JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
1, **39**, 2001

IDENTIFICATION OF MODAL MODELS OF HELICOPTERS USING IN-FLIGHT MEASUREMENTS

TADEUSZ UHL
WOJCIECH LISOWSKI

Department of Robotics and Machine Dynamics, University of Mining and Metallurgy, Kraków
e-mail: tuhl@rob.wibro.agh.edu.pl

The paper presents a new methodology of modal model identification based on in-operation system response measurements. This methodology consists in estimation of cross- and auto-correlation of acceleration acquired at all measuring points. The method was applied to helicopter modal model identification basing on in-flight measurement results. The obtained model parameters have been compared with the ground modal test results. This comparison permits evaluating of level of each mode excitation during the operation and can be applied to structure modification and diagnostics.

Key words: in-flight testing, modal analysis, helicopter structure modelling

1. Introduction

Modern design process of helicopter structures requires a very careful analysis of their dynamic properties. Two main problems should be considered: vibrations and aeroelastic stability (Bielawa, 1992). The emphasis is currently on reduction of vibration, but with close attention being paid to the fact that rotors are subject to a variety of resonance phenomena. In any practical design these phenomena must be properly understood and ways must be formulated to suppress them for all possible flight conditions (Bielawa, 1992). It should be noted that one area of concern that closely relates to structural dynamics and aeroelasticity is noise. The high noise level characteristic of most rotorcrafts has spawned the growing technical areas of rotor acoustics (far-field noise) and acoustoelasticity (structure born interior noise). Vibration problems of a helicopter are dependent on a flight condition. Vibration is a major problem with rotorcraft during a level flight. This vibration is not broadband rather, it occurs mainly at discrete frequencies related to the main and tail rotor rotational speeds. The main source of excitation of the rotorcraft vibration is aerodynamics of the main rotor even during a steady forward flight. A relatively high flexibility of the rotor and helicopter airframe causes significant interaction occurring between the rotor and the rotorcraft airframe. Due to this interaction an accurate airframe dynamic model is strongly required for design and structural modification purposes.

A modal model is the most frequently used model in structural dynamic analysis of helicopter structures. The modal model is understood in this study as a set of: natural vibration frequencies, damping ratios and natural vibration modes. These parameters allow one to evaluate dynamic properties of the structure and to predict its behaviour when subjected to excitations. The modal models are applicable to structures where linear models are adequate, the condition of reciprocity is met and the damping is small or proportional (Uhl, 1997).

The structural dynamic problem of a rotorcraft entails three general topics:

- measurement of dynamic characteristics of the system
- determination of vibration excitation
- formulation of rotorcraft structural dynamic model and estimation of its parameters.

These topics were intensively studied within EUREKA "SINOPSYS" project (see Uhl et al., 1997, 1998). This investigation focused on the choice of the optimal conditions for a structural dynamic ground-test of a helicopter airframe in order to obtain results as close as possible to the ones corresponding to flight conditions (Lisowski and Uhl, 1998). An in-flight vibration excitation is not a broadband process but a polyharmonic one. This is caused by rotation of the main and tail rotor at an approximately constant rotational speed. The aerodynamic excitation exists in real helicopter structures but its amplitude is relatively low (Lisowski and Uhl, 1998). The amplitude, spatial distribution and spectral characteristics used during the ground test should provide the tested airframe vibration response corresponding to the one present during the flight. In our case the excitation during the ground test was chosen so that the obtained response of the system at a selected point was of the same amplitude as during the flight. Such a procedure permits us to minimise errors originating in structural nonlinearities (Uhl, 1997).

In the paper a method for modal model identification of mechanical structures using measurements of a structure response to operating loads is presented. In modal model identification the measurements involve multiple channel measurements of vibrations acceleration during a helicopter flight. During the ground test it is relatively difficult to preserve boundary and loads conditions that can be observed during the flight. The method presented here allows one to simplify the identification procedure and provides the required accuracy of the parameter estimation. In case of particularly critical structures with high safety operation requirements and those that have to meet strict reliability requirements, the method may be implemented to diagnose current condition of the structure (Uhl, 1997). The theoretical bases and the computer software were developed as the part of the research project EUREKA (Uhl et al., 1997), in which the authors participated.

2. Identification of structural dynamics model based on in-operation measurements

Two structural dynamics models of mechanical structures will be considered:

- modal model of a structure
- operational model of the structure (ODS – Operational Deflection Shape).

These two models can be identified using in-operation measurements. The required measurements are the same in the considered methodology but data processing procedures required for model parameter estimation are different.

2.1. Modal model identification method based on in-operation excitations

In the classical modal analysis the identified modal parameters are determined using measurements of frequency characteristics on the tested structure. These characteristics are obtained in an active identification test, involving a controlled excitation of vibrations and measurements of the system response in the form of a vibrations acceleration spectrum. Knowing the response and the excitation spectra, we are able to identify the frequency characteristics of the structure. That procedure belongs to the frequency-domain methods (Brown et al., 1979; Uhl, 1997). These methods allow one to find the modal parameters in the neighbourhood of a single natural frequency (SDOF methods) or throughout the selected frequency band covering more than one natural frequency (MDOF methods). The second group of the methods is the time-domain methods group, which require multiple channel measurements of time characteristics of the response and excitations signals. The first step in the procedure of the time domain methods is finding the system impulse response. Once it is known, the modal parameters can be estimated using different estimation techniques (Uhl, 1997).

A slightly different approach is required when the system is subjected to immeasurable excitations due to processes taking place during machine operation (see Brown et al., 1979; Hermans, 1996, 1997a,b). An obvious advantage of those identification methods is that the excitation conditions, boundary conditions and distributions of operating loads are maintained. It is difficult, or sometimes even impossible, to meet these requirements during active tests in laboratory conditions. Identification methods using in-operation measurements can be divided into three basic categories:

- methods using auto-correlation and cross-correlation of signals (Brown et al., 1979; Hermans and Auweraer, 1997a)
- methods using the autoregression function for the response signals (Hermans et al., 1996)
- methods realised in the stochastic sub-space (Hermans et al., 1997b).

In the modal model identification of a helicopter based on an in-flight test the method involving the auto-correlation function of the response signals and cross-correlation of the response and reference signals was used as the first step. It can be proved that the correlation function can be expressed by means of damped harmonic functions for MIMO systems subjected to random excitations. To determine the modal parameters LSCE (Least Square Complex Exponential) method was applied (Brown et al., 1979; Uhl, 1997). The correlation function is approximated with the sum of decaying exponential harmonic functions. This method applied to measured impulse responses is a well-known technique in the classical experimental modal analysis, and it yields the global poles estimators. It can be proved that the cross-correlation function may be used in the modal parameter identification in the same way as the impulse response.

Accordingly, the following equation of motion of the system was considered

$$\mathsf{M}\ddot{\boldsymbol{x}} + \mathsf{C}\dot{\boldsymbol{x}} + \mathsf{K}\boldsymbol{x} = \boldsymbol{f}(t) \tag{2.1}$$

where

$\mathsf{M}, \mathsf{C}, \mathsf{K}$	–	mass, damping and stiffness matrixes
$\ddot{\boldsymbol{x}}, \dot{\boldsymbol{x}}, \boldsymbol{x}$	–	acceleration, velocity and displacement vectors
$\boldsymbol{f}$	–	vector of the excitation force.

Eq (2.1) can be transformed to the modal co-ordinates, using a transform given by the formula

$$\boldsymbol{x}(t) = \boldsymbol{\Psi}\boldsymbol{q}(t) = \sum_{r=1}^{n} \boldsymbol{\Psi}_r q_r(t) \tag{2.2}$$

where

$\boldsymbol{\Psi}$ – modal matrix with columns expressing modes of natural vibrations corresponding to a given natural frequency

q_r – denotes the modal co-ordinate.

Assuming that damping is low or proportional, after substituting (2.2) into (2.1) and multiplying by $\boldsymbol{\Psi}^\top$ we get an uncoupled system of equations of the form

$$\ddot{q}_r(t) + 2\xi_r\omega_{nr}\dot{q}_r(t) + \omega_{nr}^2 q_r(t) = \frac{1}{m_r}\boldsymbol{\Psi}_r^\top \boldsymbol{f}(t) \tag{2.3}$$

where

ω_{nr} – frequency of the natural vibrations

ξ – modal damping ratio the for rth mode of vibration

m_r – modal mass.

Assuming that the initial conditions are zero for any excitations, the solution to (2.3) may be written in the form of the following convolution

$$q_r(t) = \int_{-\infty}^{t} \boldsymbol{\Psi}_r^\top \boldsymbol{f}(\tau) g_r(t-\tau)\, d\tau \tag{2.4}$$

where

$$g_r(t) = \begin{cases} 0 & \text{for} \quad t < 0 \\ \dfrac{1}{m_r\omega_{rd}} \exp(-\xi_r\omega_{nr}t)\sin(\omega_{nr}t) & \text{for} \quad t \leqslant 0 \end{cases}$$

and $\omega_{rd} = \omega_{nr}\sqrt{1-\xi_r^2}$ is the frequency of damped natural vibrations.

Applying solution (2.4), that is valid for the modal co-ordinates, to find the solution for the generalised co-ordinates $x(t)$, we get

$$\boldsymbol{x}(t) = \sum_{r=1}^{n} \boldsymbol{\Psi}_r \int_{-\infty}^{t} \boldsymbol{\Psi}_r^\top \boldsymbol{f}(\tau) g_r(t-\tau)\, d\tau \tag{2.5}$$

where n is the number of vibration modes considered here.

For a single output and single excitation signal at the point k Eq (2.1) assumes the form

$$x_{ik}(t) = \sum_{r=1}^{n} \Psi_{ir}\Psi_{kr} \int_{-\infty}^{t} f_k(\tau) g_r(t-\tau)\, d\tau \tag{2.6}$$

where Ψ_{ir} is the ith component of the rth vibrations mode.

The impulse response induced by Dirac's impulse at the point k measured as the response at the point i has the form

$$x_{ik}(t) = \sum_{r=1}^{n} \frac{\Psi_{ir}\Psi_{kr}}{m_r\omega_{dr}} \exp(-\xi_r\omega_{nr}t)\sin(\omega_{rd}t) \tag{2.7}$$

The cross-correlation function determined for two response signals at the points i and j induced by white noise excitations at the point k has the form

$$R_{ijk}(T) = E[x_{ik}(t+T)x_{jk}(t)] \tag{2.8}$$

where E is the expected value operator.

Substituting solution (2.6) into definition of the auto-correlation function (2.8) and assuming that the excitation comes in the form of white noise, for which the correlation function is the constant α_k multiplied by the Dirac's delta $\delta(t)$, we get

$$R_{ijk}(T) = \sum_{r=1}^{n}\sum_{s=1}^{n} \alpha_k\Psi_{ir}\Psi_{kr}\Psi_{js}\Psi_{ks} \int_{0}^{\infty} g_r(\lambda+T)g_s(\lambda)\, d\lambda \tag{2.9}$$

where $\lambda = t-\tau$; the integration limits being changed because of the form of the function g and the system causality.

Applying the definition of g given by (2.4) and distinguishing between the terms dependent on T and λ, we get

$$\begin{aligned} g_r(\lambda+T) &= [\exp(\xi_r\omega_{rn}T)\cos(\omega_{rd}T)]\frac{\exp(\xi_r\omega_{rn}\lambda)\sin(\omega_{rd}\lambda)}{m_r\omega_{rd}} + \\ &+ [\exp(\xi_r\omega_{rn}T)\sin(\omega_{rd}T)]\frac{\exp(\xi_r\omega_{rn}\lambda)\cos(\omega_{rd}\lambda)}{m_r\omega_{rd}} \end{aligned} \tag{2.10}$$

Substituting (2.10) and the cross-correlation function formula for $g_s(\lambda)$, analogous to (2.9), we get

$$R_{ijk}(T) = \sum_{r=1}^{n}\Big[A_{ijkr}\exp(-\xi_r\omega_{nr}T)\cos(\omega_{rd}T) + B_{ijkr}\exp(-\xi_r\omega_{nr}T)\sin(\omega_{rd}T)\Big] \tag{2.11}$$

where A_{ijkr}, B_{ijkr} are independent of T and are the functions of the modal parameters

$$\begin{bmatrix} A_{ijkr} \\ B_{ijkr} \end{bmatrix} = \sum_{s=1}^{n} \frac{\alpha_k \Psi_{ir} \Psi_{kr} \Psi_{js} \Psi_{ks}}{m_r \omega_{rd} m_s \omega_{sd}} \int\limits_0^\infty \mathrm{e}^{-\xi_r \omega_{nr} - \xi_s \omega_{ns}} \lambda \sin(\omega_{sd}\lambda) \begin{bmatrix} \sin(\omega_{sd}\lambda) \\ \cos(\omega_{sd}\lambda) \end{bmatrix} \, d\lambda \tag{2.12}$$

Eq (2.11) represents the relationship between the auto-correlation function – as the sum of decaying exponential harmonic functions, and the impulse response function applied in the classical modal analysis to identification of modal parameters. To make direct use of the thus written correlation function, formula (2.11) can be further transformed and written as

$$R_{ij}(T) = \sum_{r=1}^{n} \frac{\Psi_{ir} G_{jr}}{m_r \omega_{rd}} \exp(-\xi_r \omega_{nr} T) \sin(\omega_{rd} T + \theta_r) \tag{2.13}$$

where the new phase shift angle θ_r and the constant G_{jr} are derived from the formula

$$\tan\theta_r = \frac{I_{rs}}{J_{rs}} \qquad\qquad G_{jr} = \frac{\Psi_{ir}}{m_r \omega_{rd}} \sum_{s=1}^{n} \sum_{k=1}^{n} \frac{\beta_{jkrs}}{I_{rs}^2 + J_{rs}^2} \tag{2.14}$$

and

$$\begin{aligned} I_{rs} &= 2\omega_{rd}(\xi_r \omega_{nr} + \xi_s \omega_{ns}) \\ J_{rs} &= \omega_{sd}^2 - \omega_{rd}^2 + (\xi_r \omega_{nr} + \xi_s \omega_{ns})^2 \\ \beta_{jkrs} & \frac{\alpha_k}{m_s} \Psi_{kr} \Psi_{js} \Psi_{ks} \end{aligned}$$

The LSCE is one of the time-domain methods used in modal analysis. The method provides a global estimate of modal parameters in the form of natural frequencies and damping ratios. This method was published first in work by Lisowski and Uhl (1998). It is a modification to the earlier CE (Complex Exponential) method. The basis for determination of the modal models is the measured variability of the transfer function. In the identification methods using the measurements of a system response to unknown excitations the transfer function is replaced with the cross correlation function. The cross-correlation function comes as a sum of decaying exponential harmonic functions (2.13). To present the estimation methods better, these functions were rewritten as

$$h_{jk}(t) = \sum_{r=1}^{2N} A_{rjk} \mathrm{e}^{s_r t} \tag{2.15}$$

where $s_r = -\omega_{rn}\xi_r + \mathrm{i}\omega_{rd}$.

By sampling the action with the constant sampling time Δt, the function $h(t)$ can be transformed into the sample series $h_0, h_1, h_2, ..., h_L$. The value of each sample can be expressed by the formula

$$h_0 = \sum_{r=1}^{2N} A_r \qquad h_1 = \sum_{r=1}^{2N} A_r V_r \qquad \dots \qquad h_L = \sum_{r=1}^{2N} A_r V_r^L \tag{2.16}$$

where A_r, V_r are the desired quantities $V_r = \mathrm{e}^{s_r \Delta t}$.

These values can be found using Prony's method (Uhl, 1997). According to that method there always exists a polynomial in V_r with real coefficients β that the following relation is satisfied

$$\beta_0 + \beta_1 V_r + \beta_2 V_r^2 + ... + \beta_L V_r^L = 0 \tag{2.17}$$

To determine the coefficients β it is necessary to solve the equation

$$\begin{bmatrix} h_0 & h_1 & \cdots & h_{2N-1} \\ h_1 & h_2 & \cdot & h_{2N} \\ \vdots & \vdots & \cdots & \vdots \\ h_{2N-1} & h_{2N} & \cdots & h_{4N-2} \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_{2N-1} \end{bmatrix} = - \begin{bmatrix} h_{2N} \\ h_{2N+1} \\ \vdots \\ h_{4N-1} \end{bmatrix} \tag{2.18}$$

The coefficients obtained from (2.18) enable one to find the roots V_r of the polynomial. Using these values V_r and the corresponding complex conjugates, we can obtain the natural frequencies and the damping factors. Having the values V_r known, we can determine the coefficients A_r, and consequently, the modal constants and the phase angles, using (2.13). The coefficients A_r can be determined when the following equation is solved

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ V_1 & V_2 & \cdots & V_{2N} \\ V_1^2 & V_2^2 & \cdots & V_{2N}^2 \\ \vdots & \vdots & \cdots & \vdots \\ V_1^{2N-1} & V_2^{2N-1} & \cdots & v_{2N}^{2N-1} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_{2N} \end{bmatrix} = - \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ \vdots \\ h_{2N} \end{bmatrix} \tag{2.19}$$

Solving Eq (2.19) for A_r, we can find the modes of natural vibrations. As it can be easily seen, these relations are derived for SISO systems; which means that only one transfer impulse function or one cross correlation function, in the case of response measurements methods, should be analysed. The LSCE is an extension of the CE method for SIMO; it enables a simultaneous analysis of all measured cross-correlation functions, thus allowing us to determine global estimators of modal parameters of the tested structure. The relevant relationships will have the form

$$\begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \vdots \\ \mathbf{h}_p \end{bmatrix} \boldsymbol{\beta} = \begin{bmatrix} \boldsymbol{h}_{G_1} \\ \boldsymbol{h}_{G_2} \\ \vdots \\ \boldsymbol{h}_{G_p} \end{bmatrix} \tag{2.20}$$

or

$$\mathbf{h}_{(2Np\times 2N)}\boldsymbol{\beta}_{(2N\times 1)} = \boldsymbol{h}_{G_{(2Np\times 1)}}$$

A solution to (2.20) for β can be found using the pseudo-inverse

$$\boldsymbol{\beta} = (\mathbf{h}^\top \mathbf{h})^{-1}\mathbf{h}^\top \boldsymbol{h}_G \tag{2.21}$$

The further procedures for finding the modal parameters are the same as in the CE method.

This algorithm for the modal parameters identification was implemented in the CADA-X system, as a part of the research project EUREKA 'SINOPSYS", in which the authors participated (Uhl et al., 1997). In the presented study the algorithm was applied to the identification of modal parameters of the helicopter SW-3 manufactured by WSK PZL Świdnik, Poland.

2.2. Measurements of the operating deflection shape of the structure

In practice of testing of engineering structures it is often enough to measure an operating deflection shape of a structure; the method is called ODS (Operating Deflection Shape) or Running Mode analysis. The $\boldsymbol{ODS}$ is defined as the structure deflection for the selected vibration frequency $\boldsymbol{X}_{ODS}(j\omega)$ or at the given time instant $\boldsymbol{x}_{ODS}(t)$ due to the external excitation force; while the motion of at least two points must be analysed. This way the structure deflection during its forced motion, understood as the relative motion of one reference point, may be determined. Since motion is a vector quantity (acceleration, velocity or deflection vectors), it has a point of origin, orientation and value, which determine the deflection shape during the structure motion. The $\boldsymbol{ODS}$ in time domain may be determined basing on various types of time responses to random, impulse or harmonic excitations. Another techniques are applied to determine the frequency-domain $\boldsymbol{ODS}$ (McHargue and Richardson, 1993). These involve mainly measurements of the response spectrum, power spectral density and frequency characteristics or the transfer frequency characteristics for any reference point, specially defined to determine the $\boldsymbol{ODS}$. The $\boldsymbol{ODS}$ differs from the modal parameters (modal vectors $\boldsymbol{\Psi}_r$) discussed in earlier paragraphs in that it depends on the type of excitation signals. The ***ODS*** will change with a change of the structure loading. On the other hand the modal vector is independent of the excitation mode. It characterises dynamic properties of the tested structure; including boundary conditions, its geometry and materials. The modes of vibrations (modal vectors) are dimensionless while the ***ODS*** is expressed in units of deflection, velocity or acceleration; depending on which units were adopted in the course of measurements. The ***ODS*** may be determined analytically or experimentally. The first method involves solving Eq (2.1) for an accepted time characteristic of the excitations. That yields the time characteristics of the response signal in the form of the vector $\boldsymbol{x}(t)$. Calculating the value of $\boldsymbol{x}(t_0)$ for any time instant and for all the co-ordinates of the vector $\boldsymbol{x}$ we get $\boldsymbol{x}_{ODS}(t)$. The ***ODS*** can also be determined experimentally, by simultaneous measurements of vibration parameters at several points. The vector obtained by selecting the amplitude for the given time instant is the time-domain ***ODS***. A similar procedure is applied to determine the ***ODS*** in a frequency domain. The system dynamics in the frequency domain can be described with the formula

$$\boldsymbol{X}(j\omega) = \mathsf{H}(j\omega)\boldsymbol{F}(j\omega) \tag{2.22}$$

where

$\boldsymbol{X}(j\omega)$	–	vector of the system response spectra
$\boldsymbol{F}(j\omega)$	–	vector of the excitation force spectra
$\mathsf{H}(j\omega)$	–	frequency characteristics matrix.

In case of linear systems, Eq (2.22) is satisfied for all frequencies throughout the considered range. The ***ODS*** in the frequency domain is defined as the system response $\boldsymbol{F}(j\omega)$ to the excitations for any frequency ω_0

$$\boldsymbol{X}_{ODS}(j\omega_0) = \mathsf{H}(j\omega_0)\boldsymbol{F}(j\omega_0) \tag{2.23}$$

It follows from (2.23) that the ***ODS*** depends on the type of excitation forces. An $\boldsymbol{x}_{ODS}(t)$ can be also determined by applying the inverse Fourier transform to (2.23)

$$\boldsymbol{x}_{ODS}(t) = \frac{1}{T} FFT^{-1}[\mathsf{H}(j\omega)\boldsymbol{F}(j\omega)] \tag{2.24}$$

wkere FFT – fast Fourier transform. In this way the ***ODS*** can be determined for those time instants for which the value of the inverse Fourier transform is calculated. The ***ODS*** in the frequency domain is determined experimentally using multiple channel measurements of response spectra and cross spectra between the measurements and reference points. Because of high costs of the measurements, the number of channels for simultaneous response measurements is limited. That is why some reference points on the structure are chosen, which do not change their position during the measurements. The other measurement points can be moved during the tests. Such a procedure is necessary as it is essential to know the phase shift angle between the system responses at the points for which the ***ODS*** is determined. When forced vibrations of systems are dominated by natural vibrations, the ***ODS*** vector will be similar to the modal vector. The degree of similarity depends on how strongly natural vibrations dominate the measured responses.

3. Case study – helicopter SW-3 in-flight modal analysis

Helicopter rotors and aerodynamic forces that arise during flight cause vibrations of its airframe. On the one hand, such vibrations disturb pilots' work and worsen passengers' comfort, on the other hand, they might cause some airframe component fatigue failure. That is why some optimisation of a helicopter design aiming at its vibration level minimisation is usually being performed. For this purpose numerical models formulated with the use of the Finite Element (FE) method are applied. Credibility of such models as well as their simulation results are assured by an updating procedure that uses experimental structural dynamics models as a reference. For identification of such models Experimental Modal Analysis (EMA) technique is used. Usually the ground modal testing is performed. During such testing neither the actual flight boundary conditions nor the rotor dynamics influence nor the rotor-fuselage coupling structural dynamics effects might be investigated. That is why more and more often the in-flight structural dynamics testing is being performed nowadays. Applied testing methodology allows one to identify this natural vibration modes that are exited during the flight.

3.1. Experiment description

A survey of experimental techniques used for the structural dynamics testing of helicopter airframes for both the ground an in-flight conditions and references can be found in the work by Lisowski and Uhl (1998). The authors conducted a series of SW-3 helicopter airframe ground modal testing for different helicopter configurations and boundary conditions. An overview of the performed tests was reported by Uhl et al. (1998). Part of the obtained results was used as a reference in the reported research. For both the ground and the in-flight tests the same set of the response acceleration measurement points was used. Location of these points on the helicopter airframe is showed in Fig.1.

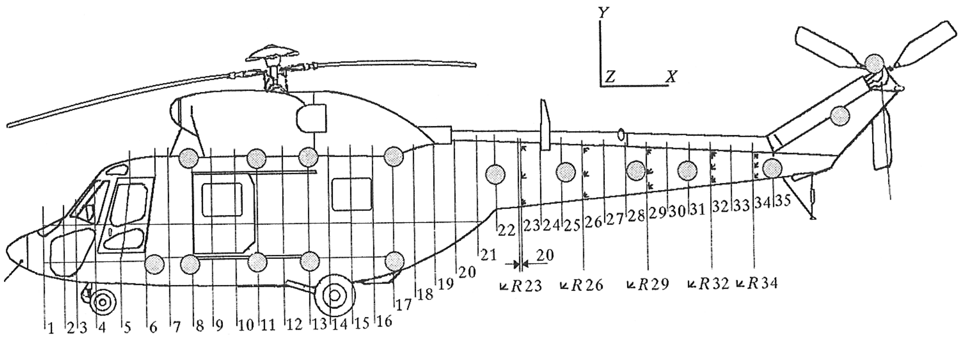

Fig. 1. Location of the measurement points

The number of the selected measurement points was limited by the flight test cost. At each point a measurement in the 3 mutually perpendicular directions was performed.

3.2. Ground testing description

Four modal experiments were performed for the ground conditions. General parameters of these experiments are summarised in Table 1.

Table 1. Description of ground modal testing

Test denotation	Boundary conditions	No. of applied exciters	Type of excitation
UFR	supported on undercarriage	1	burst random
UNR	supported on undercarriage	1	burst random
SNR	suspension	2	burst random
SNS	suspension	2	stepped sine

The following tested helicopter configuration was assumed:

- the amount of fuel as average during the in-flight testing (test UFR) or no fuel in the remaining tests
- engine and transmission shaft covers closed
- hydraulic equipment covers removed
- main rotor blades and hub disassembled

- tail rotor blades disassembled, substitute masses attached to the hub
- passenger seats disassembled and upholstery partly removed from the passenger cabin
- measurement set-up planned to be used during the in-flight testing assembled in the passenger cabin
- ballast mass distributed on pilots' seats and in the passenger cabin with aim of providing the proper location of helicopter gravity centre and the airframe balancing.

During the ground testing the LMS CADA-X software run on HP 9000 series workstation and HP 3565 multichannel dynamic signal analyser were used. The result of the testing was a database composed of Frequency Response Functions (FRF) between the applied excitation forces and the system response accelerations.

3.3. Description of in-flight testing

A sequence of flights was performed during which operational vibration response accelerations to operational excitations and flight parameters were measured.

The in-flight testing was performed for the following 10 flight conditions:

- level flight $IAS = 60$, 130 and 220 km/h
- hover IGE 3-5 m
- left and right banked turn 30°, $IAS = 220$ km/h
- left and right sideslip 30°, $IAS = 220$ km/h
- climbing with take-off power, $IAS = 130$ km/h
- steady descent, $IAS = 160$ km/h, $V_y = -6$ m/s.

For the measurement the ESAM system was applied. As the number of available measurement channels was lower than the number of the measurement signals, vibration testing in each of the flight conditions was carried out in 3 set-ups. As a result of the in-flight testing a database containing time histories of the measured acceleration signals was created.

3.4. Ground testing results

For analysis of the results of the ground modal testing the LMS CADA-X Modal Analysis software was used. Identification of the system poles was performed by making use of the polyreference Least Squares Complex Exponential (LSCE) method applied to the system impulse response function obtained from the measured FRFs via the inverse Fourier transform. For controlled and measured stationary excitations the system impulse response function can be decomposed into a sum of decaying sinusoids. Each of the extracted harmonic component has frequency and damping values corresponding to the damped natural frequency and the modal damping the tested system (Brown et al., 1979). The mode shapes were estimated by making use of a least squares curve-fitting algorithm of the measured FRFs in the frequency domain.

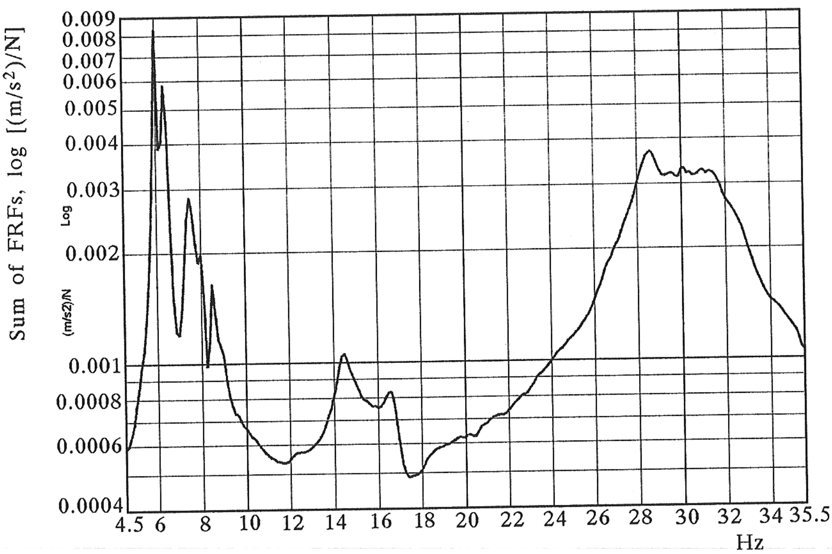

Fig. 2. An example of the measured FRFs amplitude sum for the ground modal testing

Selected results of the ground testing of the PZL-Sokół helicopter airframe were reported by Uhl et al. (1997, 1998). Generally, for 4 different tests conditions 4 different modal models of some common features were obtained. Each identified modal model comprises specific 3 groups of modes corresponding to 3 frequency ranges that can be distinguished in a measured FRFs amplitude sum – see an example presented in Fig.2. The first group of modes, which corresponds to the low frequency value sub-range, consists of the bending with torsion of the whole helicopter body modes and the boundary conditions related to rigid body modes. The second group includes modes in which the tail boom bending and torsion dominate. This group corresponds to the middle frequency value sub-range (see Fig.2). The third group is composed of modes of more complex shapes. For these modes the fuselage and the tail boom deformations are accompanied by the horizontal stabiliser and the fin bending and torsion. As the applied geometry model included only 2 points located on the tail fin and no points were selected on the horizontal stabiliser the third group of modes is not well represented in the measurement data.

The following general conclusions were formulated basing on the obtained results:

- Presence of fuel in the tank does not influence considerably the helicopter modal model in the considered frequency range
- Change of the boundary conditions between the support on the undercarriage to the suspension influences significantly the helicopter modal model, the change affects considerably less the modes in which horizontal plane bending dominates than the vertical bending modes
- The modes classified to the second group are the least sensitive to variation of the applied ground modal testing conditions
- Application of the harmonic excitation indicated the non-linear nature of structural dynamics properties of the helicopter airframe.

3.5. In-flight testing results

One of the main problems of testing structural vibration of helicopter airframes is observability of excitation forces during an experiment. If any controlled excitation cannot be effectively applied and measured the only way of testing the structural vibration is only to measure vibration signals that arise as a response to unknown excitation forces. When an assumption of the stationary white noise excitation is made the correlation functions between the measured response signals can also be expressed as sums of decaying harmonic components (James et al., 1995). Thus the Neutral Excitation Technique approach, which enables estimation of the modal model parameters from auto- and cross-correlation functions by making use of some time domain identification techniques might be applied.

In our case the auto- and cross-correlation functions were obtained directly from the measured acceleration time histories, and the polyreference Least Squares Complex Exponential time domain identification algorithm (Hermans et al., 1996; Hermans and Van der Auweraer, 1997a) was applied to identification of the tested system poles. The mode shapes were determined by using a curve-fitting algorithm applied to the auto- and cross-power functions calculated with the use of the Fourier transform. Examples of raw data and an estimated cross-correlation function as well as a calculated cross-power function are presented in Fig.3.

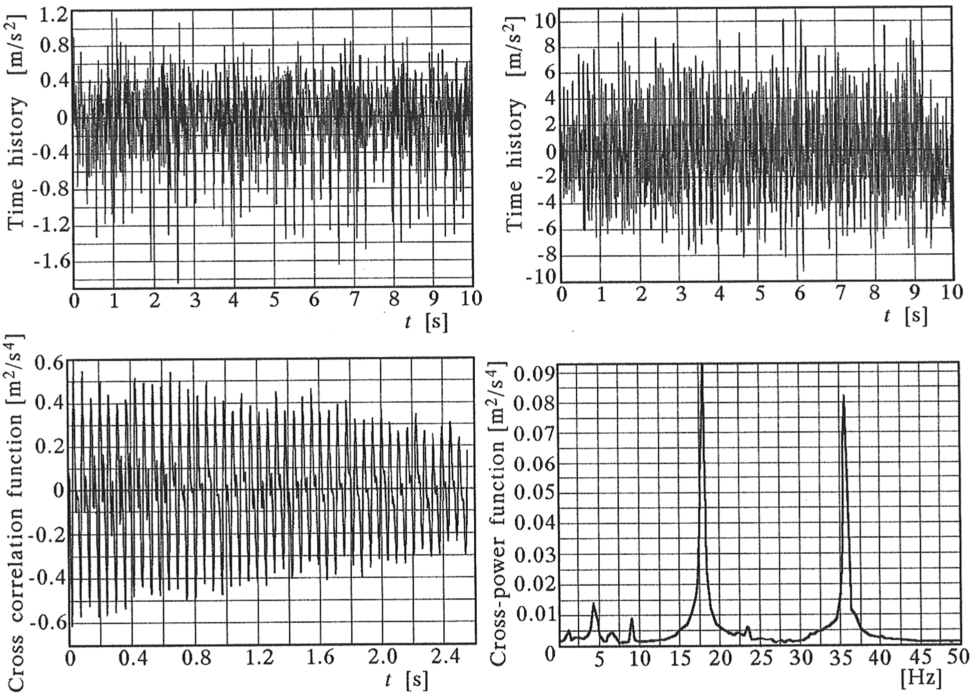

Fig. 3. Example of an acceleration time history, cross-correlation function and cross-power function for a single measurement direction

During flight aerodynamic loads for, which the assumption of the stationary white noise characteristics was made, excite a helicopter airframe. The other dominating excitation force component comes from rotor and transmission systems generating polyharmonic vibration. Frequency values of these excitation components might be calculated from the rotor speed that was recorded during an in-flight test. This is very important as the applied identification method does not allow one to distinguish directly the harmonic vibrations related to the harmonic components of the excitation forces from the structural vibration modes.

For analysis of the performed in-flight testing data the LMS CADA-X Operational Modal Analysis software was used.

The following results were obtained for 6 selected flight conditions (i.e. for level flight IAS=220 km/h, hover, left banked turn, left sideslip, climbing with take-off power and steady descent) in the frequency range of 4-20 Hz.

For all the considered flight conditions 7 modes were identified in the frequency range of interest. All the identified modes are complex. In Table 2 values of the identified natural frequency and modal damping for climbing with take-off power are listed.

Table 2. Identified natural frequency and modal damping values for climbing with take-off power

Mode denotation	Identified natural frequency value [Hz]	Identified damping value [%]
CT01	4.47	0.58
CT02	6.37	4.86
CT03	8.87	2.91
CT04	13.37	0.21
CT05	14.08	4.37
CT06	16.96	6.39
CT07	17.82	0.09

The identified modes: CT01, CT03, CT04 and CT07 are actually running modes corresponding to the harmonics of the main rotor rotation speed. This was determined basing on the knowledge of the current rotational speed of the main motor. Additionally, the damping values of these modes are smaller than those of the remaining identified modes. In Fig.4 the identified CT01 mode shape is presented.

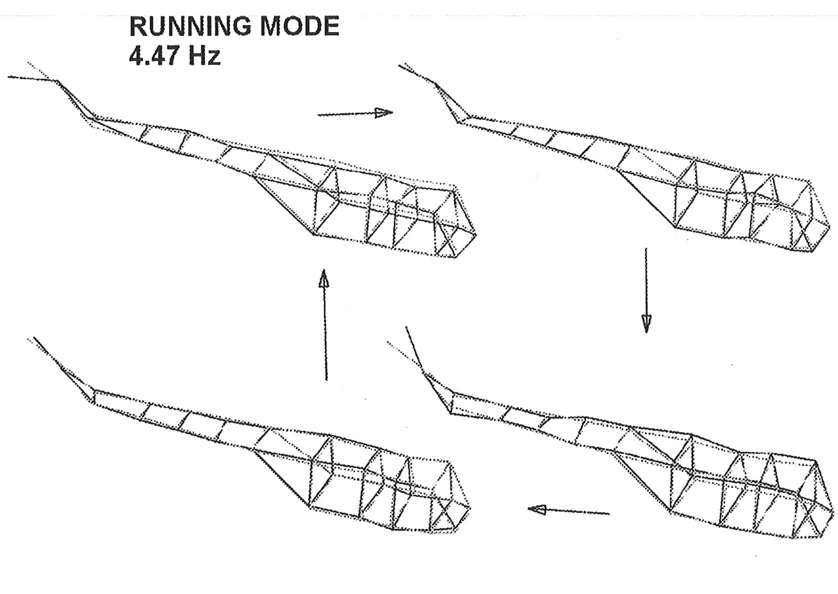

Fig. 4. Example of the identified CT01 running mode shape

When the running modes are being considered the trajectories of the measurement points, which are observed during animation, are elliptical and they do not cross the fixed reference position determined by the corresponding node of the wire-frame model.

In Fig.5 an example of the identified CT05 structural mode is presented.

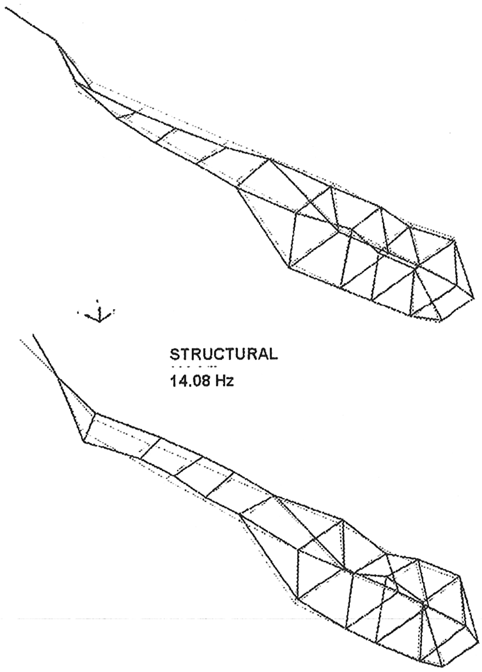

Fig. 5. Example of the identified CT05 structural mode shape

While the identified deformation pattern for all of the 3 identified structural mode shapes: CT02, CT05 and CT06 is complex it is smoother than that of the identified running mode shapes and in the area of the tail boom a distinct bending vibration component might be recognised.

The mode shapes identified for different flight conditions were compared with each other to investigate the influence of these conditions to the identification results. An example of a comparison of the running mode 01 according to the Modal Assurance Criterion (MAC) is presented in Fig.6. The comparison indicated that the identified running modes change considerably with variations of the flight conditions so they depend on an excitation forces nature, amplitude and spatial distribution.

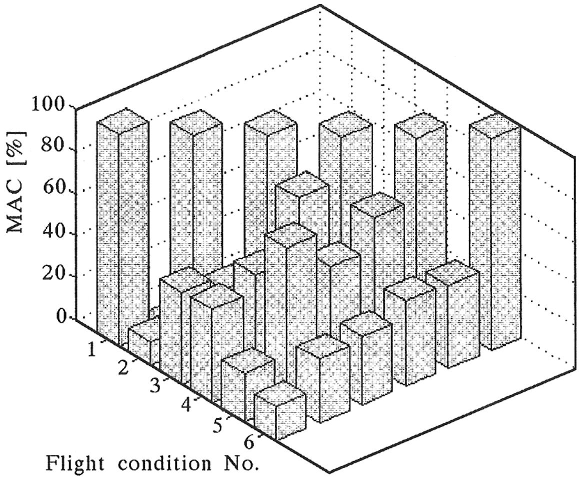

Fig. 6. Comparison of the 01 running mode identified for 6 flight conditions according to the MAC

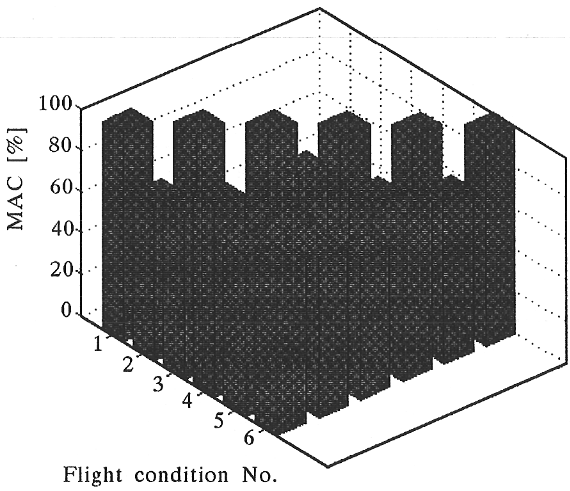

Fig. 7. Comparison of the 02 structural mode identified for 6 flight conditions according to the MAC

In Fig.7 a comparison according to the MAC of the identified for the 6 flight conditions structural mode shape 02 is shown.

This comparison shows that the correspondence between the structural modes identified for different flight conditions is high. So variations of the flight conditions do not change much the structural dynamics properties of the tested helicopter airframe considerably. The similar result was obtained from a comparison of the identified natural frequency values obtained for the selected 5 flight conditions. Results of such a comparison for the running mode 01 and the structural mode 02 are presented in Fig.8.

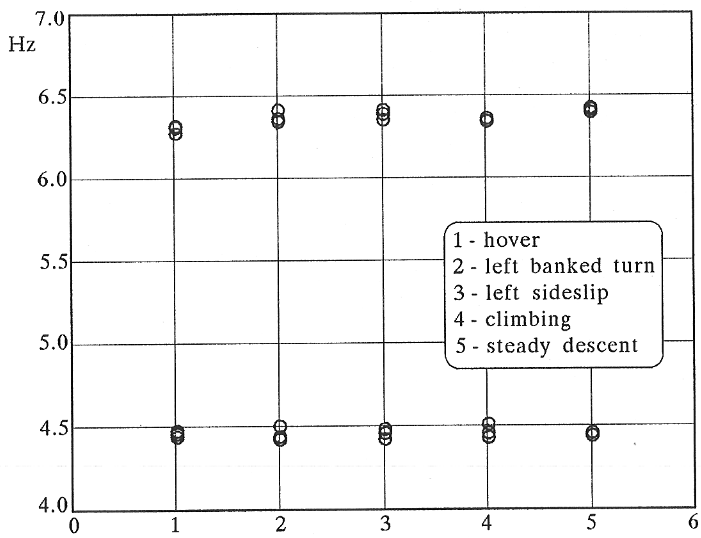

Fig. 8. Comparison of natural frequencies identified for 5 different flight conditions and 3 different set-ups for each of these conditions

Variation of the identified 'natural' frequency of the running mode 01 indicates ranges of the main rotor speed corresponding to different flights and to different flight conditions in a single flight. These ranges do not exceed 0.09 Hz and 0.08 Hz, respectively. The natural frequency of the 02 structural mode does not change by more than 0.14 Hz when different flights and the same flight condition are concerned, and by 0.06 Hz for different flight conditions during a single flight.

The conclusion of the in-flight vibration testing results is that only a few structural modes contribute considerably to the tested helicopter airframe operational vibrations. The remaining structural modes of the tested helicopter airframe were not sufficiently excited by the operational forces.

3.6. Comparison of the results of the ground and the in-flight helicopter airframe testing

The first step of the comparison consisted in checking which natural frequency identified for the ground conditions are the closest to the natural frequency identified for the in-flight data. Table 3 contains listing of the found values.

Table 3. Comparison of the ground and the in-flight natural frequency [Hz]

Climbing	UFR	UNR	SNR	SNS
6.37	6.54	6.28	6.40	6.08
14.08	14.65	14.66	14.52	14.77
16.96	16.08	16.77	16.66	16.92

The structural mode shapes identified for the in-flight test data in the considered frequency range were compared, according to the MAC, with the structural mode shapes identified for the ground testing data. Table 4 indicates the ground modes most similar to 3 in-flight structural modes identified for climbing with the take-off power conditions.

Table 4. Comparison of the ground and in-flight structural mode shapes according to the MAC

Climbing	UFR	UNR	SNR	SNS
6.37 Hz	12.09 Hz MAC 33.5%	5.76 Hz MAC 32.2%	6.40 Hz MAC 41.0%	7.04 Hz MAC 36.8%
14.08 Hz	12.09 Hz MAC 31.8%	6.28 Hz MAC 5.6%	12.17 Hz MAC 10.1%	11.67 Hz MAC 7.0%
16.96 Hz	12.09 Hz MAC 31.8%	6.28 Hz MAC 8.1%	12.17 Hz MAC 11.9%	11.67 Hz MAC 11.0%

An example of the qualitative comparison of the CT02 mode shape with the most similar mode shape identified for the ground conditions is presented in Fig.9.

The comparison showed that while there is some qualitative similarity between the corresponding mode shape pairs and there are mode pairs of quite close identified natural frequencies the correspondence of the ground and in-flight modal test results is small.

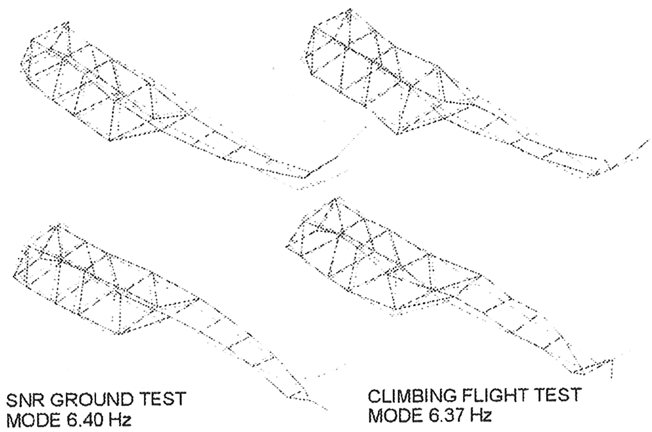

Fig. 9. An example of comparison of ground and in-flight mode shapes

4. Conclusions

The formulated identification methodology is very useful for testing of flying structures like helicopters, aeroplanes, satellites and gives possibility to test structures in conditions as close as possible to operating conditions.

The in-flight vibration test of the SW-3 helicopter airframe and comparison of the results obtained from the ground test results showed that:

- The applied experimental method of testing structural dynamics in operational conditions only basing on the response measurements, proved to be good for identification of the structural modes that are well excited during flight

- Structural dynamics properties show very little dependence on variations of the flight condition

- Contribution of the structural vibration to the overall vibration during flight is small

- Correspondence of the structural mode shapes identified for the ground and in-flight conditions is small.

Little correspondence of the ground and in-flight modal test results might be explained by a difference between the laboratory and flight testing conditions. During the ground modal testing specific laboratory boundary conditions are applied, the rotors are motionless or disassembled. Additionally, usually normal modes are identified.

During the in-flight modal testing the rotor dynamics and aerodynamic effects are significant. In operational conditions complex modes are identified.

The effectiveness of the applied identification method depends on the characteristic of the uncontrollable excitation forces. In case when the operational excitation does not excite certain modes in the considered frequency range the method is inapplicable to formulation of a comprehensive model of the structural dynamics. On the other hand the method enables identification of the structural modes that do contribute to the operational vibrations.

Acknowledgements

The authors would like to acknowledge the assistance of WSK PZL Świdnik, Poland in carrying out the vibration testing of the SW-3 helicopter.

This work was supported within EUREKA 1562 SINOPSYS grant.

References

1. BIELAWA R., 1992, *Rotary Wing Structural Dynamics and Aerolasticity*, AIAA Inc.

2. BROWN D., ALLEMANG R., ZIMMERMAN R., MERGEAY M., 1979, Parameter Estimation Techniques for Modal Analysis, *SAE Paper*, **7902221**, pp. 19

3. HERMANS L., VAN DER AUWERAER H., COPPENS P., MATHIEU L., 1996, Modal Parameter Extraction from In-Operation Data, *XV IMAC, Orlando*, 531-539

4. HERMANS L., VAN DER AUWERAER H., 1997, On the Use of Auto- and Cross Correlation Functions to Extract Modal Parameters from Output Only Data, *ISVR*, Southampton, 324-331

5. HERMANS L., VAN DER AUWERAER H., ABDELGHANI M., 1997, A Critical Evaluation of Modal Parameter Extraction Schemes for Output Only Data, *JMAC*, Tokyo, 124-131

6. JAMES G.H.III, CARNE T.G., LAUFER J.P., 1995, The Natural Excitation Technique (NexT) for Modal Parameter Extraction from Operating Structures, *Int. Journal of Analytical and Experimental Modal Analysis*, **10**, 4, 260-277

7. LISOWSKI W., UHL T., 1998, Modal Analysis of Helicopters – the Experiment Conditions, *Proceedings of the ISMA 23*, K.U. Leuven (edit.), 1509-1516

8. McHargue P.L., Richardson M.H., 1993, Operating Deflection Shapes from Time Versus Frequency Domain Measurements, *11th IMAC*, Kissimmee, FL

9. Uhl T., 1997, *Computer Assisted Identification of Mechanical* (in Polish), WNT, Warszawa

10. Uhl T., Lisowski W., Bochniak W., 1997, Application of Experimental Modal Analysis for Aircraft Structural Dynamics Investigation, *Proc. of the 2nd Seminar on RRDPAE, Institute of Aeronautics and Applied Research Bulletin*, **2**, 7, 131-136

11. Uhl T., Lisowski W., Małecki J., 1998, Strain Modal Testing of a Helicopter Tail Boom, *Proc. of 16th IMAC*, SEM USA, 1271-1277

Identyfikacja modeli modalnych smigłowców na podstawie wyników badań w locie

Streszczenie

W pracy przestawiono nową metodykę identyfikacji modeli modalnych na podstawie mierzonych w czasie eksploatacji sygnałów odpowiedzi na działające wymuszenia. Metodyka opiera się na estymacji punktowych i wzajemnych funkcji korelacji sygnałów przyspieszenia drgań mierzonych w zbiorze punktów pomiarowych. Jako przykład zastosowania opisanych algorytmów rozważono wyniki identyfikacji modelu modalnego śmigłowca zidentyfikowanego na podstawie pomiarów dokonanych w czasie lotu. Wyestymowane wartości parametrów modelu zostały porównane z odpowiadającymi im wartościami wyestymowanymi na podstawie wyników testu naziemnego. Porównanie to pozwoliło na oszacowanie stopnia, w jakim poszczególne postacie drgań są wymuszane w danych warunkach lotu. Osiągnięte wyniki identyfikacji modelu modalnego śmigłowca w warunkach lotu mogą znaleźć zastosowanie w analizie modyfikacji strukturalnych oraz diagnostyce śmigłowca.

Manuscript received October 2, 1999; accepted for print July 4, 2000