JOURNAL OF THEORETICAL
AND APPLIED MECHANICS
2, 36, 1998

ADAPTIVE BOUNDARY ELEMENT METHOD FOR ACOUSTIC SCATTERING PROBLEMS

ANDRZEJ KARAFIAT

Section of Applied Mathematics, Cracow University of Technology
e-mail: andrzej.karafiat@pk.edu.pl

The subject of the paper is an application of the boundary element method to problems of scattering of acoustic waves by an elastic solid submerged in a fluid. A model problem of linear acoustics with a simplified fluid-solid interaction on the boundary is considered. The problem is described by the Burton-Miller integral equation. The Galerkin method is used to obtain an approximate solution. Adaptive methods of approximation are discussed. Some a posteriori error estimates are given.

Key words: boundary element method, adaptive methods, acoustic scattering

1. Introduction

One of the greatest advantages of the Boundary Element Method (BEM) is the reduction from an unbounded 3D domain to a 2D surface. BEM is a discretization technique for these equations, analogous to the Finite Element Method (FEM) for simple boundary-value problems.

The presented paper is devoted to the adaptive BEM approach for a model problem of elastic scattering in linear acoustics. In the paper an equivalent variational formulation of the Boundary-Value Problem (BPV) is presented. The paper focuses on adaptation methods, which are transferred from the known techniques used in FEM.

This paper does not pretend to be a "state of the art" survey in that area. It bases mainly on earlier author's work in the group of L.Demkowicz, cooperating with Texas Institute for Computational and Applied Mathematics in Austin (Texas), in realisation of some scientific programs. The details omitted here are to be found in the paper Karafiat (1996a), and more references are given there. Theoretical formulation of the problem is analysed in the papers

Demkowicz et al. (1991a,b), Karafiat et al. (1993). Numerical experiments were presented in the papers of Demkowicz et al. (1992), Demkowicz (1991b), Karafiat et al. (1993). Practical realisation of the chosen methods was there given in details. For basic results concerning BEM we refer to Burczyński (1995). The adaptive BEM approach was a subject of at least two Ph.D. Theses of N.Heuer and M.Maischak (Heuer, 1992; Maischak, 1995). New results concerning applications of BEM to similar problems were given e.g. by Stephan et al. (1996).

2. Acoustic scattering problem

A 3D model problem of elastic scattering of acoustic waves is considered. Let $\Omega \subset I\!R^3$ be a bounded domain occupied by an elastic solid and Ω^e its complement with the boundary Γ.

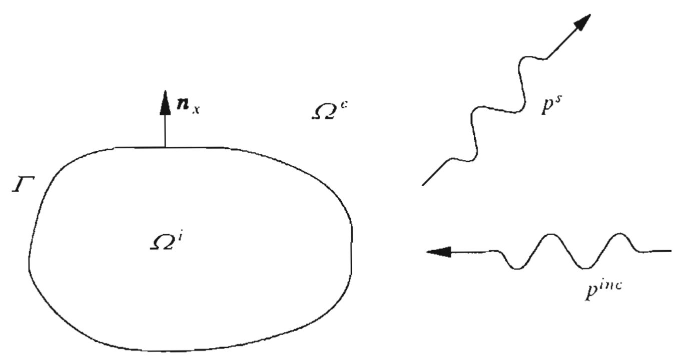

Fig. 1. Acoustic scattering problem

An incident wave propagates in a homogeneous, isotropic medium with the sound speed c (inviscid fluid). It is scattered on an elastic body, producing a scattered wave. We consider only small amplitudes of vibrations and we accept usual assumptions of the linear acoustics (cf e.g. Colton, 1992) and linear elasticity (Fung, 1965).

The time-harmonic process with frequency ω of acoustic waves may be described by one complex-valued function, being the total pressure

$$p(x) = p^{inc}(x) + p^s(x) \tag{2.1}$$

where p^{inc} is a given incident pressure and p^s an unknown scattered pressure. This function fulfils the Helmholtz equation in Ω^e

$$-\Delta p(\boldsymbol{x}) - k^2 p(\boldsymbol{x}) = 0 \qquad \boldsymbol{x} \in \Omega^e \tag{2.2}$$

where $k = \omega / c$ is the wave number. The scattered pressure satisfies the radiation (Sommerfeld) condition at infinity

$$\frac{\partial p^s}{\partial r}(\boldsymbol{x}) - ikp^s(\boldsymbol{x}) = o(r^{-1}) \qquad r = |\boldsymbol{x}| \to \infty \tag{2.3}$$

On the boundary Γ the impedance condition is imposed

$$\frac{\partial p}{\partial n_x}(\boldsymbol{x}) = \varepsilon p(\boldsymbol{x}) \qquad \forall\, \boldsymbol{x} \in \Gamma \tag{2.4}$$

where $\boldsymbol{n}_x$ is the unit outward normal vector at $\boldsymbol{x}$. For $\varepsilon = 0$ this condition corresponds to the rigid scattering. The case $\varepsilon > 0$ describes scattering by a rigid body covered with a smooth rubber coat. This kind of surface is nowadays used in submarine constructions.

In practice, the boundary-value problem (2.2)-(2.4) is replaced by a boundary integral equation. We assume that Γ is a manifold of the class $C^{1,\alpha}$, i.e. derivatives of its representation fulfil the Hölder condition

$$|\varphi(\boldsymbol{x}) - \varphi(\boldsymbol{y})| \leq c|\boldsymbol{x} - \boldsymbol{y}|^\alpha \tag{2.5}$$

with some $\alpha \in (0, 1]$.

The 3D fundamental solution of the Helmholtz equation is

$$\Phi(r) = \frac{e^{ikr}}{4\pi r} \tag{2.6}$$

The solution p of the BVP (2.2)-(2.4) satisfies the Helmholtz integral equation for all $\boldsymbol{x} \in \Gamma$

$$\frac{1}{2}p(\boldsymbol{x}) + \int_\Gamma \left[\Phi(\boldsymbol{x},\boldsymbol{y})\frac{\partial p}{\partial n_y}(\boldsymbol{y}) - \frac{\partial \Phi}{\partial n_y}(\boldsymbol{x},\boldsymbol{y})p(\boldsymbol{y})\right] dS_y = p^{inc}(\boldsymbol{x}) \tag{2.7}$$

and its derivative in the direction of the vector $\boldsymbol{n}_x$, the hypersingular equation

$$\frac{1}{2}\frac{\partial p}{\partial n_x}(\boldsymbol{x}) + \int_\Gamma \left[\frac{\partial \Phi}{\partial n_x}(\boldsymbol{x},\boldsymbol{y})\frac{\partial p}{\partial n_y}(\boldsymbol{y}) - \frac{\partial^2 \Phi}{\partial n_x \partial n_y}(\boldsymbol{x},\boldsymbol{y})p(\boldsymbol{y})\right] dS_y = \frac{\partial p^{inc}}{\partial n_x}(\boldsymbol{x}) \tag{2.8}$$

Regular solutions of Eqs (2.6) and (2.7) on Γ may be extended onto the whole external domain Ω^e by the formula

$$\int\limits_{\Gamma}\Big[\frac{\partial\Phi}{\partial n_y}(\boldsymbol{x},\boldsymbol{y})p(\boldsymbol{y})-\Phi(\boldsymbol{x},\boldsymbol{y})\frac{\partial p}{\partial n_y}(\boldsymbol{y})\Big]\,dS_y=\begin{cases}p(\boldsymbol{x}) & \text{for}\quad \boldsymbol{x}\in\Omega^e\\ \frac{1}{2}p(\boldsymbol{x}) & \text{for}\quad \boldsymbol{x}\in\Gamma\\ 0 & \text{for}\quad \boldsymbol{x}\in\Omega^i\end{cases}\tag{2.9}$$

Any solution (2.1) of the BVP (2.2)-(2.4) fulfils Eqs (2.6), (2.7). The reciprocal theorem is true for some k only; there are two sequences of k (forbidden frequencies), being eigenvalues of the corresponding internal problems, for which Eqs (2.6) or (2.7) have infinitely many solutions or none.

There are some methods for assuming the equivalence of the classical and integral formulations. One of them is the Burton-Miller approach (Burton et al., 1971), where a linear combination of the both equations is solved

$$\begin{aligned}&\frac{1}{2}p(\boldsymbol{x})+\int\limits_{\Gamma}\Big[\Phi(\boldsymbol{x},\boldsymbol{y})\frac{\partial p}{\partial n_y}(\boldsymbol{y})-\frac{\partial\Phi}{\partial n_y}(\boldsymbol{x},\boldsymbol{y})p(\boldsymbol{y})\Big]\,dS_y+\\&+\mathrm{i}\alpha\Big\{\frac{1}{2}\frac{\partial p}{\partial n_x}(\boldsymbol{x})+\int\limits_{\Gamma}\Big[\frac{\partial\Phi}{\partial n_x}(\boldsymbol{x},\boldsymbol{y})\frac{\partial p}{\partial n_y}(\boldsymbol{y})-\frac{\partial^2\Phi}{\partial n_x\partial n_y}(\boldsymbol{x},\boldsymbol{y})p(\boldsymbol{y})\Big]\,dS_y\Big\}=\\&=p^{inc}(\boldsymbol{x})+\mathrm{i}\alpha\frac{\partial p^{inc}}{\partial n_x}(\boldsymbol{x})\qquad\qquad \forall\boldsymbol{x}\in\Gamma\end{aligned}\tag{2.10}$$

which, with the Robin boundary condition

$$\frac{\partial p}{\partial n_x}(\boldsymbol{x})+\lambda(\boldsymbol{x})p(\boldsymbol{x})=g(\boldsymbol{x})\tag{2.11}$$

has, for any k, a unique solution.

Equation (2.10) with the condition (2.11) may be reformulated as a variational problem:

— Find $p\in H^{1/2}(\Gamma)$ such that

$$\begin{aligned}&\frac{1}{2}\int\limits_{\Gamma}(1+\mathrm{i}\alpha\varepsilon)p(\boldsymbol{x})q(\boldsymbol{x})\,dS_x+\int\limits_{\Gamma}\int\limits_{\Gamma}\Phi(\boldsymbol{x},\boldsymbol{y})\varepsilon p(\boldsymbol{y})q(\boldsymbol{x})\,dS_ydS_x+\\&-\int\limits_{\Gamma}\int\limits_{\Gamma}\frac{\partial\Phi}{\partial n_y}(\boldsymbol{x},\boldsymbol{y})p(\boldsymbol{y})q(\boldsymbol{x})\,dS_ydS_x-\mathrm{i}\alpha\int\limits_{\Gamma}\int\limits_{\Gamma}\Phi(\boldsymbol{x},\boldsymbol{y})\mathrm{rot}_yp(\boldsymbol{y})\mathrm{rot}_xq(\boldsymbol{x})dS_ydS_x+\\&+\mathrm{i}\alpha k^2\int\limits_{\Gamma}\int\limits_{\Gamma}\boldsymbol{n}_x\boldsymbol{n}_y\Phi(\boldsymbol{x},\boldsymbol{y})p(\boldsymbol{y})q(\boldsymbol{x})dS_ydS_x-\mathrm{i}\alpha\int\limits_{\Gamma}\int\limits_{\Gamma}\frac{\partial\Phi}{\partial n_x}(\boldsymbol{x},\boldsymbol{y})\varepsilon p(\boldsymbol{y})q(\boldsymbol{x})dS_ydS_x=\\&=\int\limits_{\Gamma}\Big[p^{inc}(\boldsymbol{x})+\mathrm{i}\alpha\frac{\partial p^{inc}}{\partial n_x}(\boldsymbol{x})\Big]q(\boldsymbol{x})\,dS_x\qquad\qquad\forall q\in H^{1/2}(\Gamma)\end{aligned}\tag{2.12}$$

where

$$\operatorname{rot} f(\boldsymbol{x}) := \boldsymbol{n}_x \times \nabla f(\boldsymbol{x}) \tag{2.13}$$

In terms of the functional analysis the above problem can be written as follows:

Problem P. Find $u \in V$ such that

$$a(p, q) = l(q) \qquad \forall q \in V \tag{2.14}$$

where $V = H^{1/2}(\Gamma)$ is a closed subspace of the Hilbert space $H = L^2(\Gamma)$, with the usual norm $\|\cdot\|$ in H. $a: H \times H - \mathbb{C}$ is a sesquilinear, continuous form given by the left-hand side of Eq (2.12) and the semilinear continuous form $l: H - \mathbb{C}$ is given by the right-hand side of the same equation. It is known, that the form a fulfils the Gärding inequality, i.e. there is a sesquilinear, compact form $c: H \times H \to \mathbb{C}$ and $\gamma \in \mathbb{R}$

$$\operatorname{Re}[a(v, v) + c(v, v)] \geq \gamma \|v\|^2 \qquad \forall v \in V \tag{2.15}$$

In the Galerkin approximation method we choose a sequence of finite-dimensional subspaces $V_h \subset V$ which approximate V and we solve an approximate equation

Problem P_h. Find $p_h \in V_h$ such that

$$a(p_h, q_h) = l(q_h) \qquad \forall q_h \in V_h \tag{2.16}$$

Basis functions e_i, $i = 1, \ldots, N$ are polynomials up to the order s. The approximate solution p_h and test function q_h are assumed to have the form

$$p_h(\boldsymbol{x}) = \sum_{i=1}^{N} p_i e_i(\boldsymbol{x}) \qquad q_h(\boldsymbol{x}) = \sum_{i=1}^{N} q_i e_i(\boldsymbol{x}) \tag{2.17}$$

and the approximate problem reduces to a system of linear equations

$$\sum_{i=1}^{N} a_{ij} p_j = b_i \qquad i = 1, \ldots, N \tag{2.18}$$

obtained by substitution of (2.17) to (2.16).

3. Adaptive boundary element method

BEM accepts general assumptions of FEM. In a given plane domain G an initial mesh of triangular or rectangular finite elements is defined. Hereinafter we restrict ourselves to the triangles (rectangles are used analogously). The elements are mapped onto Γ.

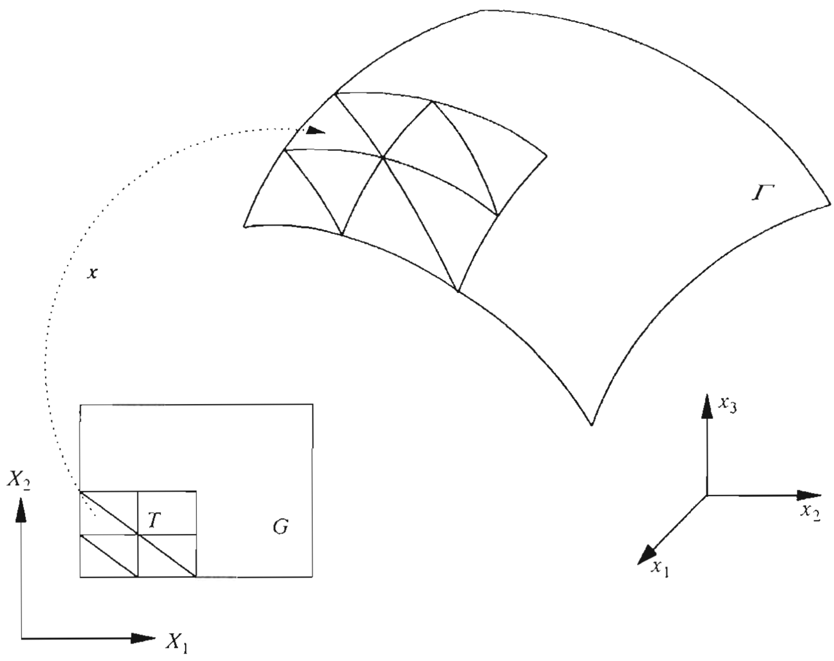

Fig. 2. Mapping of elements onto the surface

Each of the triangles $T_i \subset G$ is a range of a master element $\widehat{T}$ by an affine mapping F_i. Basis shape functions and degrees of freedom are defined on $\widehat{T}$.

The boundary element approximation on the master triangle $\widehat{T}$ coincides with that for finite elements, and the general algorithm of evaluation of the system (2.18) is analogous. The values of integrands like the fundamental solution Φ and its derivatives are taken from Γ, multiplied by corresponding shape functions with jacobians and integrated numerically on $\widehat{T}$. The values of corresponding integrals become entries of a stiffness matrix. The presence of Φ in Eq (2.12) implies that the stiffness matrix is fully populated, in contrast to finite element formulations.

To enrich the mesh by an h-refinement, the elements T_i are divided into

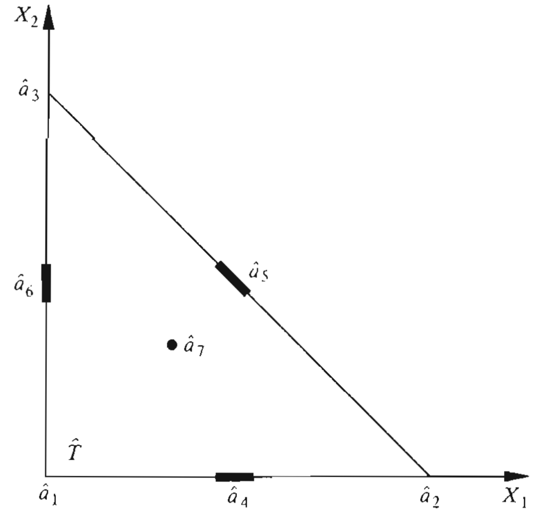

Fig. 3. Master element

two or four. In subdivision into four, a 1-irregularity rule is often assumed, i.e. an element may be subdivided only if its neighbours are equal or smaller. Otherwise the neighbours are subdivided first (Fig.4).

In the subdivision into two, where the largest side of the element is divided, there is no problem of irregularity, although shapes of elements may be changed (Fig.5a).

A combination of these subdivisions joins the both methods (Fig.5b).

In p-enrichment seven generalized nodes are introduced: vertices $\widehat{a}_1$, $\widehat{a}_2$, $\widehat{a}_3$, mid-side nodes $\widehat{a}_4$, $\widehat{a}_5$, $\widehat{a}_6$, and a central node $\widehat{a}_7$. Nodes $\widehat{a}_4 - \widehat{a}_7$ have, in general, many degrees of freedom. They may have separate orders of approximation. We adopt here a maximum rule which says that the higher polynomial degree dominates at the interface of two elements. This means that when two neighbouring elements are of different order, then the shape functions of higher order are added to those of the lower order element so as to obtain continuity of the global basis functions across the interelement boundaries.

The third adaptive method, the $hp-$ method, is a combination of the both previous ones. The method is especially effective at points, where the exact solution is singular. An exponential convergence of this method is there observed and proved.

In numerical realization of this method everyone is faced with the choice,

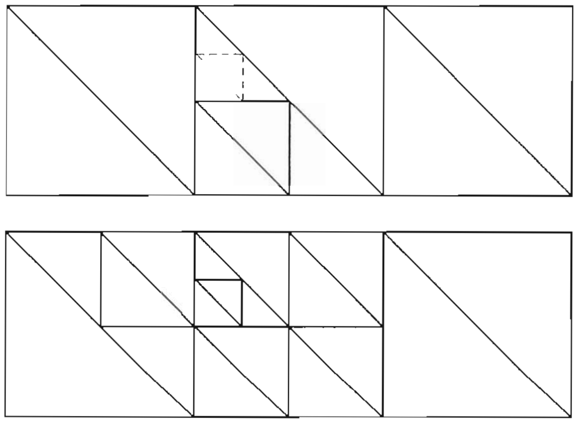

Fig. 4. 1-irregular subdivision

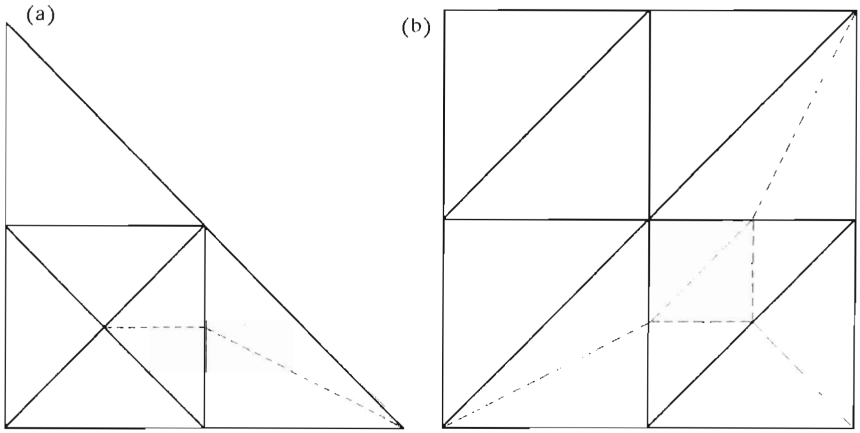

Fig. 5. (a) – Subdivision into two; (b) – combination of the both subdivisions

which method – h, p, or both – should be applied to each element separately. The quantity

$$\frac{\Delta e}{\Delta d} \tag{3.1}$$

where Δd is a number of additional degrees of freedom and Δe is an error reduction obtained with the use of these additional discrete variables, seems to be a good indicator of the refinement.

The presented approaches are based on the works of L.Demkowicz, J.T.Oden, W.Rachowicz and others (cf Demkowicz et al., 1989; Oden et al., 1989; Rachowicz et al., 1989). The adaptation techniques described above are used in approximation of a solution and in modeling of the boundary, which are independent problems, although solved often by the same algorithms and procedures. The both approximations contribute to the approximation of the whole problem.

4. Adaptive integration

In the variational formulation (2.12) of the problem considered, the functions appearing are integrated twice over the whole boundary. The integration is performed numerically over each element. According to the position of a Gaussian point $\boldsymbol{x}$ of the external integration with respect to the considered element T_j (domain of the internal integration), the following three situations are possible:

(a) $\boldsymbol{x} \in T_j$

(b) $x \in T_j,\ d(x, T_j) \leq d_0$

(c) $d(x, T_j) > d_0$

where $d(x, T_j)$ denotes the distance between $\boldsymbol{x}$ and T_j, d_0 is a fixed constant.

In the case (a) all integrands in Eq (2.12) are weakly singular. They are usually evaluated in terms of the Duffy coordinates

$$x_1 = \xi_1 \qquad x_2 = \xi_1 \xi_2 \tag{4.1}$$

the polar coordinates

$$x_1 = \rho_1 \cos \rho_2 \qquad x_2 = \rho_1 \sin \rho_2 \tag{4.2}$$

or by subtraction of the singular part. The above substitutions transform all weakly singular integrands into the bounded ones. Those are evaluated by Gaussian quadratures.

In the case (b) the regular quadratures give poor results. Adaptation based on an uniform subdivision of the integration domain (Lyness, 1978) is expensive. Adaptive methods help to get satisfactory results at a smaller expense of time and memory. The h-adaptive method uses a changeable mesh density for integration (Fig.6). By moving mesh nodes without change of its structure, we obtain an r-adaptive method (cf Karafiat, 1996b).

In the case (c) the usual Gaussian quadrature is applied.

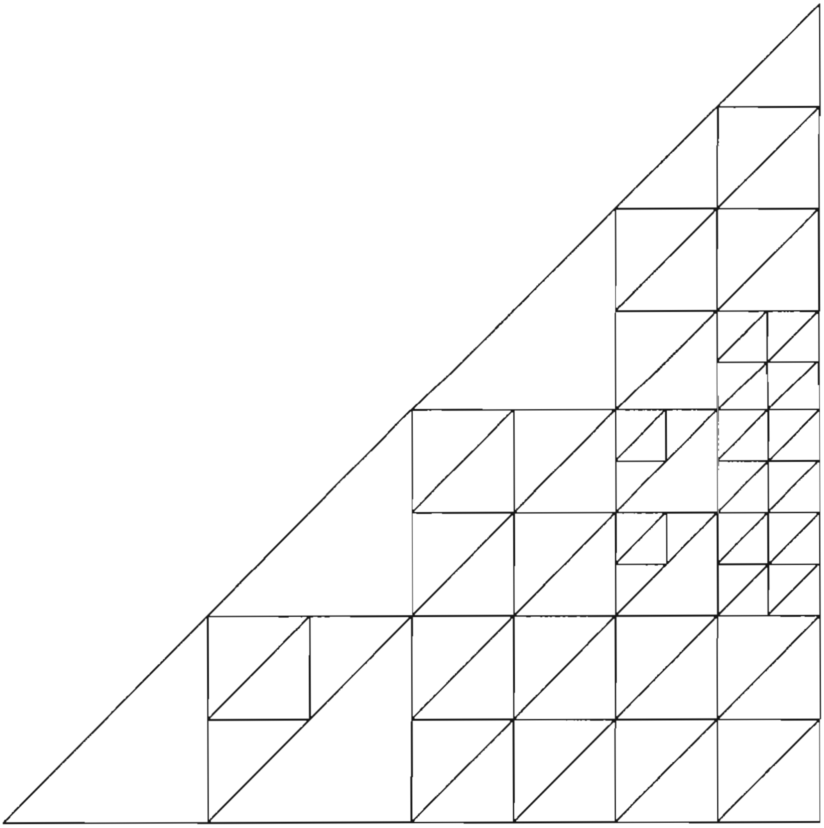

Fig. 6. Subdivision for adaptive integration of almost singular functions

5. A posteriori estimates

To reduce the error by subdividing of elements or rising the shape functions order, one has to get some means to judge the error distribution in a numerical solution. The only data available is the approximate solution itself.

The most popular technique for an a posteriori estimation is the interpolation method. It uses the interpolation theory in Sobolev spaces and values of the corresponding norms over particular elements are error indicators. The method exploits the Babuška and Mikhlin Theorems (cf Demkowicz, 1994), which extend known Cea's Lemma to some non-elliptic problems, including the problem (2.14). When Πv denotes an interpolant of a given function v, the interpolation theory yields the following error estimate

$$\|v - \Pi v\|_{1/2} \leq C h^{s+1/2} \|v\|_{s+1} \tag{5.1}$$

for the $h-$ method

$$\|v - \Pi v\|_{1/2} \leq C s^{-(m+1/2)} \|v\|_{m+1} \tag{5.2}$$

for the $p-$ method and

$$\|v - \Pi v\|_{1/2} \leq C h^{s+1/2} s^{-(m+1/2)} \|v\|_{s+1} \tag{5.3}$$

for the $hp-$ method, provided that the interpolation order s is less than m (cf Karafiat, 1997; Stephan, 1989). $\|\cdot\|_{1/2}$ denotes here the norm in the Sobolev space $H^{1/2}(\Gamma)$.

Another technique is the residual method, where the residual of numerical solution is the error indicator. Let us write the Helmholtz integral equation (2.7) in the form of operator equation

$$M(p) = p^{inc} \tag{5.4}$$

where M is an operator from $H^{1/2}(\Gamma)$ into itself and let p_h be an approximate solution of the Galerkin equation (2.16). The term

$$r = p^{inc} - M(p_h) \tag{5.5}$$

is the residual of the solution p_h. Its norm is globally related to the norm of the error. We have

$$\|r\|_{1/2} \leq \|M\| \cdot \|p - p_h\|_{1/2} \tag{5.6}$$

and, if k is not a forbidden frequency of the Helmholtz equation (2.7)

$$\|p - p_h\|_{1/2} \leq \|M^{-1}\| \; \|r\|_{-1/2} \tag{5.7}$$

Consequently, the residual norm is equivalent to the error norm. Moreover, for Γ smooth and $\varepsilon = 0$

$$\lim_{h\to 0} \frac{\|p - p_h\|_{1/2}}{\|r\|_{1/2}} = 1 \tag{5.8}$$

holds. The same inequalities are true for the usual norm in the space $L^2(\Gamma)$. See Demkowicz et al. (1992), Demkowicz et al. (1991b), where this method was explained in detail.

The most exact a posteriori estimate may be obtained solving local boundary-value problems. These subdomain-residual methods were developed for few problems only, see e.g. Yu (1991). A post-processing method, where the exact solution is replaced by its refined approximation, were considered e.g. by Rencis et al. (1989).

6. Conclusions

The variational formulation with Galerkin approximation is only one of possible approaches. More popular is the collocation method, which needs a single integration only. For external problems, however, numerical realization of singular integrals in this method may cause some troubles. Advantages of the Galerkin approach are the following:

- Symmetric stiffness matrix
- Easy elimination of singular integrals
- Mathematical analysis of the method is advanced and the convergence is proved
- Asymptotic convergence rates coincide with the interpolation ones
- Costs of the both methods with the same accuracy are of the same order.

It is possible to extend the presented research to other areas. One of them is the coupled BEM/FEM approach, which fits better to the problems of fluid-solid interaction. Another one is scattering of electromagnetic waves, which is a subject of current research at the University of Texas at Austin.

References

1. BURCZYŃSKI T., 1995, *Metoda Elementów Brzegowych w Mechanice*, Wyd. Naukowo-Techniczne, Warszawa
2. BURTON A.J., MILLER G.F., 1971, The Application of Integral Equation Methods to the Numerical Solution of Some Exterior Boundary-Value Problems, *Proc. Roy. Soc. London Ser. A.*, **323**, 201-210
3. COLTON D., KRESS R., 1992, *Inverse Acoustic and Electromagnetic Scattering Theory*, Springer, Berlin
4. DEMKOWICZ L., ODEN J.T., RACHOWICZ W., HARDY O., 1989, Toward a Universal $h-p$ Adaptive Finite Element Strategy, Part 1. Constrained Approximation and Data Structure, *Comput. Methods Appl. Mech. Engrg.*, **77**, 79-112
5. DEMKOWICZ L., KARAFIAT A., ODEN J.T., 1991a, Variational (Weak) form of the Hypersingular Formulation for the Helmholtz Exterior Boundary-Value Problems, TICOM Report 91-05, TICOM, The University of Texas at Austin, Austin, Texas

6. DEMKOWICZ L., ODEN J.T., AINSWORTH M. GENG P., 1991b, Solution of Elastic Scattering Problems in Linear Acoustics Using $h-p$ Boundary Element Methods, *J. Comput. Appl. Math.*, **36**, 29-63

7. DEMKOWICZ L., KARAFIAT A., ODEN J.T., 1992, Solution of Elastic Scattering Problems in Linear Acoustics Using $h-p$ Boundary Element Method, *Comput. Methods Appl. Mech. Engrg.*, **101**, 251-282

8. DEMKOWICZ L., 1994, Asymptotic Convergence in Finite and Boundary Element Methods: Part1. Theoretical Results, *Computers Math. Applic.*, **27**, 12, 69-84

9. FUNG Y.C., 1965, *Foundations of Solid Mechanics*, Prentice Hall, Engl Cliffs

10. HEUER N., 1992, *hp*– Versions of the Boundary Element Method, Ph.D. Thesis, Hannover (in German)

11. KARAFIAT A., ODEN J.T., GENG P., 1993, Variational Formulations and *hp* Boundary Element Approximations for Hypersingular Integral Equations for Helmholtz Exterior Boundary Value Problems in Two Dimensions, *Int. J. Engrg. Sci.*, **31**, 4, 649-672

12. KARAFIAT A., 1996a, An Analysis of the Boundary Element Method for the Acoustic Scattering Problem, The Cracow University of Technology, Kraków (in Polish)

13. KARAFIAT A., 1996b, Adaptive Integration Techniques for Almost Singular Functions in the Boundary Element Method, *Comput. Math. Appl.*, **32**, 5, 11-30

14. KARAFIAT A., 1997, On *hp*– Error Estimation in the BEM for a Three-Dimensional Helmholtz Exterior Problem, *Comput. Methods Appl. Mech. Engrg.*, **150**, 199-214

15. LYNESS J.N., 1978, Quadrature Over a Simplex: Part 2. A Representation for the Error Functional, *SIAM J. Numer. Anal.*, **15**, 870-887

16. MAISCHAK M., 1995, *hp*– Methods for Boundary Integral Equations in 3-Dimensional Problems. Theory and Implementations, Ph.D. Thesis, Hannover (in German)

17. ODEN J.T., DEMKOWICZ L., RACHOWICZ W., WESTERMANN T.A., 1989, Toward a Universal $h-p$ Adaptive Finite Element Strategy, Part 2. A Posteriori Error Estimation, *Comput. Methods Appl. Mech. Engrg.*, **77**, 113-180

18. RACHOWICZ W., ODEN J.T., DEMKOWICZ L., 1989, Toward a Universal $h-p$ Adaptive Finite Element Strategy, Part 3. Design of $h-p$ meshes, *Comput. Methods Appl. Mech. Engrg.*, **77**, 181-212

19. RENCIS J.J., JONG K.Y., 1989, Error Estimation for Boundary Element Analysis, *ASCE J. Engrg. Mechanics*, **115**, 1993-2010

20. STEPHAN E.P., SURI M., 1989, On the Convergence of the *p*-Version of the Boundary Element Galerkin Method, *Math. Comp.*, **52**, 185, 31-48

21. STEPHAN E.P., 1996, The $h-p$ Boundary Element Method for Solving 2- and 3-Dimensional Problems, *Comput. Methods Appl. Mech. Engrg.*, **133**, 183-208

22. YU D., 1991, Mathematical Foundation of Adaptive Boundary Element Methods, *Comput. Methods Appl. Mech. Engrg.*, **91**, 1237-1243

Adaptacyjna metoda elementów brzegowych w zadaniu rozpraszania fali akustycznej

Streszczenie

Przedmiotem pracy jest zastosowanie metody elementów brzegowych do zadania rozpraszania fali akustycznej w płynie przez zanurzone w niej ciało sprężyste. Przyjmuje się postulaty liniowej akustyki i teorii sprężystości i uproszczony model interakcji ciała stałego i płynu. Do rozwiązania zadania stosuje się wariacyjne sformułowanie równania całkowego Burtona-Millera, rozwiązywane w sposób przybliżony za pomocą metody Galerkina. W pracy omówiono w skrócie zastosowanie metody elementów brzegowych z różnymi rodzajami adaptacji do aproksymacji rozwiązania, modelowania brzegu i całkowania numerycznego. Podano także najczęściej stosowane metody szacowania błędu a posteriori.

Manuscript received December 19, 1997; accepted for print February 3, 1998