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MODELING OF THE LUBRICATION MECHANISM IN HUMAN JOINTS USING MICROPOLAR FLUID THEORY

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The paper aims at presentation of modeling of human joints within the framework micropolar fluid theory. A mathematical model of two converging spheres represents the biobearings. In the model the boundaries of the spheres are considered to be rigid and the lubricant (synovial fluid) is represented by a micropolar fluid. Basing on the asymptotic solution obtained for squeezing motion of converging spheres (Kucaba-Piętal, 1999), the velocity vector in a gap is determined and asymptotic values of the force are calculated. The effects of rheological constants variation on the flow field in a gap are disscussed.

Key words: synovial fluid, micropolar fluid, human joints

1. Introduction

For the last thirty years examination of the lubrication mechanisms associated with human joints has received much attention. Synovial joints are natural bearings that are subjected to wear in a similar way as their mechanical counterparts. The main obstacle the research is confronted with is that synovial joints are part of a living system and a detailed study of morphology is limited. Thus, many simplifications of such a complicated structure are necessary for construction of the mathematical model. Several types of the lubrication mechanism are belived to occur in the functioning of human joints, e.g., hydrodynamic, boundary, weeping and mixed lubrications.

The two approaches are taken in theoretical studies. The first group of papers deals with the boundary surfaces in synovial joint in a comparatively realistic way. The Newtonian model of lubricant is assumed. These papers concern lubrication which involves porosity (Nigam et al., 1982). The microelastohydrodynamic model of synovial joint was also taken into account (Jin and Dowson, 1997). In the second group of papers the rheological properties of synovial fluid are taken into account since detailed information about the joint lubricant is not available. So far, a number of non-Newtonian models of synovial fluid have been used, e.g., Sutterby's model (Pytko et al., 1993), Rivlin-Ericksen model (Wierzcholski and Pytko, 1995) and micropolar one (Prawal Sinha et al., 1981), which include non-linear effect but differ in the constitutive laws employed. This article belongs to the last group. The behaviour of synovial fluid is considered to be governed by the micropolar fluid theory proposed by Eringen (1966), within the frame of which the micro-rotational effects due to rotation of fluid molecules can be described. This approach becomes of crucial importance when considering fluid flows in narrow channels and when the fluid under investigation includes substructures. Squeezing of the synovial fluid falls into these two categories very well.

In a micopolar fluid model rigid particles contained in a small volume element can rotate about the centre of the volume element and this rotation is represented by a micro-rotation vector. This local rotation of particles superimposed with the usual rigid body motion of the entire volume element. The laws of classical continuum mechanics are augumented by the additional equations representing the conservation of microinertia moments and balance of first stress moments that arise due to the microstructure in a material. The field equations can be presented in terms of two independent kinematic vector fields – the velocity and microrotation and involve material constants. The stress tensor is not symmetric. Surveys of application of micropolar fluid theory can by found in Prohorenko and Migoun (1988) and Łukaszewicz (1999).

The first approximation of the human joint lubrication theory that uses the microcontinuum approach was presented by Prawal Sinha et al. (1981) and only few papers have been devoted to this subject, e.a., Nigam et al. (1982), Zaheeruddin (1980). The authors used the Reynolds equations in describing the flow due to the squeezing motion of flat plates representing the synovial joint.

The theoretical results obtained illustrate reasonably well that using this approach it is possible to explain such observed phenomena as:

- effective viscosity growth near the solid boundary,
- filtering action due to the porous nature of the cartilage, which could not have been explained on the basis of other models of synovial fluid.

Because the exact values of the rheological constants were not known, therefore it was impossible to compare this approach with other ones.

Rapid progress in experimental techniques brings about new information on the microscopic structure of the biolubricant. Murakami et al. (1997) investigated the effect of size and concentration of the hyaluronic acid molecules on the synovial joint behaviour in a standing position. Podsiadło et al. (1997) examined very carefully the populations of particles produced in the wear processes, presenting dimensions and shapes of the particles. Currently the comprehensive paper, which summarizes the available results on friction and wear in natural and artifical joints, allowing for estimation of the rheological coefficients that appear in the micropolar fluid theory prepared by Telega. This provides also the motivation for the present work.

The paper aims at preliminary calculation of the flow parameters occuring in human joints during the squeezing motion using estimated values of the rheological coefficients. The Stokes equations for a micropolar fluid are investigated. In the analysis it is assumed that the human joint can be approximated by a two-sphere bearing-system. The flow in a gap is depicted and a force is calculated basing on the asymptotic solution obtained for two converging spheres (Kucaba-Piętal, 1999). The rheological coefficients of micropolar fluid are estimated from the experimental data of the microstructure. In this study the calculations were made using MATHCAD 7 on the assumption that on the surfaces the microrotation vector vanishes.

The results obtained show that the effect of rheological coefficients on the flow field in the gap can be observed. The analysis of hydrodynamics parameters depending on various values of the microrotation vector on the surfaces is also very important issue. Another paper will be devoted to this problem.

It is worth mentioning that the exact values of the coefficients for synovial fluid can be determined carrying out the experiment described in the book by Prohorenko and Migoun (1988)

2. Model of the synovial joint

The synovial joint may be described as a system of two articulating bones covered with a soft tissue called cartilage and the synovial fluid filling the cavity made by these bones. Typical examples are:

- knee joint – can be represented by a cylinder arrangement that resembles a close fitting journal bearing,
- hip joint – which can be approximated by an equivalent spherical bearing, see Fig.1.

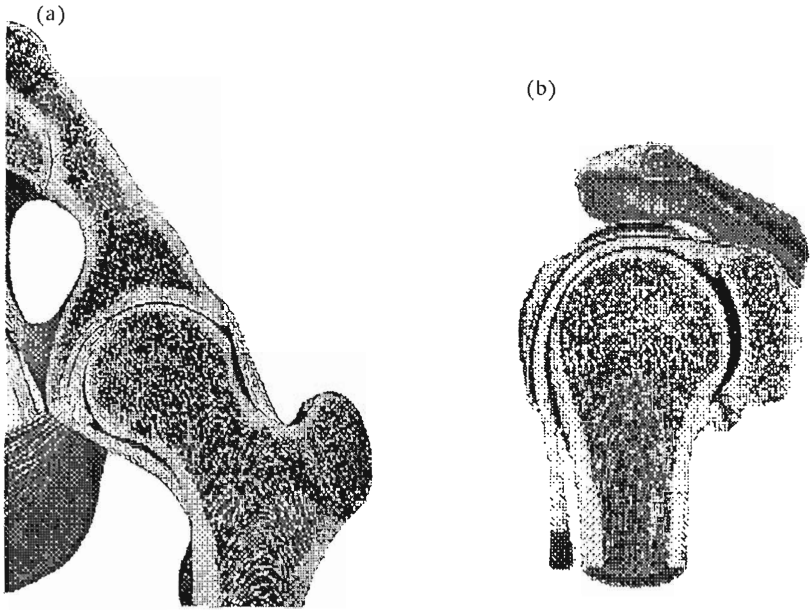

Fig. 1. Joints; (a) hip, (b) shoulder (Sobotta, 1982)

The joints contain the synovial fluid which is responsible for complex joint lubrication phenomena. The synovial fluid can be briefly described as a dialysate of blood plasma because it contains about one-third of the protein concentration of the plasma. Moreover, it contains a very important polymer known as hyaluronic acid (mucopolysaccharide) which gives the synovial fluid its viscous and elastic properties. The molecular weight of the hyaluronic acid is of the order of 10^6 and the length of the chain is of the order of $0.5 \div 5\mu$m, Dowson (1990). The salient feature of the biolubricant consists in an effective viscosity growth in the neighbourhood of the solid boundary.

The viscosity of the synovial fluid ranges from 0.01 Pa·s at low shear rates ($0.1\,s^{-1}$) to 0.02 Pa·s at much higher shear rates ($1000\,s^{-1}$).

It was shown, that the viscosity of this fluid depends on both the content and the molecular size of the hyaluronic acid. In the first paper studying this phenomenon, Negami (1964), showed that the viscosity of the synovial fluid varied almost linearly with the concentration of the hyaluronic acid (HA). Dowson (1990) put forward the concept and presented a formula describing the effect of HA concentration at low shear rate on viscosity. The latest experimental results obtained by Murakami et al. (1997) shoved that the concentration of HA primarily controled the viscous property of the lubricant. The viscous properties of saline solution of sodium hylauronate (HA, molecular weight $8.8 \cdot 10^5$) were measured by a cone/plate viscometer for various concentrations of HA which ranged from 0.1 g/dl to 2.0 g/dl. The viscosity of the fluid varied from 10^{-2} to 10^2 Pa s. Moreover, the synovial fluid contained the wear debris. In the paper by Podsiadło et al. (1997) detailed results concerning the particles which occured in synovial joints were presented. The mean values of area and length of the particles in normal joints amounted $542.6\mu m^2$ and $37.6\mu m$, respectively. The chain length of the hylauronic acid molecules and its concentration depend on the joint condition, i.e., whether it is degenerated, say rheumathoid, or functions normally. Of great interest is also the case of joints after arthroplasty. Then a great variety of wear debris is observed, since PMMA and metal particles inevitably detach from prosthesis surfaces.

These results indicate clearly, that the synovial fluid may be regarded as a micropolar fluid, in which the substructure is formed by the acid molecules and wear debris. An increase in the concentration of hylauronic acid in the synovial fluid implies apperance of the micromotions and the coupled stresses.

3. Theoretical formulation: assumptions, equations and method for solution

3.1. Formulation of the problem

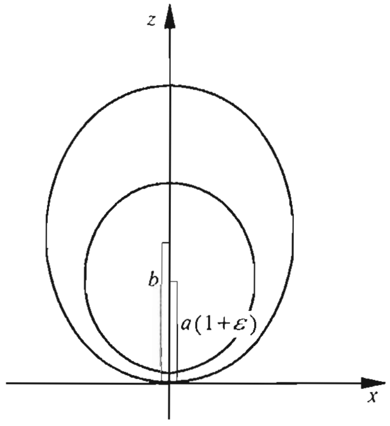

Fig. 2. Model of the hip joint

Geometrically, the synovial joint, especially the hip joint, may be represented by a two-sphere system, see Fig.2. The surfaces of the spheres are assumed to be rigid. A quasi-steady flow field of an incompressible micropolar fluid in a gap between two spheres due to a squeezing motion is considered. The sphere S_a of radius a approaches at the velocity U the other sphere S_b of radius b, which is at rest. The minimum gap of the two spheres is $a\epsilon$ with $\epsilon \leqslant 1$. In the polar coordinate system (r,θ,z) the centre of the stationary sphere lies at $(0,0,b)$.

The centre of moving spheres lies at the point $(0,0,a(1+\epsilon))$. The parameter β denotes the ratio $\beta = b/a$. The translational velocity of the sphere S_a is $(0,0,-U)$. The flow at low Reynolds numbers is examined.

The equations of motion describing the flow in the absence of body forces are the quasi-steady Stokes equations for a micropolar fluid and have the following form

$$
\begin{aligned}
&(\mu+\kappa)\nabla^2\boldsymbol{v} + \kappa\nabla\times\boldsymbol{\omega} - \nabla p = \mathbf{0}\\
&(\alpha^m+\beta^m+\gamma)\nabla\nabla\boldsymbol{\omega} - \gamma\nabla\times\nabla\times\boldsymbol{\omega} + \kappa\nabla\times\boldsymbol{v} - 2\kappa\boldsymbol{v} = \mathbf{0}
\end{aligned}
$$

The flow is incompressible, $\nabla\cdot\boldsymbol{v} = 0$, where $\boldsymbol{v}$ denotes the fluid velocity. Here $\boldsymbol{\omega}$ is the microrotation vector. The positive coefficients μ, κ, α^m, β^m, γ characterise isotropic properties of the micropolar fluid.

Because of the flow axisymmetry $\boldsymbol{\omega} = [0,\varpi,0]$ the stream function $\Psi(r,z)$ can be used and the axial and radial velocity components are expressed as follows

$$
v_r = U\frac{1}{r}\frac{\partial\Psi}{\partial z} \qquad\qquad v_z = U\frac{-1}{r}\frac{\partial\Psi}{\partial r} \tag{3.1}
$$

Substituting Eqs (3.1) into the Stokes equations yields

$$
\begin{aligned}
&-(\mu+\kappa)L_1^2\Psi + \kappa L_1(r\varpi) = 0\\
&-\gamma L_1(r\varpi) + \kappa L_1\Psi - 2\kappa r\varpi = 0
\end{aligned} \tag{3.2}
$$

where L_1 is the generalised axisymmetric Stokes operator

$$
L_1 = \frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}
$$

The above equations are linear fourth-order partial differential equations.

After elimination of the component ϖ of the microrotation vector $\boldsymbol{\omega}$ from Eqs (3.2) we arrive at

$$
L_1^2(L_1-\lambda^2)\Psi = 0 \tag{3.3}
$$

with the microrotation given by

$$\varpi = \frac{1}{2r}\Big(L_1\Psi + \frac{\gamma(\mu+\kappa)}{\kappa^2}L_1^2\Psi\Big) \tag{3.4}$$

where

$$\lambda^2 = \frac{\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}$$

To solve Eq (3.3), which is equivalent to the Stokes equations for a micropolar fluid (3.2), the boundary conditions have to be specified. For the velocities we assume the conditions of impermeability and adhesion on the surface of the spheres. For the microrotation, in nearly all relevant literature it is assumed that

$$\boldsymbol{\omega} = \mathbf{0} \tag{3.5}$$

on the surface which bounds the fluid. But recently, in a few papers particularly devoted to journal bearing or flow in a narrow channel more general conditions have been taken into account

$$\boldsymbol{\omega} = \alpha\frac{1}{2}\boldsymbol{\zeta}_0 \tag{3.6}$$

where α denotes a nonnegative constant, $\boldsymbol{\zeta}_0 = \nabla\times\boldsymbol{v}$, $\boldsymbol{\zeta}_0 = [\eta,\zeta,\xi]$. Such forms of boundary conditions follow from various liquid/wall materials utilized in modern technologies, Migoun (1984). It is clear that Eq (3.6) simplifies to the form of Eq (3.5) for $\alpha = 0$.

According to the above considerations the boundary conditions for Ψ and ϖ on the moving sphere S_a are

$$\Psi = -\frac{1}{2}r^2 \qquad \frac{\partial\Psi}{\partial z} = 0 \qquad \varpi = \alpha_1\frac{1}{2}\zeta \tag{3.7}$$

and on the stationary sphere S_b

$$\Psi = \frac{\partial\Psi}{\partial z} = 0 \qquad \varpi = \alpha_2\frac{1}{2}\zeta \tag{3.8}$$

Moreover, at infinity

$$\Psi = 0 \tag{3.9}$$

where $0 \leqslant \alpha_i \leqslant 1$ for $i = 1,2$.

The stream function Ψ that satisfies the fourth-order partial differential equation (3.3) with the corresponding boundary conditions (3.7) $\div$ (3.9) and microrotation vector (3.4) yields the flow field for the problem under consideration.

3.2. Solution

The method for solving the micropolar squeezing flow problem in the gap will be sketched here, for details the Reader is referred to the previous paper (Kucaba-Piętal, 1999). The solution process proceeds as follows:

I. We introduce non-dimensional, cylindrical coordinates by the following relations

$$R = \frac{1}{a\epsilon} r \qquad\qquad Z = \frac{1}{\sqrt{a\epsilon}} z$$

The scaled stream function Ψ is decomposed into two parts

$$\Psi = \Psi^1 + \Psi^2 \tag{3.10}$$

each of them is expanded into the power series of ϵ as follows

$$\begin{aligned}
\Psi^1(R,Z) &= a^2\epsilon\Big[\Psi_0^1(R,Z) + \epsilon\Psi_1^1(R,Z) + \epsilon^2\Psi_2^1(R,Z)\Big] + O(\epsilon^4) \\
\Psi^2(R,Z) &= a^2\epsilon\Big[(\Psi_0^2(R,Z) + \epsilon\Psi_1^2(R,Z) + \epsilon^2\Psi_2^2(R,Z)\Big] + O(\epsilon^4)
\end{aligned} \tag{3.11}$$

We set $\Psi_i = \Psi_i^2 + \Psi_i^1$.

II. Substitution Eqs (3.11) into the equations resulting from the Stokes equations and equating the terms at same powers of ϵ two sets of differential equations are obtained. These systems are effectively solved to the second order in ϵ. The solutions are polynomials in Z, include some unknown functions of R and are of the following form

$$\begin{aligned}
\Psi_0^1 &= A_0Z^3 + B_0Z^2 + C_0Z + D_0 \\
\Psi_1^1 &= A_1Z^3 + B_1Z^2 + C_1Z + D_1 - \frac{Z^5}{10}\tau A_0 - \frac{Z^4}{6}\tau B_0 \\
\Psi_2^1 &= A_2Z^3 + B_2Z^2 + C_2Z + D_2 - \frac{Z^5}{10}\Big(\tau A_1 + \frac{1}{12}\tau^2 C_0\Big) + \\
&- \frac{1}{6}Z^4\Big(\tau B_1 + \frac{1}{4}\tau^2 D_0\Big) + \frac{1}{280}Z^7\tau^2 A_0 + \frac{1}{120}Z^6\tau^2 B_0 \\
\Psi_0^2 &= E_0Z + F_0 \\
\Psi_1^2 &= E_1Z + F_1 - \frac{Z^3}{6}\tau E_0 - \frac{Z^2}{2}\tau F_0 \\
\Psi_2^2 &= E_2Z + F_2 - \frac{Z^3}{6}\tau E_1 - \frac{Z^2}{2}\tau F_1 - \frac{Z^5}{90}\tau^2 E_0 - \frac{Z^4}{24}F_0
\end{aligned} \tag{3.12}$$

where: $\tau = \frac{\partial^2}{\partial R^2} - \frac{1}{R}\frac{\partial}{\partial R}$.

We have to determine the functions depending only on R A_0, B_0, ..., F_1 which appear in the above general solutions (3.12).

III. To find the functions of R the boundary conditions on the spheres are used in the following way: first, the equations of surfaces of the spheres in the neighbourhood of the gap are expanded into powers of ϵ. In this way we obtain approaching the spheres: $H_a = 1+\frac{1}{2}R^2$ and $H_b = \frac{1}{2\beta}R^2$. That, applying the Taylor expansion and series representation of Ψ^1, Ψ^2 to the boundary conditions (3.7)$\div$(3.9) the unknown functions are determined. They are listed in Appendix. Having performed the calculation, from Eqs (3.1) the velocity field can be found and we are able to calculate the force.

IV. The components of the force acting on either of the spheres are $(0, 0, F_z)$. The asympotic formula for the force acting on the sphere S_a up to $O(\epsilon \ln \epsilon)$ has the form

$$\begin{aligned} f &= \Big(1+\frac{\kappa}{2\mu}\Big)\Big[\frac{-\beta^2}{\epsilon(1-\beta)^2} - \frac{1-7\beta+\beta^2}{5(1-\beta)^3}\ln\epsilon^{-1} + K(\beta) + \\ &- \frac{1-18\beta-29\beta^2-18\beta^3+\beta^4}{21(1-\beta)^4}\epsilon\ln\epsilon^{-1} + \epsilon L(\beta) + O(\epsilon^2)\Big] \end{aligned} \tag{3.13}$$

4. Results

Two parameters are very important when studying the problems of micropolar fluid. One of them represents the characteristic material "length" of the microstructure. The second characterises the coupling between the viscosities κ and μ (both these viscosities are characteristic for the coupled stresses) and, for the synovial fluid, may be considered as a measure of the concentration of suspended particles. In almost all papers devoted to application of the theory to lubrication and biolubrication problems such parameters are introduced and the force, wear and other quantities are calculated as a function of certain parameters. The parameters are denoted as: L (the length parameter) and N (coupling number) and defined by the formula

$$L = \frac{c}{l} \qquad l = \sqrt{\frac{\gamma}{4\mu}} \tag{4.1}$$

where c is the radial clearance of the gap, and

$$N = \sqrt{\frac{\kappa}{2\mu + \kappa}} \tag{4.2}$$

In the limiting cases as $N \to 0$ or $L \to 0$ we recover the Newtonian case.

Prohorenko and Migoun (1988) proposed an experiment to determine the values of parameters k and δ for flows in channels which enable one to determine viscosity coefficients γ, μ and κ. Parameter k characterizes the linear dimension of the microstructure. The parameter δ can be regarded as the coupling number. They are given by

$$k^2 = \lambda^2 h^2 = \frac{\kappa(2\mu + \kappa)}{\gamma(\mu + \kappa)} h^2 \qquad\qquad \delta = \frac{2\kappa(1 - \alpha)}{2(\mu + \kappa) - \kappa\alpha} \tag{4.3}$$

where h is the pipe diameter.

The material coefficients for the water were found by Kolpashchikov et al. (1981) and for the fluid used in defectoscopy by Prohorenko and Migoun (1997). It is worth mentioning that putting $\alpha = 0$ in Eq $(4.3)_2$ the parameters (δ, k) can be expressed by (L, N).

Let us discus the results obtained. For simplicity, at this initial stage it is assumed that $\alpha_1 = \alpha_2 = 0$, i.e. the microrotation vector vanishes on the surfaces. Using the experimental results of Dowson (1990) the viscosity coefficient of the synovial fluid is equal to: $\mu = 10^{-2}$ Pa s while the biobearing dimensions are: the width of the gap is $\epsilon = 150\mu$m, 300μm, $\beta = (1 + 2\epsilon)$. The values of κ and μ are estimated as

$$0.5 \leqslant \frac{\kappa}{\mu} \leqslant 4.5$$

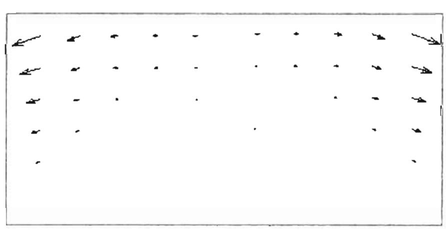

Fig. 3. Sketch of the velocity in the gap in stretched coordinates (R, Z)

In Fig.3 the velocity distribution in the gap is depicted for the parameters at the ratio $\kappa/\mu = 0.5$. This calculation requires additional rheological coefficient γ the value of which was estimated as $\gamma = 2$. The plot is presented in the scaled variables (R, Z).

The force acting on the sphere S_a as a function of κ, μ is presented in Fig.4.

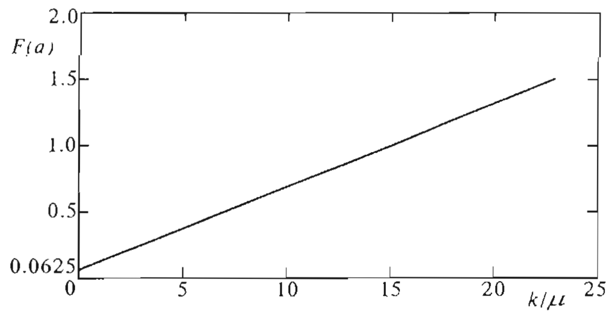

Fig. 4. Course of the asymptotic force f versus κ/μ

Summing up the following conclusions can be drawn:

- The micropolar approach to the study of lubrication offers new possibility modeling of human joints.
- Strong influence of rheological coefficients on hydrodynamic quantities of the flow in human joints can be observed.
- It would be an intersting task to conduct experiments is view of the present study to find the exact values of the constants characterizing the synovial fluid.

Acknowledgements
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A. Appendix

Applying the procedure described in Section 3.2 III the functions A_i, F_i which appear in the formula describing the stream functions, (3.12) can be obtained and have the following form

$$E_0 = \frac{d_{11}d_{22}(6A_0H_a + 2B_0) - d_{21}d_{12}(6A_0H_b + 2B_0)}{d_{12}d_{22}H}$$

$$
\begin{aligned}
F_0 &= \frac{d_{11}d_{22}(6A_0H_a+2B_0)H_b - d_{21}d_{12}(6A_0H_b+2B_0)H_a}{d_{12}d_{22}H}\\
E_1 &= -\frac{g_a - g_b}{H}\\
F_1 &= \frac{g_a+g_b}{2} - E_1(H_a+H_b)
\end{aligned}
$$

where $H = H_a - H_b$ and functions g_i $(i = 1,2)$ denote

$$
\begin{aligned}
g_1 &= \frac{d_{11}}{d_{12}}[\tau(-A_0H_a^3 - B_0H_a^2 + C_0H_a + D_0) + 6A_1H_a + B_1] +\\
&- \tau\frac{1}{6}(E_0H_a^3 + F_0H_a^2)\\
g_2 &= \frac{d_{21}}{d_{22}}[\tau(-A_0H_b^3 - B_0H_b^2 + C_0H_b + D_0) + 6A_1H_b + B_1] +\\
&- \tau\frac{1}{6}(E_0H_b^3 + F_0H_b^2)\\
A_0 &= -\frac{R^2}{H^3}\\
B_0 &= \frac{3}{2}R^2(H_a+H_b)\frac{1}{H^3}\\
C_0 &= -3R^2H_aH_b\frac{1}{H^3} - E_0\\
D_0 &= \frac{1}{2}R^2(3H_a - H_b)H_b^2\frac{1}{H^3} - F_0\\
A_1 &= (3H_a^2 + 4H_aH_b + 3H_b^2)\frac{1}{10}\tau A_0 + (H_a+H_b)\frac{1}{3}\tau B_0 +\\
&+ \frac{3}{8}(1+\beta^{-3})\frac{R^6}{H^4} + \tau\frac{1}{6}E_0\\
B_1 &= -(H_a+H_b)(H_a^2 + 3H_aH_b + H_b^2)\frac{1}{5}\tau A_0 - (H_a^2 + 4H_aH_b + H_b^2)\frac{1}{6}\tau B_0 +\\
&- \frac{3}{8}R^6H^{-4}[H_a + 2H_b + \beta^{-3}(H_b + 2H_a)] + \frac{\tau}{2}F_0\\
C_1 &= H_aH_b(4H_a^2 + 7H_aH_b + 4H_b^2)\frac{1}{10}\tau A_0 + H_aH_b(H_a+H_b)\frac{1}{3}\tau B_0 +\\
&+ \frac{3}{8}R^6H^{-4}[H_b(2H_a + H_b) + \beta-^3(2H_b + H_a)H_a] - E_1\\
D_1 &= -H_a^2H_b^2\tau\Big[(H_a+H_b)\frac{1}{5}A_0 + \frac{1}{6}B_0\Big] - \frac{3}{8}R^6H^{-4}H_bH_a(H_b - \beta^{-3}H_a) - F_1
\end{aligned}
$$

$$
\begin{aligned}
A_2 &= -(5H_a^4 + 8H_a^3H_b + 9H_a^2H_b^2 + 8H_aH_b^3 + 5H_b^4)\frac{1}{280}\tau^2A_0 + \\
&- (2H_a^3 + 3H_a^2H_b + 2H_b^3 + 3H_b^2H_a)\frac{1}{60}\tau^2B_0 + \\
&+ \frac{1}{10}(3H_a^2 + 4H_aH_b + 3H_b^2)\Big[\tau\Big(A_1 - \frac{\tau}{6}E_0\Big) + \frac{1}{12}\tau^2C_0\Big] + \\
&+ \frac{1}{3}(H_a + H_b)\Big[\tau\Big(B_1 - \frac{\tau}{2}F_0\Big) + \frac{1}{4}\tau^2D_0\Big] + \frac{3}{32}R^{10}H^{-5}(1+\beta^{-6}) + \\
&+ \frac{1}{4}R^4H^{-2}\Big[(H_a^3 - \beta^{-3}H_b^3)\tau A_0 + (H_a^2 - \beta^{-3}H_b^2)\tau B_0\Big] + \\
&+ \frac{3}{16}R^8H^{-4}(1+\beta^{-5}) - 3A_1(H_a - \beta^{-3}H_b) - B_1(1-\beta^{-3})
\end{aligned}
$$

where $H_a = 1 + \frac{1}{2}R^2$ and $H_b = \frac{1}{2\beta}R^2$ define the parabolas which approache the spheres. Symbol β denotes the ratio $\beta = b/a$.

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Modelowanie smarowania w stawach na bazie teorii płynów mikropolarnych

Streszczenie

Przedmiotem pracy jest opis smarowania w stawach na bazie teorii płynów mikropolarnych. Rozpatrywany jest ruch ciśnieniowy (squeezing). Staw (biodrowy) modelowany jest za pomoca układu dwóch kul o twardych powierzchniach. Zakłada się, że przepływ opisany jest równaniami Stokesa. Na bazie rozwiązania asymptotycznego obliczono rozkład predkości w szczelinie stawowej oraz wartość działającej asymptotycznie siły. Stałe reologiczne mazi stawowej oszacowano na podstawie danych experymentalnych.

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