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STRENGTH AND BUCKLING OF SANDWICH BEAMS WITH CORRUGATED CORE

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The subject of the paper is a sandwich beam with a crosswise or lengthwise corrugated core. The beam is made of an aluminium alloy. The plane faces and the corrugated core are glued together. Geometrical properties and rigidities of the beams are described. The load cases investigated in the work are pure bending and axial compression. The relationship between the applied bending moment and the deflection of the beam under four-point bending is discussed. The analytical and numerical (FEM) calculations as well as experimental results are described and compared. Moreover, for the axial compression, the elastic global buckling problem of the analysed beams is presented. The critical loads for the beams with the crosswise and lengthwise corrugated core are determined. The comparison of the analytical and FEM results is shown.

Key words: sandwich beam, corrugated core, strength, elastic buckling, deflection

1. Introduction

The theoretical model of sandwich structures was formulated in the middle of XX century. Reissner (1948) presented deflection of a sandwich rectangular plate. Plantema (1966) and Allen (1969) described strength and stability problems of sandwich structures. Libove and Hubka (1951) analysed properties of corrugated-core sandwich panels. Noor *et al.* (1996) and Vinson (2001) discussed elastic behaviour of sandwich structures. Luo *et al.* (1992) presented results of analytical analysis of the bending stiffness of a corrugated board and compared them with expressions given by other authors. Cheng *et al.* (2006) used the finite element method to determine an expression for the equivalent stiffness of sandwich structures with various types of cores. A more accurate expression for the stiffness of corrugated sheets was derived by Briassoulis (1986). McKee *et al.* (1963) analysed bending stiffness of a sandwich plate with a corrugated core for three-point and four-point bending tests. Gilchrist *et al.* (1999) applied the finite element method to analyse bending and twisting of a corrugated board. Buannic *et al.* (2003) presented a homogenization method for a sandwich plate with a corrugated core and compared this method with results of finite element analysis. Analytical homogenization of a corrugated cardboard under torsion was described by Abbes and Guo (2010). An equivalent stiffness of a corrugated plate under bending and torsion treated as an orthotropic plate was discussed by Liew *et al.* (2007). Rubino *et al.* (2009) presented the comparison of dynamic behaviour of a clamped monolithic plate and a clamped sandwich plate with an Y-frame and corrugated core. Seong *et al.* (2010) showed bending results of sandwich plates with bi-directionally corrugated cores. An application of a sandwich corrugated plate as a bridge structure was described by Ji *et al.* (2010).

Wittenbeck and Magnucki (2009) and Magnucki and Wittenbeck (2010) determined rigidities of a double-layered cylindrical vessel in which the internal layer was corrugated. Stability of that structure was also investigated. Magnucki *et al.* (2011, 2012) described behaviour of sandwich beams with corrugated cores. The authors presented the comparison of results of the numerical-analytical analysis and experimental investigation discussed by Wasilewicz and Jasion (2010).

The subject of the present study are sandwich beams with crosswise or lengthwise corrugated cores (Fig. 1). The simply supported sandwich beams with the length L carries two concentrated transverse forces F_1 or a compressive axial force F_0. The first configuration of forces and supports is known as four-point bending and gives a constant bending moment over the area between the forces. The distance between the forces F_1 is marked as L_0, and the distance from the support to the force F_1 is marked as L_1.

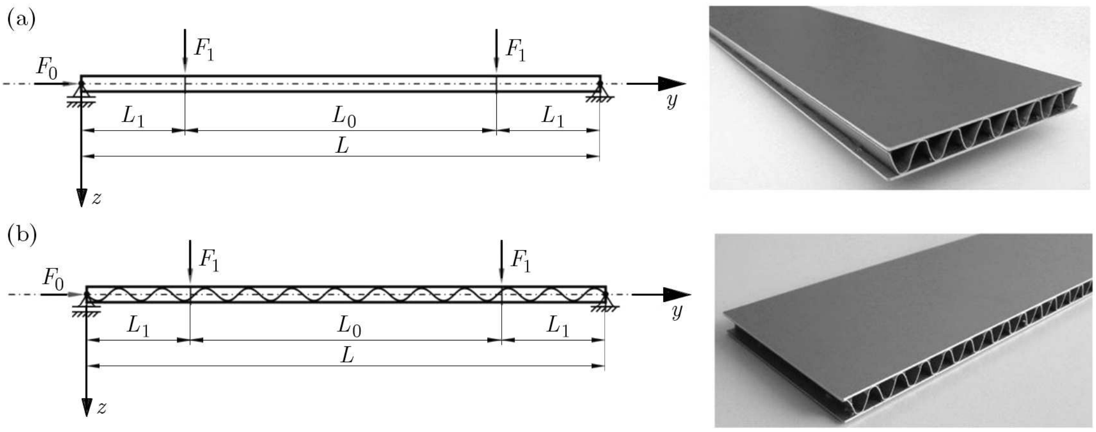

Fig. 1. Sandwich beams with lengthwise (a) and crosswise (b) corrugated cores

In the following analyses, the relationship between the bending moment and the deflection of the beam with longitudinal and transverse orientation of the corrugation is investigated. The following material and geometrical properties were assumed: $E = 69000\,\mathrm{MPa}$, $\nu = 0.3$, $L = 750\,\mathrm{mm}$, $L_1 = L_0 = 250\,\mathrm{mm}$.

2. Rigidities of the sandwich corrugated beams

The geometry of the sandwich corrugated beam is shown in Fig. 2. The beam consists of two plane faces of the thickness t_f and a corrugated core of the depth t_c and the thickness t_0. The total thickness of the beam equals h. The pitch of the corrugation equals a_0.

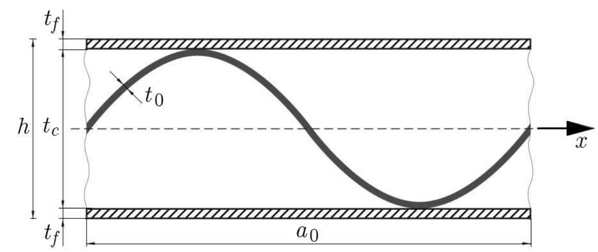

Fig. 2. The cross section of the sandwich corrugated beam

The middle surface of the corrugated core is assumed as a sine curve and is described by the following expression

$$f_c(\xi) = \frac{1}{2}t_c(1 - x_0)\sin(2\pi\xi) \tag{2.1}$$

where x_0 and ξ are dimensionless variables defined as follows

$$x_0 = \frac{t_0}{t_c} \qquad \xi = \frac{x}{a_0}$$

The dimensionless middle surface length S_0 of one pitch of the corrugated core can be written as

$$S_0 = \int\limits_0^1 \sqrt{1 + c_0^2 \cos^2(2\pi\xi)}\, d\xi \tag{2.2}$$

where $c_0 = \pi t_c(1 - x_0)/a_0$.

Expressions for the total area and the moment of inertia of the cross-section per unit length with respect to the y axis have the following form

$$A_x = (2 + k)t_f \qquad I_x = \frac{1}{12} t_c^3 [2x_1(4x_1^2 + 6x_1 + 3) + 3x_0(1 - x_0^2)S_1] \tag{2.3}$$

where

$$x_1 = \frac{t_f}{t_c} \qquad k = \frac{x_0^3}{4x_1[S_2 x_0^2 + 3(1 - x_0^2)S_3]}$$

$$S_1 = \int\limits_0^1 \sin^2(2\pi\xi)\sqrt{1 + c_0^2 \cos^2(2\pi\xi)}\, d\xi \qquad S_2 = \int\limits_0^{\frac{1}{4}} \frac{d\xi}{\sqrt{1 + c_0^2 \cos^2(2\pi\xi)}}$$

$$S_3 = \int\limits_0^{\frac{1}{4}} \sin^2(2\pi\xi)\sqrt{1 + c_0^2 \cos^2(2\pi\xi)}\, d\xi$$

The expression for the total area and the moment of inertia of the cross-section per unit length with respect to the x axis are given by

$$A_y = t_c(2x_1 + x_0 S_0) \qquad I_y = \frac{1}{12} t_c^3 \Big[2x_1(4x_1^2 + 6x_1 + 3) + \frac{x_0^3}{S_0}\Big] \tag{2.4}$$

Thus the flexural rigidity of the sandwich beam with the corrugated core can be written as

$$D_x = EI_y \qquad D_y = EI_x \tag{2.5}$$

where E is Young's modulus.

3. Analytical and numerical analysis of bending of sandwich beams

From the Euler-Bernoulli beam bending theory the relationship between the bending moment M_g and the deflection function $w(y)$ of the neutral axis, for the beam with longitudinal corrugation, is described by the following formula

$$D_y \frac{d^2 w}{dy^2} = -\frac{M_g}{a} \tag{3.1}$$

where a denotes width of the beam.

Assuming cylindrical bending, the radius of curvature R can be determined based on displacements of three arbitrary points spaced at an equal distance from each other. Here the distance is marked as c (see Fig. 3). The relationship between the bending moment M_g and the radius of curvature R is in the form

$$\frac{1}{R} = \frac{M_g}{aD_y} \tag{3.2}$$

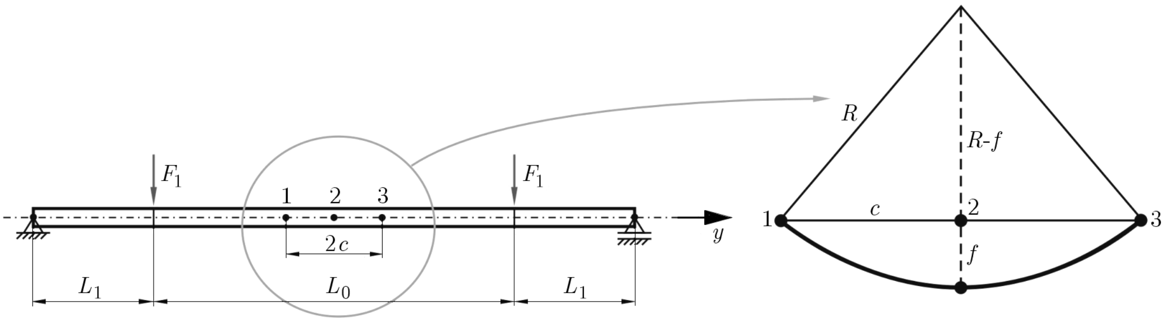

Fig. 3. Schema of the measuring procedure for beam deflection

For a small deflection, there is: $f \ll c$. Then the stiffness of the beam defined as the ratio M_g/f, can be written as

$$\frac{M_g}{f} = \frac{2aD_y}{c^2} \tag{3.3}$$

Replacement of the variable y by x in (3.2) and (3.3) gives expressions for the maximum deflection and the ratio M_g/f for transverse corrugation.

The stiffness of sandwich beams under pure bending was also analysed using the finite element method. The necessary calculations were carried out by means of ABAQUS system. The glue connection between the outer faces and the core was modelled with the use of tie constraints. The thin shell elements S4R were used to elaborate the three-dimensional model of the sandwich beam. The arrangement of loads and supports is illustrated in Fig. 1. Five cases investigated in the work, in which different combination of the face and corrugated core thicknesses are taken into account, are presented in Table 1.

Table 1. Investigated cases

No.	t_0 [mm]	t_f [mm]
1	0.3	1.0
2	0.3	1.5
3	0.3	2.0
4	0.4	1.0
5	0.5	1.0

Other parameters are: width $a = 100$ mm, depth of the core $t_c = 9.5$ mm, pitch of the corrugation $a_0 = 14$ mm, distance between the reference points $c = 50$ mm. The applied load $F_1 = 800$ N causes a constant bending moment in the central section of the beam equal to $M_g = 200$ Nm. Examples of deformation of FE models of sandwich beams with the corrugated core are shown in Fig. 4. The results of analytical and FEM calculations of the beams stiffness are given in Section5 and compared with the results of experimental tests.

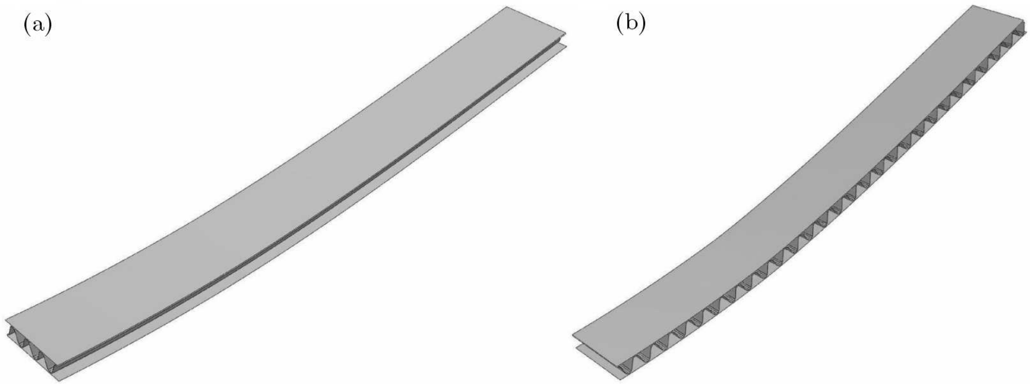

Fig. 4. Deformation of FE models of beams with lengthwise (a) and crosswise (b) corrugated core

4. Experimental bending tests of sandwich beams

Experimental investigations of stiffness of sandwich beams under four-point bending were carried out on a special test stand, which was mounted on Zwick Z100 testing machine. The view of the test stand is shown in Fig. 5. Details of experimental investigations were described by Wasilewicz and Jasion (2010).

Fig. 5. View of the test stand

The four-point bending was realized by means of a special system of tensioned beams. The load F_1 was measured by means of a dynamometer. An inductive detector of displacement HBM WA/10 mm was used to measure deflection of the beam. The distance between the inductive detectors was $c = 50$ mm. The scheme of the measurement system is presented in Fig. 6. During the tests, the data from the dynamometer and detectors were recorded. The dependence between the bending moment and displacement was plotted.

5. Comparison of the results of strength investigation of sandwich beams

The results from analytical, numerical and experimental analyses are listed in Tables 2 and 3. The dependency between the thickness of particular layers of the sandwich beam and the stiffness of the beam is shown in Figs. 7 and 8 for crosswise corrugation, and Figs. 9 and 10 for lengthwise corrugation.

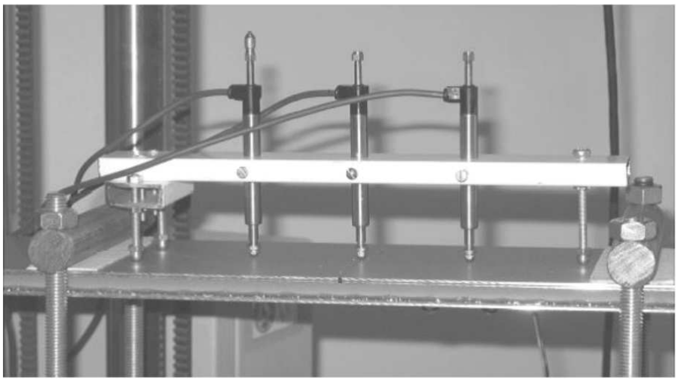

Fig. 6. Scheme of measuring detectors

Table 2. Results of investigations for the crosswise corrugated core $(M_g/f$ [Nm/mm])

No.	t_0	t_f	Analyt.	FEM	Error	Exper.	Error
1	0.3	1.0	331.22	334.16	0.88%	330.1	−0.34%
2	0.3	1.5	530.08	533.47	0.64%	–	–
3	0.3	2.0	763.42	767.16	0.49%	–	–
4	0.4	1.0	339.75	343.35	1.06%	–	–
5	0.5	1.0	348.17	352.61	1.28%	–	–

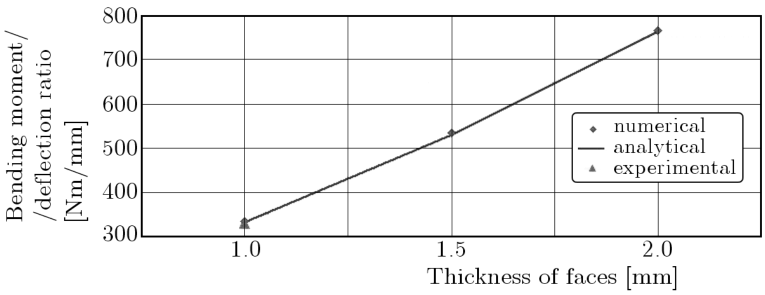

Fig. 7. Comparison of analytical, numerical and experimental results for the crosswise corrugated core (thickness of the corrugated core $t_0 = 0.3$ mm)

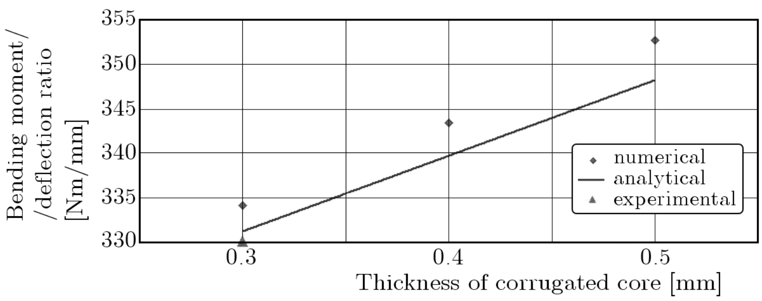

Fig. 8. Comparison of analytical, numerical and experimental results for the crosswise corrugated core (thickness of the faces $t_f = 1$ mm)

Table 3. Results of investigations for the lengthwise corrugated core (M_g/f [Nm/mm])

No.	t_0	t_f	Analyt.	FEM	Error	Exper.	Error
1	0.3	1	305.21	307.65	0.8%	304.9	0.1%
2	0.3	1.5	504.05	506.46	0.48%	–	–
3	0.3	2	737.38	738.27	0.12%	–	–
4	0.4	1	305.22	308.50	1.08%	–	–
5	0.5	1	305.24	309.21	1.29%	–	–

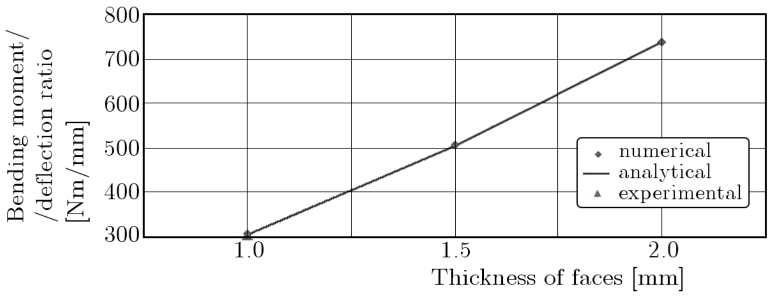

Fig. 9. Comparison of analytical, numerical and experimental results for the lengthwise corrugated core (thickness of the corrugated core $t_0 = 0.3\,\mathrm{mm}$)

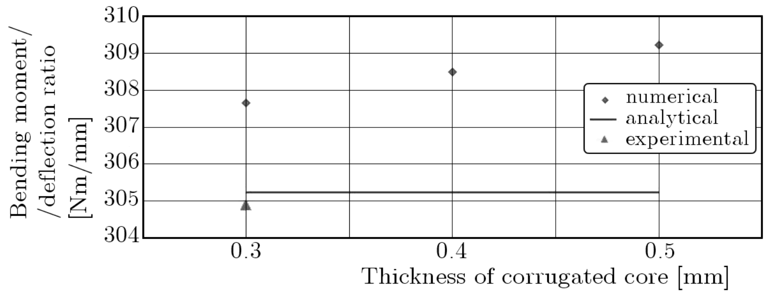

Fig. 10. Comparison of analytical, numerical and experimental results for the lengthwise corrugated core (thickness of the faces $t_f = 1\,\mathrm{mm}$)

6. Analytical and numerical FEM calculations of elastic global buckling of sandwich beams

The simply supported beam is compressed by the axial force F_0 ($F_1 = 0$, see Fig. 1). According to Euler's formula, the critical load for the beam is defined in the following form

$$F_{CR,E} = \frac{\pi^2 EI}{L^2} \tag{6.1}$$

According to that formula, examplary calculations of the critical load have been carried out for the following parameters:

- geometry of the cross section: $t_f = 9.5\,\mathrm{mm}$, $t_0 = 0.3\,\mathrm{mm}$, $t_c = 9.5\,\mathrm{mm}$, $a0 = 14\,\mathrm{mm}$
- length of the beam $L = 750\,\mathrm{mm}$
- Young's modulus $E = 69000\,\mathrm{MPa}$
- moment of inertia of the cross section
  - for the sandwich beam with the crosswise (CW) corrugated core, in accordance with Eq. $(2.3)_2$

$$I^{(CW)} = 5971.6\,\mathrm{mm}^4$$

– for the sandwich beam with yhe lengthwise (LW) corrugated core, in accordance with Eq. $(2.4)_2$

$$I^{(LW)} = 5529.0\,\mathrm{mm}^4$$

The obtained values are shown in Table 4 and compared with those given by the finite element method. The FE model of the beams was the same as that presented in Section 3. The first buckling modes for sandwich corrugated beams are shown in Fig. 11.

Table 4. Buckling loads obtained analytically and with the use of the FE method

	CW-crosswise	LW-lengthwise
$F_{CR,E}^{(Anal)}$ [N]	80688	74707
$F_{CR,E}^{(FEM)}$ [N]	79281	73314

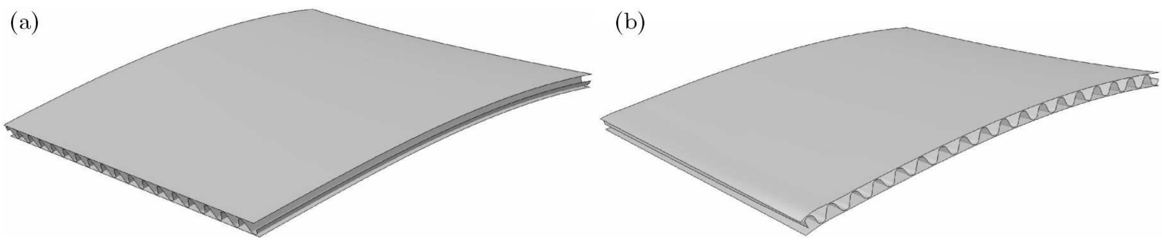

Fig. 11. The first buckling mode of beams with lengthwise (a) and crosswise (b) corrugated core

7. Conclusions

In the paper, the strength and stability problems of a sandwich beam with a crosswise and lengthwise corrugated core have been studied. The geometry of the cross section as well as rigidities of the beams have been described. For both load cases, that is for pure bending and axial compression, the analytical and FEM calculations have been carried out. The comparison of the results obtained from both methods is shown in Tables 2, 3 and 4. The biggest discrepancy in the case of pure bending is about 1.3%, and for axial compression – 1.9%.

Moreover, for the pure bending load, experimental tests have been performed. The differences between the stiffness of the beams obtained in the test and from the analytical model were less than 0.4%. It should be noted that according to Tables 2 and 3, the stiffness of the beam with the crosswise corrugated core is higher when compared to the beam with the lengthwise corrugated core. The difference varies within the range 3-12% depending on the thicknesses of particular layers of the beam.

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Wytrzymałość i stateczność belek trójwarstwowych z rdzeniem falistym

Streszczenie

Przedmiotem pracy są belki trójwarstwowe zrdzeniem falistym. Poszczególne warstwy belek wykonane są ze stopu aluminium i połączone klejem. Wyróżniono dwa kierunki pofałdowania: wzdłużny i poprzeczny. Opisano geometrię przekroju poprzecznego i wyznaczono sztywności belek. W pracy rozpatrzono dwa przypadki obciążenia. Pierwszy z nich to czyste zginanie realizowane poprzez tak zwane zginanie czteropunktowe. Dla tego typu obciążenia zbadano zależność między przyłożonym momentem gnącym a ugięciem belki. Dla drugiego przypadku obciążenia, osiowego ściskania, omówiono problem globalnej stateczności sprężystej i wyznaczono obciążenia krytyczne. Ponadto zaprezentowano porównanie wyników uzyskanych z zaproponowanego modelu analitycznego z wynikami otrzymanymi z metody elementów skończonych i z badań laboratoryjnych.

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