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FEM-BEM COUPLING IN THE MEMBRANE STRUCTURE-AIR INTERACTION PROBLEM

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This paper deals with the interaction problems between the lightweight membrane structure and surrounding air. The coupling of FEM and BEM is described and applied to analysis of aerodynamics of membrane structures. Three methods of FEM-BEM coupling are presented.

Key words: finite element method, boundary element method, membrane structure

1. Introduction

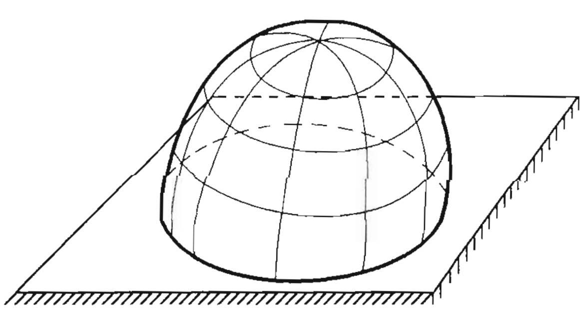

Fig. 1. Pneumatic structure

Membrane structures used in civil engineering, e.g. pneumatic structures (Fig.1) or prestressed membrane structures (Fig.2) are lightweight. They are spanned over sport facilities, exhibition halls or stores etc. Due to their light weight and large dimensions they pose the dynamic problems which are not met in traditional civil engineering objects. One of them is dynamic interaction of a lightweight structure and surrounding air. The coupling of the Finite Element Method (FEM) and the Boundary Element Method (BEM) enables the effective solution to this complicated problem to be obtained. I have dealt with the membrane structure-air interaction problems for several years. My previous papers concerning this topic are given in References.

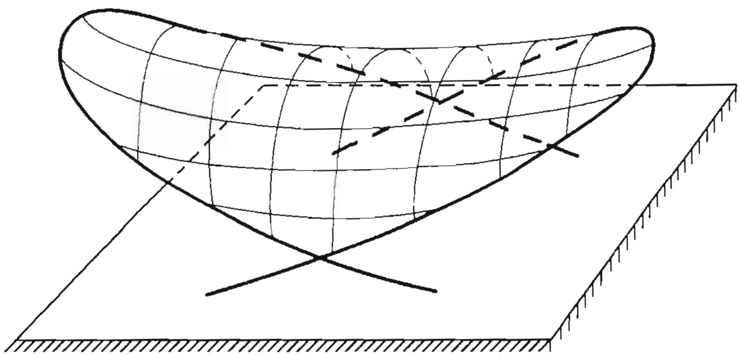

Fig. 2. Prestressed open membrane structure

2. FEM-BEM coupling methods

We consider the following three methods of the FEM-BEM coupling in aerodynamic problems of the membrane structure:

Method 1. The same finite and boundary elements (Fig.3)

Method 2. Six-node curvilinear finite elements and triangular constant-type boundary elements (Fig.4)

Method 3. Six-node curvilinear finite elements and internal collocation points in curvilinear elements takes as the boundary elements (Fig.5).

The membrane structure is discretized by six-node triangular curvilinear finite elements (Fig.3). They are used to create the stiffness and mass matrices of the membrane. After discretization the equation of small vibrations of the structure has the form

$$(\mathbf{K}_s - \omega^2 \mathbf{M}_s) u = \boldsymbol{P} \tag{2.1}$$

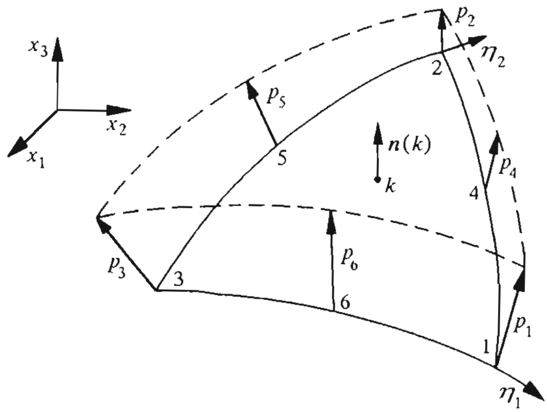

Fig. 3. Six-node triangular curvilinear finite and boundary element

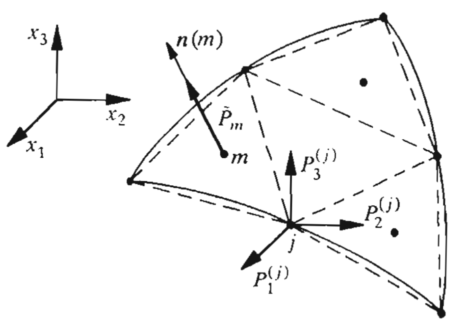

Fig. 4. Curvilinear finite element and triangular, constant-type boundary elements

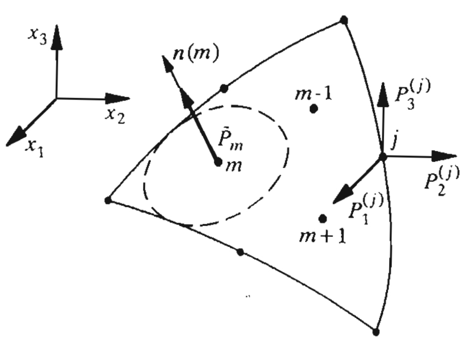

Fig. 5. Curvilinear finite element with collocation points as boundary elements

where

$\mathbf{K}_s, \mathbf{M}_s$ – stiffness and mass matrices of the structure, respectively

$\boldsymbol{u}, \boldsymbol{P}$ – vectors of displacement and aerodynamic forces amplitudes in global coordinates.

To describe the non-stationary aerodynamic pressure evoked by coupled vibrations of pneumatic structure and air, the boundary integral equations in the form of single and double potentials were used (see Sygulski, 1987, 1993). BEM was applied to solve these equations. The same boundary elements were taken for air as those used to obtain the stiffness and mass matrices of the membrane (Method 1).

Discretization of the boundary integral equations yields

$$\boldsymbol{p} = \mathbf{A}\boldsymbol{u}_n \tag{2.2}$$

where

$\mathbf{A}$ – aerodynamic matrix

$\boldsymbol{p}, \boldsymbol{u}_n$ – vectors of aerodynamic pressure and displacement amplitudes normal to the structure.

Matrix $\mathbf{A}$ is full and non-symmetric with real (incompressible air) or complex elements (compressible air). The vectors $\boldsymbol{p}$ and $\boldsymbol{u}_n$ in Eq (2.2) should be transformed into global coordinate system.

Taking advantage of the virtual work principle at the virtual displacement state it can be written (Fig.3)

$$P_r^{(j)} = \sum_{i=1}^{6} p_i \int\limits_{S_i} N_j(k) N_i(k) n_r(k)\, dS_i \qquad \begin{array}{l} j = 1,2,...,6 \\ r = 1,2,3 \end{array} \tag{2.3}$$

where

$P_r^{(j)}$ – rth component of force at the node j

$N_i(k)$ – rth shape function at the point k

$n_r(k)$ – rth component of the normal vector $\boldsymbol{n}(k)$

or in matrix form

$$\boldsymbol{P} = \mathbf{T}_1 \boldsymbol{p} \tag{2.4}$$

where

$\mathbf{T}_1$ – transformation matrix between $\boldsymbol{P}$ and $\boldsymbol{p}$ vectors.

The following relationship between $\boldsymbol{u}_n$ and $\boldsymbol{u}$ vectors is valid

$$\boldsymbol{u}_n = \mathbf{T}_2 \boldsymbol{u} \tag{2.5}$$

where

$\mathbf{T}_2$ – transformation matrix built from the components of normal vector in elements nodes.

Using the above equations one gets finally

$$P = \mathbf{T}_1\mathbf{A}\mathbf{T}_2\boldsymbol{u} \tag{2.6}$$

Introducing Eq (2.6) to (2.1) yields the equation of membrane motion with influence of surrounding air.

Method 2 of the FEM-BEM coupling was used in vibration analysis of open membrane structures (see Sygulski, 1994). The boundary elements used for discretization were the triangular constant-type ones (Fig.4). Method 3 bases on the application of internal collocation points (Fig.5) to solution to the boundary integral equation. Numerical tests (see Sygulski, 1995) proved that the use of three collocation points coinciding with the Gauss points in six-node triangular element was optimal. A circular element of constant-type corresponds to each collocation point (Fig.5). Its area is

$$S_m = G_m w_m \tag{2.7}$$

where

G_m – determinant of Jacobian at the point m

w_m – weight coefficient for the point m.

Discretization of the boundary integral equation yields

$$\widetilde{\boldsymbol{p}} = \mathbf{A}\widetilde{\boldsymbol{u}}_n \tag{2.8}$$

where

$\widetilde{\boldsymbol{p}}, \widetilde{\boldsymbol{u}}$ – vectors of aerodynamic pressure and displacement amplitudes in the direction normal to the membrane

$\mathbf{A}$ – aerodynamic matrix.

Vectors $\widetilde{\boldsymbol{p}}$ and $\widetilde{\boldsymbol{u}}$ determine the values at collocation points (Fig.5) or at geometrical centres of triangular elements (Fig.4). The resulting aerodynamic force at the point m can be determined as

$$\widetilde{P}_m = S_m\widetilde{p}_m \tag{2.9}$$

where

S_m – area of the triangle m (Fig.4) or circle area according to Eq (2.7).

Eq (2.9) can be rewritten in matrix form as follows

$$\widetilde{\boldsymbol{P}} = \mathbf{S}\widetilde{\boldsymbol{p}} \tag{2.10}$$

where $\mathbf{S} = \mathrm{diag}(S_1, S_2, ..., S_N)$.

The application of virtual principle work yields

$$P_r^{(j)} = \sum_{m=1}^{L} N_j(m)\widetilde{P}_m n_r(m) \qquad \begin{aligned} r &= 1,2,3 \\ j &= 1,2,...,6 \\ m &= 1,2,...,L \end{aligned} \tag{2.11}$$

where

$P_r^{(j)}$	–	rth component of force at the node j
$N_i(m)$	–	jth shape function at the point m
$\widetilde{P}_m$	–	resulting aerodynamic force at the point m
$n_r(m)$	–	rth component of the normal vector $n(m)$
L	–	number of collocation points in the element.

For the whole system Eq (2.11) can be rewritten in matrix form as follows

$$\boldsymbol{P} = \mathbf{T}\widetilde{\boldsymbol{P}} \tag{2.12}$$

where

$\mathbf{T}$ – transformation matrix between $\boldsymbol{P}$ and $\widetilde{\boldsymbol{P}}$ vectors.

Similarly it can be written

$$\widetilde{\boldsymbol{u}}_n = \mathbf{T}^\top \boldsymbol{u} \tag{2.13}$$

Taking advantage of Eqs (2.8), (2.10), (2.12) and (2.13) yields

$$\boldsymbol{P} = \mathbf{TSAT}^\top \boldsymbol{u} \tag{2.14}$$

Taking into account Eq (2.14) in the equation of motion for the structure (2.1) one gets the equation describing the eigenvalue problem of the membrane-air coupled system.

The analyses of free and harmonically forced vibrations of pneumatic structures and open membrane structures with air mass taken into account were presented by Sygulski (1989) ÷ (1994). The problems of pneumatic membrane and plane membrane stability in air flow were described by Sygulski (1996), (1997). The equations describing the problem, methods of their solution and the results of numerical analysis are given in theses papers. The numerical results were positively verified during experimental tests.

3. Conclusions

The FEM-BEM coupling in the problems of interaction between the structure and surrounding air is an effective way of the analysis of such complicated subject. The boundary integral techniques are attractive because they enable one to avoid the explicit imposition of the radiation condition which is embedded in the free space Green's function. Moreover, the dimension of the problem is not increased.

The numerical examples calculated with the use of air mass and radiation damping matrices enable one to draw the following conclusions:

- The effect of interaction between the surrounding air and the membrane structure made of fabric on the natural frequencies of the structure is very significant. For higher frequencies the influence of the air is less pronounced.
- The air compressibility slighthy influences the natural frequencies.
- The logarithmic decrement of radiation damping is relatively small.

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Połączenie MES i MEB w zagadnieniach interakcji powłok membranowych i powietrza

Streszczenie

Praca dotyczy problemów interakcji lekkich przekryć membranowych i otaczającego powietrza. Omawia się połączenie metody elementów skończonych (MES) i metody elementów brzegowych (MEB) do analizy zagadnień aerodynamiki powłok membranowych. Przedstawiono trzy metody połączenia MES i MEB.

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