric piezoelectric fiber laminate. The shell could be a fragment of a pipeline filled with a medium. In fact, the medium is static – it only changes its pressure in a harmonic manner: $p(x,t) = p_0 \sin \nu t$, i.e. it is constant along the shell length and sinusoidally varies with the amplitude p_0 and angular frequency ν. The shell is simply supported at its ends – see Fig. 1.

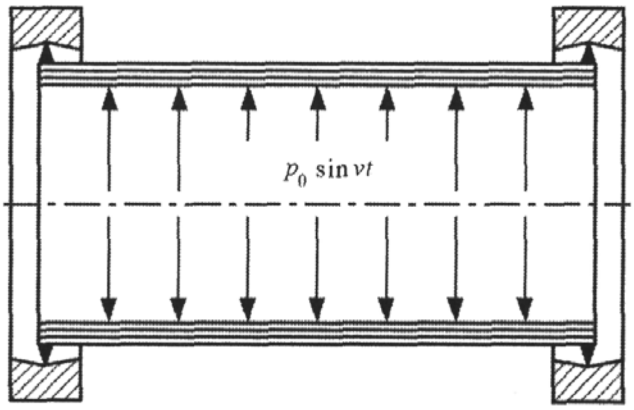

Fig. 1. Model of the considered system

The character of internal loading (pressure) and the way the shell is supported justifies, to some extent, an approximated analysis of dynamics of the system, that is an approach toward investigation of transverse vibration only (in the radial direction) without taking into account the couplings between axial, circumferential and radial vibration modes that can occur in general conditions.

Piezoelectric fibers inside a single lamina are electrodized outside the lamina. The electric field is applied via these electrodes, which are distributed along the entire external surfaces of the PFC layer. An electric signal comes from the sensor after being transformed in the control system. The sensor is assumed to be a polyvinylidene fluoride (PVDF) ring-shaped piezoelectric layer attached to the internal surface of the shell. Its location as well as the basic geometric parameters is shown in Fig. 2. PVDFs are light materials and very thin structures for sensor applications. Consistently, the inertia due to the presence of the sensing element is neglected.

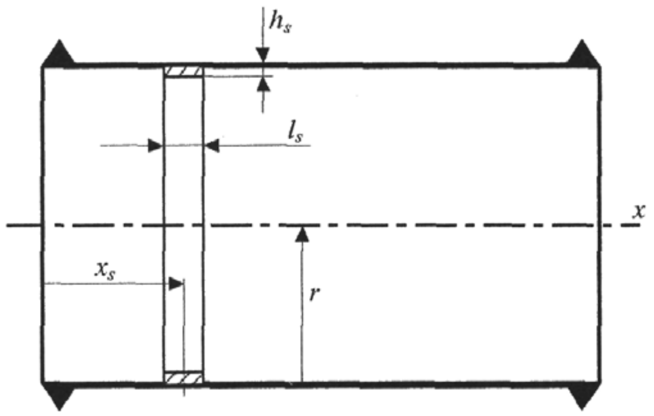

Fig. 2. Geometric parameters of the sensing element

According to Nye (1985), Damjanovič and Newnham (1992), the constitutive equation corresponding to the direct piezoelectric law, following the IEEE Standard (1978), has the form

$$D_3 = d_{31}^{(s)} \sigma_1 \tag{2.1}$$

where

- D_3 – dielectric displacement in the direction of the 3rd axis
- σ_1 – stress applied along the 1st axis (radial)
- $d_{31}^{(s)}$ – electromechanical coupling constant of the sensor.

The orientation of the coordinate system of piezoelectric material has become an international standard, where the 3rd axis always indicates the direction in which the given piezoelectric element is poled by manufacturing. In the analysed model both the sensor and PFC layers are poled as shown in Fig. 3.

The electric charge q_3 developed on the sensor surface is an area integral of the dielectric displacement D_3 given by (2.1), i.e.

$$q_3 = \int\limits_A D_3 \, dA \tag{2.2}$$

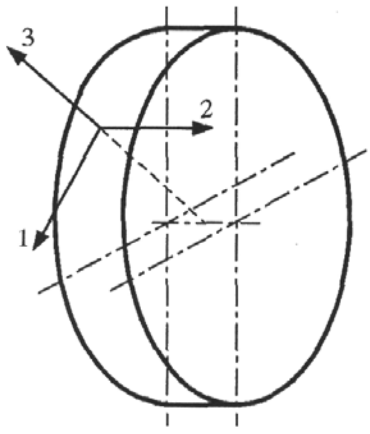

Fig. 3. Poling direction of the sensor and PFCs

As the ring-shaped sensor creates a capacitor, one can determine the voltage measured by the sensing element

$$U_S = \frac{q_3}{C} = \frac{q_3 h_s}{\epsilon_{33} A} = \frac{h_S \int\limits_{x_1}^{x_2} d_{31}^{(s)} \sigma_1 \; dA}{2\pi r l_s \epsilon_{33}} = \frac{h_s d_{31}^{(s)} Y_s}{l_s \epsilon_{33}} \int\limits_{x_1}^{x_2} \varepsilon_1 \; dx \tag{2.3}$$

where

$$x_1 = x_S - \frac{l_s}{2} \qquad x_2 = x_S + \frac{l_s}{2} \qquad \varepsilon_1 = \frac{w(x,t)}{r} \tag{2.4}$$

As can be seen in (2.3) and (2.4), the measured voltage U_S is a function of the electric charge q_3, capacity C of the PVDF ring having the dielectric permittivity ϵ_{33}, the area A, and Young's modulus Y_S. The other parameters: x_S, l_S, h_S, r are shown in Fig. 2. The transverse displacement of the shell surface is denoted by w – see the relation for the circumferential strain ε_1 in (2.4). Finally, it can be said that the measured voltage depends on the function of the transverse displacement $w = w(x,t)$ (current position), and the length and position of the sensor ring (defined by x_1, x_2 or x_S and l_S – passive parameters). It can be expressed as

$$U_S = \frac{h_s d_{31}^{(s)} Y_s}{l_s \epsilon_{33} r} \int\limits_{x_1}^{x_2} w(x,t) \; dx \tag{2.5}$$

The voltage signal from the sensor becomes then transformed into a control system according to the given control law. In the considered model the control strategy is based on a differential element in the control unit, i.e. on the velocity feedback. This implies that the actuating voltage U_A applied directly to the electrodes of the PFC laminate is the first time-derivative of U_S

$$U_A = -c\frac{dU_S}{dt} = -c\frac{h_s d_{31}^{(s)} Y_s}{l_s \epsilon_{33} r} \int\limits_{x_1}^{x_2} \frac{\partial w(x,t)}{\partial t}\, dx \tag{2.6}$$

where c stands for the gain factor in the control unit.

Derive now the equations of motion governing the behaviour of the considered PFC cylindrical shell. Dynamics of thin-walled shells is ruled by the well-known theoretical formula

$$\rho h\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2 M_x}{\partial x^2} + \frac{1}{r}N_y = p(x,t) \tag{2.7}$$

where
- ρ – mass density of the shell material
- h – thickness
- M_x – bending moment
- N_y – circumferential internal force
- p – externally applied pressure.

Explicit relationships for M_x and N_y can be found from the constitutive equations of the first order shear deformation theory completed with expressions involving piezoelectric terms (Bent et al., 1995). Disregarding, or assuming a uniform deformation in the axial and circumferential direction one obtains

$$\begin{aligned} M_x &= D_{11}\frac{\partial^2 w}{\partial x^2} + B_{12}\frac{w}{r} - R_1 \\ N_y &= B_{12}\frac{\partial^2 w}{\partial x^2} + A_{22}\frac{w}{r} - P_2 \end{aligned} \tag{2.8}$$

where

$$\begin{aligned} A_{22} &= \overline{Q}_{22}h \qquad\qquad D_{11} = \frac{h^3}{12}\overline{Q}_{11} \\ \overline{Q}_{22} &= Q_{11}\sin^4\theta + Q_{22}\cos^4\theta + 2Q_{12}\sin^2\theta\cos^2\theta + Q_{66}\sin^2 2\theta \\ \overline{Q}_{11} &= Q_{11}\cos^4\theta + Q_{22}\sin^4\theta + 2Q_{12}\sin^2\theta\cos^2\theta + Q_{66}\sin^2 2\theta \end{aligned} \tag{2.9}$$

and the coefficient B_{12} is zero for symmetric laminates (Jones, 1975; Kurnik and Tylikowski, 1997). The coefficients Q_{11}, Q_{22}, Q_{12}, Q_{66} are the elements of the stiffness matrix $\mathbf{Q}$ in the generalised Hooke's law for a single layer (two-dimensional) in the principal anisotropy axes

$$\mathbf{Q} = \begin{bmatrix} \dfrac{Y_1}{1-\nu_{12}^2\frac{Y_2}{Y_1}} & \dfrac{\nu_{12}Y_2}{1-\nu_{12}^2\frac{Y_2}{Y_1}} & 0 \\ \dfrac{\nu_{12}Y_2}{1-\nu_{12}^2\frac{Y_2}{Y_1}} & \dfrac{Y_2}{1-\nu_{12}^2\frac{Y_2}{Y_1}} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \tag{2.10}$$

where
- Y_1, Y_2 – Young's moduli
- ν_{12} – Poisson's ratio
- G_{12} – Kirchhoff's modulus of the given composite layer.

These coefficients become $\overline{Q}_{11}$, $\overline{Q}_{22}$, $\overline{Q}_{12}$, $\overline{Q}_{66}$ valued (the overbars) for a non-zero lamination angle θ. The necessary transformations are given in (2.9). As the PFC laminate is an active structure, capable of producing bending, shear, tension, etc. under application of an electric field, equation (2.8) must include terms responsible for such a electromechanical coupling (the terms P_2 and R_1).

Admittedly, for symmetric PFCs, the term R_1 disappears and the electromechanical interactions are coupled by the coefficient P_2. It is defined by

$$P_2 = h\overline{\Xi}_2 \qquad \overline{\Xi}_2 = \frac{Y_1 \sin^2\theta + Y_2(\cos^2\theta + \nu_{12})}{1 - \frac{Y_2}{Y_1}\nu_{12}^2} d_{31}^* E_3 \tag{2.11}$$

or after some simplifications

$$P_2 = (Q_{11}\sin^2\theta + Q_{22}\cos^2\theta + Q_{12})d_{31}^* E_3 h \tag{2.12}$$

where d_{31}^* is the electromechanical coupling constant of a single layer with piezoelectric fibers, E_3 – the electric field applied radially to that layer

$$E_3 = \frac{U_A}{hN} \tag{2.13}$$

where U_A is the voltage supplied by the control unit, N – number of the layers.

By substituting (2.8) together with relations (2.9)-(2.13) into equation of motion (2.7) one obtains

$$\frac{\partial^2 w}{\partial t^2} + a^2\frac{\partial^4 x}{\partial x^4} + b^2 w = \frac{1}{\rho h}p(x,t) - \beta\int\limits_{x_1}^{x_2} \frac{\partial w(x,t)}{\partial t}\, dx \tag{2.14}$$

where

$$a^2 = \frac{D_{11}}{\rho h} \qquad b^2 = \frac{A_{22}}{r^2\rho h}$$
$$\beta = c(Q_{11}\sin^2\theta + Q_{22}\cos^2\theta + Q_{12})\frac{h_s d_{31}^{(s)} d_{31}^{*} Y_S}{N l_s \epsilon_{33} r^2 \rho h}\int\limits_{x_1}^{x_2} \frac{\partial w(x,t)}{\partial t}\,dx \tag{2.15}$$

3. Analytical solution to equation of motion

The examined PFC shell is subject to harmonic excitation: $p(x,t) = p(t) = p_0 \sin \nu t$, which enables one to predict the steady-state response of the structure vibrating in the radial direction $w(x,t)$ as a product of independent spatial and time functions

$$w(x,t) = W(x)\mathrm{e}^{\mathrm{i}\nu t} \tag{3.1}$$

It shortly leads to the fourth order but ordinary differential equation

$$-\nu^2 W + a^2 W^{IV} + b^2 W = 0 \;\Rightarrow\; W^{IV} - \frac{\nu^2 - b^2}{a^2}W = 0 \tag{3.2}$$

the general solution to which is

$$W(x) = C_1\exp(-x\sqrt[4]{\xi}) + C_2\exp(x\sqrt[4]{\xi}) + C_3\exp(-\mathrm{i}x\sqrt[4]{\xi}) + $$
$$+C_4\exp(\mathrm{i}x\sqrt[4]{\xi}) \qquad \xi = \frac{\nu^2 - b^2}{a^2} \tag{3.3}$$

The constants C_1, C_2, C_3, C_4 can be determined from boundary conditions. In the analysed case the shell is simply supported, which means that the deflection and curvature at the ends of the structure equals zero

$$W(0) = 0 \qquad W(l) = 0 \qquad W''(0) = 0 \qquad W''(l) = 0 \tag{3.4}$$

The eigenforms resulting from (3.3) and the corresponding vibration modes become simple sinusoidal functions

$$W_i(x) = \sin i\frac{\pi x}{l} \qquad W_i^{IV}(x) = W_i(x)\frac{\pi^4}{l^4}i^4 \qquad i = 1,2,3,\dots \tag{3.5}$$

By expanding the solution to the equation of motion into an infinite series based on eigenforms (3.5), i.e.

$$w(x,t) = \sum_{i=1}^{\infty} W_i(x)T_i(t) \tag{3.6}$$

and substituting it into (2.14), one obtains

$$\begin{aligned}&\sum_{i=1}^{\infty}\Big(W_i(x)\ddot{T}_i(t) + a^2W_i^{IV}(x)T_i(t) + b^2W_i(x)T_i(t) + \beta\dot{T}_i(t)\int\limits_{x_1}^{x_2} W_i(x)\Big) = \\ &= \frac{1}{\rho h}\sum_{i=1}^{\infty} p_i(t)W_i(x)\end{aligned} \tag{3.7}$$

where the internal pressure $p(x,t)$ has been expanded into the infinite series with respect to the eigenmodes $W_i(x)$ as well. In particular

$$p_i(t) = \frac{\int\limits_0^l p(x,t)W_i(x)\,dx}{\int\limits_0^l W_i^2(x)\,dx} = p_0\sin\nu t\frac{\int\limits_0^l \sin i\frac{\pi x}{l}\,dx}{\int\limits_0^l \sin^2 i\frac{\pi x}{l}\,dx} = p_0\sin\nu t\varGamma_i \tag{3.8}$$

The properties of the integral appearing in (3.8) make the coefficient $\varGamma_i$ zero for any even number of $i \in N$. It can be easily checked that

$$\sum_{i=1}^{\infty} W_i(x)\varGamma_i = \sum_{i=1}^{\infty}\sin\frac{\pi i x}{l}\frac{4(1-\cos i\pi)}{2i\pi - \sin 2i\pi} \to H(x) - H(x-l) \tag{3.9}$$

where $H(\cdot)$ is Heaviside's step function, and $H(x)-H(x-l)$ is a unit function within the interval $x \in [0,l]$, i.e. where the pressure $p(x,t)$ is applied.

Some troubles introduces the term responsible for the action of the control system, expressed by the integral standing next to β in (3.7). To make (3.7) a series of linearly independent functions corresponding to any $W_i(x)$, this term must be expressed with respect to $W_i(x)$ as well. Namely

$$\int\limits_{x_1}^{x_2} W_i(x)\,dx = \sum_{j=1}^{\infty}\Big(\int\limits_{x_1}^{x_2} W_i(x)\,dx\Big)_j W_n(x) \tag{3.10}$$

where

$$\Big(\int\limits_{x_1}^{x_2} W_i(x)\,dx\Big)_j = \int\limits_{x_1}^{x_2} W_i(x)\,dx\frac{\int\limits_0^l W_j(x)\,dx}{\int\limits_0^l W_j^2(x)\,dx} = \varGamma_j\int\limits_{x_1}^{x_2} W_i(x)\,dx \tag{3.11}$$

Rewriting (3.7) by making use of (3.10) and (3.11) in the form

$$\sum_{i=1}^{\infty}\Big(\ddot{T}_i(t)+\omega_i^2T_i(t)+\beta\dot{T}_i(t)\int\limits_{x_1}^{x_2}W_i(x)\sum_{j=1}^{\infty}\Gamma_jW_j(x)\Big)W_i(x)= \\ =\frac{1}{\rho h}\sum_{i=1}^{\infty}p_i(t)W_i(x) \tag{3.12}$$

with the eigenfrequencies

$$\omega_i^2=b^2+a^2i^4\frac{\pi^4}{l^4} \tag{3.13}$$

the following set of simultaneous second order ordinary differential equations is obtained

$$\begin{aligned}
&\ddot{T}_1+\beta_{11}\dot{T}_1+\beta_{12}\dot{T}_2+\beta_{12}\dot{T}_2+\beta_{13}\dot{T}_3+\beta_{14}\dot{T}_4+\beta_{15}\dot{T}_5+\ldots+\omega_1^2T_1=q_1\sin\nu t\\
&\ddot{T}_2+\omega_2^2T_2=0\\
&\ddot{T}_3+\beta_{31}\dot{T}_1+\beta_{32}\dot{T}_2+\beta_{32}\dot{T}_2+\beta_{33}\dot{T}_3+\beta_{34}\dot{T}_4+\beta_{35}\dot{T}_5+\ldots+\omega_3^2T_3=q_3\sin\nu t\\
&\ddot{T}_4+\omega_4^2T_4=0\\
&\vdots
\end{aligned} \tag{3.14}$$

Note that the even-numbered equations lack the first time-derivatives $\dot{T}_i$, corresponding to dissipation of the vibration energy. This follows from the above-mentioned fact that Γ_i is zero for any even $i \in N$, and from the definitions

$$\beta_{ij}=\beta\Gamma_ic_j \qquad q_i=\frac{p_0}{\rho h}\Gamma_i \qquad i,j=1,2,3,... \tag{3.15}$$

where

$$c_j=c_j(x_1,x_2)=\int\limits_{x_1}^{x_2}W_j(x)\,dx=\int\limits_{x_1}^{x_2}\sin j\frac{\pi x}{l}\,dx=\frac{l}{\pi j}\Big(\cos j\frac{\pi x_1}{l}-\cos j\frac{\pi x_2}{l}\Big) \tag{3.16}$$

While searching for the steady-state response of the structure, it is convenient to assume zero initial conditions for (3.14) in order to exclude any transient behaviour – the more so since it vanishes in time because of natural internal damping present in the material of the structure. Additionally, this implies that the solution to all even equations in (3.14) is trivial: $T_i(t) \equiv 0$ since the even modes are not excited by the assumed type of pressure loading: $q_i \equiv 0$ for even $i \in N$. Hence, by applying the zero initial conditions

$$\begin{aligned} T_1(0) = T_2(0) = \ldots = T_n(0) = \ldots = 0 \\ \dot{T}_1(0) = \dot{T}_2(0) = \ldots = \dot{T}_n(0) = \ldots = 0 \end{aligned} \tag{3.17}$$

set of equations (3.14) assumes the following final form

$$\begin{aligned} &\ddot{T}_1 + \beta_{11}\dot{T}_1 + \beta_{13}\dot{T}_3 + \beta_{15}\dot{T}_5 + \beta_{17}\dot{T}_7 + \ldots + \omega_1^2 T_1 = q_1 \sin\nu t \\ &\ddot{T}_3 + \beta_{31}\dot{T}_1 + \beta_{33}\dot{T}_3 + \beta_{35}\dot{T}_5 + \beta_{37}\dot{T}_7 + \ldots + \omega_3^2 T_3 = q_3 \sin\nu t \\ &\ddot{T}_5 + \beta_{51}\dot{T}_1 + \beta_{53}\dot{T}_3 + \beta_{55}\dot{T}_5 + \beta_{57}\dot{T}_7 + \ldots + \omega_5^2 T_5 = q_5 \sin\nu t \\ &\ddot{T}_7 + \beta_{71}\dot{T}_1 + \beta_{73}\dot{T}_3 + \beta_{75}\dot{T}_5 + \beta_{77}\dot{T}_7 + \ldots + \omega_7^2 T_7 = q_7 \sin\nu t \\ &\vdots \end{aligned} \tag{3.18}$$

4. Numerical simulations

As the equations appearing in (3.18) are in fact simple non-homogeneous second-order ordinary differential ones with constant coefficients, the particular solution to them can be easily found, e.g. in a complex form: $T_i(t) = \widetilde{A}_i e^{i\nu t}$, where $\widetilde{A}_i$ is the complex amplitude corresponding to the ith vibration mode. By substituting the thus set-forth partial solution into (3.18), the differential equations can be transformed into algebraic ones

$$\begin{Vmatrix} A_1 & i\nu\beta_{13} & i\nu\beta_{15} & i\nu\beta_{17} & \cdots \\ i\nu\beta_{31} & A_3 & i\nu\beta_{35} & i\nu\beta_{37} & \cdots \\ i\nu\beta_{51} & i\nu\beta_{53} & A_5 & i\nu\beta_{57} & \cdots \\ i\nu\beta_{71} & i\nu\beta_{73} & i\nu\beta_{75} & A_7 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{Vmatrix} \begin{vmatrix} \widetilde{A}_1 \\ \widetilde{A}_3 \\ \widetilde{A}_5 \\ \widetilde{A}_7 \\ \vdots \end{vmatrix} = \begin{vmatrix} q_1 \\ q_3 \\ q_5 \\ q_7 \\ \vdots \end{vmatrix} \sin\nu t \tag{4.1}$$

where $A_k = \omega_k^2 - \nu^2 + i\beta_{kk}$, $k = 1, 3, 5, 7, ...$
– very convenient for further numerical applications.

The numerical investigations are concentrated on the dynamic behaviour of the considered system. The dynamics is examined with the help of amplitude-frequency characteristics found for chosen structural and control parameters of the system. The piezoelectric fibers are embedded into carbon/epoxy laminae, which possess highly anisotropic properties. This makes the shell particularly sensitive to the lamination angle with respect to resonant frequencies. Location of the resonances as a function of the lamination angle (from 0° up to 90°) is shown in Fig. 4. As can be seen, the ply angles close to 90° shift the resonances towards higher frequencies (the structure stiffens) and collect them, which is obviously undesirable.

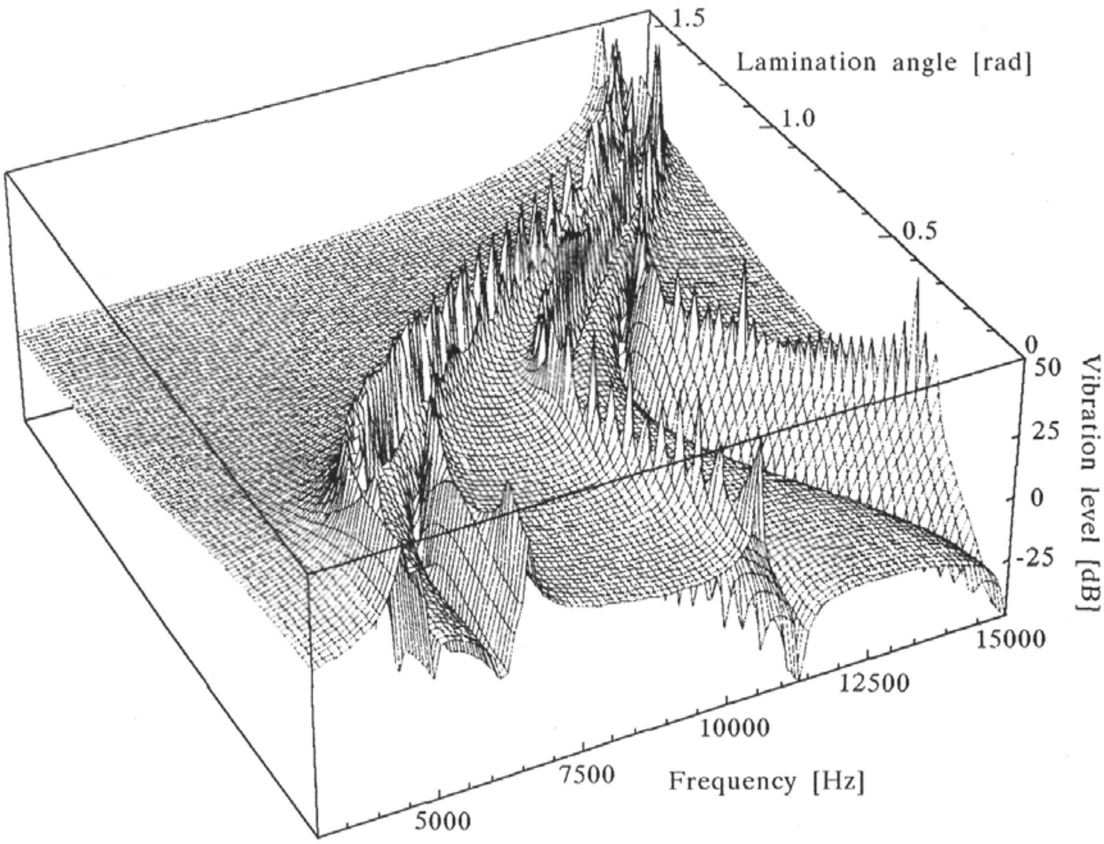

Fig. 4. Effect of lamination angle on resonance characteristics of the PFC shell

The dynamic characteristics presented in Fig. 4 correspond to the analysed shell without applying any control to it (the gain factor c is zero). The effect of active damping can be clearly illustrated in plane sections of the resonant characteristics for some selected lamination angles. This is done in Fig. 5 to Fig. 7, where the dashed lines (activated control for a moderate values of the gain factor) are drawn against the background of the continuous curves (no active damping).

Numerical simulations prove the advantageous effect brought about by employing a simple control strategy to the system on the lowering of the transverse vibration excited by the harmonically varying internal pressure. The drop in the vibration amplitude is observed in all cases of geometric arran-

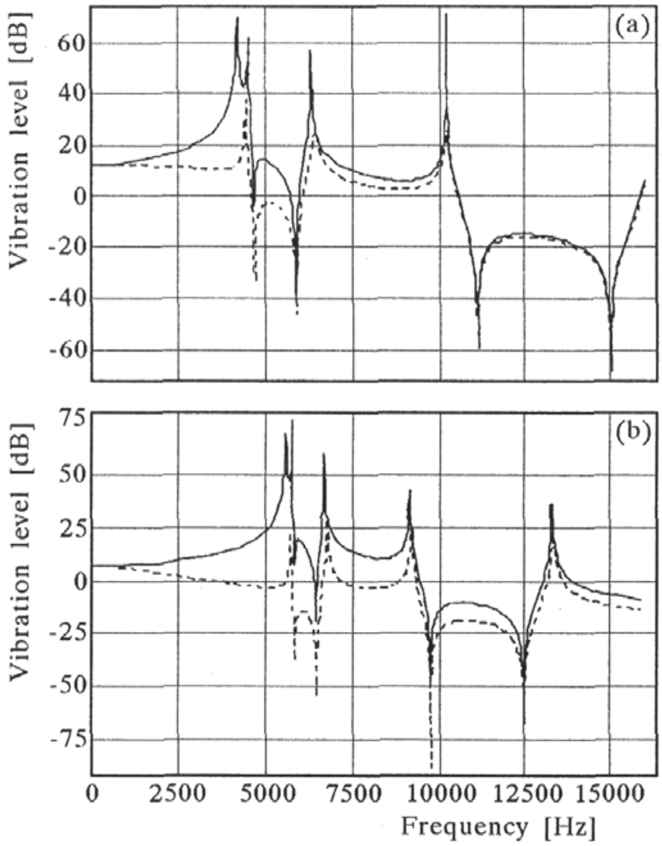

Fig. 5. Effect of active damping for the cross-ply (0°) and angle-ply (30°) PFC shell

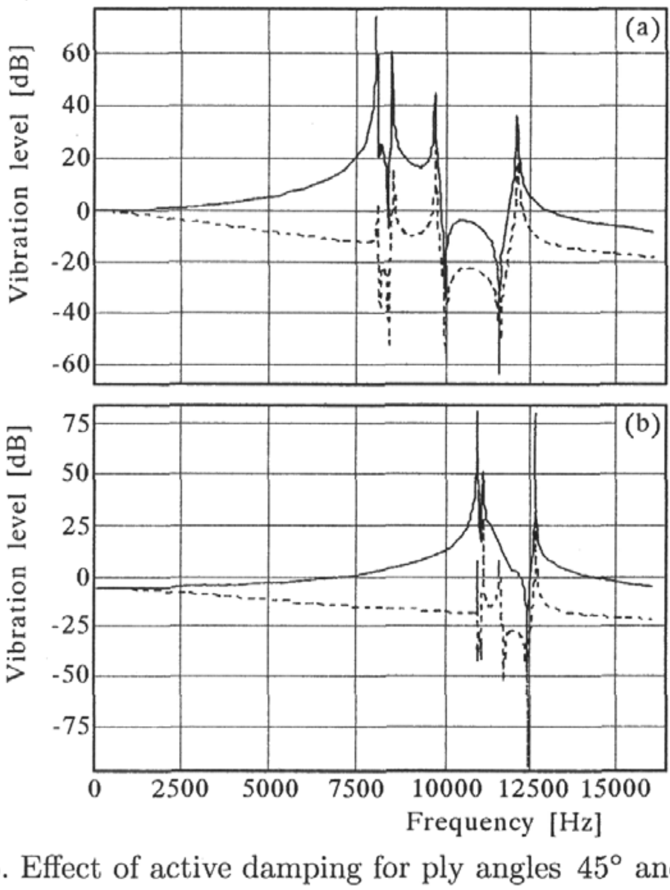

Fig. 6. Effect of active damping for ply angles 45° and 60°

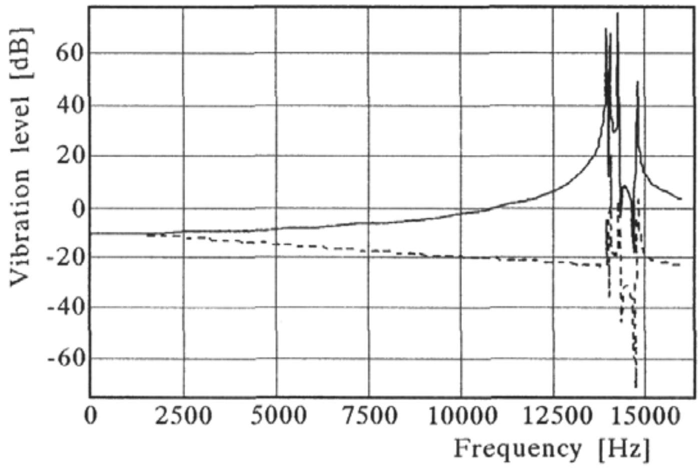

Fig. 7. Effect of active damping for the 90° – cross-ply PFC shell

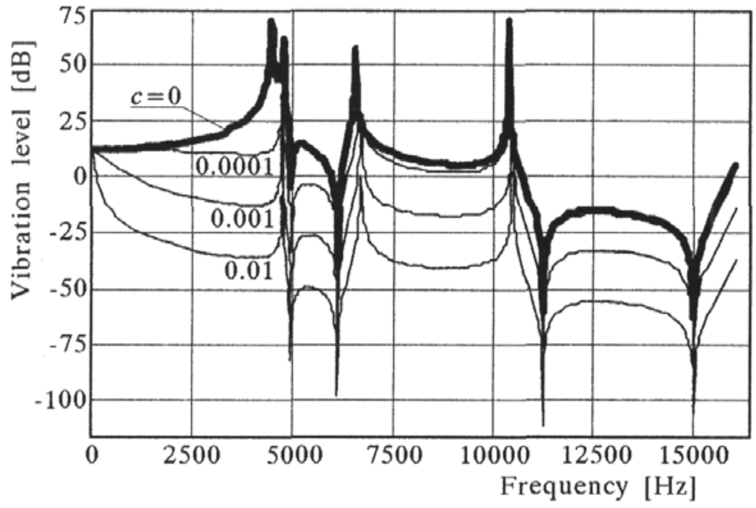

Fig. 8. Resonance curves for various gain factors in the control system

gement of the shell active layers. Expectedly, an increase in the gain factor value entails further lowering of the vibration level. Some exemplary resonance curves corresponding to the zero-degree-cross-ply PFC shell undergoing the velocity-feedback control for different gain factors is presented in Fig. 8. Undoubtedly, value of the gain factor cannot be raised freely. The electric strength (the so-called breakdown voltage) of the piezoceramic fibers is the limit. An excessive electric field resulting from over-regulation would surely destroy the whole effort. Incorporation of another control technique could be a way out in that case.

5. Conclusions

Piezoelectric fiber composites (PFCs) represent a new approach toward active damping in mechanical systems and are promising candidates among the state-of-the-art structural materials. The actuating capability of integrated fiber architectures allows an access to a new generation of smart materials. The present paper discusses the efficiency of incorporating PFCs to a cylindrical shell subject to a dynamic pressure inside the shell. The pressure-induced radial vibration proves to be very sensitive to the angle of lamination – which gives a designer the possibility of passive adjustment of the structure to the a priori known operating conditions. In the case when they are not known or unpredictable, the application of a suitably driven electric field will significantly decrease the vibration level. The method however, is limited by the dielectric strength of the piezoceramic fibers.

Admittedly, the paper describes a simplified model in which any mechanical coupling between different vibration modes in various directions (radial, circumferential and axial) are neglected. The future studies should take them all into account to let the model be applicable to more complex examples of external excitation, e.g. including the case when the medium is flowing through the shell. A more detailed insight into the problem of sensing the resulting vibration will be necessary as well. These would enable the model to deal with the problem of lowering acoustic emission from short segments of pipelines.

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Drgania promieniowe cylindrycznej powłoki wykonanej z kompozytu zawierającego włókna piezoelektryczne

Streszczenie

W pracy podjęto problem aktywnego tłumienia drgań poprzecznych cylindrycznej powłoki wykonanej z laminatu zintegrowanego z włóknami piezoelektrycznymi (PFC). Aktywne laminaty PFC otwierają nowe możliwości na polu aktywnego sterowania drganiami w różnych elementach konstrukcyjnych. W pracy zajęto się drganiami powłoki wywołanymi pulsującym ciśnieniem wewnątrz tej powłoki. Przedstawiono uproszczony model, w którym ze względu na charakter obciążenia zrezygnowano z uwzględnienia sprzężeń pomiędzy promieniowymi, osiowymi i obwodowymi postaciami drgań, koncentrując się na efektywności aktywnego tłumienia w powłoce PFC w przypadku drgań poprzecznych. W wyniku symulacji numerycznych pokazano, że charakterystyki rezonansowe są bardzo czułe na kąt laminowania, co stanowić może narzędzie dla konstruktora podczas pasywnego dostosowywania obiektu do znanych warunków eksploatacyjnych, w jakich ten obiekt będzie pracować. Główny nacisk położono jednak na wpływ aktywnego sterowania na profil charakterystyk amplitudowo-częstotliwościowych. Stwierdzono, że zastosowanie sterowania ze sprzężeniem prędkościowym daje wyraźne obniżenie poziomu drgań, co uznano za wynik obiecujący i zachęcający do udoskonalenia modelu.

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