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MODELLING AND COMPUTATION OF LAMINAR UNSTEADY FLOW PAST OSCILLATING AND ROTATING CIRCULAR CYLINDERS

LÁSZLÓ BARANYI

Department of Fluid and Heat Engineering, University of Miskolc, Miskolc, Hungary
e-mail: arambl@gold.uni-miskolc.hu

In this paper a finite difference solution is presented for the 2D, unsteady incompressible Navier-Stokes equations for laminar flow about fixed, oscillating or rotating cylinders. Equations are transformed to a non-inertial system fixed to the cylinder. Convective terms are handled by a third order upwind difference, other space derivatives by fourth order central differences, and time derivatives by forward differences. The computed Strouhal numbers for fixed cylinders compare well with experimental results. The variation of time mean and root- mean-square values of lift and drag coefficients with rotation parameter α is also shown for a rotating cylinder for two different grids. Amplitude bounds of locked-in vortex shedding due to crossflow cylinder oscillation were determined for $\mathrm{Re} = 180$.

Key words: bluff body, forced vibration, rotation, lock-in, laminar flow, finite difference

1. Introduction

The vibration of structures in a fluid flow has received much experimental and numerical study due to its practical importance. Numerical studies of vortex shedding have dealt with the flow of a uniform stream normal to a fixed cylinder, e.g., Karniadakis and Triantafyllou (1989). If the cylinder is vibrating, either in forced or natural motion, a non-linear interaction occurs as the cylinder frequency approaches that of vortex shedding. In this case vortex shedding occurs at the cylinder vibration frequency over a range of flow velocities, a phenomenon called lock-in. Among the several papers dealing with the numerical simulation of lock-in are Hulbrut et al. (1982), Meneghini and Bearman (1995), Baranyi and Shirakashi (1999).

When the cylinder is rotating, it can be a means of boundary layer control. Hyung et al. (1995) carried out experimental tests on the flow past a rotating cylinder in uniform shear flow. Cheng et al. (1997) developed a hybrid vortex scheme for flow past rotating cylinders.

The present study, based on the finite difference method, transforms the Navier-Stokes equations to a non-inertial reference frame fixed to the moving cylinder. Computational and experimental results for flow about fixed cylinders are compared. Amplitude bounds of locked- in vortex shedding due to crossflow oscillation of a circular cylinder are determined for Re = 180. Computational results are presented for lift and drag coefficients of a rotating cylinder as well.

2. Problem formulation

Incompressible laminar flow past a circular cylinder undergoing in-line and crossflow oscillation and rotation is considered. The two components of the non-dimensional Navier-Stokes equations in a non-inertial or relative system fixed to the cylinder are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{1}{\mathrm{Re}}\Big(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\Big)-a_{0x}+4\alpha v+4\alpha^2 x+2\frac{d\alpha}{dt}y$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+\frac{1}{\mathrm{Re}}\Big(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\Big)-a_{0y}-4\alpha u+4\alpha^2 y-2\frac{d\alpha}{dt}x \tag{2.1}$$

The equation of continuity has the form

$$\Theta=\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{2.2}$$

The body force due to gravity is included in the pressure p. Here Re is Reynolds number based on cylinder diameter d; x, y are Cartesian co-ordinates; u, v and a_{0x}, a_{oy} are the x, y components of velocity and cylinder acceleration, respectively; Θ is dilation; t is time; α is the rotation parameter defined as the ratio of the peripheral velocity and freestream velocity $\alpha = \Omega d/(2U)$. Here Ω is the angular velocity of the cylinder, positive when the rotation is counter-clockwise.

Roache (1982) recommends using a separate equation for pressure p, obtainable by taking the divergence of the Navier-Stokes equations, and neglecting all but one terms of dilation Θ, giving the Poisson equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 2\Big(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\Big) - \frac{\partial \Theta}{\partial t} + 4\alpha\Big(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\Big) + 8\alpha^2 \tag{2.3}$$

Equations (2.1) $\div$ (2.3) remain valid for flows when the cylinder is just rotating $\alpha_{0x} = \alpha_{0y} = 0$, just oscillating $\alpha = 0$, or fixed $\alpha_{0x} = \alpha_{0y} = 0$ and $\alpha = 0$.

2.1. Boundary, initial conditions and mapping

In the relative system it looks as if the parallel flow were rotating at the physical plane (Fig.1) with non-dimensional angular velocity $-\Omega$. At time t the freestream velocity U includes an angle of $\vartheta = \Omega t$ with the x-axis.

Boundary conditions (BCs) on the surface of the cylinder R_1 (see Fig.1):
— velocity: no-slip condition

$$u = v = 0$$

— pressure

$$\frac{\partial p}{\partial n} = \frac{1}{\mathrm{Re}}\nabla^2 v_n - a_{0n} + 2\alpha^2$$

where n refers to components in the direction of the outer normal.

BCs far from the cylinder R_2:
— velocity: uniform flow in the inertial system

$$\begin{aligned} u &= [U - u_0(t)]\cos(2\alpha t) - v_0(t)\sin(2\alpha t) - 2\alpha R_2 \sin\varphi \\ v &= -v_0(t)\cos(2\alpha t) - [U - u_0(t)]\sin(2\alpha t) - 2\alpha R_2 \cos\varphi \end{aligned}$$

where
u_0, v_0 – x, y components of cylinder velocity
φ – polar angle measured in the relative system; zero along the positive x-axis, and is increasing in the clockwise direction.

— pressure

$$\frac{\partial p}{\partial n} \cong 0$$

It should be noted that the assumption of uniform flow along R_2 is reasonable except for the narrow wake ,since the outer boundary of the physical domain is very far from the cylinder.

Initial conditions for the whole domain

$$\begin{aligned} u &= U - u_0(0) - 2\alpha R \sin\varphi \\ v &= -v_0(0) - 2\alpha R \cos\varphi \end{aligned}$$

and pressure p is considered to be constant at $t = 0$.

The physical domain and governing equations are transformed into a computational plane (see Fig.1). Since a boundary-fitted co-ordinate system is used, BCs can be imposed accurately, and interpolation, often leading to poor solutions, can be omitted.

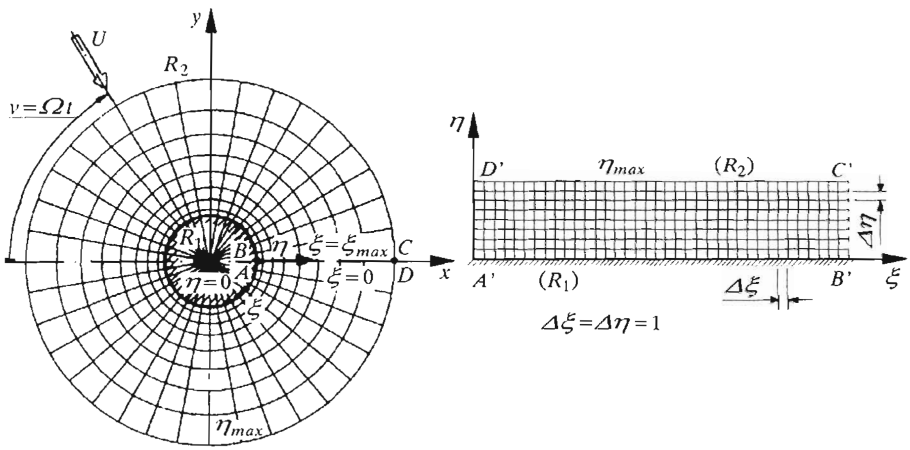

Fig. 1. Physical and computational planes

A unique, single-valued relationship between the co-ordinates on the computational domain (ξ, η, τ) and the physical co-ordinates (x, y, t) is given as

$$x(\xi,\eta) = R(\eta)\cos[g(\xi)] \qquad y(\xi,\eta) = -R(\eta)\sin[g(\xi)] \qquad t = \tau \tag{2.4}$$

where τ is time on the computational plane, and the dimensionless radius is

$$R(\eta) = R_1 \exp[f(\eta)] \tag{2.5}$$

This mapping assures that the grid is orthogonal on the physical plane for arbitrary functions $f(\eta)$ and $g(\xi)$ and can provide a very fine grid in the vicinity of the cylinder and a coarse grid far from the body. Transformations (2.4) and (2.5) are unique and single-valued only for a non-vanishing Jacobian. Both the co-ordinate system and the grid are fixed to the accelerating cylinder. As the mapping is given by analytic functions, the metric parameters and co-ordinate derivatives can be computed in closed forms leading to high accuracy solutions.

3. Results and discussion

A computational code was developed based on the finite difference method. The time derivatives are approximated by forward differences, and the fourth order central difference scheme is used for the diffusion terms and for pressure derivatives. The modified third order upwind scheme by Kawamura (1984) proved to be successful in handling the convective terms. The Navier-Stokes equations are integrated explicitly, giving the velocity distribution at each time step. After determining the velocity distribution in an arbitrary time step, the pressure is calculated from the transformed Poisson equation by using the successive over-relaxation (SOR) method. The condition of $\Theta = 0$ was imposed at each time step. The computational grids used were 145×79 and 241×131 O-grids. The number of grid points was chosen to assure the conformal property of the transformation. The diameter of the outer boundary of computation was $30d$. Dimensionless time steps used were 0.001 or 0.0005.

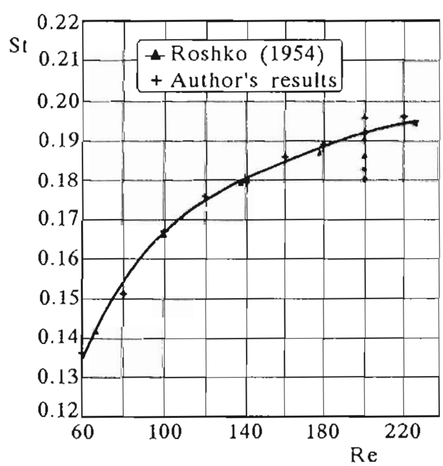

Fig. 2. Strouhal number vs. Reynolds number

Computations were carried out for the flow around a fixed circular cylinder for different Re numbers. The calculated Strouhal numbers St and the results of Roshko's (1954) experiments compare well, as seen in Fig.2. Several other quantities are calculated, e.g., instantaneous lift and drag coefficients; the distribution of velocity, vorticity, pressure and stream function; the location of the front stagnation point, the lower and the upper separation points changing with time. By applying the Fast Fourier Transform (FFT) to the oscillating signals, their spectra can be obtained, and the frequency of vortex shedding can thus be determined.

The amplitude bounds A of locked-in vortex shedding due to forced crossflow oscillation of a circular cylinder for $\mathrm{Re} = 180$ were investigated as a function of the dimensionless frequency of cylinder oscillation $\mathrm{St}_c/\mathrm{St}$, and the results are shown in Fig.3. Here St is the Strouhal number for fixed cylinder at $\mathrm{Re} = 180$, St_c is the Strouhal number based on the frequency of cylinder oscillation.

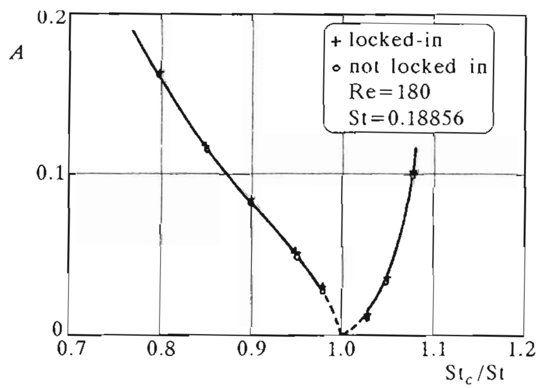

Fig. 3. Amplitude threshold values for crossflow cylinder oscillation

Computations were done for a rotating cylinder at $\mathrm{Re} = 180$ with 145×79 and 241×131 grids for different α values. Fig.4a shows the mean values for lift and drag coefficients against α for the two grids, and remarkably good agreement was obtained. It can be seen in the figure that the absolute value of $\overline{C}_L$ increases almost linearly with α. It is known that for frictionless ideal fluid-flow around a rotating cylinder, the lift coefficient $C_{L\,id}$ is also a linear function

$$C_{L\,id} = -2\pi\alpha \tag{3.1}$$

It can be seen that at this Re number and in the investigated α domain, the real lift (Fig.4a) is only about 40% of that predicted by (3.1). Fig.4b shows the variation of the root-mean square (r.m.s.) values of lift and drag coefficients with α for the two grids mentioned. While $C_{D\,r.m.s.}$ values agree very well over a wide range of α domain, the $C_{L\,r.m.s.}$ values for the two grids differ from each other over the whole α range. It looks as if the accurate prediction of the r.m.s. values of the lift coefficient is a challenging test of

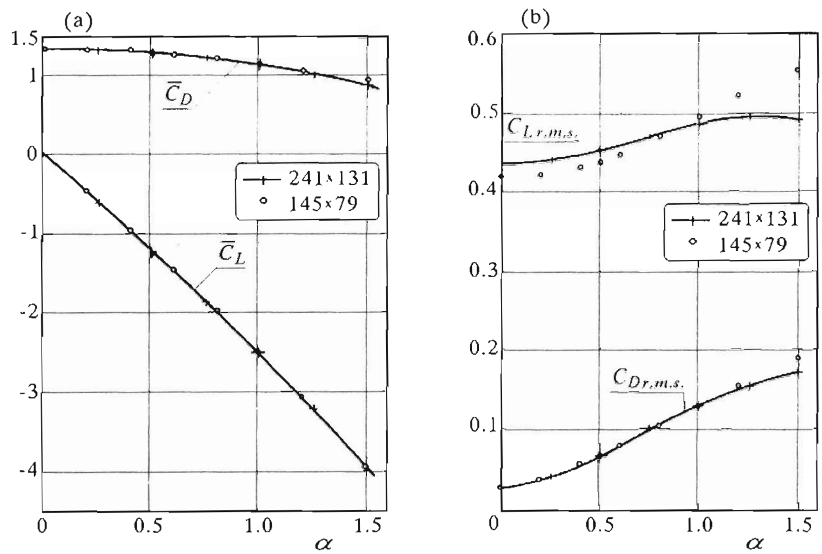

Fig. 4. Time-mean and root-mean square values of lift and drag coefficients vs. rotational parameter α

the computational method not only for fixed or oscillating cylinders but for rotating cylinders as well.

4. Conclusions

The finite difference method has been applied for the numerical simulation of unsteady, laminar incompressible fluid flow past fixed, oscillating and rotating circular cylinders.

By introducing boundary-fitted co-ordinates in the non-inertial system fixed to the moving cylinder, more accurate computational results were obtained.

Agreement between experimental and computational results for fixed cylinders up to $\mathrm{Re} = 200$ was found to be excellent, suggesting that extension of the computation to oscillating cylinder is a promising approach.

Amplitude threshold values for locked-in vortex shedding due to forced crossflow cylinder oscillation were determined for $\mathrm{Re} = 180$.

Computations carried out for rotating cylinders showed that (a) real lift is only about 40% of that predicted by the Magnus effect, and (b) a coarse grid is adequate for prediction of $\overline{C}_L$, $\overline{C}_D$ and $C_{D\,r.m.s.}$, but a fine mesh is required for the r.m.s. value of the lift coefficient.

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5. References

1. BARANYI L., SHIRAKASHI M., 1999, Numerical Solution for Laminar Unsteady Flow about Fixed and Oscillating Cylinders, *Journal of Computer Assisted Mechanics and Engineering Sciences*, Warsaw, **6**, 263-277

2. CHENG M., CHEW Y.T., LUO S.C., 1997, A Hybrid Vortex Method for Flows Over a Bluff Body, *Int. Journal for Numerical Methods in Fluids*, **299**, 253-274

3. HULBRUT S.E., SPAULDING M.L., WHITE F.M., 1982, Numerical Solution for Laminar Two Dimensional Flow About a Cylinder Oscillating in a Uniform Stream, *Journal of Fluids Engineering*, **104**, 214-222

4. HYUNG J.S., CHONG K.C., JAE M.H., 1995, Experimental Study of Uniform Shear Flow Past a Rotating Cylinder, *Journal of Fluids Engineering*, **117**, 62-67

5. KARNIADAKIS G.E., TRIANTAFYLLOU G.S., 1989, Frequency Selection and Asymptotic States in Laminar Wakes, *Journal of Fluid Mechanics*, **199**, 441-469

6. KAWAMURA T., 1984, Computation of High Reynolds Number Flow Around a Circular Cylinder with Surface Roughness, *Proceedings of the 22nd Aerospace Sciences Meeting*, Reno, Nevada, AIAA-84-0340, 1-11

7. MENEGHINI J.R., BEARMAN P.W., 1995, Numerical Simulation of High Amplitude Oscillatory Flow About a Circular Cylinder, *Journal of Fluids and Structures*, 435-455

8. ROACHE P.J., 1982, *Computational Fluid Dynamics*, Hermosa Publishers, Albuquerque

9. ROSHKO A., 1954, On the Development of Turbulent Wakes from Vortex Streets, *NACA Rep.*, 1191

Modelowanie i analiza nieustalonego przepływu laminarnego wokół drgających i wirujacych cylindrów kołowych

Streszczenie

W pracy przedstawiono uzyskane metodą różnic skończonych rozwiązanie równań Naviera-Stokesa opisujących dwuwymiarowy, nieściśliwy przepływ laminarny wokół cylindrów. Równania przetransformowano do inercyjnego układu współrzędnych związanych z cylindrem. Człony konwekcyjne przedstawiono za pomocą różnic wstecznych trzeciego rzedu, inne pochodne przestrzenne – różnicami centralnymi czwartego rzędu, a pochodne czasowe – różnicami progresywnymi. Obliczone w ten sposób liczby Strouhala dla cylindrów utwierdzonych są zgodne z wynikami doświadczalnymi. Zmienność czasowych współczynników sił nośnych i oporu w zależności od parametru rotacji α przedstawiono również dla wirującego cylindra i dwóch siatek podziału.

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