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EFFECT OF STRAIN HARDENING AND NORMAL ANISOTROPY ON ADMISSIBLE VALUES OF STRAIN AND STRESS IN PIPE-BENDING PROCESSES

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The paper aims at discussing the effect of hardening exponent and normal anisotropy index values on admissible values of bending angle and strain in cold bending of thin- and thick-walled steel pipes. A condition of distributed instability onset was adopted as a limiting criterion for the process. The analysis reveals that higher indexes of hardening and material anisotropy result in larger values of admissible strain and bending angle. The neutral axis position corresponding to the critical instability onset is shown to be shifted toward compressed layers of the pipe. The fracture problem associated with the bending process is presented.

Key words: plasticity of metals, pipe bending, deformational instability

1. Introduction

A series of papers by Śloderbach (1998, 1999) have presented a method for analytical determination of particular deformation parameters arising in plastic bending of thin- and thick-walled pipes. A set of formulas for determining: wall thickness distribution across the bending zone, admissible value of bending angle α_b, admissible values of logarithmic strains $\varphi_1, \varphi_2, \varphi_3$ and equivalent strain φ_i (strain intensity) (Marciniak, 1971; Marciniak and Kołodziejski, 1983) were derived. All the quantities were found to be dependent on: bending radius R, geometry of pipe $d_{ext} \times g_0$, internal radius r_{int}, angle of bending α_b, angular parameters α, β defining a particular section within the bending zone and coefficient k defining the bending zone extent along the bend arc. The process of pipe bending using suitable machinery (benders) was found to be a combination of tensile elongation and stretch drawing (especially if mandrels are applied) for stretched layers of the pipe. The deformation of the compressed layers could be seen as combined compression and upsetting.

The present paper aims at discussing the effect of parameters of hardening and normal anisotropy on admissible values of bending angle, strain and stress involved in the process of cold bending of thin- and thick-walled steel pipes. The ranges of admissible angle of bending and equivalent strain were assigned their top limits by adopting the criterion of distributed instability onset (Moore and Wallace, 1964/1965; Gabryszewski and Gronostajski, 1991).

Only tensile fibres of the tube were considered since the vast available experimental material confirms that damage starts primarily at the apex point of the stretched layer. The data presented by Dobosiewicz and Wojczyk (1998), Seyna and Ginalski (1987, 1989) show that lifetime of power pipeline bends can be reduced by as much as 300% and the creep strength – by 30% when compared with straight sections. The final part of the paper deals with displacement of the neutral axis for particular instances of plastic instability and the angle of bending $\alpha_b = 180°$.

2. Basic assumptions and relationships

As noted before, the admissible values of bending angle and equivalent strain are determined as those referred to the instant of instability onset (see e.g. Gabryszewski and Gronostajski, 1991; Śloderbach and Sawicki, 1984). The extent of plastic strain that can be utilized in metal-working processes is limited by material flow instability or fracture. It is usually the former that arises first but certain combinations of material properties, deforming conditions and technical particularities may reverse the sequence of events. The present paper deals with the case when a limiting instance is the onset of distributed material instability (Gabryszewski and Gronostajski, 1991) in uniaxial tension, the other assumption adopted being the plane state of strain onset under plane stress conditions. It is further assumed that the bending process is accomplished at an ambient temperature (cold working) and is quasi-static. Thermal effects associated with large plastic deformations are neglected.

The following form of the strain hardening curve is assumed

$$\sigma_p = C(\varphi_0 + \varphi_i)^n \tag{2.1}$$

where

σ_p – current yield stress
C – material constant
φ_0 – logarithmic initial strain
n – strain-hardening exponent.

For majority of metals used in industry the value of n ranges from 0 to 0.5.

The pipe bending operation may be roughly regarded as a combination of uniaxial tension/compression and biaxial stretch drawing/upsetting under plane stress. Up to date there is no general criterion available for determining deformational instability onset under such complex circumstances, so the following two special cases will be considered.

2.1. Uniaxial tension case (Marciniak, 1971; Gabryszewski and Gronostajski, 1991)

In this case the equivalent strain value corresponding to the adopted type of instability is given by

$$\varphi_{i(1)} = n - \varphi_0 \tag{2.2}$$

2.2. Plane state of strain (Moore and Wallace, 1964; El-Sebaie and Mallor, 1972)

For the plane strain conditions

$$\varphi_{i(2)} = nZ - \varphi_0 \tag{2.3}$$

where

$\varphi_{i(2)}$ – value of equivalent strain corresponding to the adopted type of instability
Z – critical subtangent value determining the stress and strain values at the instant of instability under biaxial stress and strain conditions

$$Z = \frac{1+r}{\sqrt{1+2r}} \tag{2.4}$$

Parameter r is a measure of normal anisotropy (Marciniak, 1971) given by the following formula

$$r = \frac{\varphi_1}{\varphi_3} = \frac{\ln\frac{l}{l_0}}{\ln\frac{g}{g_0}} \tag{2.5}$$

where l_0, l; g_0, g – initial/final specimen gauge length and thickness, respectively.

It results from the above formula that an increase in the value of r is accompanied by a lower area reduction, i.e. stronger resistance arises against thinning of walls in a bent pipe. The coefficient r has been found to lie between 1 and 2.5 for majority of steels used in the pipe manufacture.

As noted earlier, the process of pipe bending on a bender and especially the behaviour of tensioned fibres may be considered as a combination of uniaxial elongating and plastic stretch drawing under local plane stress within the bending zone. It is possible, therefore, to assume that the admissible value of equivalent strain $\varphi_{i\,all}$ corresponding to the onset of distributed instability in a bent pipe will be contained within the following limits

$$\varphi_{i(1)} \leqslant \varphi_{i\,all} \leqslant \varphi_{i(2)} \tag{2.6}$$

On substituting Eq (2.2) and (2.3) into Eq (2.6) one gets

$$n - \varphi_0 \leqslant \varphi_{i\,all} \leqslant \frac{1+r}{\sqrt{1+2r}}n - \varphi_0 \tag{2.7}$$

For isotropic materials $r = 1$, so

$$n - \varphi_0 \leqslant \varphi_{i\,all} \leqslant \frac{2}{\sqrt{3}}n - \varphi_0 \tag{2.8}$$

Relationships between bending angle α_b and strain intensity may be given in either of the three forms below (a general one and two simplified ones, respectively) as obtained by Śloderbach (1999). The formulas were found to be in very good agreement with the experimental data presented by Korzemski (1971) and, especially, Franz (1961)

$$\begin{aligned}
\cos\Big(k\frac{\alpha_{b\,cr}}{2}\Big) &= \frac{R + r_i - R\exp\sqrt{1.5\varphi_i^2 - (\varphi_2^2 + \varphi_3^2)}}{r_i} = \frac{R + r_i - R\exp\varphi_1}{r_i}\\
\cos\Big(k\frac{\alpha'_{b\,cr}}{2}\Big) &= \frac{R + r_{ext} - R\exp\sqrt{1.5\varphi_i'^2 - (\varphi_2'^2 + \varphi_3'^2)}}{r_{ext}} = \frac{R + r_{ext} - R\exp\varphi'_1}{r_{ext}}\\
\cos\Big(k\frac{\alpha''_{b\,cr}}{2}\Big) &= \frac{R + r_{ext} - R\exp\sqrt{1.5\varphi_i''^2 - \varphi_3''^2}}{r_{ext}} = \frac{R + r_{ext} - R\exp\varphi''_1}{r_{ext}}
\end{aligned} \tag{2.9}$$

where

$\alpha_{b\,cr}, \alpha'_{b\,cr}, \alpha''_{b\,cr}$ – critical values of the bending angle resulting from the general method and the two approximated ones, respectively. They also correspond to the admissible values of strain intensities φ_i, φ'_i and φ''_i as defined by Eqs (2.6) ÷ (2.8)

k – actual bending zone extent within the bending arc length determined from actual tests. It can be generally said that the coefficient value is dependent on a particular technique of bending being used, bending radius, type of material and geometry of a bent pipe. The results presented by Franz (1961) and Korzemski (1971) served the authors as a basis for inferring that k should range from one to three, the limits being determined by the types of materials used there. This particular question was discussed in detail elsewhere (Śloderbach, 1999) and, consequently, the admissible range in this study was adopted as $k \in [1, 10]$ to account for a wider variety of materials

R – bending radius

r_i – small current radius within the bending zone

r_{ext} – external radius of the bent pipe

$\varphi_1, \varphi_2, \varphi_3$ – logarithmic strains

φ_i – strain intensity (equivalent strain).

In addition, the following two obvious relationships hold

$$r_i = r_{int} + g_i \qquad\qquad r_{ext} = r_{int} + g_0 \tag{2.10}$$

where

r_{int} – internal pipe radius

g_i – current wall thickness within the bending zone

g_0 – initial wall thickness,

for elongated fibres $i = 1$, so $g_i = g_1$.

The single-prime (′) and the double-prime (″) superscripts refer to the terms that were obtained from the first and second order approximation, respectively (Śloderbach, 1999).

The geometry of the pipe bending process is shown in Fig.1.

3. Critical values of strain and stress

Eqs (2.6), (2.7) and (2.8) giving critical values of strain and stress arising in the cold pipe bending will now be used together with the equations determining

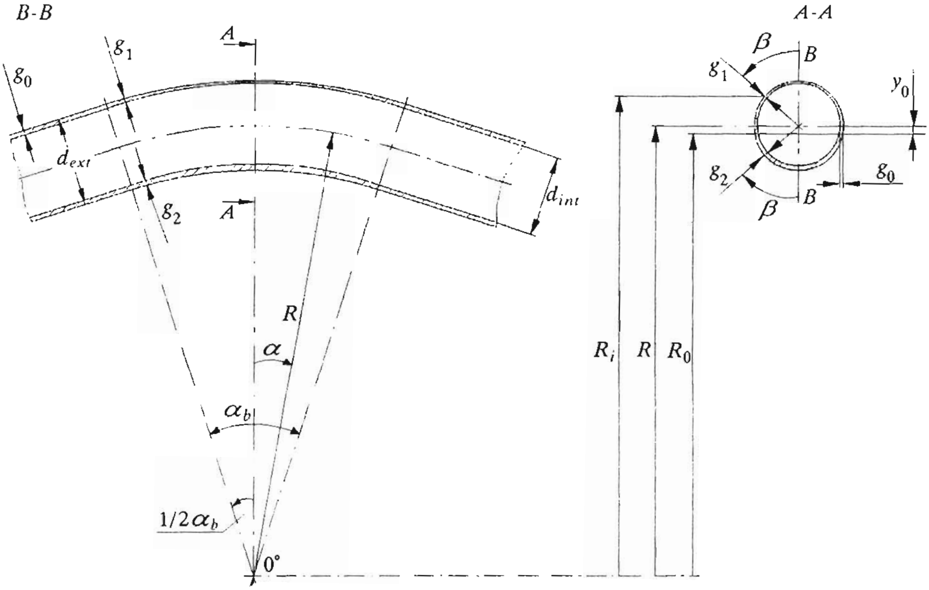

Fig. 1. Geometrical quantities involved in the pipe bending process description

the onset of the plane state of strain under plane stress conditions. The problem as applied to the uniaxial tension case has been covered by many authors and will be illustrated here by a single working example.

Śloderbach and Strauchold (1999) obtained the formulas for determination of particular stress components arising in a certain plane state of strain which resulted from adopting the second-order approximation of the exact solution given by Śloderbach (1999).

The logarithmic plastic strains and the equivalent strain have the following form

$$
\begin{aligned}
\varphi_1'' &= \ln \frac{R + r_{ext}\cos\beta\Big(\cos k\alpha - \cos k\frac{\alpha_b}{2}\Big)}{R} \\
\varphi_2'' &= \ln \frac{r_{ext}}{r_{ext}} = 0 \qquad\qquad \varphi_3'' = \ln \frac{g_1''}{g_0} \\
\varphi_i'' &= \sqrt{\frac{2}{3}(\varphi_1''^2 + \varphi_3''^2)}
\end{aligned}
\tag{3.1}
$$

By comparing the above expression for $\varphi_{i\,all}''$ with the right-hand side of

Eq (2.7) the following relationship is obtained

$$\varphi''_{i\,all} = \frac{1+r}{\sqrt{1+2r}}n - \varphi_0 \tag{3.2}$$

Inserting the last expression into Eq $(2.9)_3$ one gets a critical value of the bending angle $\alpha''_{b\,cr}$ corresponding to the distributed instability onset. This type of material instability may arise around the apex point $(\alpha = \beta = 0°)$ of the elongated fibre area if a pipe is cold bent using a mandrel.

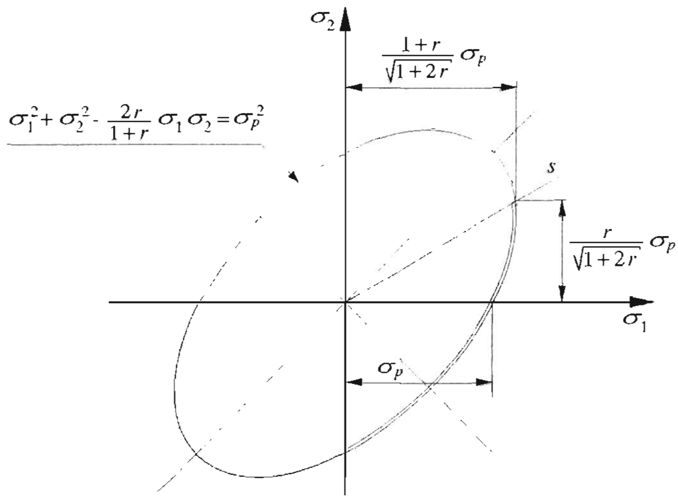

Fig. 2. Stress conditions derived from the yield ellipsis at the instant of stability loss

In practice, the above relationships will be used as follows: first, for a given material one finds $\varphi''_{i\,all}$ from Eq (3.2) and then for given parameters of the bending operation determines the angle $\alpha''_{b\,cr}$ from Eq $(2.9)_3$ and corresponding logarithmic strains φ''_1 and φ''_3 from Eqs (3.1). The equivalent critical yield stress value $\sigma_{p\,cr}$ corresponding to condition (3.2) can be obtained from Eq (2.1) on inserting the $\varphi''_{i\,all}$ value. The corresponding stress components may be determined from Fig.2 as those representing point S on the yield ellipsis. According to associated rules of plastic flow the point determines the onset of plane state of strain under plane stress conditions. The fundamental set of quasi-linear partial differential equations for characteristics and the Huber-von Mises yield criterion at this point are of parabolic type (Hill, 1950). Thus, the stress components can be written in the form

$$\sigma''_1 = \frac{1+r}{\sqrt{1+2r}}\sigma''_p \qquad \sigma''_2 = \frac{r}{\sqrt{1+2r}}\sigma''_p \tag{3.3}$$

It is worth noting that for $\alpha = 0°$ (the apex point of the bending zone) and for $k\alpha_b = 180°$ $(\cos(k\alpha_b/2) = 0$, maximum deformation attained within the bend arc length) Eq $(3.1)_1$ coincides with $\varepsilon'' = d_{ext}/(2R)$ recommended by both the Polish and European Union codes of practice for determining strains in pipe bending operations (Zdankiewicz, 1998).

4. Initial and boundary conditions of the bending problem

The relationships presented in Section 3 satisfy the following initial and boundary conditions of the bending process:

i. if $\alpha = \alpha_b/2 = 0$ – bending starts (no bending yet)

ii. if $\alpha = \alpha_b/2 \neq 0$ – end points of the bending zone

iii. if $\beta = 90°$ – location of neutral surface

(and then: $g_1'' = g_0$, $\varphi_1'' = \varphi_2'' = \varphi_3'' = 0$, $\varphi_i'' = 0$)

iv. if $k\alpha_b = 180°$ and $\alpha = \beta = 0°$ – apex point of the bending zone

and then

$$
\begin{aligned}
g_1'' &= g_{1\,min} & \qquad \varphi_1'' &= \ln\frac{R + r_{ext}}{R} \\
\varphi_2'' &= 0 & \qquad \varphi_3'' &= \ln\frac{g_{1\,min}''}{g_0}
\end{aligned}
\tag{4.1}
$$

With these conditions being recalled and the flow chart shown in Fig.3 it will be easier to analyse the working examples below. The sample problems are solved to show critical parameter values (stress and bending angle values) for the two instability cases mentioned as Eqs (2.2) and (2.3). In all the example problems the same material characteristics were adopted. The pipe diameter was chosen to be in accordance with the reference tests performed by Franz (1961).

4.1. Example 1

Eqs (3.2) and $(2.9)_3$ may be used for determining the onset of distributed instability. On exceeding these critical values plastic deformation may become localized and eventually end up in fracture. So, if $\varphi_i'' = \varphi_{i\,all}''$, then $\alpha_b'' = \alpha_{b\,all}''$.

Assume a length of OD 44.5 × 4.5 (OD = outside diameter) pipe is given. It is made of K10 pipe steel (designation acc. to Polish Standards) with the

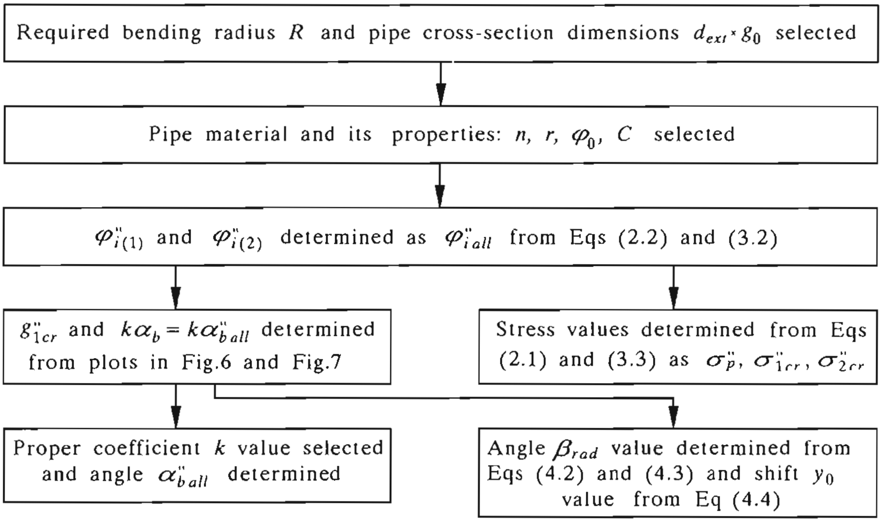

Fig. 3. Flow chart showing the computation stages

following material characteristics: $n = 0.2$, $r = 1.5$, $\varphi_0 = 0.016$, $C = 550\,\text{MPa}$ (see (2.1) and (2.5)). It follows then from Eq (3.2) that $\varphi''_{i\,all} = 0.234$.

From plots in Fig.6 and Fig.7 one finds for $R = 80\,\text{mm}$ ($R = 1.8 \times 44.5$) that: $k\alpha''_{b\,all} = 158°$ for $k = 1$, $\alpha''_{b\,all} = 79°$ ($k = 2$), $\alpha''_{b\,all} = 63°$ ($k = 2.5$), $\alpha''_{b\,all} = 53°$ ($k = 3$) and $g''_{1\,cr} = 3.68\,\text{mm}$. The $\varphi''_{1\,all}$ value found from Eq $(3.1)_1$ is $\varphi''_{1\,all} = 0.203$. The yield stress determined from Eq (2.1) is $\sigma''_{p\,cr} = 417\,\text{MPa}$ and the corresponding components of the plane state of stress as calculated from formulas (3.3) become $\sigma''_{1\,cr} = 521\,\text{MPa}$ and $\sigma''_{2\,cr} = 313\,\text{MPa}$.

An approximate location of the neutral line of plastic bending may be found from general properties of this line (Jakubowski and Orłoś, 1970; Krzyś and Życzkowski, 1962): first, the line divides the cross-section into two parts of equal area, secondly, the neutral axis and the central axis no longer coincide, the former being shifted toward the centre of curvature. The proof is as follows.

The approximate condition of equal areas can be written as

$$2\beta_{rad} r_{2m}^2 + \pi r_{1m}^2 \cong \pi r_{ext}^2 \tag{4.2}$$

where

β_{rad} – angle (in radians) at a radial plane section ($\alpha = 0°$)
r_{1m} – mean radius of the tensioned layer cross-section area
r_{2m} – mean radius of the compressed layer cross-section area and

$$r_{1m} = \frac{r_{ext} + r_{int} + g''_1}{2} \qquad\qquad r_{2m} = \frac{r_{ext} + r_{int} + g''_2}{2} \tag{4.3}$$

Since $g_1'' = 3.66\,\mathrm{mm}$ and $g_2'' = 5.84\,\mathrm{mm}$ (Śloderbach and Strauchold, 1999), then $r_{1m} = 21.83\,\mathrm{mm}$ and $r_{2m} = 22.92\,\mathrm{mm}$.

Inserting these values into Eq (4.2) one gets

$$\beta_{rad} = 0.0533 \qquad (\beta_0 = 3°10')$$

$$y_0 = r_{2m} \sin\beta_0 = 1.268\mathrm{mm} \tag{4.4}$$

where $y_0 = R - R_0$ is the distance by which the neutral axis has been shifted toward the centre of curvature $(R_0 = 78.732\,\mathrm{mm})$. Finally, one gets $y_0/r_{ext} = 0.05$.

4.2. Example 2

On obtaining the state of deformation given by formula (2.2), one finds $\varphi_{i(1)} = 0.184$. The same value (0.18) was reported by Franz (1961; p.23) for similar geometrical conditions of bending, pipe dimensions and material. The corresponding critical value of bending angle determined from Fig.7 is $k\alpha''_{b\,all} = 140°$. Consequently, $g_{1(1)} = 3.86\,\mathrm{mm}$ and $g_{2(1)} = 5.66\,\mathrm{mm}$ (Śloderbach and Strauchold, 1999). We then have: $r_{1m} = 21.93\,\mathrm{mm}$, $r_{2m} = 22.83\,\mathrm{mm}$ and $\sigma_{p\,cr} = 398.6\,\mathrm{MPa}$. From Eq (4.2): $\beta_{rad} = 0.042$ and $\beta_0 = 2°27'$. Finally, the following values are found: $y_0 = 0.976\,\mathrm{mm}$, $R_0 = 79.024\,\mathrm{mm}$ and $y_0/r_{ext} = 0.044$.

4.3. Example 3

If $k\alpha_b = 180°$, then $g_1'' = 3.521\,\mathrm{mm}$, $r''_{1m} = 21.761\,\mathrm{mm}$ and $g_2 = 6.234\,\mathrm{mm}$ (Śloderbach and Strauchold, 1999), $r''_{2m} = 23.117\,\mathrm{mm}$.

From Eq (4.2) one gets $\beta''_{rad} = 0.0633$, $\beta''_0 = 3°40'$ and further $y_0 = 1.462\,\mathrm{mm}$, $R_0 = 78.538\,\mathrm{mm}$. Finally $y_0/r_{ext} = 0.066$.

The above computational results prove that the neutral axis gets displaced toward the centre of curvature, the shift being dependent on the angle of bending.

4.4. Example 4. Fracture problem

The reported minimum value of the total plastic elongation after fracture for K10 steel is $\varepsilon_f = 0.25$ and the corresponding logarithmic longitudinal strain is $\varphi''_{1\,cr} = \ln(1 + \varepsilon_f) = 0.223$. The resulting critical angle of bending (see Eq $(3.1)_1$) is then $k\alpha''_{b\,cr} = 168°$. It means that, as expected, fracture will occur later than distributed instability, the latter being determined by Eq (2.7) for a steel with the normal anisotropy index $r = 1.5$ as in Example 1, since

$\varphi''_{1\,all}$ from Example 1 is lower than $\varphi''_{1\,cr}$. We conclude that if $\varphi''_{1\,all} < \varphi''_{1\,cr}$, then $\alpha''_{b\,all} < \alpha''_{b\,cr}$.

The admissible value of strain intensity corresponding to the adopted value of ε_f calculated from Eq $(3.1)_4$ is $\varphi''_i = 0.2575$ and the corresponding admissible angle of bending determined from Eq $(2.9)_3$ or the plot in Fig.7 is also $k\alpha''_{b\,cr} = 168°$. Fracture will, therefore, follow instability, since a bending angle of $168°$ is larger than an angle of $158°$ found in Example 1. If $\varphi''_i = \varphi''_{i\,cr}$, then $\sigma''_p = \sigma''_{p\,cr} = (UTS)''$ (ultimate tensile strength value). It follows from Eq (2.1) on inserting the suitable data that $(UTS)'' \cong 424.4$ MPa. The maximum UTS value for the steel in question is equal to 440 MPa (see [17]). The calculated value of 424.4 MPa compares favourably with the above since $(UTS)'' \leqslant (UTS)_{max}$.

5. Analysis of the results

The plots in Fig.4 and Fig.5 present the equivalent strain values corresponding to the onset of distributed instability under plane strain and plane stress conditions. As can be seen, $\varphi_{i(2)}$ increases with increasing normal anisotropy index r and hardening exponent n. It means that higher values of the two indexes have a beneficial effect on the cold pipe bending process since the admissible level of deformation becomes higher and, in practical terms, larger values of admissible bending angle $\alpha_{b\,all}$ can be adopted. It can be said therefore that materials with higher values of r and n are better suited to plastic bending, the effect of n being slightly stronger of the two. The relationship of r and n to structural nonuniformity and grain size of material has been discussed extensively in the literature (Cahn and Haasen, 1983).

Fig.6 shows variation of the bend wall thickness at the apex point of the bending zone with the increase of bending angle α_b for various bending radii R from the range $(1-5) \times d_{ext}$ according to the Polish Standard [17]. Our computational analysis involved the OD 44.5×4.5 pipe frequently encountered in the literature (Korzemski, 1971; Franz, 1961). As can be seen from the plots the wall thickness decreases with increasing bending angle α_b, the rate of change being, however, dependent on the bending radius R.

Fig.7 shows variation of φ''_i as determined at the apex point of the tensioned area from the formula presented in Section 3 with the increase of bending angle α_b and for various bending radii R (as before, $R \in (1-5) \times d_{ext}$, OD 44.5×4.5 pipe). As expected, the φ''_i value increases with increasing α_b,

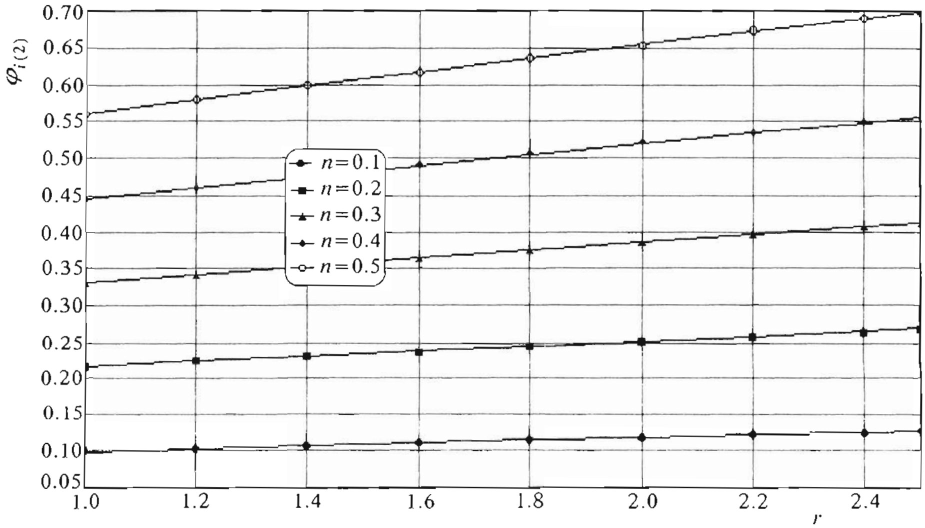

Fig. 4. Admissible equivalent strain as a function of the normal anisotropy index for selected values of the hardening exponent

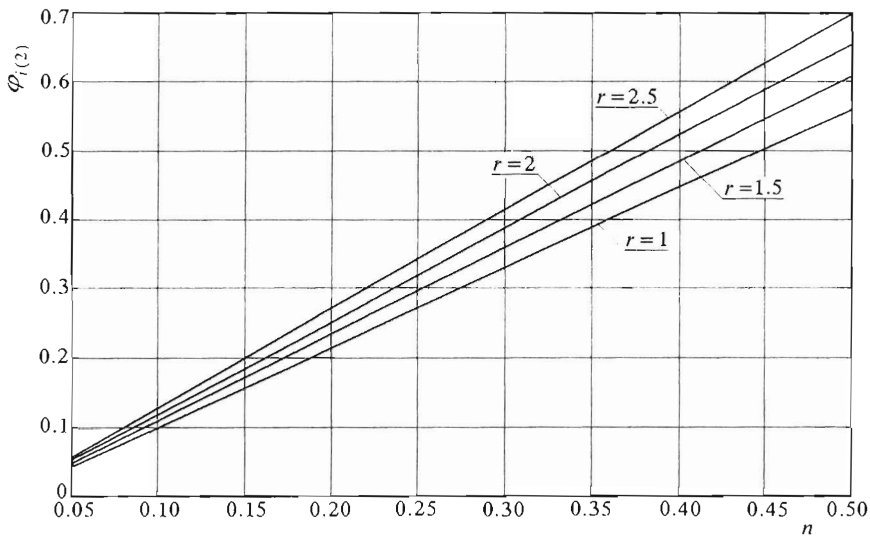

Fig. 5. Admissible equivalent strain as a function of the hardening exponent for selected values of the normal anisotropy index

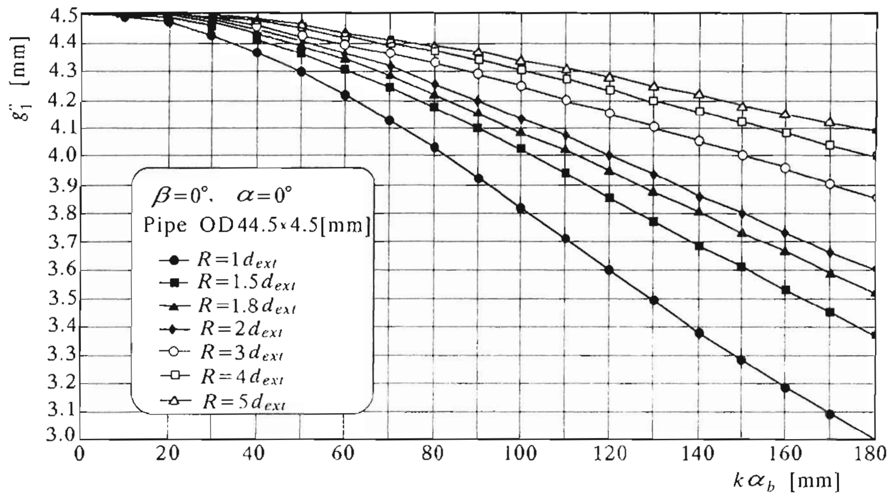

Fig. 6. Wall thickness at the bend apex as a function of bending angle for selected values of bending radius [17]

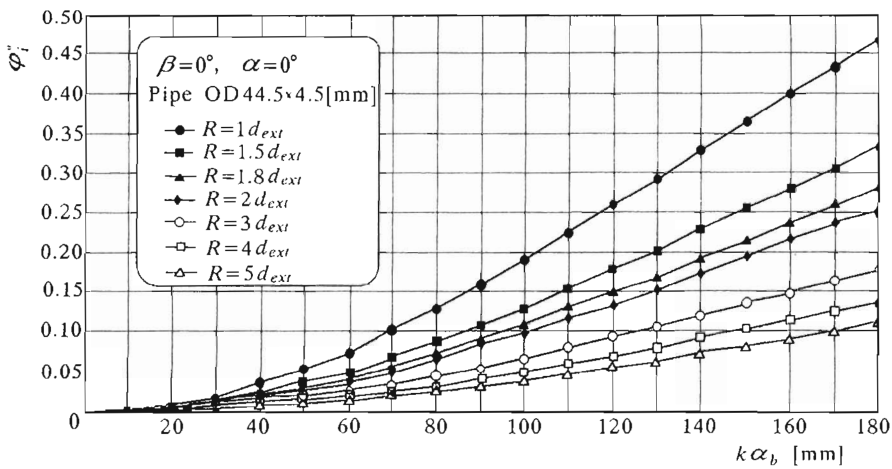

Fig. 7. Equivalent strain value at the bend apex as a function of bending angle for selected values of bending radius [17]

the rate of increase being lower for larger values of R. If the radius R tends to infinity, then φ_i'' tends to zero and therefore g_1 approaches g_0, and that means no bending ($\alpha_b = 0°$). The above results and observations are in agreement with the list of initial and boundary conditions presented in Section 4.

To sum up this section, any numerical research in the pipe bending field has to be juxtaposed with the classic experimental results presented by Franz (1961). His tremendous work could be only done with support from powerful industrial sources and can be hardly reproduced by an academic researcher. Our logarithmic measures of strain (3.1) are in good agreement with his results, so we could presume that the instability criteria are pretty sound, too. In spite of being simplified they are the only ones presently available.

6. Final remarks and conclusions

- Higher indexes of normal anisotropy and strain hardening result in higher intensity of admissible deformation in cold bending of thin- and thick-walled steel pipes.
- The condition of distributed instability onset in the case of biaxial state of strain under plane stress conditions leads to higher values of permissible strain intensity than for the case of instability in uniaxial uniform tension. In this latter case the permissible strain intensity value is roughly equal to the hardening exponent value for a given material.
- The results obtained make the authors think that the deformational instability condition (2.3) is better suited to actual pipe bending processes, especially those using a mandrel, than condition (2.2) adopted intuitively by Franz (1961).
- Hot or semi-hot pipe bending process models must involve other forms of the constitutive equation than Eq (2.1). Also, different instability criteria are required because of temperature effects. This latter observation can be easily explained: the hardening exponent value decreases with increasing temperature, enhanced ductility of material allows higher admissible strains to be utilized.
- The neutral axis in plastic bending undergoes a shift toward the centre of curvature. The shift value is of the one-millimetre order and increases

with increasing angle of bending.

- The whole variety of bending process variables could be conveniently accounted for by nomograms and in this way be available to manufacturing designers.

  Worthwhileness of heat treatment after plastic working depends on the type of process (cold or hot), and in the case of cold bending – on the amount of plastic strain induced.

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Wpływ umocnienia i anizotropii normalnej na dopuszczalne wartości odkształceń i naprężeń w procesie gięcia rur

Streszczenie

W niniejszej pracy analizowano wpływ parametrów wzmocnienia i anizotropii normalnej materiału na dopuszczalne (krytyczne) wartości kąta gięcia i wartości odkształcenia podczas gięcia na zimno cienko- i grubościennych rur metalowych w zakresie kąta gięcia $\alpha_g \in [0° \div 180°]$. Jako kryterium ograniczające proces gięcia przyjęto warunek rozproszonej utraty stateczności materiału dla przypadku jednoosiowego rozciągania i płaskiego stanu naprężenia. W pracy wykazano, że wzrost współczynników wzmocnienia i anizotropii normalnej zwiększa wartość dopuszczalnego odkształcenia zastępczego i kąta gięcia. W pracy obliczono także wartości przesunięcia osi obojętnej gięcia odpowiadające stanom rozproszonej utraty stateczności oraz wykazano, że przy założonych warunkach procesu gięcia jest to przesunięcie w kierunku warstw ściskanych łuku i którego wartość zwiększa się wraz ze wzrostem kąta gięcia. Rozpatrzono także problem pękania w procesie gięcia rur, w którym czynnikiem oceny jest wskaźnik techniczny plastyczności A_5.

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