JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
4, **39**, 2001

GENETIC ALGORITHMS IN FATIGUE CRACK DETECTION

Marek Krawczuk
Arkadiusz Żak
Wiesław Ostachowicz

Institute of Fluid Flow Machinery, Polish Academy of Sciences, Gdańsk
e-mail: mk@imppan.pg.gda.pl

This paper presents results on the identification of fatigue cracks in beams via genetic search technique and changes in natural frequencies. The location and size of the crack are determined by minimisation of an error function involving the difference between the calculated and "measured" natural frequencies. The simulation studies indicate that the changes in the natural frequencies and genetic algorithm allows one to estimate the fatigue crack parameters (location and size) very accurately and fast.

Key words: fatigue crack detection, vibration methods, genetic algorithms

1. Introduction

Damage detection via changing of modal parameters has been a topic of extensive research over the past few decades, see Fu-Kuo Chang (1997, 1999). Damage will cause local changes in the stiffness of a structure, which will lead to changes in its dynamic response. The changes in natural frequencies, mode shapes or amplitudes of forced vibrations are most frequently considered as the damage indicator. Two distinct methodologies have been applied to identify the damage parameters (location and size) in a structure using vibration data. The first method is based on finite element model updating and error localisation (Friswell and Mottershead, 1995; Kaouk and Zimmerman, 1993). The second one assumes a candidate set of possible damage scenarios (i.e. type of damage, location and size). The calculated changes in dynamic characteristics for all damage scenarios are compared with the measured ones and the closest damage case is chosen (Cawley et al., 1978; Krawczuk and Ostachowicz, 1997).

This paper presents a study on the identification of a fatigue crack in beams using genetic search technique and changes in natural frequencies. The location and size of the fatigue crack are estimated by the minimisation of an output error with criteria based on changes in natural frequencies. The method is demonstrated using a model of a cracked cantilever beam. The results obtained from simulation studies indicate the applicability of the presented approach to damage detection in structures.

2. Genetic algorithm

The genetic algorithm is a search technique based on ideas from the science of genetics and the process of natural selection. A simple genetic algorithm consists of three basic operations: reproduction, crossover and mutation. The algorithm starts with a randomly generated initial population. Members of this population are usually binary strings (called chromosomes). Particular elements of the chromosomes are called the genes. In these strings values of a variable or variables are coded, which can be a solution to the examinined problem in the search space. These variables are then used to evaluate the corresponding fitness value, which is the objective function. In the next step the chromosomes are selected for reproduction. Selected processes can be realised in many ways (Goldberg, 1989), nevertheless the number of selected members is a function of their fitness. Thus, the individuals with a higher fitness will receive more copies. In order to minimise premature convergence of the initial populations special scaling methods are applied (Goldberg, 1989). One of the most widely applied method is the linear scaling proposed by Bagley (1967). After reproduction the process of crossover is realised. There are many ways of implementing this idea (Goldberg, 1989). Generally, crossover with one or many crossover points can be used. The crossover points are selected randomly, usually using roulette wheel. This way, by exchanging some portions between the selected chromosomes (called parents), two new strings (called children) are created. The final process is mutation. Here, a particular gene in a particular chromosome is randomly changed. It means that O is changed to 1, and vice versa. The process of mutation in nature is very rare and, for this reason, in a genetic algorithm the probability of mutation in a chromosome is kept on a very low level.

3. Objective function

Objective function used in the present paper is based on the changes in natural frequencies found from "measurements". The changes in natural frequencies can be called the classical damage indicators. They are without any doubt the most widely damage indicators in the past and nowadays. The main reason for their great popularity is that natural frequencies are rather easy to determine with a relatively high level of accuracy. In fact, one sensor placed on a structure and connected to a frequency analyser gives estimates for several natural frequencies. Further, natural frequencies are sensitive to all kinds of damage – local and global damage.

The form of objective function is based on the proposed by Messina et al. (1992) Damage Location Assurance Criterion (DLAC)

$$\mathrm{DLAC}(s) = \frac{|(\delta\boldsymbol{\Omega})^\top \delta\boldsymbol{\omega}_s|^2}{\Big((\delta\boldsymbol{\Omega})^\top \delta\boldsymbol{\Omega}\Big)\Big((\delta\boldsymbol{\omega}_s)^\top \delta\boldsymbol{\omega}_s\Big)} \tag{3.1}$$

where $\delta\boldsymbol{\Omega}$ is the trial "experimental" frequency change vector and $\delta\boldsymbol{\omega}_s$ is the theoretical frequency change for the damage at the location s.

DLAC values lie in the range 0 to 1, with 0 indicating no correlation and 1 indicating the exact match between the patterns of the frequency changes. The value of s, giving the highest DLAC values, determines the predicted damage location and size.

4. Model of cracked beam

The model of cracked, cantilever beam is presented in Fig. 1

For each part of the presented beam the following equation of transverse vibration can be written. In this equation the shear effect is omitted

$$EJ\frac{\partial^4 y_i(x,t)}{\partial x^4} + \rho A\frac{\partial^2 y_i(x,t)}{\partial t^2} = 0 \qquad i = 1,2 \tag{4.1}$$

where

E – Young modulus
J – moment of inertia of the beam cross-section
ρ – material density
A – cross-sectional area of beam.

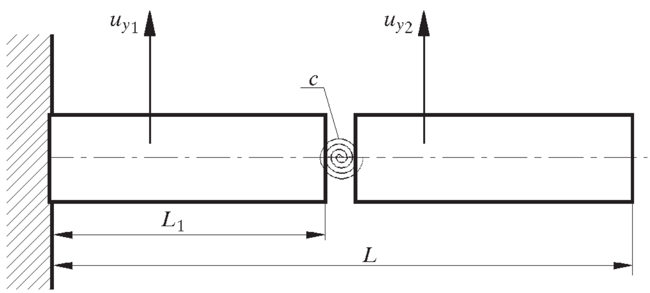

Fig. 1. Dimensions of the cracked cantilever beam

The solution to equation (4.1) can be expressed in the following form

$$y_i(x) = A_i \sin(kx) + B_i \cos(kx) + C_i \sinh(kx) + D_i \cosh(kh) \qquad i = 1,2 \tag{4.2}$$

where the parameters k_i correspond to the natural frequencies of the analysed beam.

The boundary conditions for the analysed structure can be expressed in the following form:

— fixed end $(x = 0)$

$$y_1(0) = 0 \qquad y_1'(0) = 0$$

— crack $(x = e = L_1/L)$

$$\begin{array}{ll} y_1(e) = y_2(e) & y_2'(e) - y_1'(e) = cy_2''(e) \\ y_1''(e) = y_2''(e) & y_1'''(e) = y_2'''(e) \end{array}$$

— free end $(x = 1.0)$

$$y_2''(1) = 0 \qquad y_2''(1) = 0$$

where c is the stiffness of the elastic element modelling the crack, see Ostachowicz and Krawczuk (1991).

Taking into account the boundary conditions and form of the solution to equation of motion (4.1) the characteristic equation of the problem can be formulated. This equation allows one to determine the natural frequencies of the cracked beam

$$\det\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ \mathrm{ch}(ke) & \mathrm{sh}(ke) & \cos(ke) & \sin(ke) & -\mathrm{ch}(ke) & -\mathrm{sh}(ke) & -\cos(ke) & -\sin(ke) \\ \mathrm{sh}(ke) & \mathrm{ch}(ke) & -\sin(ke) & \cos(ke) & A_1 & A_2 & A_3 & A_4 \\ \mathrm{ch}(ke) & \mathrm{sh}(ke) & -cos(ke) & -\sin(ke) & -\mathrm{ch}(ke) & -\mathrm{sh}(ke) & \cos(ke) & \sin(ke) \\ \mathrm{sh}(ke) & \mathrm{ch}(ke) & \sin(ke) & -cos(ke) & -\mathrm{sh}(ke) & -\mathrm{ch}(ke) & -\sin(ke) & \cos(ke) \\ 0 & 0 & 0 & 0 & \mathrm{ch}k & \mathrm{ch}k & -\cos k & -\sin k \\ 0 & 0 & 0 & 0 & \mathrm{sh}k & \mathrm{ch}k & \sin k & -cosk \end{bmatrix} = 0 \tag{4.3}$$

where: sh $=$ sinh, ch $=$ cosh, $A_1 = ck\cosh(ke) - \sinh(ke)$, $A_2 = ck\sinh(ke) - \cosh(ke)$, $A_3 = -ck\cos(ke) + \sin(ke)$, $A_4 = -ck\sin(ke) + \cos(ke)$.

From the above equation it arises that changes in the natural frequencies will be functions of the crack depth and location along the beam.

5. Numerical examples

The numerical calculations were carried out for a cracked, cantilever beam. The length of beam was 0.4 m, width 0.02 m and height 0.02 m. The beam was made of steel with Young's modulus $2.1 \cdot 10^{11}\,\mathrm{N/m^2}$ and mass density $7860\,\mathrm{kg/m^3}$.

Two cases were tested:

Case 1 – the crack had depth up to 5% of the total beam height, and was located 0.04 m from the fixed end ($e = L_1/L = 0.1$),

Case 2 – the crack had depth up to 2% of the total beam height, and was located 0.12 m from the fixed end ($e = L1/L = 0.3$).

In all cases the population had 10 members. One member had 30 bits (15 for each variable). During numerical calculations it was assumed that the probability of crossover was 95% and the probability of mutation was 0.05%. The crack depth and location were identified using the eigensensitivity approach described in Section 3. The first four natural frequencies were used in the numerical tests. Only one run of the genetic algorithm was used for each case. The results of numerical calculations are presented in Fig. 2 to Fig. 5.

From Figures 2-5 it arises that the genetic algorithm correctly locates the damage and also correctly estimates its size. The convergence to proper results was obtained after no more than 110 populations in the first case and 85 populations in the second one.

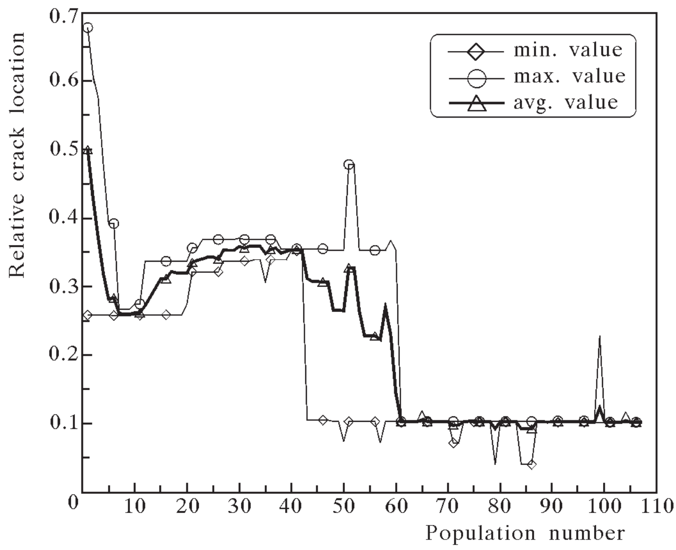

Fig. 2. Crack depth as a function of the number of populations – case 1

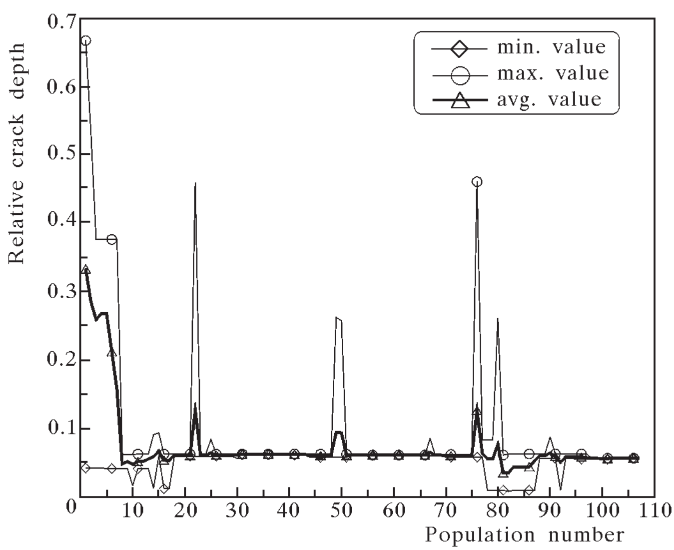

Fig. 3. Crack location as a function of the number of populations – case 1

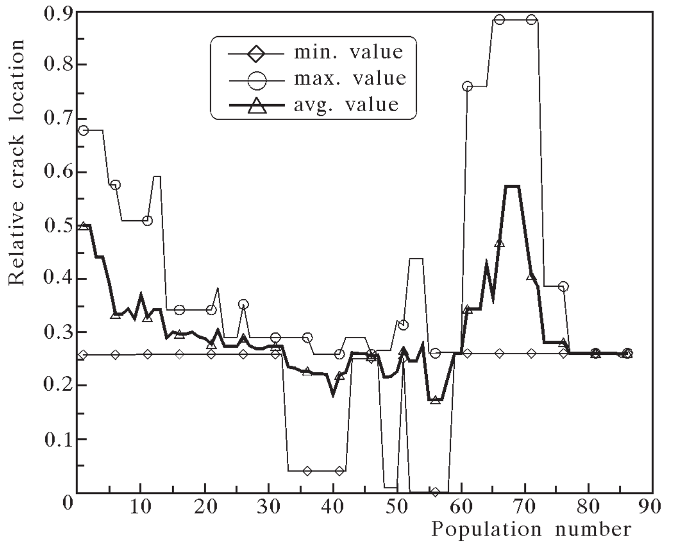

Fig. 4. Crack depth as a function of the number of populations – case 2

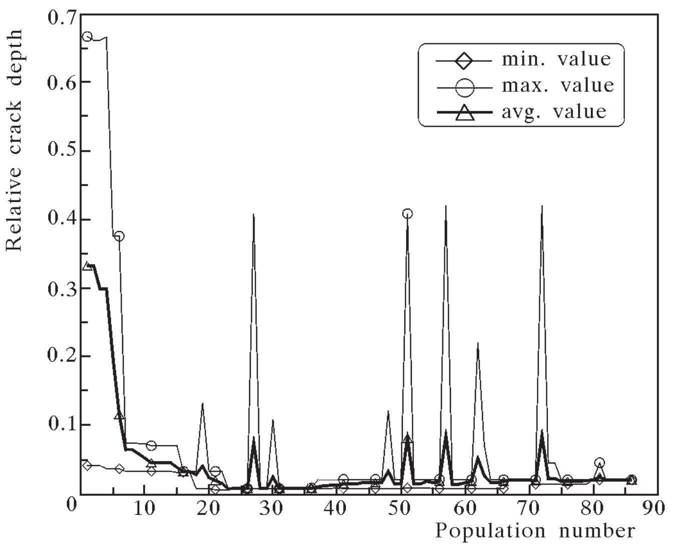

Fig. 5. Crack location as a function of the number of population – case 2

6. Conclusions

The combined genetic algorithm and the eigensensitivity criterion DLAC as an objective function has been used to identify the location and size of the crack from "experimental" vibration data. The genetic algorithm presented in this paper is very simple. Nevertheless, the obtained results are promising. The number of calculations which are needed for damage detection is much less than that in classical search algorithms. For this reason, the time of numerical calculations is shorter.

Future works should be devoted to implementation of processes which are observed in nature to this algorithm. For example elitism, where the best solution is always passed on to the next generation, is a particular feature for which good results have been reported (Goldberg, 1989). It is also planned to check other vibration criteria being applied to structural health monitoring and based on changes in mode shapes and amplitudes of forced vibrations as objective functions. Such comparative analysis should explain which damage indicator is most sensitive to changes in stiffness of a structure due to damage.

References

1. Adams R.D., Cawley P., 1979, The Localisation of Defects in Structures from Measurements of Natural Frequencies, *Journal of Strain Analysis*, **14**, 2, 49-57
2. Bagley J.D., 1967, The Behavior Addaptive Systems which Employ Genetic and Correlation Algorithms, Ph.D. Thesis, University of Michigan
3. Cawley P., Adams R.D., Pye C.J., Stone B.J., 1978, A Vibration Technique for Non- Destructivly Assessing the Integrity of Structures, *Journal of Mechanical Engineering Sciences*, **20**, 2, 93-100
4. Friswell M.I., Mottershead J.E., 1995, *Finite Element Model Updating in Structural Dynamics*, Kluwer Academic Publishers
5. Fu-Kuo Chang, 1997, *Structural Health Monitoring: Current Status and Perspectives*, Technomic Pub., Basel
6. Fu-Kuo Chang, 1999, *Structural Health Monitoring: Current Status and Perspectives*, Technomic Pub., Basel
7. Goldberg D., 1989, *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley Publishing Company, Reading

8. Kaouk M., Zimmerman D.C., 1993, Evaluation of the Minimum Rank Update in Damage Detection: An Experimental Study. *Proc. of the 11th International Modal Analysis Conference*, 1061-1068

9. Krawczuk M., Ostachowicz W., 1997, Damage Indicators for Diagnostic of Fatigue Cracks in Structures by Vibration Measurements – a Survey, *Journal of Theoretical and Applied Mechanics*, **34**, 2, 307-326

10. Messina A., Jones I.A., Williams E.J., 1992, Damage Detection and Localisation Using Natural Frequency Changes, *Proc. of the 1st Conference on Identification*, 67-76

Algorytmy genetyczne w detekcji pęknięć zmęczeniowych

Streszczenie

W pracy przedstawiono wyniki identyfikacji położenia i wielkości pęknięć zmęczeniowych metodą algorytmów genetycznych z wykorzystaniem zmian częstości drgań własnych. Położenie i wielkość pęknięcia poszukiwano minimalizując funkcję celu wykorzystującą różnice między częstościami mierzonymi i obliczanymi. Wyniki symulacji wskazują, że zmiany częstości drgań własnych i algorytm genetyczny pozwalają wyznaczać parametry pęknięcia zmęczeniowego (położenie i wielkość) szybko i dokładnie.

Manuscript received April 28, 2000; accepted for print December 17, 2000