THE HWKK RHEOLOGICAL MODEL FOR RESINS

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A rheological model for epoxy and polyester resins, denoted with the symbol HWKK, has been developed and positively validated for selected loading programmes. This model describes the first-rank reversible creep of amorphous thermohardened polymers with good accuracy. The mechanical HWKK model consists of four elements connected in series, formulated by Hooke, Wilczyński and Kelvin. Constitutive equations of viscoelasticity for the HWKK model have been formulated, with fractional and normal exponential functions used as the generating ones. An algorithm for identification of the material constants, based on creep experiments of bar samples uniaxially tensioned, has been developed. The material constants have been identified for epoxy and polyester resins.

Key words: polymers, resins, constitutive equations of viscoelasticity, material constants

1. Introduction

Epoxy and polyester resins, belonging to amorphous thermohardened polymers, are widespread in engineering practice, especially in aeronautics, as matrices of fibre-reinforced laminates. These materials exhibit different creep properties, first of all dependent on the stress level and temperature. When the creep of an uniaxially tensioned bar sample is caused by the Heaviside loading programme, three types of the creep may appear, i.e. the first-rank creep (with creep velocity decreasing in time), the second-rank creep (steady-state creep) and the third- rank creep yielding non-linear strains and failure

(Skrzypek, 1986). The second- and third-rank creep phenomena take place at high levels of tension stresses and/or high temperatures.

Constitutive equations of viscoelasticity of polymers can be reflected by mechanical models, composed of the Hooke (H), Kelvin (K), Newton (N) and Wilczyński (W) elements (Klasztorny et al., 2001). The H, K and N elements are defined clasically, whereas the W element is a modified Kelvin's element, with time-dependent damping properties, described by a Mittag-Leffler's fractional exponential function. A fractional exponential function, as a generating function in viscoelasticity of polymers, was originally applied by Rabotnov (1977). This approach was developed by Wilczyński (1978), who replaced a weakly convergent Mittag-Leffler's series with an integral form.

The nature of viscoelastic behaviour of high-molecular materials was explained by Ferry (1970). In the short-lasting creep interval (SLC) fast kinks of molecules are observed, weakly described by single Kelvin's element. This process gradually passes into the long-lasting creep interval (LLC) in which scrolls of molecules are created. For modelling the first-rank reversible creep in both intervals, Ferry developed the $H \infty K$ model, composed of a single Hooke's element and an infinite number of Kelvin's elements, connected in series. An assessment of accuracy of the $H \infty K$ model is difficult, since Ferry did not analyse directional creep strains in a semi-logarithmic scale, which is neccessary to reveal deviation from the HK model. Moreover, Ferry investigated experimentally a large number of polymers, but not resins.

Wilczyński and Klasztorny (2000) modelled the viscoelastic behaviour of resins with the HW model, separately for the shear and bulk creep, which is described by 2 elastic and 6 viscoelastic constants. Klasztorny et al. (2001) developed this model to the HWKN form, described by 2 elastic and 12 viscoelastic constants. However, the validation experiments have not confirmed either HW or HWKN model.

In practice, laminates work in a linear viscoelastic range. Normal stress levels ($\sigma \leqslant 0.3R_m$, where R_m is the tensile strength of a resin) and temperatures ($10°C \leqslant T \leqslant 30°C$) protect the first-rank reversible creep of resins (Rabotnov, 1977).

2. Constitutive equations of viscoelasticity for resins

On the basis of the creep experiments, Rabotnov (1977) pointed that:

- Resins can be treated as isotropic materials.

- The linear heredity theory, invariant in time, is very close to reality. This theory is described by constitutive equations of Volterra's type.

- The first-rank creep of uniaxially tensioned samples cannot be reflected by the HK standard model.
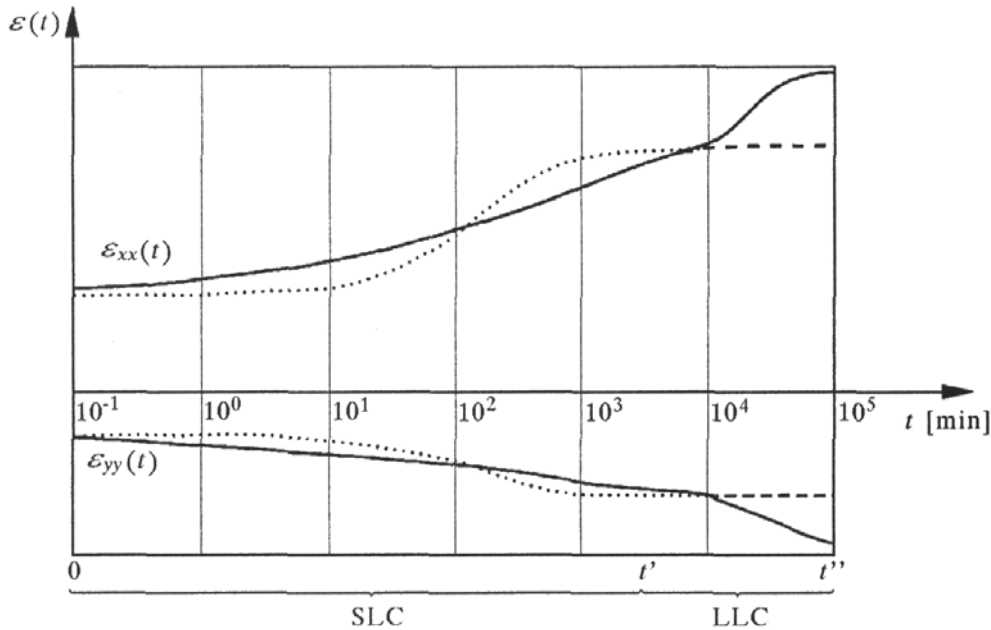

Fig. 1. A simplified scheme illustrating the short-lasting creep (SLC) and the long-lasting creep (LLC) of unidirectionally tensioned bar samples, made of epoxy or polyester resin: dotted line – HK model; broken line – HWK model; solid line – HWKK model

The first-rank creep experiments recently performed on uniaxially tensioned bar samples, made of epoxy or polyester resin, realised at Laboratory of Strength of Materials, Faculty of Mechanical Engineering, Military University of Technology, Warsaw, pointed that (vide Fig. 1):

- The creep histories can be divided into two intervals, corresponding to the short-lasting creep (SLC) and the long-lasting creep (LLC).

- The SLC creep can be simulated with good accuracy with the Wilczyński and Kelvin elements, connected in series.

- The SLC directional strains, $\varepsilon_{xx}(t)$, $\varepsilon_{yy}(t)$, are quasi-proportional, with x – the direction of tension, yz – the plane perpendicular to the x axis.

- The LLC creep can be simulated with good accuracy with an additional Kelvin's element.

- The LLC directional strains, $\varepsilon_{xx}(t)$, $\varepsilon_{yy}(t)$, are also quasi-proportional.

These features of the first-rank creep of resins can be reflected by a rheological model presented in Fig. 2. This model, denoted with the symbol HWKK, takes into account the elastic strains (the H element), the SLC viscoelastic strains (the W and K elements) and the LLC viscoelastic strains (the second K element).
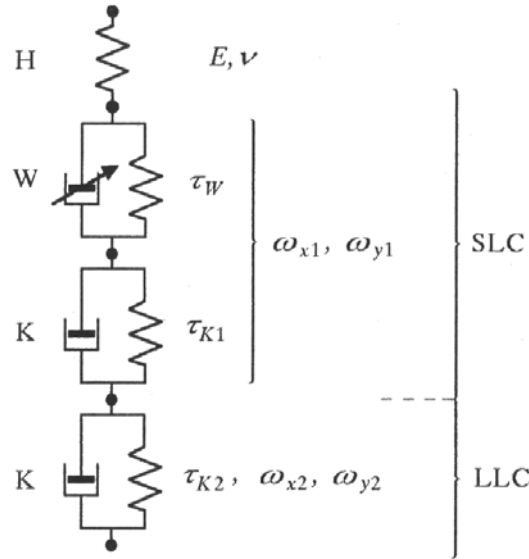

Fig. 2. Mechanical scheme of the HWKK rheological model for resins

In a matrix notation, the constitutive equations of linear viscoelasticity, governing the HWKK model, have the form (cf Wilczyński and Klasztorny, 2000; Klasztorny et al., 2001)

$$\boldsymbol{\varepsilon}(t) = \mathbf{S}(t) \otimes \boldsymbol{\sigma}(t) \tag{2.1}$$

where

$$\boldsymbol{\varepsilon} = \mathrm{col}\,(\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{yz}, \varepsilon_{xz}, \varepsilon_{xy})$$

$$\boldsymbol{\sigma} = \mathrm{col}\,(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{yz}, \sigma_{xz}, \sigma_{xy}) \tag{2.2}$$

$$\mathbf{S} = \begin{bmatrix} S_x & S_y & S_y & 0 & 0 & 0 \\ & S_x & S_y & 0 & 0 & 0 \\ & & S_x & 0 & 0 & 0 \\ & & & S_x - S_y & 0 & 0 \\ & & & & S_x - S_y & 0 \\ \text{symm.} & & & & & S_x - S_y \end{bmatrix}$$

The symbols used in Eqs (2.1) and (2.2) denote:

xyz — an optional cartesian coordinate system

$\varepsilon, \boldsymbol{\sigma}$ — time-dependent strain and stress tensors (in the matrix notation)

$\mathsf{S}(t)$ — a matrix of time-dependent compliances

$S_x(t), S_y(t)$ — time-dependent directional compliances of an isotropic material

t — time variable

$\otimes$ — convolution operator.

The time-dependent compliances are defined by the formulae

$$S_x(t) = S_x^0 \left[1 + \omega_{x1} \int_0^t \Phi_1(t - \vartheta) \, d\vartheta + \omega_{x2} \int_0^t F_2(t - \vartheta) \, d\vartheta \right]$$

$$S_y(t) = S_y^0 \left[1 + \omega_{y1} \int_0^t \Phi_1(t - \vartheta) \, d\vartheta + \omega_{y2} \int_0^t F_2(t - \vartheta) \, d\vartheta \right]$$

(2.3)

where

$$S_x^0 = \frac{1}{E} \qquad\qquad S_y^0 = -\frac{\nu}{E}$$

$$\Phi_1(t) = \gamma \Phi(t) + (1 - \gamma) F_1(t) \qquad\qquad 0 < \gamma < 1$$

$$\Phi(t) = \alpha \int_0^\infty e^{-\alpha \xi t} \xi \Lambda(\xi; \mu) \, d\xi \qquad\qquad \alpha = \frac{1}{\tau_W}$$

$$\Lambda(\xi; \mu) = \frac{\sin \pi \mu}{\pi} \frac{\xi^{\mu-1}}{1 + 2\xi^\mu \cos \pi \mu + \xi^{2\mu}} \qquad 0 < \mu < 1$$

$$F_i(t) = \alpha_i e^{-\alpha_i t} \qquad\qquad \alpha_i = \frac{1}{\tau_{Ki}} \quad i = 1,2$$

$$\tau_{K2} \gg \tau_W, \tau_{K1}$$

(2.4)

The symbols used in Eqs (2.3) and (2.4) have the following physical interpretation:

S_x^0, S_y^0 — directional elastic compliances

E, ν — constants of elasticity of an isotropic material (Young's modulus, Poisson's ratio)

$\Phi_1(t)$ — a generating function related to the SLC

$\Phi(t), F_1(t)$ — fractional and normal exponential functions, related to SLC

$F_2(t)$	–	a generating function related to LLC (normal exponential function)
ω_{x1}, ω_{y1}	–	viscoelastic constants determining the directional long-lasting compliances, related to the SLC component
ω_{x2}, ω_{y2}	–	viscoelastic constants determining the directional long-lasting compliances, related to the LLC component
μ	–	a fraction defining the elastic-sequence-time distribution $\Lambda(\xi; \mu)$
γ	–	a fraction defining a linear combination of the functions $\Phi(t)$, $F_1(t)$
$\tau_W, \tau_{K1}, \tau_{K2}$	–	the elastic-sequence-times (EST) of the viscoelastic elements of the HWKK model, respectively related to functions $\Phi(t)$, $F_1(t)$, $F_2(t)$.

3. An algorithm for identification of the material constants describing the HWKK model

Material constants describing the HWKK model can be estimated from creep experiments, performed on uniaxially tensioned cylindrical test specimens. A bar sample is tensioned in the x direction, coinciding with the bar axis. The loading programme for the classical creep is described by the formula

$$\sigma_{xx}(t) = \sigma_{xx}^0 H(t) \tag{3.1}$$

where σ_{xx}^0 is a constant tensioning stress, and $H(t)$ is the Heaviside function. The remaining components of the stress tensor are equal to zero. Putting Eq. (3.1) into Eq. (2.1), taking into consideration Eqs (2.2)-(2.4), one derives the formulae describing the directional creep, i.e.

$$\varepsilon_{xx}(t) = \Big[1 + \omega_{x1}\varphi_1(t) + \omega_{x2}\varphi_2(t)\Big]\varepsilon_{xx}^0$$

$$\varepsilon_{yy}(t) = \Big[1 + \omega_{y1}\varphi_1(t) + \omega_{y2}\varphi_2(t)\Big]\varepsilon_{yy}^0 \tag{3.2}$$

where

$$\varepsilon_{xx}^0 = S_x^0 \sigma_{xx}^0 = \frac{1}{E}\sigma_{xx}^0 \qquad\qquad \varepsilon_{yy}^0 = S_y^0 \sigma_{xx}^0 = -\frac{\nu}{E}\sigma_{xx}^0$$

$$\varphi_1(t) = 1 - \gamma \int\limits_0^\infty e^{-\alpha\xi t} \Lambda(\xi;\mu)\, d\xi - (1-\gamma)e^{-\alpha_1 t} \tag{3.3}$$

$$\varphi_2(t) = 1 - e^{-\alpha_2 t}$$

with $0 < \varphi_i(t) < 1$, $i = 1,2$, $t > 0$ and

$\varepsilon_{xx}^0, \varepsilon_{yy}^0$ –   directional elastic strains

$\varphi_1(t), \varphi_2(t)$ –   creep functions corresponding to the SLC, LLC.

Note that $\varphi_1(t), \varphi_2(t) \in [0;1]$ and $\varphi_1(t) \to 1$, $\varphi_2(t) \to 1$ for $t \to \infty$. The remaining components of the strain tensor satisfy the conditions

$$\varepsilon_{zz}(t) = \varepsilon_{yy}(t) \qquad\qquad \varepsilon_{yz}(t) = \varepsilon_{xz}(t) = \varepsilon_{xy}(t) = 0$$

Introductory numerical tests have pointed out that the approximation of the directional strain histories with the generating functions $\Phi(t)$, $F_1(t)$, $F_2(t)$ is very flexible. Assuming the relative deviation of the simulated creep strains from the experimental results $\delta \leqslant 2\%$, one obtains a large number of sets of the values of γ, μ, τ_W, τ_{K1} satisfying this condition, with $0.4 < \gamma < 0.6$, $0.4 < \mu < 0.6$. This fact enables one to prefix the parameters γ, μ at the average values $\gamma = 0.5$, $\mu = 0.5$ and to estimate the remaining parameters τ_W, τ_{K1} from a non-linear optimisation task.

Summing up, the HWKK rheological model for resins, simulating the first-rank reversible creep, is described by two elastic constants (E, ν) and seven viscoelastic constants $(\omega_{x1}, \omega_{x2}, \omega_{y1}, \omega_{y2}, \tau_W, \tau_{K1}, \tau_{k2})$.

The elastic constants of a given matrix (epoxy or polyester) are calculated from well-known simple formulae equivalent to Eqs (3.3)$_{1,2}$, i.e.

$$E = \frac{\sigma_{xx}^o}{\varepsilon_{xx}^o} \qquad\qquad \nu = -\frac{\varepsilon_{yy}^o}{\varepsilon_{xx}^o} \tag{3.4}$$

In the SLC interval $\varphi_2(t) \approx 0$, thus the viscoelastic constants τ_W, τ_{K1} can be estimated from the following condition

$$\sum_{i=1}^m |\varepsilon_{xx}(t_i) - \varepsilon_{xx}^e(t_i)| = \min \tag{3.5}$$

where

t_i –   collocation points, selected quasi-uniformly in a logarithmic scale in the interval $[0, t']$, $i = 1, ..., m$

$\varepsilon_{xx}(t_i)$ –   the simulated discrete value of the longitudinal strain

$\varepsilon_{xx}^e(t_i)$ –   the experimental discrete value of this strain.

Equation (3.5) states the objective function of the non-linear optimisation task. In order to find the optimal values of the decision variables, τ_W, τ_{K1}, a method of systematic searching has been applied

$$\tau_W = \tau_{W,\min} + j\Delta\tau_W \qquad j = 0, 1, 2, ..., N_W$$

$$\tau_{K1} = \tau_{K1,\min} + k\Delta\tau_{K1} \qquad k = 0, 1, 2, ..., N_{K1}$$

In each searching step, the relative deviation of the simulated creep curve from the experimental collocation values is calculated from the formula

$$\delta_1 = \frac{\sum_{i=1}^{m} |\varepsilon_{xx}(t_i) - \varepsilon_{xx}^e(t_i)|}{\sum_{i=1}^{m} |\varepsilon_{xx}^e(t_i)|} \cdot 100\% \qquad (3.6)$$

Moreover, in each searching step the constant ω_{x1} is updated from the collocation condition at $t = t_m$, i.e. (vide Eq. (3.2)$_1$)

$$\varepsilon_{xx}^e(t_m) = [1 + \omega_{x1}\varphi_1(t_m)]\varepsilon_{xx}^o \quad \Rightarrow \quad \omega_{x1} \qquad (3.7)$$

The next stage of the identification process is based on the LLC interval, described by Eq. (3.2)$_2$, now with $\varphi_2(t) \neq 0$. The elastic-sequence-time τ_{K2} is estimated from the condition

$$\sum_{i=1}^{n} |\varepsilon_{xx}(t_{m+i}) - \varepsilon_{xx}^e(t_{m+i})| = \min \qquad (3.8)$$

using a method of systematic searching, for

$$\tau_{K2} = \tau_{K2,\min} + k\Delta\tau_{K2} \qquad k = 0, 1, 2, ..., N_{K2}$$

and calculating the relative deviation in the LLC interval, defined as

$$\delta_2 = \frac{\sum_{i=1}^{n} |\varepsilon_{xx}(t_{m+i}) - \varepsilon_{xx}^e(t_{m+i})|}{\sum_{i=1}^{n} |\varepsilon_{xx}^e(t_{m+i})|} \cdot 100\% \qquad (3.9)$$

In each searching step the constant ω_{x2} is updated from the collocation condition at $t = t_{m+n}$, i.e. (vide Eq. (3.2)$_1$)

$$\varepsilon_{xx}^e(t_{m+n}) = [1 + \omega_{x1}\varphi_1(t_{m+n}) + \omega_{x2}\varphi_2(t_{m+n})]\varepsilon_{xx}^o \quad \Rightarrow \quad \omega_{x2} \qquad (3.10)$$

Finally, one calculates the remaining viscoelastic constants from the formulae based on the condition $\varphi_2(t) \approx 0$ in the SLC interval, i.e. (vide Eq. (3.2)$_2$)

$$\varepsilon_{yy}^e(t_m) = [1 + \omega_{y1}\varphi_1(t_m)]\varepsilon_{yy}^o \quad \Rightarrow \quad \omega_{y1}$$

$$(3.11)$$

$$\varepsilon_{yy}^e(t_{m+n}) = [1 + \omega_{y1}\varphi_1(t_{m+n}) + \omega_{y2}\varphi_2(t_{m+n})]\varepsilon_{yy}^o \quad \Rightarrow \quad \omega_{y2}$$

Taking into account the accuracy of the measurements of stresses and strains (three digits), the following values of the EST increments have been assumed in the computations

$$\Delta\tau_W = 0.01\,\text{h} \qquad \Delta\tau_{K1} = 0.1\,\text{h} \qquad \Delta\tau_{K2} = 10\,\text{h}$$

4. Identification of the material constants for selected resins

The experiments were performed on the DST 5000 creep machine, located at the Laboratory of Strength of Materials, Institute of Material Technology and Applied Mechanics, Military University of Technology, Poland. The directional strain histories were measured using strain gauges RL 120/10, at a constant temperature of 18°C. A computer aided measuring track contained an LC-020-3212 device, allowing registration of the strains with errors smaller than $\Delta\varepsilon = 0.00007$. It gives the relative errors in the longitudinal strains smaller than 3%. The fundamental sampling frequency was determined by the following time steps: $\Delta t = 1$ secound for the first 3 minutes; $\Delta t = 1$ minute for the next 6 hours; $\Delta t = 30$ minuts for next 21 days.

The samples have been made of EPIDIAN 53 epoxy resin and of POLIMAL 109 polyester resin. In order to check the repeatibility of the creep processes, the experiments were performed on 4 epoxy and 4 polyester samples. The samples were shaped as cylinders of 60 mm in length and 23 mm in diameter. The samples were manufactured in accordance with the standard technology and trained with an introductory loading/unloading process.

In practice, an instant loading of the Heaviside type may cause dynamic effects and fracture of the samples. In the experiments, the tensioning force was increasing or decreasing quasi-linearly in the intervals lasting 30 or 15 seconds.

The test samples were tensioned uniaxially (in the x direction), under normal stress equal to $\sigma_{xx} = 0.3R_m$, resulting in $\sigma_{xx} = 15.6\,\text{MPa}$ for EPIDIAN 53 epoxy resin and $\sigma_{xx} = 10.5\,\text{MPa}$ for POLIMAL 109 polyester resin.

All registered discrete creep histories were subject to data preprocessing, containing left- hand cutting, giving the response to Heaviside's loading programme and smoothing, which eliminates random measuring errors. The smoothing of the experimental creep histories was performed with the following simple formula

$$\varepsilon_i := \frac{1}{7}\sum_{j=-3}^{j=3}\varepsilon_{i+j} \qquad i = 4, 11, 18, \ldots \tag{4.1}$$

where $\varepsilon_k = \varepsilon_{xx}^e(t_k)$ or $\varepsilon_k = \varepsilon_{yy}^e(t_k)$. The discrete values of strains on the right side of Eq. (4.1) correspond to equidistant time points in a natural scale. Hence, each smoothed creep history contains a set of points decreased by seven times in comparison with the original creep history. After the data preprocessing, the directional creep strains were renumbered $(i = 1, 2, 3, ...)$.

The identification algorithm developed in Section 3 was programmed in PASCAL, and the following paths were created:

- Data preprocessing of the registered directional strain histories (graphical visualisation of the original creep histories; left-hand cutting; smoothing; graphical visualisation of the smoothed creep histories).

- Elastic and viscoelastic modelling of a resin (calculating E, ν; estimating the viscoelastic constants; graphical visualisatin of viscoelastic Poisson's ratio; graphical visualisation of the simulated creep histories, backgrounded with the experimental values in the collocation points).

A qualitative assessment of the creep of a given resin can be based on viscoelastic Poisson's ratio, defined clasically by the formula

$$\tilde{\nu}(t) = -\frac{\varepsilon_{yy}^e(t)}{\varepsilon_{xx}^e(t)} \tag{4.2}$$

The identification of the material constants was performed for the samples exhibiting the most regular creep histories. The identified values of the material constants for the selected resins are collected in Table 1. At these values the simulated creep histories deviate from the experimental strains in the collocation points by errors $\delta_1 = 0.32\%$, $\delta_2 = 0.82\%$ for the epoxy, and by $\delta_1 = 0.36\%$, $\delta_2 = 1.13\%$ for the polyester.

Table 1. Material constants of elasticity and viscoelasticity for selected resins

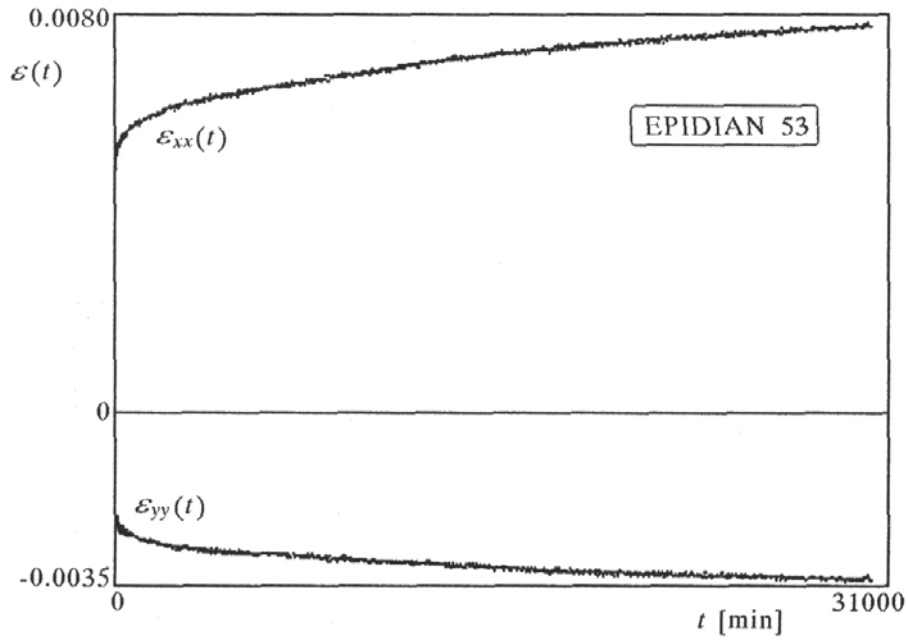| Constant	EPIDIAN 53	POLIMAL 109
E [GPa]	3.14	4.28
ν	0.418	0.363
ω_{x1}	0.309	0.793
ω_{y1}	0.350	0.923
τ_W [h]	2.18	0.84
τ_{K1} [h]	30.5	32.5
ω_{x2}	1.25	1.35
ω_{y2}	1.19	1.87
τ_{K2} [h]	2060	1930

Fig. 3. Original experimental directional creep strain histories for EPIDIAN 53 epoxy. Natural scale
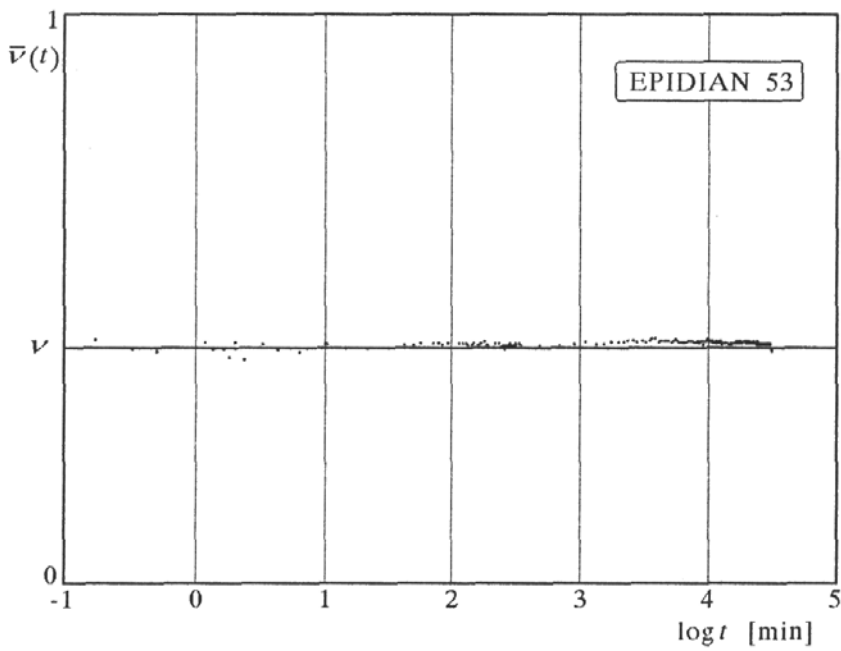

Fig. 4. Original experimental viscoelastic Poisson's ratio, related to creep of EPIDIAN 53 epoxy. Semi-logarithmic scale
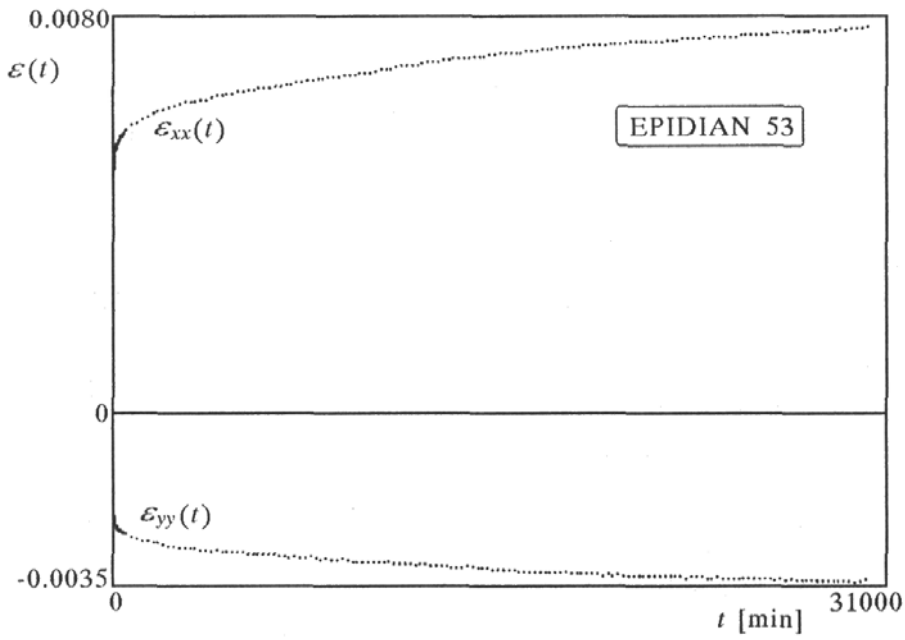
Fig. 5. Smoothed experimental directional creep strain histories for EPIDIAN 53
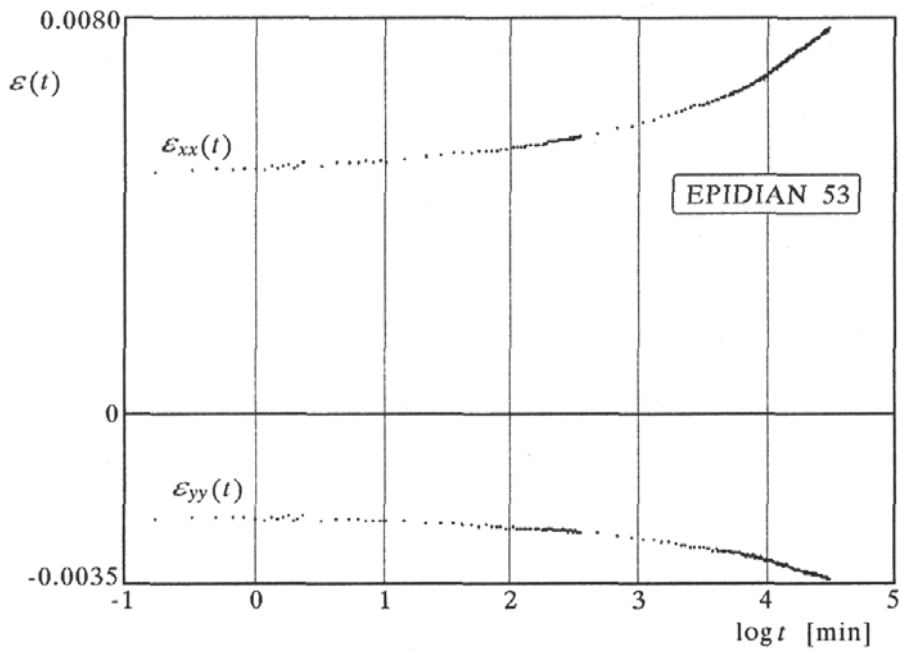epoxy. Natural scale

Fig. 6. Smoothed experimental directional creep strain histories for EPIDIAN 53
epoxy. Semi-logarithmic scale

The results of viscoelastic modelling for the first examined resin (epoxy), are presented in Fig. 3 ÷ Fig. 7. The original experimental creep histories are presented in Fig. 3. These histories are affected by random measuring errors resulting from the apparatus of an older type used in the experiments and from temperature fluctuations. Figure 4 reflects original experimental discrete values of Poisson's ratio $\bar{\nu}(t)$ versus time, in a semi-logarithmic scale. Since $\bar{\nu}(t)$ is quasi-constant, the directional creep histories are quasi-proportional.
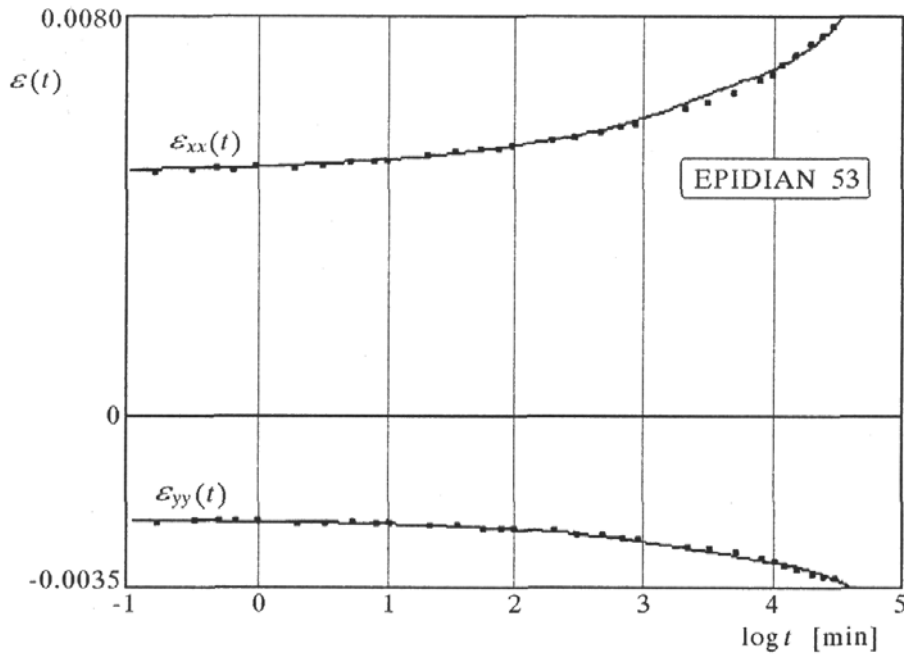
Fig. 7. Simulated directional creep strain histories, backgrounded with the experimental values in the collocation points, for EPIDIAN 53 epoxy

After eliminating random measuring errors, with formula (4.1), one obtains smoothed experimental directional creep histories for the epoxy, in a natural scale, presented in Fig. 5. In order to expose the deviation of this process from the one obtained for the classic HK model (see Fig. 1), Figure 6 presents smoothed experimental creep histories in a semi-logarithmic scale. The simulated directional creep histories for the epoxy, calculated from Eqs (3.2) and backgrounded with a discrete set of the experimental values in the collocation points, are drawn in Fig. 7.

The results of viscoelastic modelling for the second examined resin (polyester), are presented in Fig. 8 ÷ Fig. 12, in a way analogous to that for the epoxy.

Final results for both resins (Fig. 7 and Fig. 12) confirm good agreement of simulated creep histories with the experimental data.
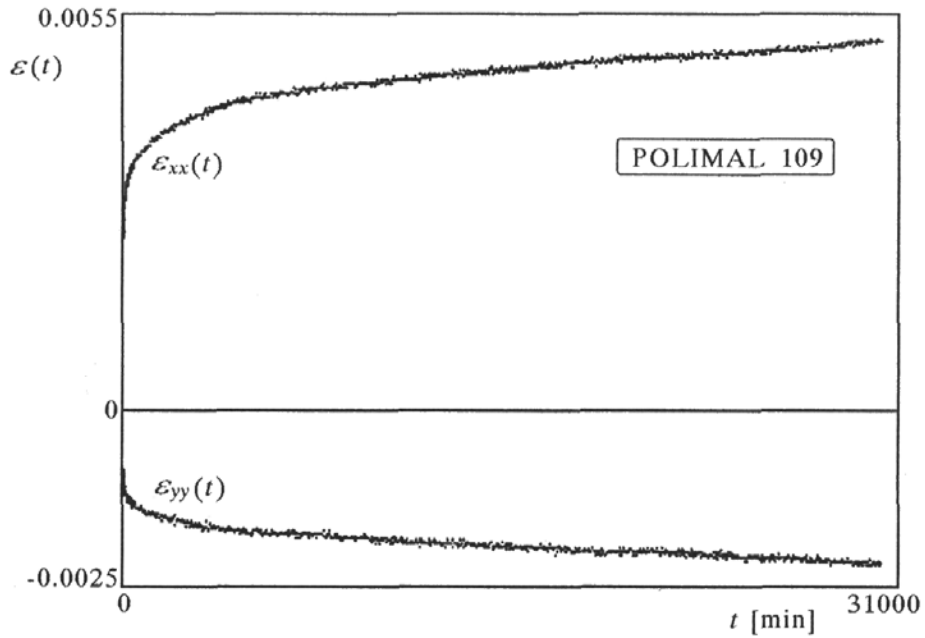
Fig. 8. Original experimental directional creep strain histories for POLIMAL 109
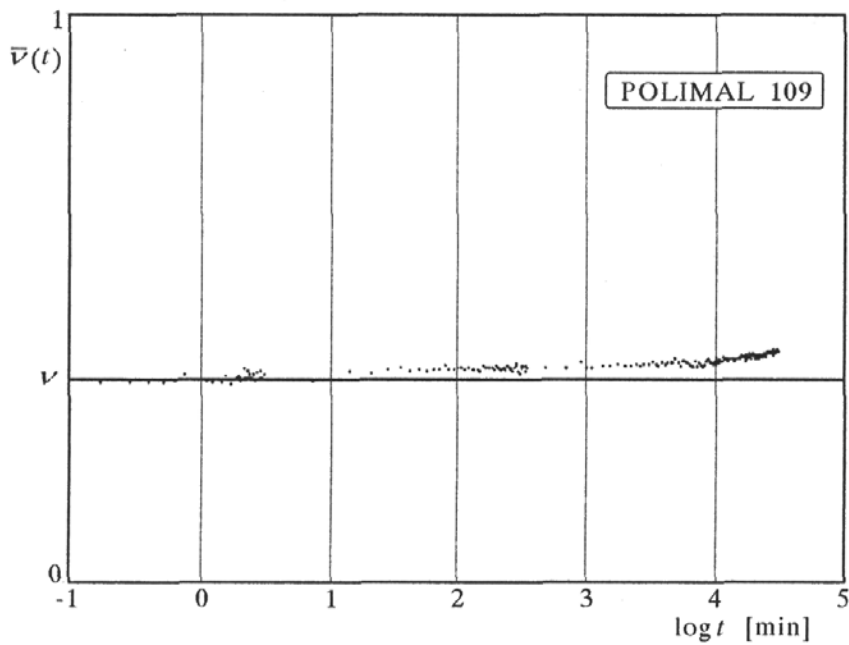polyester. Natural scale

Fig. 9. Original experimental viscoelastic Poisson's ratio, related to creep of
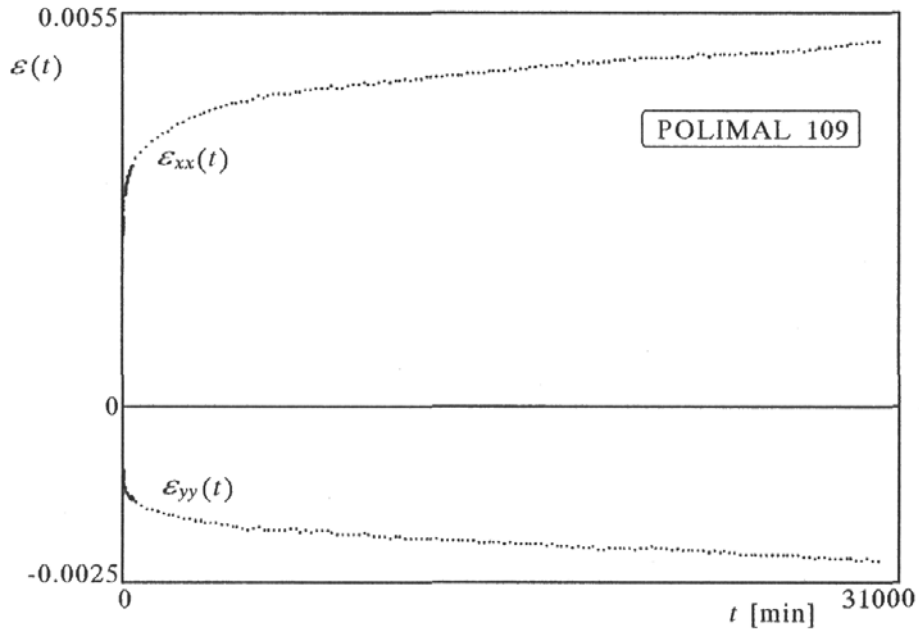POLIMAL 109 polyester. Semi-logarithmic scale

Fig. 10. Smoothed experimental directional creep strain histories for POLIMAL 109 polyester. Natural scale
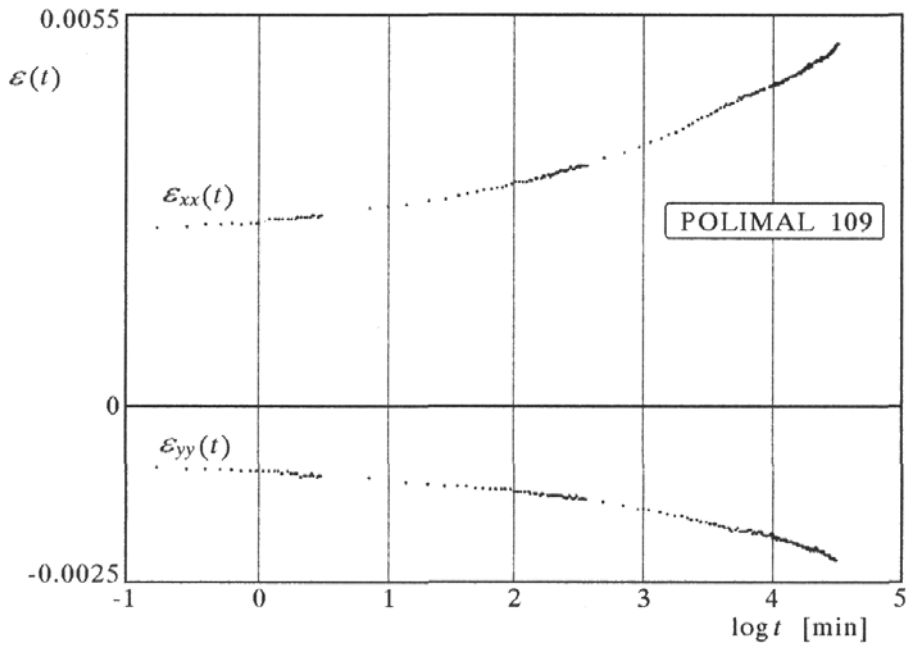

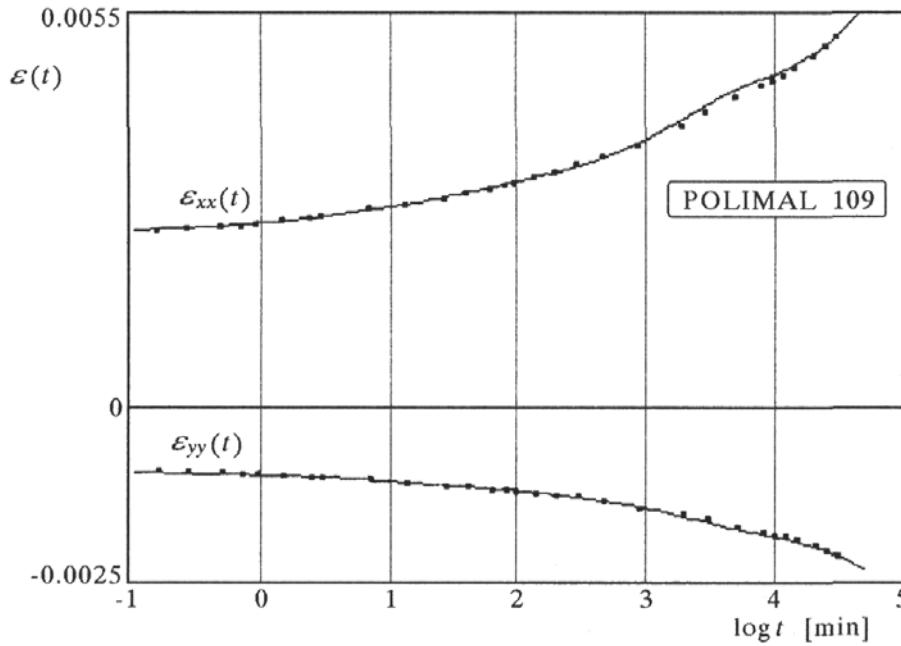Fig. 11. Smoothed experimental directional creep strain histories for POLIMAL 109 polyester. Semi-logarithmic scale

Fig. 12. Simulated directional creep strain histories, backgrounded with the experimental values in the collocation points, for POLIMAL 109 polyester

5. Validation of the HWKK model

The application of the HWKK model into engineering practice must be preceded with validation performed for selected loading programmes. In this study, two pulsating loading programmes have been applied.
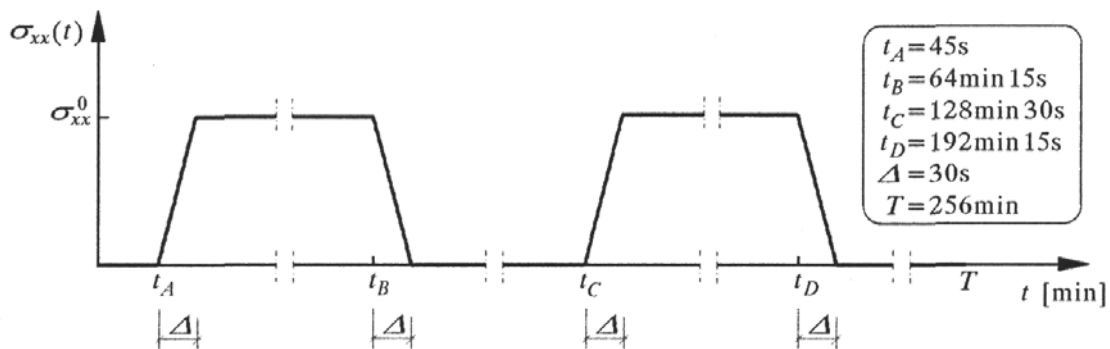

Fig. 13. Loading programme for validation of the HWKK model for EPIDIAN 53 epoxy

Epoxy resin (EPIDIAN 53)

The loading programme is presented in Fig. 13. This programme consists of two trapezoidal loading inervals and two unloading intervals, each lasting

60 minutes. The loading programme is described by the formulae

$$\sigma_{xx}(t) = \begin{cases} 0 & t \in [0, t_A] \cup (t_B + \Delta, t_C] \cup (t_D + \Delta, T] \\[2mm] \sigma_{xx}^o & t \in (t_A + \Delta, t_B] \cup (t_C + \Delta, t_D] \\[2mm] \dfrac{1}{\Delta}\sigma_{xx}^o(t - t_A) & t \in (t_A, t_A + \Delta] \\[2mm] \sigma_{xx}^o\left[1 - \dfrac{1}{\Delta}(t - t_B)\right] & t \in (t_B, t_B + \Delta] \\[2mm] \dfrac{1}{\Delta}\sigma_{xx}^o(t - t_C) & t \in (t_C, t_C + \Delta] \\[2mm] \sigma_{xx}^o\left[1 - \frac{1}{\Delta}(t - t_D)\right] & t \in (t_D, t_D + \Delta] \end{cases}$$

(5.1)

where $\sigma_{xx}^o = 15.6\,\mathrm{MPa}$, $\Delta = 30\,\mathrm{s}$, $T = 256\,\mathrm{min}$.

The directional viscoelastic strains are simulated directly from Eqs (2.1), i.e.

$$\begin{aligned} \varepsilon_{xx}(t) &= S_x(t) \otimes \sigma_{xx}(t) = \\ &= S_x^o[\sigma_{xx}(t) + \omega_{x1}\gamma\lambda(t) + \omega_{x1}(1 - \gamma)\lambda_1(t) + \omega_{x2}\lambda_2(t)] \end{aligned}$$

(5.2)

$$\begin{aligned} \varepsilon_{yy}(t) &= S_y(t) \otimes \sigma_{xx}(t) = \\ &= S_y^o[\sigma_{xx}(t) + \omega_{y1}\gamma\lambda(t) + \omega_{y1}(1 - \gamma)\lambda_1(t) + \omega_{y2}\lambda_2(t)] \end{aligned}$$

where

$$\lambda(t) = \int\limits_0^t \Phi(t - \vartheta)\sigma_{xx}(\vartheta)\,d\vartheta$$

(5.3)

$$\lambda_i(t) = \int\limits_0^t F_i(t - \vartheta)\sigma_{xx}(\vartheta)\,d\vartheta \qquad i = 1, 2$$

Integrals (5.3) were calculated numerically, using the simplest one-point Gauss quadrature. For the first integral one obtains

$$\lambda(t_N) = \int\limits_0^{t_N} \Phi(t_N - \vartheta)\sigma_{xx}(\vartheta)\,d\vartheta = \begin{cases} h\sum\limits_{i=1}^{N} \Phi_{N+1-i}\sigma_{xx,i} & \text{or} \\ h\sum\limits_{i=1}^{N} \Phi_i\sigma_{xx,N+1-i} \end{cases}$$

(5.4)

with $\Phi_i = \Phi(t_i - h/2)$, $\sigma_{xx,i} = \sigma_{xx}(t_i - h/2)$, $t_i = ih$, $h = \Delta\vartheta$.
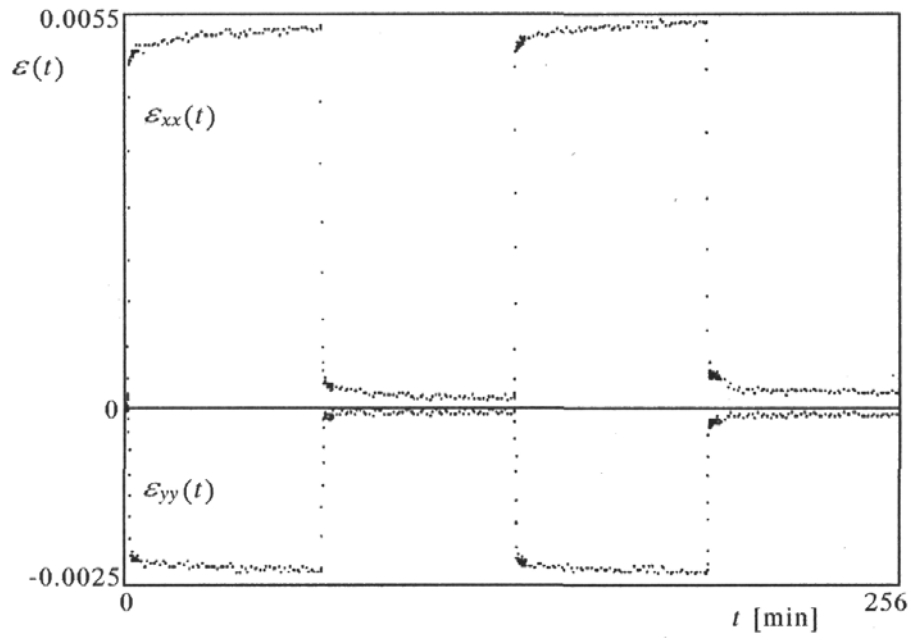
Fig. 14. Original experimental directional strain histories for EPIDIAN 53 epoxy, for the loading programme shown in Fig. 13
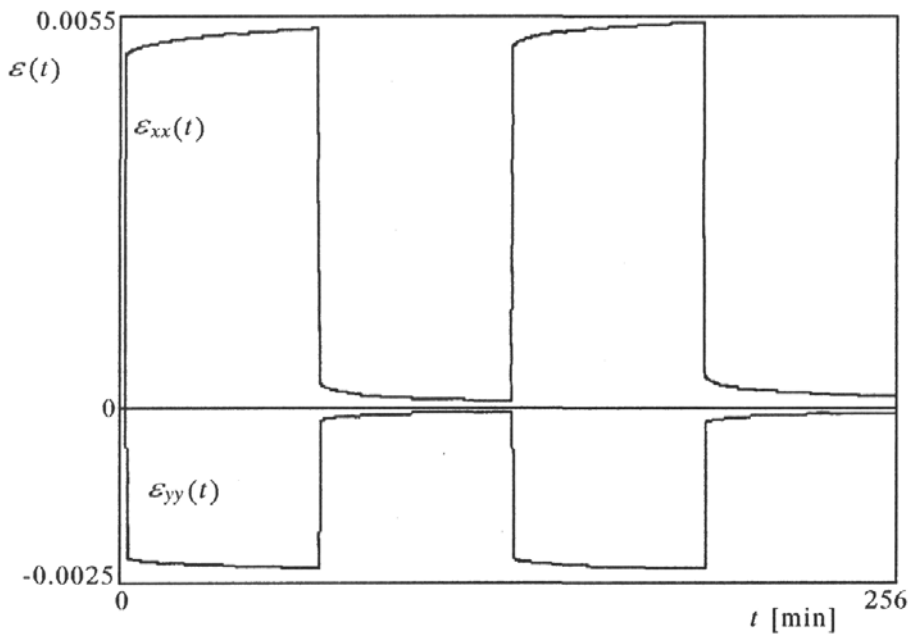

Fig. 15. Simulated directional strain histories for EPIDIAN 53 epoxy, for the loading programme shown in Fig. 13

The experimental original discrete creep histories, induced by the loading programme given in Fig. 13, are plotted in Fig. 14. Measuring errors, resulting from both the measuring apparatus and the temperature fluctuations, are observed. The simulated creep histories, calculated from Eqs (5.2) for the values of material constants taken from Table 1, are presented in Fig. 15. These results, coinciding with the experimental data with good accuracy, positively validate the HWKK model.

Polyester resin (POLIMAL 109)

The loading programme for the polyester specimens is shown in Fig. 16. In this case, the prediction of the viscoelastic directional strains can be done directly from the Boltzmann superposition rule, i.e.

$$\varepsilon_{xx}(t) = [f_x(t) - f_x(t - t_A)H(t - t_A) + f_x(t - t_A - t_B)H(t - t_A - t_B)]\varepsilon_{xx}^o$$
$$(5.5)$$
$$\varepsilon_{yy}(t) = [f_y(t) - f_y(t - t_A)H(t - t_A) + f_y(t - t_A - t_B)H(t - t_A - t_B)]\varepsilon_{yy}^o$$

with

$$f_x(t) = 1 + \omega_{x1}\varphi_1(t) + \omega_{x2}\varphi_2(t)$$
$$(5.6)$$
$$f_y(t) = 1 + \omega_{y1}\varphi_1(t) + \omega_{y2}\varphi_2(t)$$

Three jumps at $t = 0$ min, $t = 30000$ min and $t = 60000$ min were distributed quasi-linearly in the intervals of 15 seconds. This effect has been neglected in the computations, according to Eqs (5.5).
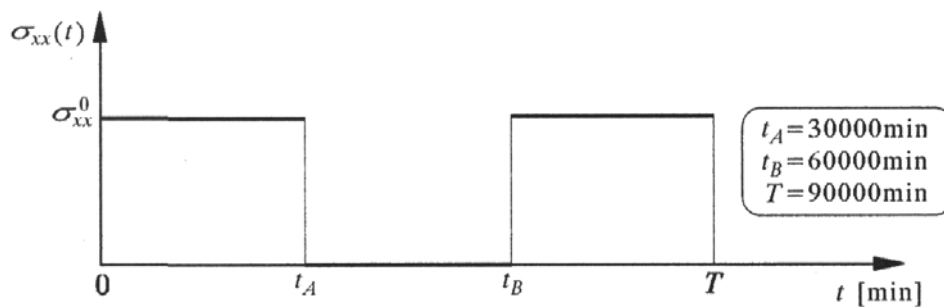

Fig. 16. Loading programme for validation of the HWKK model for POLIMAL 109 polyester

The experimental original discrete creep histories, induced by the loading programme given in Fig. 16, are plotted in Fig. 17. Measuring errors, resulting from both the measuring apparatus and the temperature fluctuations, are
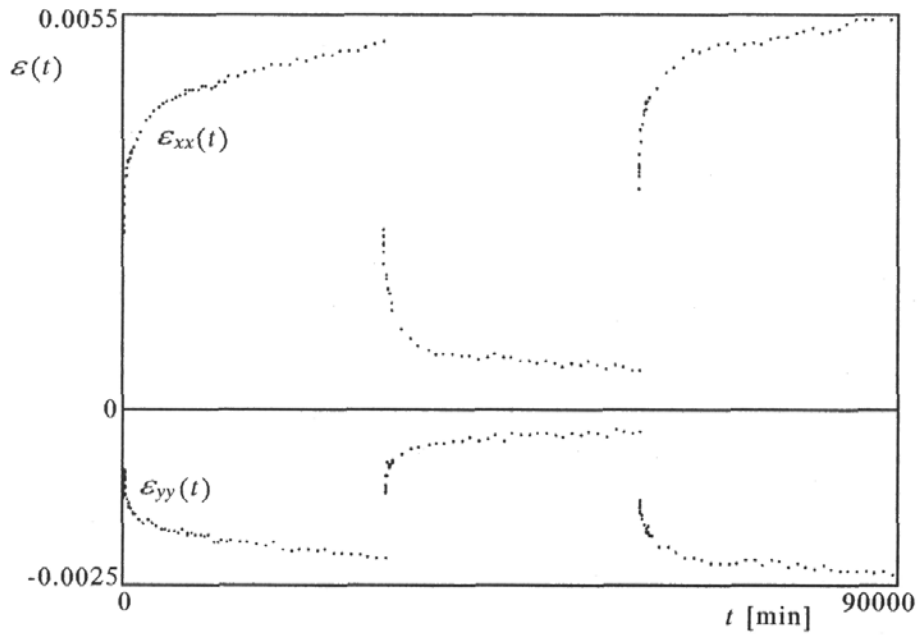
Fig. 17. Original experimental directional strain histories for POLIMAL 109
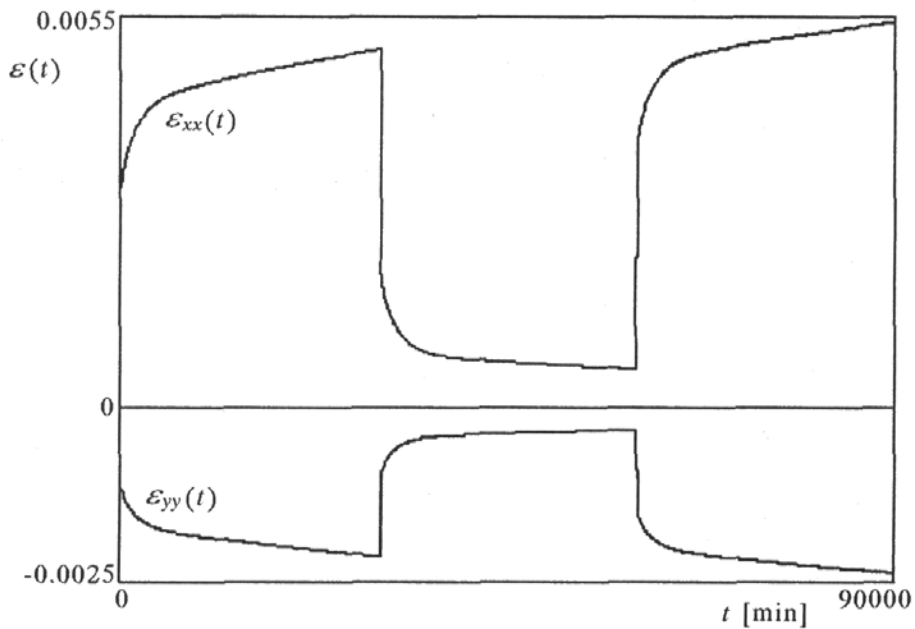polyester, for the loading programme shown in Fig. 16

Fig. 18. Simulated directional strain histories for POLIMAL 109 polyester, for the
loading programme shown in Fig. 16

also observed. The simulated creep histories, calculated from Eqs (5.5) for the values of material constants taken from Table 1, are shown in Fig. 18. These results, coinciding with the experimental data with good accuracy, also positively validate the HWKK model.

6. General conclusions

The HWKK rheological model for resins, epoxy and polyester, describes the first-rank reversible creep with good accuracy, acceptable in engineering practice. In authors' opinion, a number of the material constants (2 elastic and 7 viscoelastic constants) is minimal in the problem undertaken. Each constant has clear physical interpretation. The HWKK model has been positively validated in selected tests.

The creep experiments have been performed on 4 bar samples for each resin. Acceptable repeatability of the directional strain histories has been confirmed. However, the total creep time (500 h) has appeared too short, for both resins, to estimate with high accuracy the viscoelastic constants desribing the LLC interval.

Smoothing formula (4.1) is very effective for nearly total elimination of the measuring errors. The identification algorithm is also very effective and provides synonymous solutions.

The generating functions describing the HWKK model yield fully analytical formulae for the so-called complex compliances of a resin used as a matrix in fibre-reinforced composites. It permits the incorporation of the HWKK model into fully analytical modelling of such composites (Klasztorny and Wilczyński, 2002).

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Model reologiczny HWKK dla żywic

Streszczenie

Opracowano nowy model reologiczny żywicy epoksydowej i poliestrowej, oznaczony symbolem HWKK. Przeprowadzono walidację tego modelu dla wybranych programów obciążenia, uzyskując pozytywne wyniki. Model HWKK opisuje z dobrą dokładnością pełzanie pierwszorzędowe odwracalne polimerów amorficznych termoutwardzalnych. Model ten jest odwzorowany przez cztery elementy mechaniczne połączone szeregowo typu Hooke'a, Wilczyńskiego i Kelvina.

Sformułowana równania konstytutywne lepkosrężystości i modelu HWKK stosując jako funkcje tworzące jedną funkcję wykładniczą ułamkową oraz dwie funkcje wykładnicze zwykłe. Opracowano algorytm identyfikacji stałych materiałowych opisujących model HWKK, bazujący na eksperymentach pełzania próbek prętowych rozciąganych osiowo. Stałe te zidentyfikowano dla żywicy epoksydowej i poliestrowej.