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APPLICATION OF STATISTICAL LINEARIZATION TECHNIQUES TO DESIGN OF QUASI-OPTIMAL ACTIVE CONTROL OF NONLINEAR SYSTEMS

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A comparison between the applicability of statistical linearization methods with moment criteria and criteria in probability density space to the determination of quasi-optimal external control for the nonlinear dynamic system excited by a coloured Gaussian noise and the mean square criterion is discussed in this paper. To determine the quasi-optimal control two modified versions of a standard iterative procedure are proposed, where three versions of statistical linearization with moment criteria and two versions with criteria in the probability density space were combined with the optimal control method for linear systems with the mean square criterion. The detailed considerations are given for a nonlinear 2-degree-of-freedom system with external control force excited by a coloured Gaussian noise which is treated as an output of 2D linear filter. The control is assumed as a linear feedback. The obtained results are illustrated by a numerical example.

Key words: statistical linearization, stochastic optimal control, active suspensions

1. Introduction

Considerable progress has been made over the last three decades in active control applications in mechanical and civil engineering. In a general case, for nonlinear stochastic models the problem of the determination of optimal control remains unsolved (Stengel, 1986; Zhu et al., 1999). Therefore, several approximate approaches were proposed. One of the most attractive was application of the statistical linearization and Linear Quadratic Gaussian (LQG) theory to an iterative procedure (Beaman, 1984; Heess, 1970; Yoshida, 1984).

The linearization coefficients were determined from the mean square criterion. Since in vibration analysis of stochastic systems in mechanical and structural engineering several criteria of statistical and equivalent linearization were proposed, the objective of this paper is to compare these approaches when applied to the study of optimal stochastic control for nonlinear systems. A simple comparison between three statistical linearization techniques applied application to determination of quasi-optimal active control of a nonlinear 2-degree-of-freedom vehicle model was given by Socha (1999b).

The objective of this paper is to study applicability of a new linearization technique, namely, the statistical linearization with probability density criteria and LQG theory to the determination of quasi-optimal control. In this approach the linearization coefficients are found basing on the criterion of linearization which is a probabilistic metric in probability density space. The elements of this space are found as probability density fuctions of random variables resulting from linear and nonlinear transformations of one-dimensional Gaussian variables. This linearization technique was presented by the author (see Socha, 1999a). In contrast to the mean square and other moment criteria determination of the linearization coefficients using criteria in the probability density space requires an application of a minimization procedure.

A nonlinear 2-degree-of-freedom system is considered in detail, subjest to an external control force excited by coloured Gaussian noise treated as an output of a 2D linear filter. The mean square criterion is proposed when solving the optimal control problem and the control is assumed as linear feedback. To find the coefficients which determine the quasi-optimal control first, the statistical linearization is applied and next the standard LQG procedure is used (cf Kwakernak and Sivan, 1972). The solution is obtained from an iterative procedure, where the Riccatti and Lyapunov equations are solved and a standard minimization procedure is applied. The obtained results are illustrated by numerical example.

2. Short review of statistical linearization techniques

The earliest works ofin the theory of statistical linearization applied to control engineering were conducted independently by Botton (1954) and Kazakov (1956). The method consists in nonlinear elements appearing in the model by linear forms, while the coefficients of linearization can be found basing on a specific criterion of linearization.

To review these approaches and other linearization techniques we consider a nonlinear stochastic model of dynamic system described by the Ito differential vector equation

$$d\boldsymbol{z}(t) = \boldsymbol{\Phi}(\boldsymbol{z})dt + \sum_{k=1}^{M} G_k d\xi_k(t) \tag{2.1}$$

where

$\boldsymbol{z}$	–	state vector, $\boldsymbol{z} = [z_1, ..., z_n]^\top$
$\boldsymbol{\Phi}$	–	vector nonlinear function such that $\boldsymbol{\Phi}(0) = \mathbf{0}$, $\boldsymbol{\Phi} = [\Phi_1, ..., \Phi_n]^\top$
G_k	–	deterministic vectors, $G_k = [G_{k1}, ..., G_{kn}]^\top$
ξ_k	–	independent standard Wiener processes.

We assume that a unique solution of Eq (2.1) exists.

The objective of statistical linearization is to find for nonlinear elements of vector $\boldsymbol{\Phi}(\boldsymbol{x})$ the corresponding equivalent ones "in some sense" but in a linear form. Let x_j be a linear combination of elements z_l, $l = 1, ..., n$ of the state vector $\boldsymbol{z}$, i.e.

$$x_j = \sum_{l=1}^{n} \alpha_{jl} z_l \qquad j = 1, ..., n \tag{2.2}$$

where α_{jl} are constant parameters. Then the substitution can be done for nonlinear elements into Eq (2.1)

$$Y_j = \psi_j(x_j) \qquad j = 1, ..., n \tag{2.3}$$

using a linearized form

$$Y_j = c_j x_j \qquad j = 1, ..., n \tag{2.4}$$

To obtain the linearization coefficients of the linearization techniques may be employed. They can be devided into the two groups; namely, statistical linearization with moment criteria and statistical linearization with criteria in probability density function space. Hereinafter we quote these approaches.

2.1. Moment criteria

The following criteria for scalar functions $\psi_j(x_j)$, $j = 1, ..., n$ are considered:

Criterion 1. Equality of second order moments (Kazakov, 1956)

$$E\Big[(c_{1j}x_j)^2\Big] = E\Big[(\psi_j(x_j))^2\Big] \tag{2.5}$$

Criterion 2. Mean square error of displacements (Kazakov, 1956)

$$E\Big[(c_{2j}x_j - \psi_j(x_j))^2\Big] \to \min \tag{2.6}$$

Criterion 3. Mean square error of potential energies (Elishakoff and Zhang, 1992)

$$E\Big[\Big(\int_0^{x_j} [c_{3j}v - \psi_j(v)]\; dv\Big)^2\Big] \to \min \tag{2.7}$$

where c_{rj}, $r = 1,2,3$, $j = 1,...,n$ are the linearization coefficients.

In these criteria the expectation does not depend on the linearization coefficients and the corresponding probability density functions depend on 1D variables. We note that the linearization coefficients c_{rj} are nonlinear functions of variances of variables x_j which can be calculated from second order moments of coordinates of the state vector $\boldsymbol{z}$. It follows from the equation

$$\sigma_{x_j}^2 = E\Big[\Big(\sum_{l=1}^{n} \alpha_{jl}z_l\Big)^2\Big] \qquad j = 1, ..., n \tag{2.8}$$

2.2. Criteria in the probability density functions space

Bellow the two equivalence criteria in probability density space for scalar functions $\psi_j(z_j)$, $j = 1,...,n$ are presented

Criterion 4. Square probability metric (Socha, 1999a)

$$I_{d_j} = \int_{-\infty}^{+\infty} [g_N(y_j) - g_L(y_j)]^2\; dy_j \tag{2.9}$$

where $g_N(y)$ and $g_L(y)$ are the probability density functions of the variables defined by Eqs (2.3) and (2.4), respectively.

Criterion 5. Pseudo-moment probability metric (Socha, 1999a)

$$I_{e_j} = \int_{-\infty}^{+\infty} |y_j|^{2l} |g_N(y_j) - g_L(y_j)|\; dy_j \qquad l = 1, 2, ... \tag{2.10}$$

If we assume that the input processes are the Gaussian ones with the mean values $m_{x_j} = 0$ for $j = 1, ..., n$ and the probability density functions

$$g_I(x_j) = \frac{1}{\sqrt{2\pi}\sigma_{x_j}} \exp\Big(-\frac{x_j^2}{2\sigma_{x_j}^2}\Big) \tag{2.11}$$

where $\sigma_{x_j}^2 = E[x_j^2]$, then the output processes Y_j for $j = 1, ..., n$ from the static linear elements defined by Eq (2.4) are also Gaussian and the corresponding probability density functions are given by

$$g_L(y_j) = \frac{1}{\sqrt{2\pi} c_j \sigma_{x_j}} \exp\Big(-\frac{y_j^2}{2c_j^2\sigma_{x_j}^2}\Big) \tag{2.12}$$

To apply the proposed criteria, Eqs (2.9) and (2.10), we have to find the probability density functions $g_N(y_j)$. Unfortunately, except for some special cases it is impossible to find them in an analytical form. It is well known that one of these special cases is a scalar strictly monotonically increasing or decreasing function

$$Y_j = \psi_j(x_j) \qquad j = 1, ..., n \tag{2.13}$$

with continuous derivatives $\psi_j'(x_j)$ for all $x_j \in R$. Then the probability density function of the output variable (2.13) is given by

$$g_Y(y_j) = g_I(h(y_j))|h'(y_j)| \qquad j = 1, ..., n \tag{2.14}$$

where $g_I(x_j)$, $j = 1, ..., n$ are the probability density functions of the input variables and h_j are the inverse functions to $\psi_j(x_j)$

$$x_j = h_j(Y_j) = \psi_j^{-1}(Y_j) \qquad j = 1, ..., n \tag{2.15}$$

In a general case when the nonlinear functions $\psi_j(x_j)$ are not strongly monotonically increasing or decreasing or not differentiable everywhere the approximation methods have to be used (Pugacev and Sinicyn, 1985).

In contrast to the standard statistical linearization with criteria in the state space one can not find the formulae for linearization coefficients in an analytical form. However, in some special cases some analytical considerations can be done. For instance, for the criterion I_{d_j} defined by Eq (2.9) and for an input Gaussian process with the mean equal to zero the necessary condition of minimum can be derived for $j = 1, ..., n$ in the following form

$$\frac{\partial I_{d_j}(t)}{\partial c_j} = 2 \int_{-\infty}^{+\infty} [g_N(y_j, t) - g_L(y_j, t)] \frac{1}{c_j} \Big(1 - \frac{y_j^2}{c_j^2 q^2}\Big) g_L(y_j, t)\, dy_j = 0 \tag{2.16}$$

To apply the proposed linearization technique to the determination of the linearization coefficient k and response characteristics one may use an iterative procedure involving minimization of one of the proposed criteria and a solution of the Lyapunov differential equation. A proposition of such a procedure will be given in Section 5.

3. Vehicle model

Consider the linear two-degree-of-freedom vehicle model shown in Fig.1 with one nonlinear suspension spring between the masses m_1 and m_2. $u(t)$ denotes the active suspension force (acting independently of the forces in the passive elements).

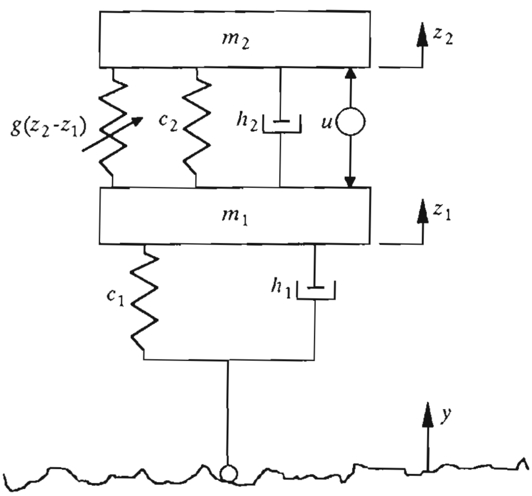

Fig. 1. Two-degree-of-freedom vehicle model

The equations of motion of the system after a simple transformation can be written as

$$
\begin{aligned}
dx_1 &= x_3 dt \qquad\qquad dx_2 = x_4 dt \\
dx_3 &= \frac{1}{m_1}\Big\{-c_1x_1 - h_1x_3 + c_2x_2 + h_2x_4 + g(x_2) - u + \\
&+ m_1[a_1a_2x_5 + (a_1 + a_2)x_5]\Big\}dt - qd\xi
\end{aligned}
\tag{3.1}
$$

$$
\begin{aligned}
dx_4 &= \frac{1}{m_2}[-c_2x_2 - h_2x_4 - g(x_2) + u]dt + \\
&+ \frac{1}{m_1}[-c_1x_1 + h_1x_3 - c_2x_2 - h_2x_4 - g(x_2) + u]dt \\
dx_5 &= x_6 dt \qquad\qquad dx_6 = [-a_1a_2x_5 - (a_1 + a_2)x_6]dt + qd\xi
\end{aligned}
$$

where the state variables are defined by

$$
\begin{array}{lll}
x_1 = z_1 - y & x_2 = z_2 - z_1 & x_3 = \dot{z}_1 - \dot{y} \\
x_4 = \dot{z}_2 - \dot{z}_1 & x_5 = y & x_6 = \dot{y}
\end{array}
\tag{3.2}
$$

and

c_1, c_2 – stiffness constants
h_1, h_2 – damping constant parameters
ξ – standard Wiener process
a_1, a_2, q – constant parameters of the linear filter defined by

$$
a_1 = a_1^* v \qquad a_2 = a_2^* v \qquad q = q^* \sqrt{a_1 a_2 v}
\tag{3.3}
$$

where a_1^*, a_2^* and q^* are constant parameters of the random road profile and v is a constant speed of the vehicle.

4. Performance index and optimal active control

The active control u aims at minimization of the joint performance index I defined by the stationary response characteristics of the system (3.1)

$$
I = \rho_1 I_1 + \rho_2 I_2 + \rho_3 I_3 + \rho_4 I_4
\tag{4.1}
$$

where the partial performance indexes:

I_1 – the measure of a ride comfort
I_2 – limit of the space required for suspension
I_3 – avoids loosing contact between the wheel and road
I_4 – limits the control force
ρ_i – weight coefficients, $(i = 1, ..., 4)$.

This criterion is a modified version of the criterion given for a linear model by Hać (1985). In new state variables the performance index I has the form

$$
I = \frac{\rho_1}{m_2^2} E\Big[\Big(c_2x_2 + h_2x_4 + g(x_2) - u\Big)^2\Big] + \rho_2 E[x_1^2] + \rho_3 E[x_1^2] + \rho_4 E[u^2]
\tag{4.2}
$$

Here the stationary moments are considered. If the following linearized form can substitute the nonlinear stiffness $g(x_2)$ in

$$g(x_2) = \alpha k x_2 \tag{4.3}$$

where α is the constant parameter and k is the linearization coefficient, then the optimal control problem can be transformed to the standard one

$$\begin{aligned} d\boldsymbol{x} &= [\mathsf{A}(k)\boldsymbol{x} + \boldsymbol{B}u]dt + \boldsymbol{G}d\xi \\ I &= E[\boldsymbol{x}^\top \mathsf{Q}(k)\boldsymbol{x} + 2\boldsymbol{x}^\top \boldsymbol{N}(k)u + ru^2] \to \min \end{aligned} \tag{4.4}$$

where

$\boldsymbol{x}$	–	state vector, $\boldsymbol{x} = [x_1, ..., x_6]^\top$
$\mathsf{A}(k), \mathsf{Q}(k)$	–	matrices dependent on linearization coefficient k
$\boldsymbol{B}, \boldsymbol{G}, \boldsymbol{N}(k)$	–	vectors
r	–	scalar defined by the parameters of Eqs (3.1).

In this paper we compare three methods of statistical linearization for the Gaussian excitations corresponding to the following moment criteria:

Criterion 1. Equality of second order moments (Kazakov, 1956)

$$E\Big[(k_a x_2)^2\Big] = E\Big[\Big(g(x_2)\Big)^2\Big] \tag{4.5}$$

Criterion 2. Mean square error of displacements (Kazakov, 1956)

$$E\Big[\Big(k_b x_2 - g(x_2)\Big)^2\Big] \to \min \tag{4.6}$$

Criterion 3. Mean square error of potential energies (Elishakoff and Zhang, 1992)

$$E\Big[\Big(\int\limits_0^{x_2} [k_c v - g(v)]dv\Big)^2\Big] \to \min \tag{4.7}$$

where c_{rj}, $r = 1,2,3$, $j = 1, ..., n$ are linearization coefficients.

and two criteria in the probability density functions space:

Criterion 4. Square probability metric (Socha, 1999a)

$$\int\limits_{-\infty}^{+\infty} [g_Y(y) - g_L(y, k_d)]^2 dy \to \min \tag{4.8}$$

where $g_N(y)$ and $g_L(y)$ are the probability density functions of variables defined by Eqs (2.3) and (2.4), respectively.

Criterion 5. Pseudo-moment probability metric (Socha, 1999a)

$$\int_{-\infty}^{+\infty} |y|^2 |[g_Y(y) - g_L(y, k_d)]| \, dy \to \min \tag{4.9}$$

where the 1D nonlinearity Y is defined by

$$Y = \psi(x_2) = c_2 x_2 + g(x_2) \tag{4.10}$$

In this case the linearized form substitutes the nonlinearity $\psi(x_2)$

$$\psi(x_2) = k^* x_2 \tag{4.11}$$

where k^* is the linearization coefficient. Then the optimal control problem can be transformed to the problem represented by Eqs (4.4) for $k = k^*$. We note that if $g(0) = 0$ then k_a, k_b and k_c are nonlinear functions of the stationary second order moments $k_i = k_i(E[x_2^2])$ for a, b and c. As an example of nonlinear function $g(x_2)$ we consider

$$g(x_2) = \alpha x_2^3 \tag{4.12}$$

One can show (cf Elishakoff and Zhang, 1992; Kazakov, 1956) that the corresponding linearization coefficients have the form

$$k_a = \sqrt{15}\, E[x_2^2] \qquad k_b = 3E[x_2^2] \qquad k_c = 2.5E[x_2^2] \tag{4.13}$$

and in the case of criteria 4 and 5 the nonlinear function $\psi(x_2)$ and the corresponding probability density function for a nonlinear variable have the form

$$\begin{aligned} &\psi(x_2) = c_2 x_2 + \alpha x_2^3 \\ &g_Y(y) = \frac{1}{\sqrt{2\pi}\sigma_L} \exp\Big[-\frac{(v_1+v_2)^2}{2\sigma_L^2}\Big] \frac{1}{6a\varepsilon}\Big(\frac{a+y}{v_1^2} + \frac{a-y}{v_2^2}\Big) \end{aligned} \tag{4.14}$$

where

$$\begin{aligned} &v_1 = \sqrt[3]{\frac{y}{2\alpha} + \sqrt{\frac{y^2}{4\alpha^2} + \frac{c_2^3}{27\alpha^3}}} \qquad v_2 = \sqrt[3]{\frac{y}{2\alpha} - \sqrt{\frac{y^2}{4\alpha^2} + \frac{c_2^3}{27\alpha^3}}} \\ &a = \sqrt{y^2 + \frac{4c_2^3}{27\alpha}} \end{aligned} \tag{4.15}$$

The probability density of the linearized variable

$$y = k^* x_2 \tag{4.16}$$

has the form

$$g_L(y, k^*) = \frac{1}{\sqrt{2\pi} k^* \sigma_L} \exp\Big(- \frac{y^2}{2k^{*2}\sigma_L^2} \Big) \tag{4.17}$$

where $\sigma_L^2 = E[x^2]$ is the variance of the input Gaussian variable, k^* is equal to k_d or k_e on criterion 4 or 5.

To determine the optimal cotrol for a nonlinear system employing nonlinear criterion we accept the idea proposed in the literature; e.g., by Beaman (1984), Yoshida (1984) consisting in application the statistical linearization and stochastic optimal control method to a linear system with mean square criterion. These two standard approaches are basic steps in the two iterative procedures which are given in the next section.

The linearization coefficient results from minimization of criterion (4.8) or (4.9). To obtain the quasi-optimal control one can use the iterative procedures A and B proposed in Section 5.

5. Iterative procedures

The difference between both the approaches i.e. statistical linearization techniques using the moment criteria and criteria in the probability density space, respectively, implies differences between the iterative procedures for the determination of quasi-optimal controls for the Nonlinear Quadratic Gaussian problem. In the case of statistical linearization using moment criteria determination of the linearization coefficients requires a modified version of the standard iterative procedure (cf Yoshida, 1984) while in the case of statistical linearization using criteria in the probability density function space a new iterative procedure can be applied. The following two procedures are proposed.

5.1. Procedure A (for criteria $1 \div 3$)

Step 1. Assume that one of the linearization coefficiets is equal to zero, for instance, $k = k_a = 0$.

Step 2. Calculate $\mathsf{A} = \mathsf{A}(k)$ and $\boldsymbol{N} = \boldsymbol{N}(k)$ in Eq $(4.4)_1$ and then solve the algebraic Riccati equation

$$\mathsf{P}\Big(\mathsf{A} - \frac{1}{r}\boldsymbol{B}\boldsymbol{N}^\top\Big) + \Big(\mathsf{A} - \frac{1}{r}\boldsymbol{B}\boldsymbol{N}^\top\Big)^\top \mathsf{P} - \mathsf{P}\boldsymbol{B}\mathsf{R}^{-1}\boldsymbol{B}^\top\mathsf{P} + \Big(\mathsf{Q} - \frac{1}{r}\boldsymbol{N}\boldsymbol{N}^\top\Big) = \mathbf{0} \tag{5.1}$$

The solution is a symmetric positive definite matrix P.

Step 3. Find the optimal control and the matrix C

$$u(t) = -\mathsf{C}\boldsymbol{x}(t) = -\frac{1}{r}(\boldsymbol{N}^\top + \boldsymbol{B}^\top\mathsf{P})\boldsymbol{x}(t) \tag{5.2}$$

Next, substitute for C, $\mathsf{A}(k)$ and $\boldsymbol{N}(k)$ into the covariance equation

$$(\mathsf{A} - \boldsymbol{B}\mathsf{C})V_L + V_L(\mathsf{A} - \boldsymbol{B}\mathsf{C})^\top + \boldsymbol{G}\boldsymbol{G}^\top = \mathbf{0} \tag{5.3}$$

and solve the equation. The solution of Eq (5.3) is V_L.

Step 4. Substitute for the element of covariance matrix $E[x_2^2] = V_{L_{22}}$ obtained in step 3 into the linearization coefficient k_a defined by Eq (4.13).

Step 5. Calculate P, u and V_L using Eqs (5.1) $\div$ (5.3) and the linearization coefficient k obtained in the last step.

Step 6. Iterate steps $2 \div 5$ until V_L and P converge.

Step 7. Calculate the optimal value of criterion I_{opt} using the solution of the Riccati equation obtained in step 5

$$I_{opt} = \operatorname{tr}(\mathsf{P}\boldsymbol{G}\boldsymbol{G}^\top) \tag{5.4}$$

5.2. Procedure B (for criteria $4 \div 5$)

Step 1. Assume $k^* = c_2$.

Step 2. Calculate $\mathsf{A} = \mathsf{A}(k^*)$ and $\boldsymbol{N} = \boldsymbol{N}(k^*)$ in Eq $(4.4)_1$ and then solve Eq (5.1). The solution of Eq (5.1) is P.

Step 3. Substitute P obtained in step 2 into Eq (5.2) and find the matrix C. Next, substitute for C and $\mathsf{A}(k^*)$ into Eq (5.3) and solve the equation. The solution of Eq (5.3) is V_L.

Step 4. For C obtained in the previous step calculate for linearized element the variance $\sigma_{x_2}^2 = E[x_2^2]$ of the input Gaussian variable and next the corresponding probability density functions given by Eqs $(4.14)_2$ and (4.17), respectively.

Step 5. For nonlinear element find the linearization coefficient k^* which minimizes, for instance, Criterion 4.

Step 6. Substitute to the linearization coefficient k^* from step 5 into Eq (5.3) and then solve the equation.

Step 7. If the error of accuracy is greater then a given parameter ε_1 then repeat steps $4 \div 6$ until V_L converges.

Step 8. Calculate new $\mathbf{A}(k^*)$ and $\boldsymbol{N}(k^*)$ and next using these matrices calculate $\mathbf{P}$, $\mathbf{C}$ and V_L using Eqs $(5.1) \div (5.3)$.

Step 9. Iterate steps $4 \div 8$ until V_L and $\mathbf{P}$ converge.

Step 10. Calculate the optimal value of criterion I_{opt} substituting the solution of Riccati equation obtained in step 8 into Eq (5.4).

To determine the probability density function from Eq (2.12) we have to solve moment equations higher order for the whole nonlinear dynamic system using one of closure techniques and then to calculate higher order moments for variables from the domain of nonlinear elements.

6. Numerical results

To illustrate the results obtained a comparison of the criterion I_{opt} defined by Eq (5.4) versus parameter α has been shown. In this comparison three moment criteria i.e. equality of second order moments of nonlinear and linearized elements, mean square error of the displacement, mean square error of the potential energies and two criteria in the probability density function space i.e. square probability metric and pseudomoment probability metric are considered. The numerical results denoted by lines with squares, stars, circles crosses and triangels, respectively, are presented in Fig.2. The parameters selected for calculations and simulations are $m_1 = 100$, $m_2 = 500$, $c_1 = 100$, $c_2 = 50$, $h_1 = 1$, $h_2 = 5$, $a_1^* = 0.025$, $a_2^* = 0.075$, $q^* = \sqrt{0.0067}$, $v = 20$, $\rho_1 = 1$, $\rho_2 = 1000$, $\rho_3 = 10000$, $\rho_4 = 1$.

Fig.3 shows the dependence of the criterion I_{opt} upon the speed of vehicle as it changes from 10^0 to 10^2. The other parameters are the same $\rho_2 = 100$, $\rho_3 = 1000$ and $\alpha = 20$.

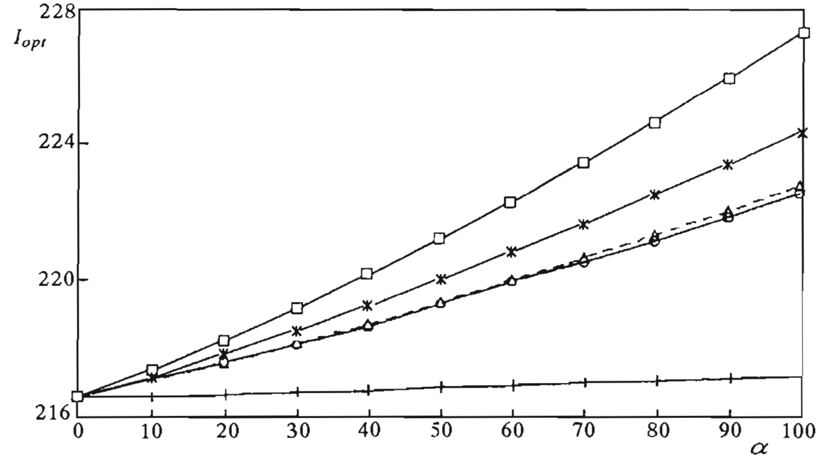

Fig. 2. Comparison between optimal criteria obtained by application of different statistical linearization techniques versus parameter α; □ – crit. 1, $*$ – crit. 2, ○ – crit. 3, + – crit. 4, △ – crit. 5 (broken line)

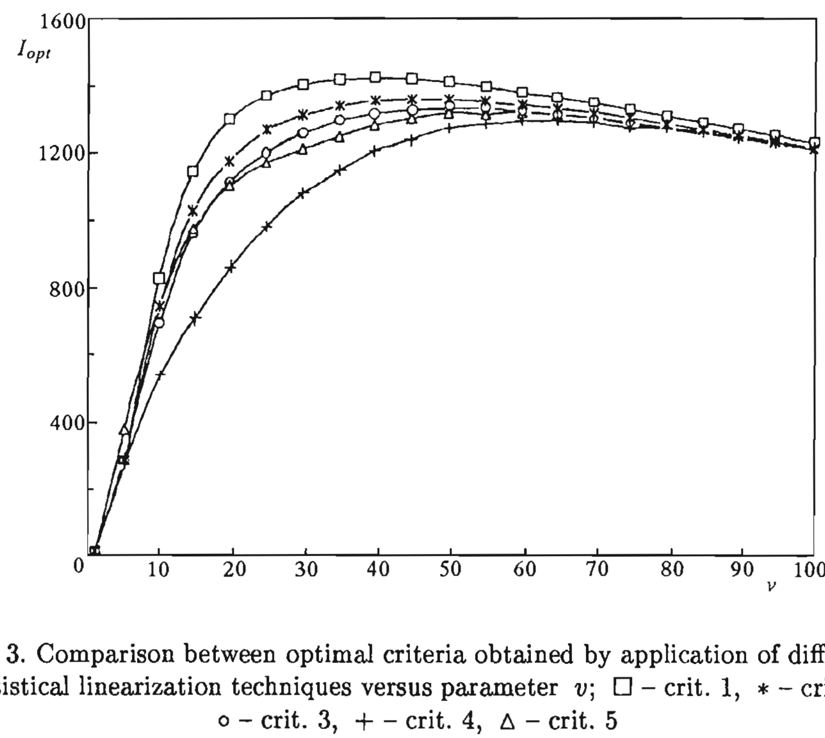

Fig. 3. Comparison between optimal criteria obtained by application of different statistical linearization techniques versus parameter v; □ – crit. 1, $*$ – crit. 2, ○ – crit. 3, + – crit. 4, △ – crit. 5

7. Conclusions and final remarks

In this paper the problem of quasi-optimal control for nonlinear 2-degree-of-freedom vehicle suspension under external white noise excitations with nonlinear criterion has been studied. To determine the quasi-optimal control modified versions of the standard iterative procedure were modified. The three versions of statistical linearization methods with moment criteria and two versions with criteria in probability density functions space were combined with optimal control method for linear system with mean square criterion.

The convergence of the proposed iterative procedures was established in numerical studies.

From the numerical results obtained it follows that for theverified numerically mean square criterion of minimization (5.4) for a control problem there are no significant differences between the considered linearization methods. We note that the smallest value of criterion (5.4) was obtained by the application of the statistical linearization using criterion 4 i.e. the square probability metric. However, to draw general conclusions regarding the applicability of the discussed linearization methods to control of nonlinear stochastic systems further examples should be studied.

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References

1. BEAMAN J., 1984, Nonlinear Quadratic Gaussian Control, *Int. J. Control*, **39**, 343-361
2. BOTOON R.C., 1954, Nonlinear Control Systems with Random Inputs, *IRE Transactions Circuit Theory*, **1**, 32-34
3. ELISHAKOFF I., ZHANG R., 1992, Comparison of the New Energy – Based Version of Stochastic Linearization Technique, in N.Beilimo and F.Casciati (edit.), *Nonlinear Stochastic Mechanics*, Springer, Berlin, 201-212
4. HAĆ A., 1985, Suspension Optimization of 2-DOF Vehicle Model Using a Stochastic Optimal Control Technique, *J. Sound and Vibr.*, **100**, 343-357
5. HEESS G., 1970, Application of State Space Statistical Linearization to Optimal Stochastic Control of Non-Linear Systems, *Int. J. Control*, **11**, 697-701
6. KAZAKOV I., 1956, Approximate Probabilistic Analysis of the Accuracy of Operation of Essentially Nonlinear Systems, *Avtomat. i Telemekhan.*, **17**, 423-450

7. KWAKERNAK S., SIVAN R., 1972, *Linear Optimal Control Systems*, Wiley-Interscience, New York

8. PUGACEV W.S., SINICYN I.N., 1985, *Stochastic Differential Systems*, Wiley, Chichester

9. SOCHA L., 1999, Statistical and Equivalent Linearization Techniques with Probability Density Criteria, *Journal of Theoretical and Applied Mechanics*, **37**, 369-382

10. SOCHA L., 1999, Active Control of Nonlinear 2-Degrees of Freedom Vehicle Suspension under Stochastic Excitations, in J.Holnickic-Szulc and J.Rodellar (edit.), *Smart Structures*, Kluwer Academic Publ., The Netherlands, 321-327

11. STENGEL R.F., 1986, *Stochastic Optimal Control*, John Wiley and Sons, New York

12. YOSHIDA K., 1984, A Method of Optimal Control of Non-Linear Stochastic Systems with Non-Quadratic Criteria, *Int. J. Control*, **39**, 279-291

13. ZHU W.Q., YING Z.G., SOONG T.T., 1999, Optimal Nonlinear Feedback Control of Structures under Random Loading, in: B.F.Spencer, Jr and E.A.Johnson (edit.), *Stochastic Structural Dynamics*, A.A. Balkema, Rotterdam, Brookfield, 141-148

Zastosowanie statystycznej linearyzacji w nieliniowych układach do wyznaczania kwazi-optymalnego aktywnego sterowania

Streszczenie

W pracy przedstawiono porównanie zastosowania kilku metod statystycznej linearyzacji z kryteriami momentowymi oraz z przestrzeni gęstości prawdopodobieństw do wyznaczania kwazi-optymalnego, addytywnego sterowania w układach nieliniowych z wymuszeniem w postaci kolorowego Gaussowskiego szumu i ze średnio kwadratowym kryterium optymalizacji. W celu wyznaczenia kwazi-optymalnego sterowania zaproponowano dwie zmodyfikowane wersje standardowej procedury iteracyjnej, gdzie trzy wersje statystycznej linearyzacji z momentowymi kryteriami i dwie wersje z kryteriami w przestrzeni gęstości prawdopodobieństw wykorzystano łącznie ze standardowym algorytmem wyznaczania sterowania optymalnego w układach liniowych ze średniokwadratowym wskaźnikiem jakości. Szczegółowa analiza została przedstawiona dla układów o dwu stopniach swobody z zewnetrznym wymuszeniem w postai Gaussowskiego szumu kolorowego traktowanego jako wyjście z dwuwymiarowego filtru liniowego. Sterowanie przyjęto jako liniowe sprzężenie zwrotne. Otrzymane wyniki zilustrowano na przykładzie numerycznym.

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