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THE ANALYSIS OF PROPAGATION OF SUBSPAN OSCILLATIONS THROUGH THE GROUPS OF SPACERS IN CONDUCTOR BUNDLES

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The paper presents a theoretical model of wave propagation through groups of spacers in a multiconductor bundle. The important quantities; such as, energy of passed and reflected waves, respectively and the energy dissipated in the spacers have been determined. It is shown that oscillations can be reduced by a proper design of groups of spacers. The theoretical analysis is illustrated with numerical results.

Key words: subspan oscillations, wave propagation, multiconductor bundles

1. Introduction

Multiconductor bundles are used in many high voltage power transmission lines. The characteristic construction features of bundles favour the formation of subspan oscillations. They are located within sectors – between the groups of spacers.

The subspan oscillations are caused by aerodynamic shielding of leeward-lying conductors by the windward ones (Claren et al., 1974). They occur within the range of wind velocity from 7 to 18 m/s. The corresponding frequency is associated with the subspan length. The wave length is usually twice the subspan length. For the bundles with subspans $30 \div 70$ meters long, and the tensions of conductors used in practice, the subspan oscillations occur at frequencies lying in the range $1 \div 3$ Hz. The conductors of upper pair in subspans oscillate in phase opposition. The amplitudes of motion can be so large that the collision of conductors can take place.

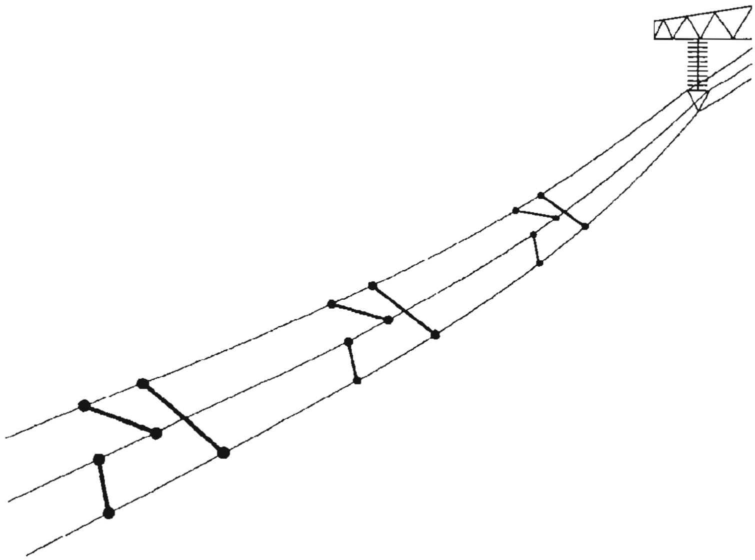

Fig. 1. Three-conductor bundle

The subspan oscillations were described by Diana et al. (1990) and [5,6]. Both the occurrence and decay of oscillations depend on conditions determined by the wind flow and construction parameters (mainly the number of conductors, the number of spacers and their configuration in groups, different lengths of subspans) (Markiewicz and Snamina, 1995). Special visco-elastic spacers are used to dissipate the energy of motion. The variety of atmospheric conditions causes electric lines being often in favourable circumstances to form subspan oscillations. It is, therefore, essential to design the bundles in which the oscillations would be cut down quickly by the dissipation of energy in spacers and conductors. Hence, it is a basic problem to investigate the propagation of wave arising in a subspan through the groups of spacers (Mead, 1973).

The characteristic quantities: the energy of passed and reflected waves, and the dissipated energy are used for a proper selection of construction parameters of the spacers.

2. Model of a spacer

Spacers can differ in construction details, but the basic elements are common. In all constructions there is the rigid element of a spacer separated from conductors by flexible elements. The model of a spacer is general and can be applied to any two-conductor spacer. It is assumed that the spacers are connected in the planes perpendicular to conductors and they undergo plane motion (Fig.2). The local co-ordinate system $\xi\eta$ is connected with the equilibrium state of a spacer. The vector $\overrightarrow{A^0B^0}$, which connects the origin with the end of a spacer in the state of equilibrium is parallel to the versor $\widehat{\boldsymbol{\xi}}$, and the point A^0 coincides with the origin of the local co-ordinate system.

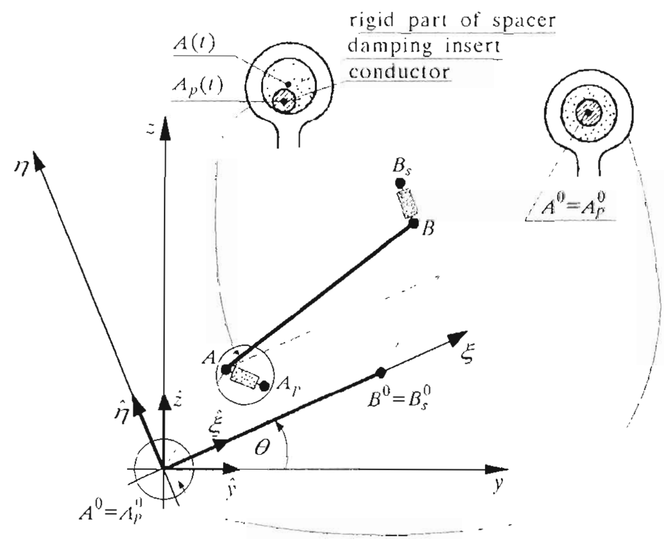

Fig. 2. Physical model of a spacer

The origin and the end of a spacer in the state of equilibrium coincide with the points A_p and B_s of the respective conductors. The points A^0, B^0, A_p^0, B_s^0 stand for the positions A, B, A_p, B_s in the equilibrium state.

The motion of the conductors and spacer causes the points A, B of spacer and A_p, B_s of conductors to displace.

Assuming small angles of rotation (Fig.3) we obtain the linear equations of motion around the static equilibrium in the local co-ordinate system

$$\begin{aligned} m\ddot{\delta}_{C\xi} &= H_{A\xi} + H_{B\xi} \\ m\ddot{\delta}_{C\eta} &= H_{A\eta} + H_{B\eta} \\ I\ddot{\delta}_{\varphi} &= H_{B\eta}(b-d) - H_{A\eta}d + (H_{A\xi} + H_{B\xi})a \end{aligned} \tag{2.1}$$

where

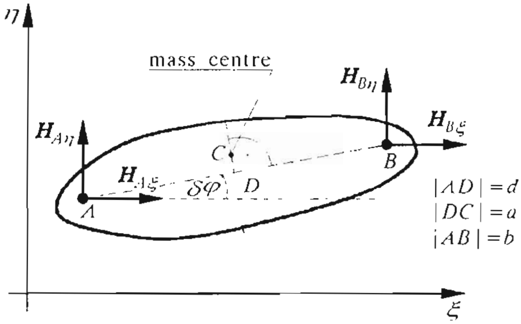

Fig. 3. Rigid element of a spacer

m	–	mass of the spacer
I	–	moment of inertia of the spacer
b	–	spacer length
d, a	–	distances defining the position of the spacer centre of mass
$\delta_{C\xi}, \delta_{C\eta}$	–	local components of the centre of mass displacement vector
$\delta\varphi$	–	angle of rotation

and $H_{A\xi}, H_{A\eta}, H_{B\xi}, H_{B\eta}$ – local components of the reaction forces from conductors.

For small rotations the following relations hold

$$\begin{aligned} \delta_{A\xi} &= \delta_{C\xi} + \delta_{\varphi}a & \qquad \delta_{B\xi} &= \delta_{C\xi} + \delta_{\varphi}a \\ \delta_{A\eta} &= \delta_{C\eta} - \delta_{\varphi}d & \qquad \delta_{B\eta} &= \delta_{C\eta} + \delta_{\varphi}(b-d) \end{aligned} \tag{2.2}$$

where $\delta_{A\xi}$, $\delta_{A\eta}$, $\delta_{B\xi}$, $\delta_{B\eta}$ are the local components of the displacement vectors of the origin and the end of spacer (index A stands for the quantities associated with the spacer origin, index B – with its end).

Damping elements are described by the general linear physical law in the form

$$\widehat{\mathsf{L}}\boldsymbol{H} = \widehat{\mathsf{M}}\boldsymbol{\Delta} \tag{2.3}$$

where

$\boldsymbol{H}$	–	vector of the forces acting on the origin $\boldsymbol{H}_A$ or the end $\boldsymbol{H}_B$ of spacer
$\boldsymbol{\Delta}$	–	relative displacement vector of the origin $\boldsymbol{\Delta}_A$ or the end $\boldsymbol{\Delta}_B$ of spacer

$\widehat{\mathsf{L}}, \widehat{\mathsf{M}}$ – matrices of differential operators

$$\widehat{\mathsf{L}} = \{\widehat{L}_{rt}\}_{r,t=1,2} \qquad \widehat{\mathsf{M}} = \{\widehat{M}_{rt}\}_{r,t=1,2}$$

with the follows differential operators are $(r, t = 1, 2)$

$$\begin{aligned} \widehat{L}_{rt} &= p_0^{(rt)} + p_1^{(rt)} \frac{d}{dt} + ... + p_{n_1^{(rt)}}^{(rt)} \frac{d^{n_1^{(rt)}}}{dt^{n_1^{(rt)}}} \\ \widehat{M}_{rt} &= q_0^{(rt)} + q_1^{(rt)} \frac{d}{dt} + ... + q_{n_2^{(rt)}}^{(rt)} \frac{d^{n_2^{(rt)}}}{dt^{n_2^{(rt)}}} \end{aligned} \tag{2.4}$$

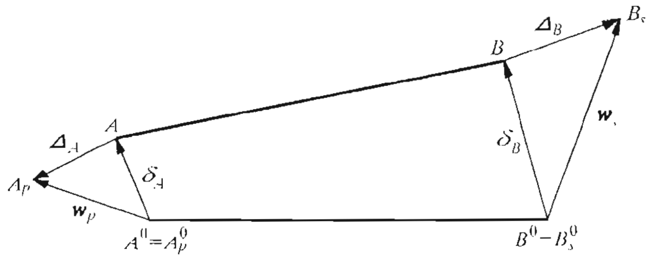

Fig. 4. Displacement vectors

The physical law for each spacer is defined in the local co-ordinate system. Writing down the components of forces $\boldsymbol{H}_A$, $\boldsymbol{H}_B$ and the components of relative displacement vectors $\boldsymbol{\Delta}_A$, $\boldsymbol{\Delta}_B$ in the local co-ordinate system, we obtain from Eq (2.3)

$$\begin{aligned} \widehat{\mathsf{L}}^A \begin{bmatrix} H_{A\xi} \\ H_{A\eta} \end{bmatrix} &= \widehat{\mathsf{M}}^A \begin{bmatrix} \Delta_{A\xi} \\ \Delta_{A\eta} \end{bmatrix} \\ \widehat{\mathsf{L}}^B \begin{bmatrix} H_{B\xi} \\ H_{B\eta} \end{bmatrix} &= \widehat{\mathsf{M}}^B \begin{bmatrix} \Delta_{B\xi} \\ \Delta_{B\eta} \end{bmatrix} \end{aligned} \tag{2.5}$$

where $\widehat{\mathsf{L}}^A$, $\widehat{\mathsf{M}}^A$, $\widehat{\mathsf{L}}^B$, $\widehat{\mathsf{M}}^B$ stand for the matrices of differential operators (Eq (2.4)).

In accordance with the notations in Fig.4 we have

$$
\begin{bmatrix} \Delta_{A\xi} \\ \Delta_{A\eta} \end{bmatrix} = \begin{bmatrix} w_{p\xi} \\ w_{p\eta} \end{bmatrix} - \begin{bmatrix} \delta_{A\xi} \\ \delta_{A\eta} \end{bmatrix}
$$

$$
\begin{bmatrix} \Delta_{B\xi} \\ \Delta_{B\eta} \end{bmatrix} = \begin{bmatrix} w_{s\xi} \\ w_{s\eta} \end{bmatrix} - \begin{bmatrix} \delta_{B\xi} \\ \delta_{B\eta} \end{bmatrix} \tag{2.6}
$$

The transformation between the local co-ordinate system associated with the spacer and the global one is represented by the transfer matrix

$$
\mathbf{\Phi} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{2.7}
$$

thus we obtain

$$
\begin{bmatrix} w_{p\xi} \\ w_{p\eta} \end{bmatrix} = \mathbf{\Phi} \begin{bmatrix} w_{py} \\ w_{pz} \end{bmatrix}
$$

$$
\begin{bmatrix} w_{s\xi} \\ w_{s\eta} \end{bmatrix} = \mathbf{\Phi} \begin{bmatrix} w_{sy} \\ w_{sz} \end{bmatrix} \tag{2.8}
$$

and

$$
\begin{bmatrix} H_{Ay} \\ H_{Az} \end{bmatrix} = \mathbf{\Phi}^{-1} \begin{bmatrix} H_{A\xi} \\ H_{A\eta} \end{bmatrix}
$$

$$
\begin{bmatrix} H_{By} \\ H_{Bz} \end{bmatrix} = \mathbf{\Phi}^{-1} \begin{bmatrix} H_{B\xi} \\ H_{B\eta} \end{bmatrix} \tag{2.9}
$$

3. Propagation of the wave travelling through the group of spacers

According to experimental results [5] we consider small vibrations and describe the physical and geometrical characteristics of the structure by linear differential and algebraic equations.

The equation of motion for the conductor lateral vibrations can be written as follows

$$
\mu\frac{\partial^2 w}{\partial t^2} - N\Big(\frac{\partial^2 w}{\partial x^2} + \alpha\frac{\partial^3 w}{\partial x^2 \partial t}\Big) = 0 \tag{3.1}
$$

where

$w(x,t)$ – function representing the lateral displacement
μ – linear mass density of the conductor
N – tensile force
α – dimensional coefficient of dissipation.

The waves which propagate along the power transmission line must satisfy the constraints resulting from the connection of conductors with spacers. The analytical form of these conditions depends on the approach used for description the structure motion. We use the superposition principle, which is applied here to travelling harmonic waves generated by all forces in the structure. We also assume that, apart from the external original sources, the forces in spacers are also the secondary sources of travelling harmonic waves entering the superposition. We can describe the waves by making use of unknown forces in spacers. These waves must satisfy the conditions resulting from the physical characteristics of spacers.

The basic formula used in the method is the expression for a harmonic wave travelling on each side of the point at which the steady state force is applied. From the elementary equilibrium conditions for a cable sector it appears that the force

$$\boldsymbol{F}(t) = \boldsymbol{F}_0 \mathrm{e}^{\mathrm{i}\nu t} \tag{3.2}$$

applied at the point x_0 of the cable generates a wave given by the formula

$$\boldsymbol{\psi}(x,t) = \frac{\boldsymbol{F}_0}{2\mathrm{i}kN(1+\mathrm{i}\alpha\nu)} \mathrm{e}^{-\mathrm{i}k|x-x_0|} \mathrm{e}^{\mathrm{i}\nu t} \tag{3.3}$$

where

$\boldsymbol{F}_0$ – complex vector of the force amplitude
ν – circular frequency of the wave
k – complex wave vector, the real part of which is connected with the wave length by the relation $\lambda = 2\pi/\Re(k)$, and the imaginary part is responsible for the decrease in the wave amplitude with the growing distance from the source

$$k = \sqrt{\frac{\mu}{N}\Big(\frac{\nu^2 - \mathrm{i}\alpha\nu^3}{1+\alpha^2\nu^2}\Big)}$$

The superscript f stands for the quantities associated with the fth spacer, while the indices s and p are equal to the conductor number joined with the end and with the origin of the fth spacer, respectively.

From Eqs (2.2), (2.6), (2.8) we obtain

$$
\begin{bmatrix} \Delta^f_{A\xi} \\ \Delta^f_{A\eta} \end{bmatrix} = \mathbf{\Phi}^f \begin{bmatrix} w^f_{py} \\ w^f_{pz} \end{bmatrix} - \begin{bmatrix} \delta^f_{C\xi} + \delta^f_{\varphi} a^f \\ \delta^f_{C\eta} - \delta^f_{\varphi} d^f \end{bmatrix}
$$

$$
\begin{bmatrix} \Delta^f_{B\xi} \\ \Delta^f_{B\eta} \end{bmatrix} = \mathbf{\Phi}^f \begin{bmatrix} w^f_{sy} \\ w^f_{sz} \end{bmatrix} - \begin{bmatrix} \delta^f_{C\xi} + \delta^f_{\varphi} a^f \\ \delta^f_{C\eta} + \delta^f_{\varphi}(b^f - d^f) \end{bmatrix} \tag{3.4}
$$

The components of displacement vectors of a cable are determined as a superposition of deflections caused by the propagation of harmonic waves

$$
\begin{bmatrix} w^f_{sy}(t) \\ w^f_{sz}(t) \end{bmatrix} = -\frac{1}{2\mathrm{i}kN(1+\mathrm{i}\alpha\nu)} \Bigg\{ \sum_{l=1}^{m} \Bigg(\begin{bmatrix} F^l_{Ay} \\ F^l_{Az} \end{bmatrix} q_{sl} +
$$

$$
+ \begin{bmatrix} F^l_{By} \\ F^l_{Bz} \end{bmatrix} (1 - q_{sl}) \Bigg) Q_{sl} \mathrm{e}^{-\mathrm{i}k|x^f - x^l|} - \sum_{l=1}^{n} \begin{bmatrix} P^l_y \\ P^l_z \end{bmatrix} G_{sl} \mathrm{e}^{-\mathrm{i}k|x^f - x^l_p|} \Bigg\} \mathrm{e}^{\mathrm{i}\nu t} \tag{3.5}
$$

In the above relation

x^l	–	co-ordinate of the lth spacer section
x^l_P	–	co-ordinate of section where the lth harmonic force is applied
P^l_y, P^l_z	–	amplitudes of the complex components of the lth force
$F^l_{Ay}, F^l_{Az}, F^l_{By}, F^l_{Bz}$	–	amplitudes of the complex components of forces acting at the origin and the end of the lth spacer
m	–	number of spacers
n	–	number of external dynamic harmonic sources (forces)
q_{sl}, Q_{sl}, G_{sl}	–	transformation coefficients defined as follows:

$$
q_{sl} = \begin{cases} 1 & \text{when the } s\text{th cable is connected with the origin of the } l\text{th spacer} \\ 0 & \text{when the } s\text{th cable is connected with the end of the } l\text{th spacer} \\ 2 & \text{when the } s\text{th cable is not connected with the } l\text{th spacer} \end{cases}
$$

$$
Q_{sl} = \begin{cases} 1 & \text{when the } s\text{th cable is connected with the } l\text{th spacer} \\ 0 & \text{when the } s\text{th cable is not connected with the } l\text{th spacer} \end{cases}
$$

$$G_{sl} = \begin{cases} 1 & \text{when the } l\text{th external force is applied to the } s\text{th cable} \\ 0 & \text{when the } l\text{th external force is not applied to the } s\text{th cable} \end{cases}$$

The coefficients q_{sl} and Q_{sl} determine in Eq (3.5) the forces marked with the symbol A (associated with the origins of spacers) or the forces marked with the symbol B (associated with the ends of spacers).

We define the complex amplitudes

$$\begin{bmatrix} \delta^f_{C\xi}(t) \\ \delta^f_{C\eta}(t) \\ \delta^f_{\varphi}(t) \end{bmatrix} = \begin{bmatrix} A^f_\xi \\ A^f_\eta \\ A^f_\varphi \end{bmatrix} e^{i\nu t}$$

$$\begin{bmatrix} w^f_{sy}(t) \\ w^f_{sz}(t) \end{bmatrix} = \begin{bmatrix} W^f_{sy} \\ W^f_{sz} \end{bmatrix} e^{i\nu t} \qquad \begin{bmatrix} w^f_{py}(t) \\ w^f_{pz}(t) \end{bmatrix} = \begin{bmatrix} W^f_{py} \\ W^f_{pz} \end{bmatrix} e^{i\nu t} \tag{3.6}$$

and the operators

$$L_{rt} = \sum_{j=0}^{n_1^{(rt)}} (i\nu)^j p_j^{(rt)} \qquad M_{rt} = \sum_{j=0}^{n_2^{(rt)}} (i\nu)^j q_j^{(rt)} \tag{3.7}$$

Substituting Eq (3.4) into Eq (2.5), making use of Eq (2.9) in Eq (3.5) and using Eqs (3.6) and (3.7), we obtain the system of equations with unknown complex amplitudes $F^l_{A\xi}$, $F^l_{A\eta}$, $F^l_{B\xi}$, $F^l_{B\eta}$ of forces and complex amplitudes A^l_ξ, A^l_η, A^l_φ $(l = 1, 2, ..., m)$ of displacements

$$\begin{aligned}
&-m^f \nu^2 A^f_\xi = F^f_{A\xi} + F^f_{B\xi} \\
&-m^f \nu^2 A^f_\eta = F^f_{A\eta} + F^f_{B\eta} \\
&-I^f \nu^2 A^f_\varphi = F^f_{B\eta}(b^f - d^f) - F^f_{A\eta} d^f + (F^f_{A\xi} + F^f_{B\xi}) a^f \\
&\mathsf{L}^{Af} \begin{bmatrix} F^f_{A\xi} \\ F^f_{A\eta} \end{bmatrix} = \mathsf{M}^{Af} \left\{ \mathbf{\Phi}^f \begin{bmatrix} W^f_{py} \\ W^f_{pz} \end{bmatrix} - \begin{bmatrix} A^f_\xi + A^f_\varphi a^f \\ A^f_\eta - A^f_\varphi d^f \end{bmatrix} \right\} \\
&\mathsf{L}^{Bf} \begin{bmatrix} F^f_{B\xi} \\ F^f_{B\eta} \end{bmatrix} = \mathsf{M}^{Bf} \left\{ \mathbf{\Phi}^f \begin{bmatrix} W^f_{sy} \\ W^f_{sz} \end{bmatrix} - \begin{bmatrix} A^f_\xi + A^f_\varphi a^f \\ A^f_\eta + A^f_\varphi (b^f - d^f) \end{bmatrix} \right\}
\end{aligned} \tag{3.8}$$

$$f = 1, 2, ..., m \qquad s = s(f) \qquad p = p(f)$$

where

$$
\begin{bmatrix} W^f_{sy} \\ W^f_{sz} \end{bmatrix} = -\frac{1}{2\mathrm{i}kN(1+\mathrm{i}\alpha\nu)} \Bigg\{ \sum_{l=1}^{m} [\boldsymbol{\Phi}^l]^{-1} \Bigg(\begin{bmatrix} F^l_{A\xi} \\ F^l_{A\eta} \end{bmatrix} q_{sl} +
+ \begin{bmatrix} F^l_{B\xi} \\ F^l_{B\eta} \end{bmatrix} (1-q_{sl}) \Bigg) Q_{sl} \mathrm{e}^{-\mathrm{i}k|x^f - x^l|} - \sum_{l=1}^{n} \begin{bmatrix} P^l_y \\ P^l_z \end{bmatrix} G_{sl} \mathrm{e}^{-\mathrm{i}k|x^f - x^l_p|} \Bigg\} \tag{3.9}
$$

The formulae for W^f_{py}, W^f_{pz} can be obtained from Eqs (3.8) only by replacing indices s with p.

In Eqs (3.8) $\mathbf{L}^{Af}$, $\mathbf{M}^{Af}$, $\mathbf{L}^{Bf}$, $\mathbf{M}^{Bf}$ stand for the matrices of algebraic operators defined as in Eqs (3.7), associated with the origin (symbol A) and with the end (symbol B) of the fth spacer. Substituting Eq (3.9) into Eqs (3.8) we get the system of $7m$ equations in $7m$ unknown quantities. Such a system must be written for each spacer in a group. After solving these equations we use complex force amplitudes to calculate the complex amplitudes of travelling waves. Then energy of the passed waves, energy of the reflected waves and energy dissipated in the spacers can be found.

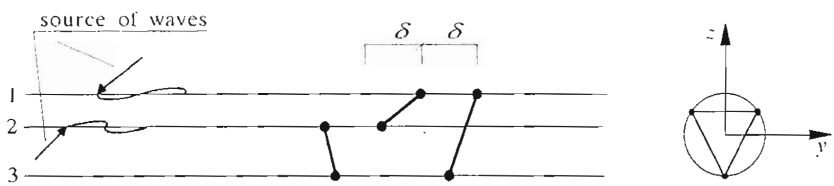

Fig. 5. Group of spacers

When analysing the propagation of harmonic wave through a group of spacers we have assumed that damping in a cable was negligible $\alpha = 0$. As the external sources of waves we consider two harmonic forces having the same amplitudes and opposite phases. They are parallel to the y-axis of the global co-ordinate system and are applied at a cross-section on the left-hand side of spacers to cables No. 1 and 2 (upper pair – Fig.5).

The following dimensionless quantities have been introduced

$$
\widetilde{x}^l = \frac{x^l}{\lambda} \qquad \text{(in the same way :} \widetilde{A}^l_\xi, \widetilde{A}^l_\eta, \widetilde{x}^l_P, \widetilde{W}^l_{sy}, \widetilde{W}^l_{sz}, \widetilde{b}^l, \widetilde{a}^l, \widetilde{d}^l)
$$

$$
\widetilde{A}^l_\varphi = A^l_\varphi
$$

$$\widetilde{F}^l_{A\xi} = \frac{F^l_{A\xi}}{N} \qquad \text{(in the same way :}\widetilde{F}^l_{A\eta}, \widetilde{F}^l_{B\xi}, \widetilde{F}^l_{B\eta}, \widetilde{P}^l_y, \widetilde{P}^l_z)$$

$$\left.\begin{array}{ll} \widetilde{q}^{(rt)}_j = \dfrac{q^{(rt)}_j \lambda}{N} \dfrac{1}{T^j} & j = 0,1,2,...,n^{(rt)}_2 \\ \widetilde{p}^{(rt)}_j = p^{(rt)}_j \dfrac{1}{T^j} & j = 0,1,2,...,n^{(rt)}_1 \qquad r,t = 1,2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{ \begin{array}{l} L_{rt} = \widetilde{L}_{rt} = \displaystyle\sum_{j=0}^{n^{(rt)}_1} (2\pi\mathrm{i})^j \widetilde{p}^{(rt)}_j \\ M_{rt} = \dfrac{N}{\lambda} \widetilde{M}_{rt} = \dfrac{N}{\lambda} \displaystyle\sum_{j=0}^{n^{(rt)}_2} (2\pi\mathrm{i})^j \widetilde{q}^{(rt)}_j \end{array} \right.$$

$$\widetilde{m}^l = \frac{m^l}{\mu\lambda} \qquad \widetilde{I}^l = \frac{I^l}{\mu\lambda^3}$$

where $T = 2\pi/\nu$ stands for the wave period.

Eqs (3.8) take the form

$$\begin{aligned} &-4\pi^2 \widetilde{m}^f \widetilde{A}^f_\xi = \widetilde{F}^f_{A\xi} + \widetilde{F}^f_{B\xi} \\ &-4\pi^2 \widetilde{m}^f \widetilde{A}^f_\eta = \widetilde{F}^f_{A\eta} + \widetilde{F}^f_{B\eta} \\ &-4\pi^2 \widetilde{I}^f \widetilde{A}^f_\varphi = \widetilde{F}^f_{B\eta}(\widetilde{b}^f - \widetilde{d}^f) - \widetilde{F}^f_{A\eta}\widetilde{d}^f + (\widetilde{F}^f_{A\xi} + \widetilde{F}^f_{B\xi})\widetilde{a}^f \\ &\widetilde{\mathbf{L}}^{Af} \begin{bmatrix} \widetilde{F}^f_{A\xi} \\ \widetilde{F}^f_{A\eta} \end{bmatrix} = \widetilde{\mathbf{M}}^{Af} \left\{ \mathbf{\Phi}^f \begin{bmatrix} \widetilde{W}^f_{py} \\ \widetilde{W}^f_{pz} \end{bmatrix} - \begin{bmatrix} \widetilde{A}^f_\xi + \widetilde{A}^f_\varphi \widetilde{a}^f \\ \widetilde{A}^f_\eta - \widetilde{A}^f_\varphi \widetilde{d}^f \end{bmatrix} \right\} \\ &\widetilde{\mathbf{L}}^{Bf} \begin{bmatrix} \widetilde{F}^f_{B\xi} \\ \widetilde{F}^f_{B\eta} \end{bmatrix} = \widetilde{\mathbf{M}}^{Bf} \left\{ \mathbf{\Phi}^f \begin{bmatrix} \widetilde{W}^f_{sy} \\ \widetilde{W}^f_{sz} \end{bmatrix} - \begin{bmatrix} \widetilde{A}^f_\xi + \widetilde{A}^f_\varphi \widetilde{a}^f \\ \widetilde{A}^f_\eta + \widetilde{A}^f_\varphi (\widetilde{b}^f - \widetilde{d}^f) \end{bmatrix} \right\} \\ &f = 1,2,...,m \qquad s = s(f) \qquad p = p(f) \end{aligned} \tag{3.10}$$

where

$$\begin{bmatrix} \widetilde{W}^f_{sy} \\ \widetilde{W}^f_{sz} \end{bmatrix} = -\frac{1}{4\pi \mathrm{i}} \left\{ \sum_{l=1}^{m} [\mathbf{\Phi}^l]^{-1} \left(\begin{bmatrix} \widetilde{F}^l_{A\xi} \\ \widetilde{F}^l_{A\eta} \end{bmatrix} q_{sl} + \right.\right.$$

$$\left.\left. + \begin{bmatrix} \widetilde{F}^l_{B\xi} \\ \widetilde{F}^l_{B\eta} \end{bmatrix} (1 - q_{sl}) \right) Q_{sl} \mathrm{e}^{-\mathrm{i}2\pi|\widetilde{x}^f - \widetilde{x}^l|} - \sum_{l=1}^{2} \begin{bmatrix} \widetilde{P}^l_y \\ \widetilde{P}^l_z \end{bmatrix} G_{sl} \mathrm{e}^{-\mathrm{i}2\pi|\widetilde{x}^f|} \right\} \tag{3.11}$$

In numerical computations we assume that all spacers are made of the same material and their construction is the same. The flexible elements of spacers are represented by the standard model of visco-elasticity (Fig.6).

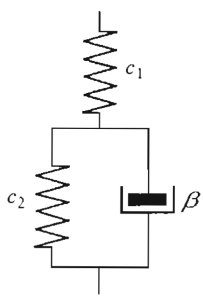

Fig. 6. Standard model of spacer holders

The algebraic operators have the following form $(f = 1, 2, ..., m)$

$$L^{Af}_{12} = L^{Af}_{21} = L^{Bf}_{12} = L^{Bf}_{21} = 0$$

$$M^{Af}_{12} = M^{Af}_{21} = M^{Bf}_{12} = M^{Bf}_{21} = 0$$

$$L^{Af}_{11} = L^{Af}_{22} = L^{Bf}_{11} = L^{Bf}_{22} = p_0 + (\mathrm{i}\nu)p_1$$

$$M^{Af}_{11} = M^{Af}_{22} = M^{Bf}_{11} = M^{Bf}_{22} = q_0 + (\mathrm{i}\nu)q_1$$

where

$$p_0 = 1 \qquad\qquad q_0 = \frac{c_2 c_1}{c_2 + c_1}$$

$$p_1 = \frac{\beta}{c_2 + c_1} \qquad\qquad q_1 = \frac{\beta c_1}{c_2 + c_1}$$

The dimensionless parameters of the standard model have the form

$$\widetilde{c}_1 = \frac{c_1 \lambda}{N} \qquad \widetilde{c}_2 = \frac{c_2 \lambda}{N} \qquad \widetilde{\beta} = \frac{\beta \lambda}{N} \frac{1}{T}$$

The dimensionless distance between the spacers in a group is additionally introduced

$$\widetilde{\delta} = \frac{\delta}{\lambda}$$

Rigid part of each spacer is a homogeneous rod with the centre of mass lying in its geometrical centre $(a^f = 0;\ d^f = b^f/2)$.

The results of computations are shown in Fig.7 and Fig.8. The powers shown in the graphs are dimensionless. The power of incidence waves is equal to 1. They run from the left-hand side of the group of spacers.

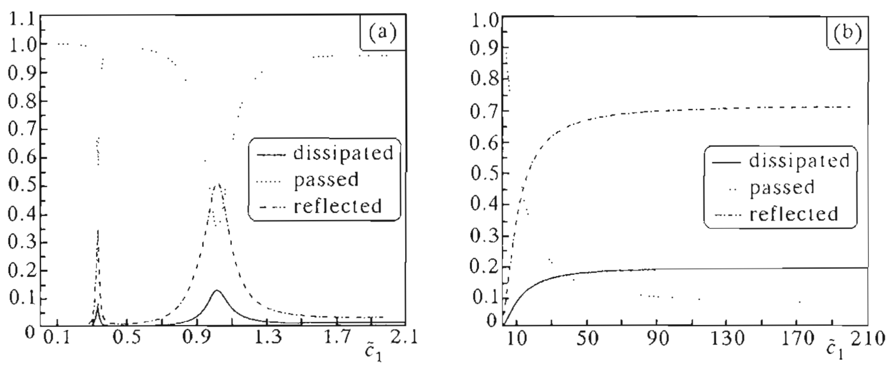

Fig. 7. Power dissipated, power of the passed waves and power of the reflected waves; $\widetilde{c}_2 = 20$, $\widetilde{\beta} = 16$, $\widetilde{\delta} = 0.1$, $\widetilde{b} = 0.01$, $\widetilde{m} = 0.05$

4. Optimal choice of damping coefficients

As an example we consider optimization of the spacer with respect to maximum of dissipated energy. Numerical calculations of the optimal damping have been made the following method. The damping coefficients of spacers β_1, β_2, β_3 are the parameters of optimization. We assume $c_1 = \infty$ and the same c_2 in all spacers of the group. The numerical program calculates optimal damping for arbitrary c_2, using the sequence quadratic programming method. To calculate the direction of the solution improvement the quadratic programming problem is solved. The Hesjan is updated by the variable metric method.

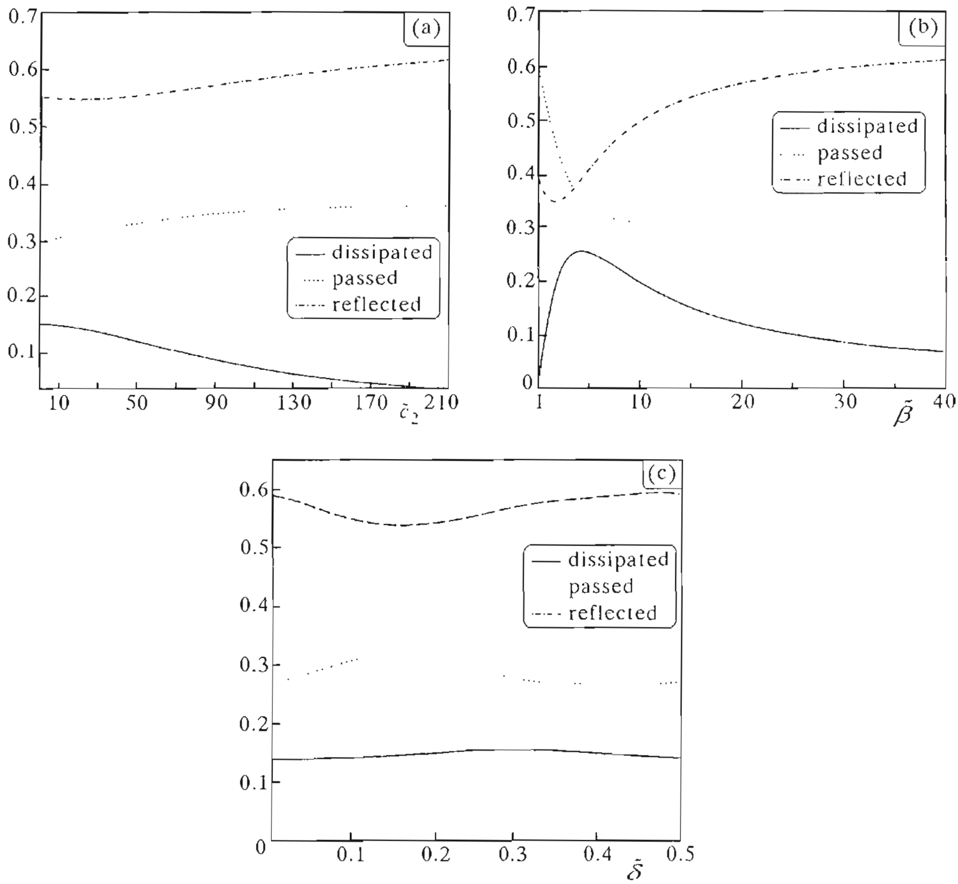

Fig. 8. Power dissipated, power of the passed waves and power of the reflected waves; (a) $\widetilde{c}_1 = 20$, $\widetilde{\beta} = 16$, $\widetilde{\delta} = 0.1$, $\widetilde{b} = 0.01$, $\widetilde{m} = 0.05$;
(b) $\widetilde{c}_1 = 20$, $\widetilde{c}_2 = 20$, $\widetilde{\delta} = 0.1$, $\widetilde{b} = 0.01$, $\widetilde{m} = 0.05$;
(c) $\widetilde{c}_1 = 20$, $\widetilde{c}_2 = 20$, $\widetilde{\beta} = 16$, $\widetilde{b} = 0.01$, $\widetilde{m} = 0.05$

After determining the direction of solution improvement of the maximization in this direction is realized.

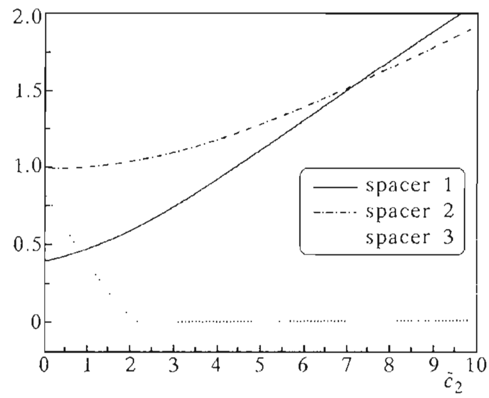

Fig. 9. Optimal damping parameters

In Fig.9 the optimal damping parameters β_1, β_2, β_3 versus c_2 are shown.

5. Conclusions

The following conclusions can be drawn from the analysis of the diagrams presented:

- Power of the passed waves varies with the stiffness parameter c_1. For $c_1 = 0$ the conductors in a bundle are independent of each other and the source generates travelling waves in the two upper conductors, the power of the passed waves is equal to 1. There are resonances of the system for small values of c_1. The power of the passed waves is nearly constant for great values of c_1.
- The curve representing the dependence of energy dissipated in the spacers on the damping parameter β reveals a maximum. The parameter β increase kinematically stiffens the spacers causing the decrease in the motion amplitudes of the spacer holders.
- There are optimal damping parameters of the spacers with respect to the maximum of dissipated energy for arbitrary stiffness parameters.

The algorithm shown in this paper may be used in the designing of the group of spacers. The energy of the passed waves and the dissipated energy can be used as a criterion when choosing the optimal group of spacers.

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Analiza propagacji drgań odcinkowych przez grupy odstępników w wiązkach przewodów

Streszczenie

W pracy przedstawiono teoretyczny model przejścia fali biegnącej przez grupy odstępników w wiązce przewodów elektroenergetycznych. Wyznaczono energię fali padającej i fali odbitej oraz energię rozproszoną w odstępnikach. Pokazano, że przy odpowiedniej konstrukcji odstępników można obniżyć drgania odcinkowe przewodów.

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