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APPLICATION OF THE BOUNDARY ELEMENT METHOD IN RADIATION

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Two applications of the boundary element technique to radiative analysis are discussed: radiation in participating media and the presence of concave, self irradiating cavities in heat conducting bodies. When compared with classic Hottel's zoning approach, BEM leads to shorter computing times. This is achieved because the most time consuming procedure of Hottel's approach, volume integration, can be avoided. Additionally, instead of multiple integrals only a single integration ought to be performed. The paper presents a general, self adaptive quadrature capable of evaluating integrals over the elements located close to the observation point. The paper shows also an original technique of coupling heat radiation and conduction. Both problems are solved by using BEM. Static condenstation of linear unknowns before entering the iterative loop, makes this approach very efficient. Numerical examples are included.

Key words: heat radiation, boundary elements

1. Introduction

The Boundary Element Method (BEM) has become a well-established tool in solving various engineering problems. Conductive heat transfer, elastostatics, acoustics and electromagnetics are but a few examples where BEM can be competitive to the Finite Elements Method (FEM) or Finite Differences Method (FDM). The basic idea behind BEM consists in transformation the original boundary-value problem into an equivalent integral equation. The latter is then discretized and the resulting set of algebraic equations is solved yielding the desired values of the unknown function and its derivative.

Heat radiation is a phenomenon ruled by an integral equation. Therefore, FEM and the FDM, being developed to solve differential equations, require serious modifications to cope with the heat radiation. BEM, as a technique of discretization of integral equations, seems to be the proper tool to handle heat radiation problems. It is not only the type of equation that make BEM suitable for radiative analysis. Białecki (1993) showed that the kernel functions arising in integral equations of heat radiation and heat conduction have the same asymptotic singular behaviour. Both kernels depend on the distance between the observation and current points in the power of -2. All these features cause that the idea of using BEM to solve heat radiation problems arise in a natural way.

The present paper shows some advantages of BEM when applied to solving two types of heat radiation problems: heat transmission in optically active media and heat conduction in bodies whose boundaries can mutually irradiate themselves.

2. Radiation in emitting-absorbing medium

2.1. Governing equations

Consider a volume V containing participating medium and bounded by a diffuse surface S. The governing equations of heat radiation in this system link the following four functions:

- radiative heat flux q^r defined as a net radiative energy gain of an elemental surface bounding the domain of interest,
- blackbody emissive power $e_b(T)$ of the same surface,
- radiative heat source q_v^r being the net radiative energy gain of an elemental volume of the medium,
- blackbody emissive power of the medium $e_b(T^m)$,

where T, T^m are the temperatures of the bounding surface and medium, respectively. The equations contain two material properties: ϵ, being the emissivity of the surface and a, the absorption coefficient of the medium. Let $\boldsymbol{r}$, stand for the current point placed on the walls bounding the medium, $\boldsymbol{r}'$ denote the current point placed within the medium and $\boldsymbol{p}$ stand for the observation point. The transmissivity $\tau(\boldsymbol{r},\boldsymbol{p})$ is defined as a fraction of the energy emitted at $\boldsymbol{r}$ that reaches $\boldsymbol{p}$. Transmissivity can be calculated from the relationship

$$\tau(\boldsymbol{r},\boldsymbol{p}) = \exp\Big[-\int\limits_{L_{rp}} a(\boldsymbol{r}')\, dL_{rp}(\boldsymbol{r}')\Big] \tag{2.1}$$

with integration carried out along the line linking $\boldsymbol{r}$ and $\boldsymbol{p}$. Another important function, radiosity is defined as the net flux of radiative energy leaving an infinitesimal surface. Radiosity is a sum of the emitted and reflected energy and is a linear combination of the blackbody emissive power and the radiative heat flux $b = e_b + q^r(1-\epsilon)/\epsilon$. Using this notation the set of equations describing radiative heat transfer in emitting-absorbing media can be written as

$$\begin{aligned} &q^r(\boldsymbol{p}) + \epsilon(\boldsymbol{p})e_b[T(\boldsymbol{p})] = \epsilon(\boldsymbol{p})\int\limits_S b(\boldsymbol{r})\tau(\boldsymbol{r},\boldsymbol{p})K(\boldsymbol{r},\boldsymbol{p})\, dS(\boldsymbol{r}) + \\ &+\epsilon(\boldsymbol{p})\int\limits_V a(\boldsymbol{r}')e_b[T^m(\boldsymbol{r}')]\tau(\boldsymbol{r}',\boldsymbol{p})K_p(\boldsymbol{r}',\boldsymbol{p})\, dV(\boldsymbol{r}') \\ &q_v^r(\boldsymbol{p}) + 4a(\boldsymbol{p})e_b[T^m(\boldsymbol{p})] = a(\boldsymbol{p})\int\limits_S b(\boldsymbol{r})\tau(\boldsymbol{r},\boldsymbol{p})K_r(\boldsymbol{r},\boldsymbol{p})\, dS(\boldsymbol{r}) + \\ &+a(\boldsymbol{p})\int\limits_V a(\boldsymbol{r}')e_b[T^m(\boldsymbol{r}')]\tau(\boldsymbol{r}',\boldsymbol{p})K_0(\boldsymbol{r}',\boldsymbol{p})\, dV(\boldsymbol{r}') \end{aligned} \tag{2.2}$$

where the kernel functions are defined as

$$\begin{aligned} K(\boldsymbol{r},\boldsymbol{p}) &= \frac{\cos\phi_r \cos\phi_p}{\pi|\boldsymbol{r}-\boldsymbol{p}|^2} \qquad & K_p(\boldsymbol{r}',\boldsymbol{p}) &= \frac{\cos\phi_p}{\pi|\boldsymbol{r}'-\boldsymbol{p}|^2} \\ K_r(\boldsymbol{r},\boldsymbol{p}) &= \frac{\cos\phi_r}{\pi|\boldsymbol{r}'-\boldsymbol{p}|^2} \qquad & K_0(\boldsymbol{r}',\boldsymbol{p}) &= \frac{1}{\pi|\boldsymbol{r}'-\boldsymbol{p}|^2} \end{aligned} \tag{2.3}$$

where ϕ_r, ϕ_p are the angles made by the line connecting the current and observation points and the normal at $\boldsymbol{r}$ and $\boldsymbol{p}$, respectively. Symbol $|\boldsymbol{r}-\boldsymbol{p}|$ stands for the distance between the $\boldsymbol{r}$ and $\boldsymbol{p}$.

The surface integrals appearing in the above equations are associated with the radiation of the walls bounding the enclosure under consideration. The volume integrals are due to the radiation of the emitting-absorbing medium filling the enclosure. When using spectral material properties, spectral blackbody emissive power and fluxes, Eqs (2.2) are valid for a selected wavelength. The same equations can be written for a given spectral band or the entire spectrum. For the sake of simplicity only the last situation will be addressed in the present paper. The approach has been also tested successfully for the band model, Białecki (1993).

BEM can be regarded as a variant of the zoning approach. The main drawback of the classic version of the powerful zonal technique lies in the excessive computing times. Another difficulty is due to the poor accuracy of calculations of the entries of the final matrix. It will be shown how using BEM these difficulties can be eliminated.

2.2. Acceleration of the computations

Hottel's method (Hottel and Sarofim, 1973) can be interpreted as a Galerkin technique of discretization of the governing equations, Białecki (1993). The classic zonal technique uses interpolating and weighting functions constant within surface and volume zones and vanishing outside these zones. Several FEM procedures described in the literature (cf Razzaque et al., 1983) can be seen as a generalization of the Galerkin technique with interpolation and weighting functions being locally based polynomials of higher order.

2.2.1. Avoiding volume integration

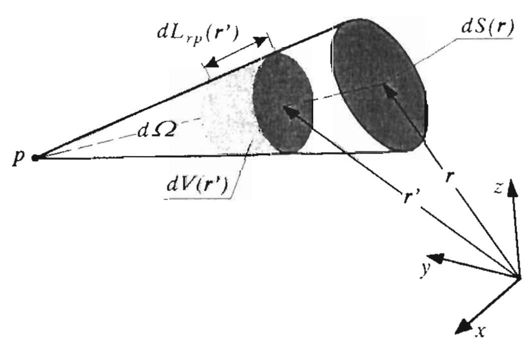

Fig. 1. Elemental volume in the spherical coordinate system

Entries of the system matrices in the zonal methods are expressed in terms of line, surface and volume integrals. The bottleneck of these approaches is the discretization of volume integrals. BEM offers a possibility of avoiding this step by converting the volume integral into equivalent, iterated surface integral from a line integral. The idea is to carry out the integration over the entire volume in a spherical system of coordinates. This is accomplished by first integrating along the line of sight (radius) and then integrating the result over the projected infinitesimal surface area of the bounding surface. These operations make use of the obvious relationship (cf Fig.1)

$$dL_{rp}(\boldsymbol{r}')\,d\Omega = dL_{rp}(\boldsymbol{r}')\frac{dS(\boldsymbol{r})\cos\phi_r}{|\boldsymbol{r}-\boldsymbol{p}|^2} = \frac{dV(\boldsymbol{r}')}{|\boldsymbol{r}'-\boldsymbol{p}|^2} \tag{2.4}$$

The resulting form of Eqs (2.2) is then

$$\begin{aligned} q^r(\boldsymbol{p}) + \epsilon(\boldsymbol{p})e_b[T(\boldsymbol{p})] &= \epsilon(\boldsymbol{p})\int_S b(\boldsymbol{r})\tau(\boldsymbol{r},\boldsymbol{p})K(\boldsymbol{r},\boldsymbol{p})\,dS(\boldsymbol{r}) + \\ &+\epsilon(\boldsymbol{p})\int_S\Big\{\int_{L_{rp}} a(\boldsymbol{r}')e_b[T^m(\boldsymbol{r}')]\tau(\boldsymbol{r}',\boldsymbol{p})\,dL_{rp}(\boldsymbol{r}')\Big\}K(\boldsymbol{r},\boldsymbol{p})\,dS(\boldsymbol{r}) \\ q_v^r(\boldsymbol{p}) + 4a(\boldsymbol{p})e_b[T^m(\boldsymbol{p})] &= a(\boldsymbol{p})\int_S b(\boldsymbol{r})\tau(\boldsymbol{r},\boldsymbol{p})K_r(\boldsymbol{r},\boldsymbol{p})\,dS(\boldsymbol{r}) + \\ &+a(\boldsymbol{p})\int_S\Big\{\int_{L_{rp}} a(\boldsymbol{r}')e_b[T^m(\boldsymbol{r}')]\tau(\boldsymbol{r}',\boldsymbol{p})\,dL_{rp}(\boldsymbol{r}')\Big\}K_r(\boldsymbol{r},\boldsymbol{p})\,dS(\boldsymbol{r}) \end{aligned} \tag{2.5}$$

Although the integrals arising in classic and BEM formulations are of the same dimensionality, BEM discretization is much cheaper. This aspect of numerical implementation will be discussed later.

2.2.2. Avoiding multiple integrals

The Galerkin technique requires double integration over surface and volume zones. Thus, to calculate the entries of the matrices, standard zoning methods require expensive integration in four, five or six dimensions. The time of matrix generation can be considerably reduced when, instead of the Galerkin technique, another weighted residual approach, namely, the collocation method, is used. The weighting function here is the Dirac function distribution. The disadvantage of this approach is, that contrary to the Galerkin discretization, the resulting matrices are nonsymmetric. As the savings in computer times are much greater than the gains due to the symmetry of the final matrix of the Galerkin formulation, collocation is more frequently used.

2.3. Discertization of integrals

The first step of the discretization of the integral equations is subdivision of the surface bounding the domain under consideration into a set of surface elements. This is analogous to the standard FEM approach. Additionally, the entire volume of the medium is subdivided into volume cells. It should be stressed that the volume mesh is completely independent of the surface mesh. Thus, it is possible to use fine surface elements next to coarse volume cells. This feature might be useful in practical computations.

2.3.1. Line integrals

The governing integral equations contain two types of line integrals: one is the transmissivity, Eq (2.1). The second is the due to the internal integration arising in Eqs (2.5) is defined as

$$J_2 = \int\limits_{L_{rp}} a(\boldsymbol{r}')e_b[T^m(\boldsymbol{r}')]\tau(\boldsymbol{r}',\boldsymbol{p})\, dL_{rp}(\boldsymbol{r}') \tag{2.6}$$

Assuming that the temperature and the absorption coefficient within each volume cell is constant, both line integrals can be evaluated analytically. Let d_l denote the length of the line of sight within a single cell and a_l and T_l^m stand for the absorption coefficient and temperature of the medium filling this cell, respectively. Then, the transmissivity can be calculated analytically as

$$J_1 = \tau(\boldsymbol{r},\boldsymbol{p}) \approx \tilde{\tau}(1, I_{rp}) = \exp\Big(-\sum_{l=1}^{I_{rp}} a_l d_l\Big) \tag{2.7}$$

where I_{rp} is the number of cells intersected by a ray travelling from $\boldsymbol{r}$ to $\boldsymbol{p}$.

The second integral can be also expressed in terms of the same quantities as

$$J_2 \approx \sum_{l=1}^{I_{rp}} e_b(T_l^m)A_l\tilde{\tau}(l+1, I_{rp}) \tag{2.8}$$

where

A_l – self absorption, $A_l = 1 - \exp(-a_l d_l)$

$\tilde{\tau}$ – transmissivity from the border of the lth element to $\boldsymbol{p}$ placed on the bounding surface, and

$$\tilde{\tau}(l+1, I_{rp}) = \exp\Big(-\sum_{m=l+1}^{I_{rp}} a_m d_m\Big)$$

2.3.2. Surface integrals

The final set of equations has the form

$$\begin{aligned} \mathbf{A}\boldsymbol{e}_b(T) + \mathbf{B}\boldsymbol{q}^r + \mathbf{C}\boldsymbol{e}_b(T^m) &= \mathbf{0} \\ \boldsymbol{q}_v^r + \mathbf{D}\boldsymbol{e}_b(T) + \mathbf{E}\boldsymbol{q}^r + \mathbf{F}\boldsymbol{e}_b(T^m) &= \mathbf{0} \end{aligned} \tag{2.9}$$

where $\mathbf{A}$ through $\mathbf{F}$ are known matrices depending on geometry and material properies, vectors $\boldsymbol{e}_b(T)$, $\boldsymbol{e}_b(T^m)$ store the values of blackbody emissive powers at points located on the walls and within the medium, respectively. Vector $\boldsymbol{q}^r$ contains the nodal values of the radiative heat flux whereas the entries of the $\boldsymbol{q}_v^r$ vector are the values of the radiative heat sources at points located within the medium.

With the values of the line integration known, the entries of the matrices $\mathbf{A}$ through $\mathbf{F}$ can be expressed as integrals over single surface elements. These integrals are evaluated using standard FEM and BEM approach, *i.e.* approximating both the geometry and distribution of the functions by locally based polynomials (shape functions). The next step of this procedure is the transformation of the arbitrary shaped surface elements into unit squares. Finally, the integrals over these squares are evaluated using Gaussian quadratures. It should be noted that to discretize the volume integral the following values should be known:

- Lengths of ray intercepted by the limits of the volume cells. These quantities are already determined when discretizing the integral associated with surface radiation
- The values of the kernel function, shape function and Jacobian at the Gaussian nodes. Also these quantities are found when discretizing the first integral.

Thus, the discretization of the volume integral, being the most time consuming step of zoning approaches, is in BEM carried out as a *by-product* of the discretization of the surface integral. The time of discetization of the volume integral is therefore practically negligible.

2.4. Accuracy of integration

The entries of the matrices resulting from all kinds of zoning discretization are expressed in integrals of the kernel function. This function is proportional to the squared inverse of the distance between the source and observation points. Standard quadratures are based on polynomial interpolation. Thus, by their nature, the quadratures cannot approximate very steep functions arising when the observation point is located close to the surface element over which the integration is carried out. Poor accuracy of determining the entries of the final set of equations has been reported in the heat radiation literature (Larsen and Howell, 1986; Vercammen and Fromment, 1980). To overcome this difficulty, some authors suggest using least square smoothing (Larsen and Howell, 1986; Vercammen and Fromment, 1980). As the integration error is not randomly distributed, this does not seem to be a proper methodology.

The same loss of accuracy arises when calculating the entries of BEM matrices for heat conduction, elastostatics etc. BEM literature describes several efficient techniques of handling this problem known as the quadrature of a *nearly singular function*. As the asymptotic behaviour of the integrands in BEM applied to potential and structural problems is the same as that of the heat radiation kernel function, these methods can be used without any modification in radiation codes.

The method used in this paper is based on *a priori* asymptotic error formula originally developed by Lachat and Watson (1977). The technique has been further improved by Białecki et al. (1994). The relative error ε of the integration can be expressed in terms of the number of Gaussian nodes G_ξ placed along the local coordinate ξ, minimum distance L_{min} between the element and source point and the length D_ξ of the element in ξ direction. The formula reads

$$2\frac{(2G_\xi+1)!}{(2G_\xi)!}\left[\frac{D_\xi}{4L_{min}}\right]^{2G_\xi} \leq \varepsilon \tag{2.10}$$

Similar equation holds for the second local coordinate defined over the element. Eq (2.10) enables one to find the minimum number of Gaussian nodes that keeps the integration error below the prescribed threshold. It should be stressed that there are two self adaptive strategies of increasing the accuracy of integration: increasing the number of Gaussian nodes and subdividing the element into subelements. The first approach is less efficient in the case the relative distance between the source point and the element is less than 0.25.

When using this adaptive integration, the accuracy of matrix generation is limited solely by the precision of the computer arithmetic. Therefore there is no need to use any smoothing procedures to satisfy the summation relation of the matrix entries. Numerical tests show that using single precision and 32 bit real number representation, the total error in the summation relation can be held below 1E-7. It should be stressed that the aforementioned technique is not restricted to the collocation method. The self adapting quadratures are also applicable to any zoning method including FEM and standard Hottel's approach.

2.5. Numerical example

A temperature field in a small test combustion chamber has been measured, Nadziakiewicz (1989). The resulting radiative heat fluxes are depicted in Fig.2. The results were obtained using 522 surface elements and 800 volume cells.

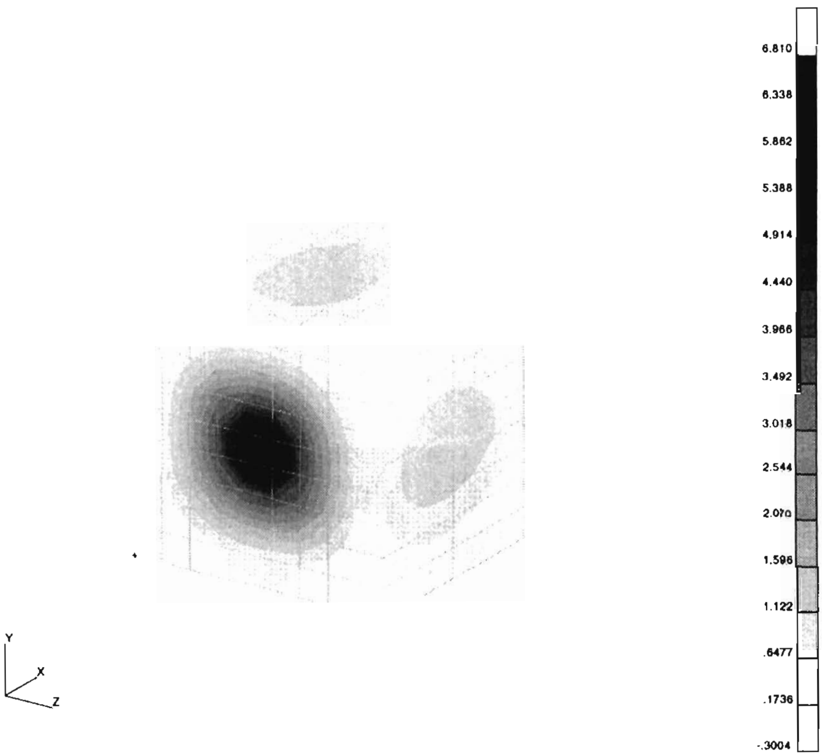

Fig. 2. Radiative heat fluxes in kW/m^2 on a wall of a combustion chamber

3. Coupled radiation, conduction and convection

Another area of BEM is application the situation when a heat conducting body contains radiating enclosures or cavities. In this case, the boundary conditions on concave boundaries contain the radiative heat flux, being a solution of an integral equation. Such boundary conditions are non local and require a special treatment.

The problem analysed here is a case of steady state heat conduction and transparent medium filling the radiating enclosures and cavities. The application of BEM to the solution of such problems will be described for the special case when the conductive portion of the analysis is carried out using BEM. The coupling between radiation and conduction can also be modelled when the conduction in the solid is solved by other numerical techniques, e.g. FEM.

3.1. Heat conduction

Discretization of the steady state heat conduction problem yields a set of algebraic equations of the form

$$\mathbf{H}\boldsymbol{T} = \mathbf{G}\boldsymbol{q} + \mathbf{Z}\boldsymbol{q}^r \tag{3.1}$$

where $\mathbf{H}$, $\mathbf{G}$, $\mathbf{Z}$ stand for matrices of constant, known coefficients. Vectors $\boldsymbol{T}$, $\boldsymbol{q}$, $\boldsymbol{q}^r$ contain nodal values of temperatures, conductive and radiative heat fluxes, respectively. Owing to BEM discretization used, all these nodal values are placed solely on the boundary of the body.

3.2. Heat radiation

Radiative heat analysis of the energy exchange for the special case of transparent medium and gray walls gives a set of equation linking the radiative heat fluxes and fourth powers of the temperature

$$\mathbf{A}\boldsymbol{e}_b(T^r) + \mathbf{B}\boldsymbol{q}^r \tag{3.2}$$

where $\boldsymbol{e}_b(T^r)$ is a vector containing the blackbody emissive powers (fourth powers of temperature) at the nodes placed on the radiating, concave boundaries. The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined as in the case of emitting medium taking the absorption coeffcient $a = 0$.

3.3. Coupling condition

The coupling of the equations of radiation and convection is enforced by the boundary condition on the radiating boundary. In more realistic problems, the heat transported to the boundary by conduction within the body is removed by both radiation and convection. This can be written as

$$q = h(T^r - T_f) + q^r \tag{3.3}$$

where

h	–	convective heat transfer coefficient
T_f	–	temperature of the fluid filling the enclosure (cavity).

3.4. Solution strategy

Substitution of all prescribed boundary values at the non-radiating boundaries and Eq (3.3) into Eq (3.2) yields a set of equations having a form

$$\mathbf{K}\boldsymbol{x} = \boldsymbol{f} + \mathbf{Z}\boldsymbol{q}^r \tag{3.4}$$

where the square matrix $\mathbf{K}$ is composed of these columns of $\mathbf{H}$ and $\mathbf{G}$ matrices that correspond to the unknown conductive fluxes and temperatures. Vector $\boldsymbol{x}$ contains these fluxes and temperatures. Vector $\boldsymbol{f}$ contains products of values prescribed in the boundary conditions and appropriate columns of $\mathbf{H}$ and $\mathbf{G}$ matrices.

Eq (3.4) can be solved for $\boldsymbol{x}$ using the standard Gaussian elimination. During the elimination, columns of the matrix $\mathbf{Z}$ are treated as multiple right-hand side vectors. The result of the elimination can be written in the form

$$\boldsymbol{x} = \boldsymbol{f}_G + \mathbf{Z}_G\boldsymbol{q}^r \tag{3.5}$$

where the subscript G denotes the result of Gaussian elimination.

From all equations forming Eq (3.5) those solved with respect to the temperatures at the radiating boundary are then sorted out. This subset can be written as

$$\boldsymbol{T}^r = \boldsymbol{f}_G^r + \mathbf{Z}_G^r\boldsymbol{q}^r \tag{3.6}$$

where the subscript r is attached to the equations solved with respect to temperatures at radiating boundaries.

Solution of Eq (3.2) with respect to radiative heat fluxes can be achieved using the standard Gaussian elimination. Substitution of the result of the elimination into Eq (3.6) yields a set of equations containing solely unknown temperatures at radiating boundaries. Only this set needs to be solved iteratively. The Newton Raphson solver has proved to be a very efficient tool in accomplishing this task. Once the temperatures are calculated. the remaining unknows can be readily determined by multiplication of the matrices generated at previous steps of elimination and appropriate vectors of unknowns. The details of this procedure can be found in Bialecki [2] where also the case of nonlinear material properties is discussed. The technique has been used to solve a simple problem of rectangular cavity open to the environment and formed by a heat conducting solid.

3.5. Numerical example

The heat conducting rectangular body with rectangular cavity whose walls irradiate themselves and are open to radiating environment has been considered. The stady state, 3D temperature field has been calculated. The heat conductivity was assumed temperature dependent.

4. Conclusions

Although BEM itself is a numerical technique of discretization of integral equations, it has been used so far to solve problems ruled by differential equations. The heat radiation, a phenomenon governed by integral equations is a promissing area of BEM application. The technique is a variant of the zoning method, developed 40 years ago by Hottel. In comparison with this classical approach and their recent modifications, BEM offers much shorter times of matrix generation, being the bottleneck of all zoning approaches. It has been shown that BEM does not require volume integration and multiple integrals. Moreover, the accuracy of matrix calculation can be maintained arbitrary high level.

BEM is also a powerful tool in solving coupled conduction-radiation problems. The configuration coefficients arising in such problems can be cheaply and exactly evaluated using single integration and adaptive quadratures. Problems of this type, when solved by FEM, require different numerical techniques applied to the conductive and radiative portion of analysis. Using BEM in solving both problems simplifies the coding and makes the solution procedure consistent.

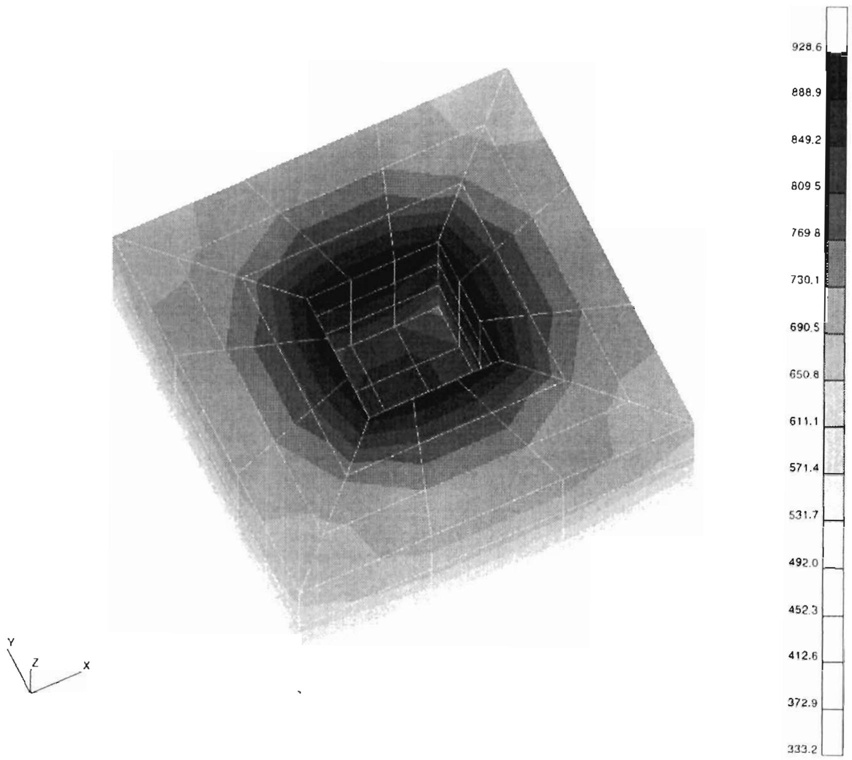

Fig. 3. Temperature field of a solid with self irradiating cavity

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Zastosowanie metody elementów brzegowych do rozwiązywania zagadnień promieniowania ciepła

Streszczenie

Rozpatrzono dwa zastosowania metody elementów brzegowych (MEB) w zagadnieniach promieniowania ciepła: transmisja promieniowania w ośrodku optycznie czynnym i przewodzenia ciepła w ciałach zawierających promieniujące wnęki Czas obliczeń przy zastosowaniu MEB jest krótszy niż w klasycznej metodzie strefowej Hottela. W MEB unika się całkowania po objętości, czyli najbardziej czasochłonnego etapu metody Hottela. Dodatkowo, w miejsce całek wielokrotnych obliczanych w metodzie strefowej, w MEB oblicza się całki jednokrotne. W artykule zaprezentowano ogólną, adaptacyjną kwadraturę do obliczeń całek po elementach leżących blisko punktu obserwacji. Przedstawiono także oryginalną procedurę sprzęgania zadań promieniowania we wnękach i przewodzenia w ściankach tworzących tę wnękę. Oba problemy cząstkowe rozwiązywane są metodą elementów brzegowych. Przed rozpoczęciem procesu iteracyjnego rozwiązywania równań bilansu ciepła, eliminowane są liniowe niewiadome. Wykorzystuje przy tym technikę statycznej kondensacji. Procedura taka prowadzi do znacznego skrócenia czasu obliczeń. Artykuł zawiera przykłady numeryczne.

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