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CONTACT, MOTION AND WEAR IN RAILWAY MECHANICS

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One of the most intriguing problems in railway mechanics is the modeling of contact between a rail and wheel. On the one hand, in the *contact zone* the friction forces are determined – which are essential for the behavior of railway vehicles as *multibody systems*, see Kalker (1990), True (1993). On the other hand, the power extended by those frictional forces on the surfaces in contact leads to abrasion of particles and thus to wear phenomena, see Brommundt (1996), Frischmuth (1996), Langemann (1999).
The aim of this paper is to present recent results on the coupling between motion of a railway vehicle and evolution of contact surfaces due to wear. We concentrate on a wheel with initially perfect geometry, which changes due to frictional power dissipation during rolling on a perfect track.

Key words: vehicle dynamics, friction, wear, railway mechanics, contact problems, multibody systems

1. Motion

Typically, vehicles are described as multibody systems – we include embs (Elastic Multi Body Systems) in this notion. Positions of all masses which constitute the system under consideration are defined in terms of a finite number of coordinates y (or sometimes $p = p(t)$). Those coordinates may be (preferably) independent, in that case their number – the dimension of y – is referred to as *degrees of freedom*. Otherwise *constraints* may be imposed, e.g. in the case of so-called *kinematically closed loops*. In the first case the temporal changes of y are governed by ordinary differential equations of second order

$$m(y(t))\ddot{y}(t) = f\Big(t, y(t), \dot{y}(t), u(t, y(t), \dot{y}(t))\Big) \tag{1.1}$$

where the temporal derivatives are denoted by the dot. The *mass matrix* m represents the mass distribution within the described system, the function f comprises all (generalized) forces acting on the system. The function u is introduced to allow for a control motion, e.g. imposing a desired speed on the center of mass.

A constrained system can be transformed, in principle, into such a setting using Lagrange formalism. However, from a numerical point of view this is rather not desirable. A direct approach to a *differential-algebraic system*

$$\begin{aligned} m(y(t))\ddot{y}(t) &= f\Big(t, y(t), \dot{y}(t), \lambda, u(t, y(t), \dot{y}(t))\Big) + \nabla g^{\top}\lambda \\ 0 &= g\Big(t, y(t), u(t, y(t), \dot{y}(t))\Big) \end{aligned} \tag{1.2}$$

leads to more efficient algorithms, see Gear et al. (1985), Hairer and Wanner (1991), Simeon et al. (1991), Arnold and Frischmuth (1998).

For a rigid railway wheel, for instance, we have six degrees of freedom. However, assuming that the wheel is in continuous contact with a rigid rail, the vertical shift (elevation) of the mass center can be skipped. Similarly, for a rigid wheelset with both wheels in contact with rails the roll angle ϕ can be evaluated for a given lateral shift y and yaw angle ψ.

In both cases rather complicated geometrical calculations are required to obtain the redundant position variables from the degrees of freedom. To speed up the calculations, especially for simulations in real time, one prefers to do those calculations off-line during preprocessing, see Arnold and Frischmuth (1998). Of course, this works best if the bodies involved have symmetries and do not change with time.

If the elasticity of the bodies in contact cannot be neglected then the constraints are not adequate, in which case we have equations of motion of the form (1.1). If we allow for temporary separation of the contact couple we may have to switch between different descriptions.

The above amounts to the fact that the modeling of motion of a railway vehicle is quite a complicated task. There exists a number of successful simulation packages where the mentioned problems have been appropriately simplified and solved. It is worthwhile to mention that depending on the motivation some rather simple models proved most interesting, see e.g. Kaas-Petersen (1986), True (1993).

For the purpose of studying wear phenomena it is obvious that we need to be able to cover relatively long time intervals. This excludes too complicated models. On the other hand, we need some minimum spatial resolution, thus a certain level of complication cannot be avoided.

2. Contact

Most important for the mbs models introduced in the previous section is certainly the definition of the force function f, and in the case of constraints also of the function g. For ideal constraints, in the case of the formulation $(1.2)_1$, we have *constraint forces* in the normal direction to the *constraint manifold* (of unknown intensity, determined by *Lagrange multipliers* λ). In the presence of friction *tangential forces* depend on *normal forces*, i.e. on λ.

Moreover, the forces depend on such details of the contact geometry as the curvatures of the surfaces in the contact zone. This dependence is complicated enough for reasonably smooth surfaces which have contact in exactly one spot. In railway mechanics, unfortunately, we have to face multiple contact and high derivatives.

Typically, the determination of contact forces is carried out in several steps. First, the *geometrical contact point* is determined. This is done in a purely geometric way, given the actual geometry of the contact partners and their positions in terms of mbs coordinates. For unworn profiles this dependence is well studied and available e.g. in the form of tables. Second, the so-called *normal problem* is solved.

3. Wear

For this paper we assume that, given the multibody position vector $y(t)$ and the generalized velocities $\dot{y}(t)$, we are able to calculate the distributions of wear-relevant factors over the contact surfaces. The most frequently used wear model assumes the removed mass to be proportional to the dissipated energy. All we need in this case is local slip and tangential stresses in the contact region. Their scalar product, multiplied by the wear coefficient β, is the local speed of surface retreat.

Direct numerical time integration of this quantity, for each point of the involved surfaces, in order to obtain the current positions of the worn surfaces, fails for the obvious reason that integration time is too long and the integrand too oscillating. A frequently used technique is to amplify the real wear coefficient (by several orders), and shorten the integration time in the same way. This approach was criticized by Langemann (1999). Here we follow a different route. We postulate an evolution problem for the wear surface under consideration of the general form

$$\dot{x}(t,\sigma) = \mathcal{F}(t,\sigma,x(t,\cdot))\vec{n}(t,\sigma) \tag{3.1}$$

where

σ – surface parameter
$\vec{n}$ – outer normal and $-\mathcal{F}$ is the wear speed.

The latter may depend on time and position on the surface, but the most essential is the dependence on the actual surface as a whole.

In the previous work we have made efforts to express this dependence, at least approximately, in an analytic way. Simulations with a nonlinear regularized inverse diffusion equation have led to instabilities of the wear process similar to the observed behavior. In this paper, however, we want to evaluate $\mathcal{F}$ by numerical simulation of a multibody system described by (1.1). Roughly speaking, the evaluation of $\mathcal{F}$ consists in calculating the integral mean values over time of the local wear for a mbs with frozen geometry. It turns out that – for the systems we studied – such mean values converged to unique limits, depending on the control motion (e.g. speed) and actual geometry $x(t,\cdot)$, but independent of the initial values assumed for the degrees of freedom y in (1.1). This unique limit is assumed to be the postulated wear speed $-\mathcal{F}(t,\cdot,x(t,\cdot))$. Efforts to give analytical representations of such a mapping

$$x(t,\cdot) \rightarrow \dot{x}_n(t,\cdot) \tag{3.2}$$

acting essentially from a function space defined on the surface to itself, remained restricted to very simple setups, cf Brommundt (1996), Langemann (1999).

Numerical evaluation of $\mathcal{F}$ depends 'only' on our ability to describe the geometry and forces for worn surfaces represented by $x(t,\cdot)$, to integrate equations of motion (1.1) and to integrate along the factors determining the wear.

The remaining part is now the integration of evolution equation (3.1). Doing this by Euler's explicit method is much like the naive algorithms based on amplification factors. This factor is the ratio between simulation time and step size of Euler's method. However, given formulation (3.1) we have now the choice of the whole wealth of known integration methods. In particular, we get a much better control of artificial oscillations.

4. Numerical study

In the case of railway mechanics typically force and, if any, constraint functions are given in the form of hundreds of lines of a source code. This makes analytical considerations extremely difficult, the more so as useful properties are not to be expected.

In this paper our main interest is focused on the numerical integration of wear equation (3.1). To do this we choose a dynamical model that is reasonably realistic, but on the other hand simple enough to allow quick and reliable integration of the auxiliary problems. We base this single wheelset model on the work by True (1993) but introduce some modifications in order to get feedback from the geometrical changes in the profiles.

For the wear part, we choose the simplest possible law, assuming the loss of mass proportional to the dissipated frictional energy.

We review now briefly the important ingredients of the dynamical model, stressing the modifications that come with time-dependent profiles.

To start with, let us have a look at the standard rail and wheel profiles (UIC 60/S1002), cf Fig. 1, as they are being met in Europe on new tracks, respectively vehicles. Each of the profiles is defined piecewise, second derivatives suffer jumps where the pieces meet. The rail profile is concave, the wheel profile has changing signs of curvature. Most important, the distance between the rail and wheel, which depends on their relative displacement, is essentially non-convex. In general, it has lots of local minima. Gravity causes, as long as it dominates the dynamical forces, the minimum of the distance function to be zero, the contact forces are applied at the spot where this minimum is assumed. Hence, obviously, the distance function, and its minimizer are essential constituents of the problem formulation (1.1), respectively (1.2).

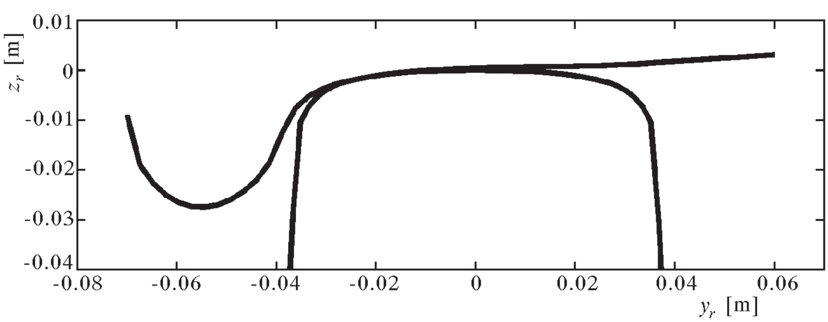

Fig. 1. Rail and wheel profiles

By design, for all reasonable lateral and angular displacements, there should be a reasonably large region where the rail and wheel profiles come close to each other, cf Fig. 1. Otherwise the stresses would exceed the yield stress. Zooming in shows, cf Fig. 2, that for chosen position coordinates, there is a unique minimizer at $-2\,\mathrm{cm}$ from the origin in the lateral wheel surface direction. This point x_c is called the *point of geometrical contact*. Under load, however, there is an *elastic approach* between the rail and wheel of the order 10^{-4} m. The bottom of the graph of the actual distance function is flat for a patch of an extension of the order 10^{-2} m. Obviously, the geometric contact point x_c will not be exactly in the center of such a patch.

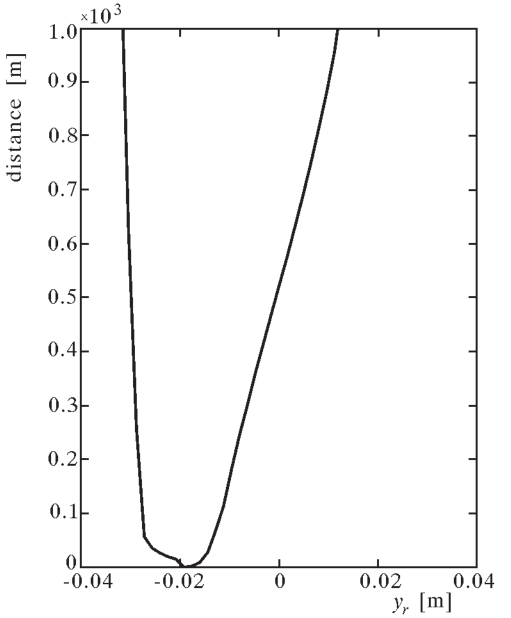

Fig. 2. Distance function

We have to stress that the point of contact and related quantities can be expressed in terms of the surface parameters σ of the wheel or railhead, or in coordinates of the 3D physical space. In the plots we indicate by the subscripts r, w or ws whether we refer to the rail, wheel or wheelset. The meaning of x and y depends on the context, e.g. y_2 is the generalized mbs coordinate of the rolling wheelset, y_r is the lateral parameter of the railhead frame.

The point of geometrical contact is determined by the generalized coordinates defining the position of the wheel in the system of the railhead. For symmetry reasons the lateral displacement, roll and yaw angles alone define the contact point and the vertical displacement (compliance neglected), cf Arnold and Netter (1998). For frozen yaw angle the dependence is depicted in Fig. 3. Note the step regions in the plot which correspond to the jumps of the contact point, e.g. from the tread to flange. This very unpleasant effect is absent in the case of conical wheels. While numerically attractive, especially for the dae-approach, this case is far too unrealistic for a serious study of wear. In fact, the lack of the flange leads to large lateral displacements so that the dissipated energy would spread over a region much wider than the actual wheel is.

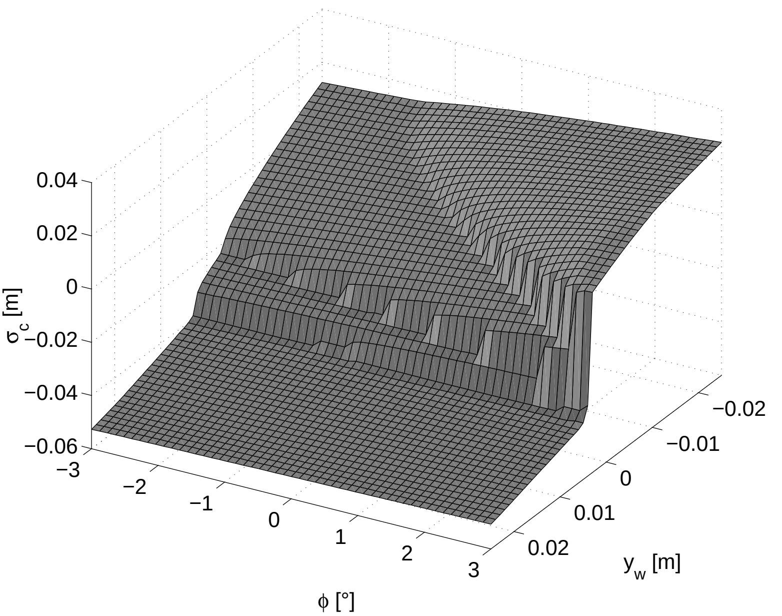

Fig. 3. Point of geometrical contact

Note that for each yaw angle ψ we get a different dependence. Moreover, like the location of the global minimum this characteristic depends in a sensitive way on the underlying profiles. Usually, these geometric data are calculated during preprocessing and interpolated at running time. For wear calculations, however, the procedure has to be repeated at each time step of the integration of (3.1).

For a wheelset with a rigid axle the permanent contact to rails, see Fig. 4, introduces a constraint, which eliminates the roll angle ϕ from the equations. In fact, shifting the wheelset to the left or right lifts the corresponding wheel

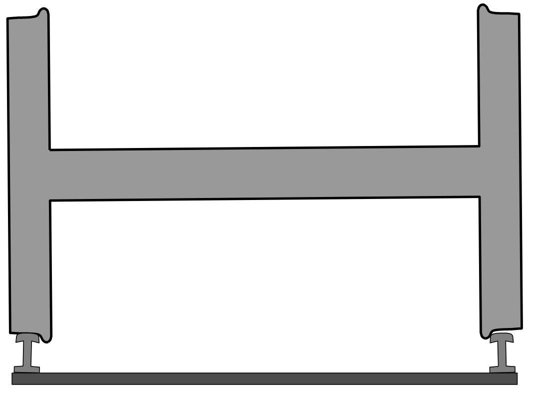

Fig. 4. Wheelset on rails

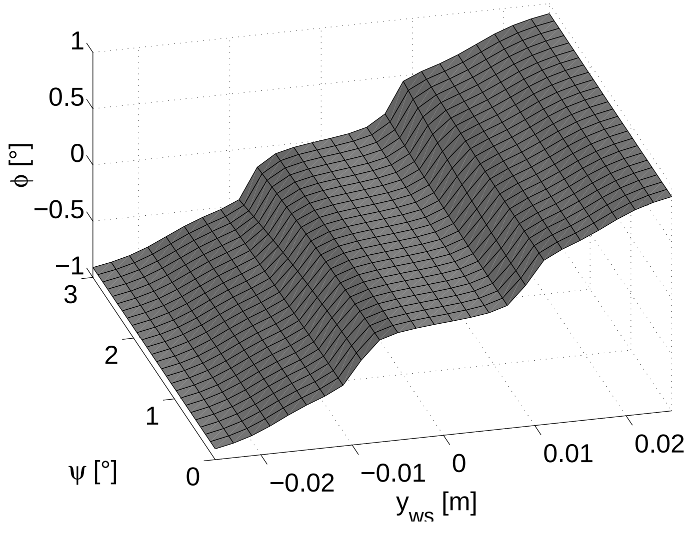

Fig. 5. Roll angle of wheelset

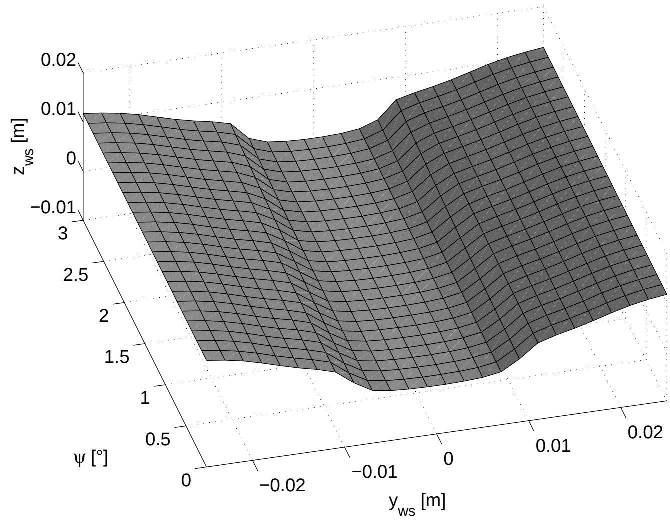

Fig. 6. Vertical displacement

up. Thus we get a dependence of the roll angle (Fig. 5) and the center of mass (Fig. 6) on the lateral shift y and yaw angle ψ of the rigid wheelset.

Multiplied by the axle load, this gives potential of the conservative forces acting on the wheelset. We have a stabilizing effect like for the motion in a half-pipe. However, the width of this half-pipe is finite, there is no extension of this potential beyond the interval $[-0.07, 0.06]$. For proper exploitation the trajectories should mostly stay within the flat bottom of the slide, cf Meinke (2000).

The most delicate part of the dynamical model, however, is the calculation of frictional forces. In terms of multibody system coordinates we have the load and rigid body slip as the input quantities. It turns out that locally, within the contact patch, the slip velocity is very inhomogeneous. The patch splits into a stick and slip region the size of which is essential for the calculation of the resultant tangential force, which is needed for time-integration of the mbs-coordinates. This interference of a finer scale of resolution is usually treated with a level of accuracy dictated by the computational goals. There are reasonable algebraic approximations (Vermeulen Johnson), one step discretized methods (Fastsim) or full numerical solutions (Contact), cf Kalker (1990), Kik and piotrowski (1996). Our choice follows the method by True (1993), but with the input variables taken from the actual worn profiles instead of the frozen values of the original model.

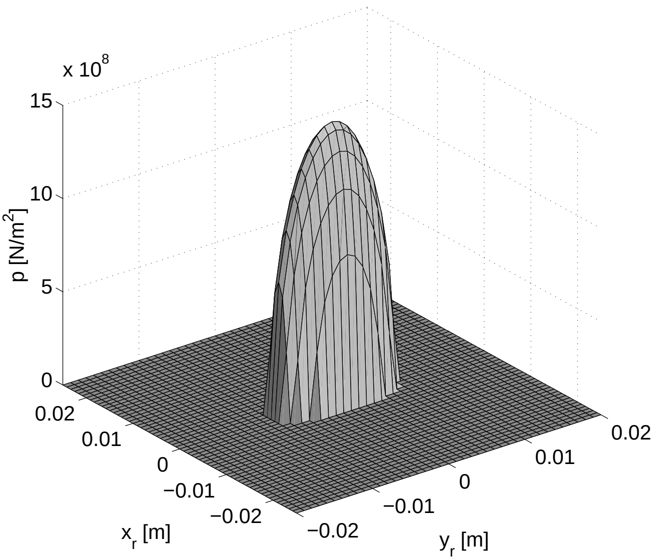

Fig. 7. Normal stress in contact region

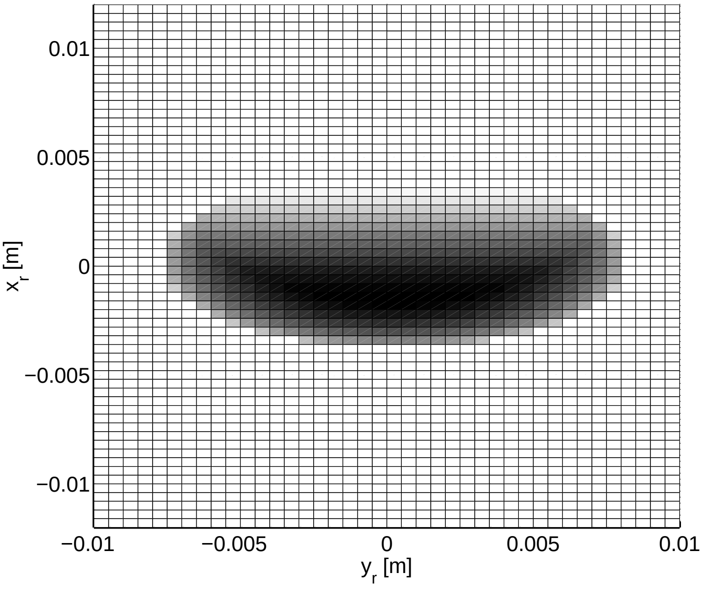

Fig. 8. Tangential stress in contact region

In the first step, we calculate the distribution of the normal stress p (pressure), see Fig. 7, following Hertz's theory, see Hertz (1982). Thus we disregard the tangential stresses and the known fact that the distance function is assumed to be quadratic in Hertz's theory. Obviously, a quadratic approximation is not valid in a domain big enough for the given order of normal loads and hence elastic approaches, cf Fig. 2 again. However, better results come at a much higher numerical cost, so for our present purpose we accept the error in the normal component.

In the consequent step, on the basis of the obtained normal stress and without any feedback, the local slip velocity and tangential stresses are approximated. A sample result is presented in Fig. 8. Only in a part of the patch we have slip, and thus, full contribution of μp to the tangential forces q. For a very high rigid body slip the whole patch becomes the slip region, then we have saturation and the friction force becomes μP with load L. For a moderate slip we get just a fraction of that limit value. For quick calculations a spline approximation to the full solution has to suffice.

Now we have – at least in a very abbreviated way – described the major contributions to the wheelset model being sensitive to geometric changes of the wheel profiles. Let us assume given initial conditions for all free position variables and their time-derivatives. As the control motion we assume a prescribed longitudinal position of the center of mass (or of the end of a spring pulling the center of mass) of the wheelset. Then we have a well-posed initial value problem for the system (1.1). For a reasonable load, prescribed travel speed and initial conditions the solution exists for all the time (stays in the half-pipe). Stationary solutions are unstable above the so-called critical speed, solutions with flange contact are shown in Fig. 9. and Fig. 10, cf Frischmuth et al. (1996). For certain speeds chaos may be observed, cf Kaas-Petersen (1986).

In terms of multibody variables, given those solutions, we can calculate the dissipated power. In order to study wear, we need again the higher resolution of the local distribution within the contact patch. The scalar product of the local slip velocity and tangential stress gives the local surface intensity of the power dissipation.

Obviously, this is a highly concentrated and quickly varying quantity. Fortunately, it turns out that taking an integral mean value over integration time tends very fast to a constant limit distribution, under given constant conditions. Within this paper, we assume that the wheel stays round. For out of round wheels we refer the Reader to works by Frischmuth (1996, 1997), Frischmuth and Langemann (1998). In this case all we need is the curve along the profile giving the average dissipated power per unit length and unit time,

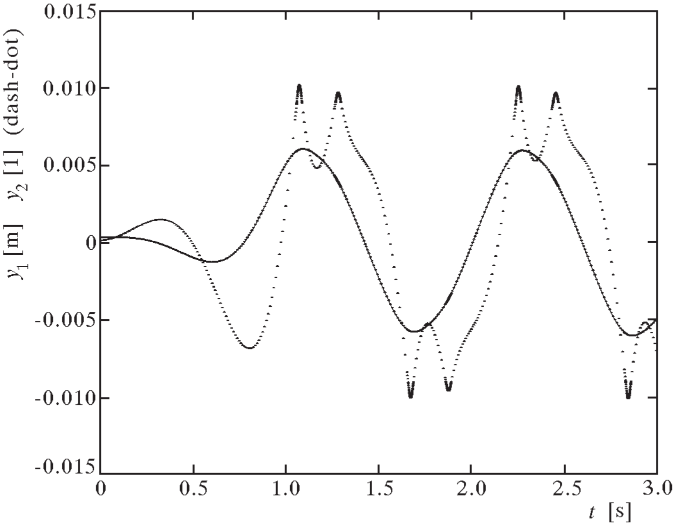

Fig. 9. Hunting wheelset

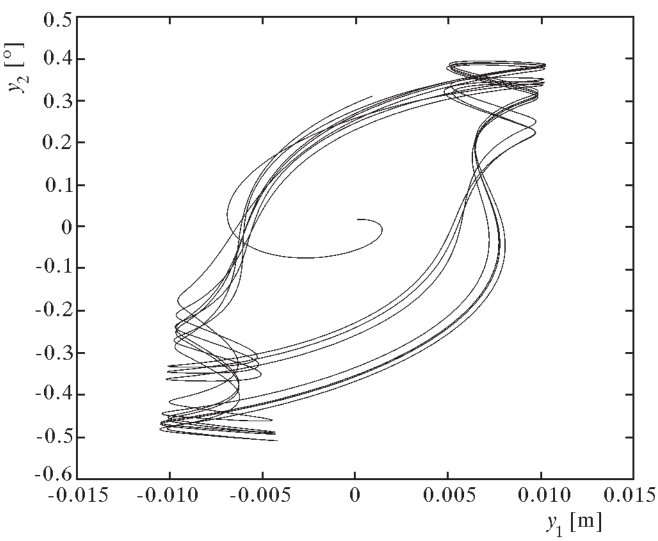

Fig. 10. Yaw angle versus lateral displacement

cf Chudzikiewicz (2000), Frischmuth (2000). Multiplied by the constant wear factor β this is equivalent to the prescribed normal speed $-\mathcal{F}$ at which the profile is abrased.

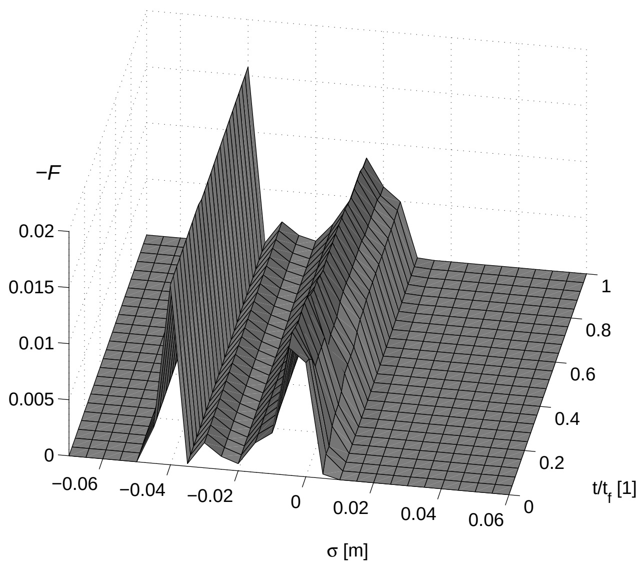

Fig. 11. Power of dissipated energy

Now, eventually, we have full description of the numerical evaluation of the wear speed $-\mathcal{F}$, i.e. the operator on the right-hand side of evolution equation (3.1) for the wheel geometry.

The integration time of (3.1) turns out to be sensitive, even in a 1D case, to the profile, having assumed symmetry of the wheel. Usually, explicit difference methods tend to oscillations which are easy to understand: abrasion of the material in the contact region leads to increasing of the distance function, hence the contact region is going to move away. Too large time steps together with too fine resolution of the profile geometry leads to alternating phases of the abrasion in the neighboring regions. For stable and fast methods for solving (3.1) we refer to Sethian (1996).

Fig. 11 presents the power dissipation over the lifetime of the wheel. The change in the wheel profile is not really visible, it is still of the order of the elastic approach. Nonetheless, given the steep dependence of the contact point on the profile data, this leads to clear variation of the contact conditions during the wear process. In particular, due to wear of initially exposed spots, the wear eventually reaches places that are not initially in contact whatever the mbs positions. The actual wear can be approximated from the data presented in Fig. 11 by integrating along the time axis, multiplying with the constant β and modifying the profile in the normal direction to the reference configuration.

5. Conclusions

Railway mechanics is a very wide field of research, the problem of rail-wheel contact is just one of many of its aspects, but certainly one of the most complicated. It cannot be uncoupled from other sub-problems, like models of the track, the subgrade, the whole vehicle. In this paper we assumed fargoing simplifications in order to concentrate on the numerical strategy for solving the surface evolution. Other important effects remain unaccounted for, e.g. plastic deformations, damage below the surface, vibrations, cf Bogacz and Dżula (1993). Our main point is that wear should be modeled as an evolution problem, and appropriate numerical methods should be used to calculate time dependence of the surfaces exposed to abrasive wear.

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Kontakt, ruch i zużycie w mechanice kolejowej

Streszczenie

Jednym z najbardziej ciekawych zagadnień mechaniki kolejowej jest modelowanie kontaktu między kołem a szyną. Z jednej strony w strefie kontaktu określone są siły tarcia – które są istotne dla zachowania pojazdów kolejowych jako układów wielu ciał, patrz Kalker (1990), True (1993). Z drugiej strony zaś moc dyssypowana przez siły tarcia na powierzchniach kontaktowych prowadzi do usuwania cząsteczek i tym samym do zjawisk zniszczenia, patrz Brommundt (1996), Frischmuth (1996), Langemann (1999).

Celem niniejszej pracy jest prezentacja aktualnych wyników na temat sprzężenia między ruchem pojazdu kolejowego a ewolucją powierzchni kontaktu wskutek zużycia. Dla ustalenia uwagi rozpatrujemy koło o początkowo idealnej geometrii, która zmienia się pod wpływem mocy sił tarcia dyssypowanej podczas toczenia po perfekcyjnym torze.

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