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DRYING PROCESSES IN CONTEXT OF THE THEORY OF FLUID SATURATED POROUS MATERIALS

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The paper presents a thermomechanical model of drying of fluid-saturated capillaric porous materials. The governing equations developed on the basis of balance equations and thermodynamics of irreversible processes are presented. Their solution allows for prediction of the evolution of moisture content distribution, deformation of the dried body, and stress induced by drying. The strains and stresses are examined in a prismatic bar dried convectively (2D initial-boundary-value problem). The influence of different drying conditions and various shapes of the bar cross-section on the strains and stresses is discussed using the linear and non-liner models, respectively.

Key words: thermomechanical model of drying, deformation of dried body, drying induced stresses

1. Introduction

Modelling of the drying processes of fluid-saturated capillaric porous materials involves some mechanical problems because they tend to deform materials and cause their cracking. This paper presents a theory that describes fully coupled multiphase transport in deformable fluid-saturated capillary-porous media. Such a theory allows for predictions of the evolution of moisture content distribution, deformation of the dried body, and stresses induced during drying. This coupled theory also allows for analysis of the dried materials response to the alteration of drying conditions and thus the control of the drying induced stresses. Carrying out a computer simulation of the process on the basis of this theory one can state that when the stresses rise until they reach a critical state at some points, there is a possibility to reduce them by reducing the rate of drying. This can be done by altering the drying parameters (e.g. temperature or humidity of the drying medium).

The present considerations are entirely based on the concept of balance equations of mass, momentum, energy and entropy, and thermodynamics of irreversible processes with the use of Caratheodory's principle. The constitutive and rate equations are formulated which together with the balance equations create the thermomechanical model of drying.

The strains and stresses are examined in a prismatic bar dried convectively (2D initial-boundary value problem). The influence of different drying conditions and various shapes of the bar cross-section on the strains and stresses is discussed using the linear and non-linear models.

Our considerations have resulted in some conclusions of practical importance. One of them is that the moisture potential reaches local maximum close to the boundary during intensive heating. This causes blocking of the moisture flow from the interior of the body towards its boundary and makes the boundary layer dry very quickly. The second conclusion is that the thermal and shrinkage stresses sum up in some periods of drying and neutralize each other in other periods. Another conclusion states that the shape of dried body and the grooves placed on its surface influence considerably the deformations and stress distribution in the dried body. Optimization of the computer simulation of the drying processes was carried out and one states that the optimal drying process is that for which the drying rate corresponds to the maximal stress equal to the permissible stress. For lower drying rates the stresses do not violate the strength of the material but the time of drying are much longer.

The paper presents the numerical results in the form of diagrams drawn automatically by computer.

2. Balance equations

To capture the major features of a real dried body, let us assume that the body consists of a deformable porous solid with uniformly distributed pores filled with a mixture of water and humid air.

Using of the principles of balance of mass, momentum, and energy for each individual constituent α $\{\alpha = S(\text{solid}),\ L(\text{liquid}),\ V(\text{vapour}),\ A(\text{air})\}$ yields in the following relations (see Kowalski, 1994)

$$
\begin{aligned}
&\rho'_\alpha + \rho_\alpha \operatorname{div} \boldsymbol{v}_\alpha = \widehat{\rho}_\alpha && \widehat{\rho}_S \equiv 0 \qquad \widehat{\rho}_A \equiv 0 \qquad \sum_\alpha \widehat{\rho}_\alpha = 0\\
&\operatorname{div} \mathbf{T}_\alpha + \rho(\boldsymbol{g} - \boldsymbol{a}_\alpha) + \widehat{\boldsymbol{p}}_\alpha = \boldsymbol{0} && \sum_\alpha (\widehat{\boldsymbol{p}} + \widehat{\rho}_\alpha \boldsymbol{v}_\alpha) \equiv 0\\
&\rho_\alpha e'_\alpha = \operatorname{tr}(\mathbf{T}_\alpha \mathbf{D}_\alpha) - \operatorname{div} \boldsymbol{q}_\alpha + \rho_\alpha r_\alpha + \widehat{e}_\alpha\\
&\sum_\alpha \Big[\widehat{e}_\alpha + \widehat{\boldsymbol{p}}_\alpha \cdot \boldsymbol{v}_\alpha + \widehat{\rho}\Big(e_\alpha + \frac{1}{2}\boldsymbol{v}_\alpha \cdot \boldsymbol{v}_\alpha\Big)\Big] = 0
\end{aligned}
\tag{2.1}
$$

for $\alpha = S, L, V, A$. In the above equations: ρ_α is the partial mass density, $\boldsymbol{v}_\alpha$ and $\boldsymbol{a}_\alpha$ denote the velocity and acceleration, respectively, of the constituent α, $\mathbf{T}_\alpha$ is the Cauchy stress tensor, $\boldsymbol{g} = -\operatorname{grad} \mu_{grav}$ is the gravitational force and μ_{grav} is the gravitational potential, e_α is the specific internal energy, $\mathbf{D}_\alpha$ denotes the strain rate tensor, $\boldsymbol{q}_\alpha$ is the heat flux, ρ_α is the specific external heat supply, $\widehat{\rho}_\alpha, \widehat{\boldsymbol{p}}_\alpha, \widehat{e}_\alpha$ represent the mass, momentum, and energy production terms associated with the exchange of mass, momentum and energy from other constituents by means of phase transition, mechanical interactions, and energy exchange, and symbol $(\cdot)'$ denotes the material derivative with convection velocity $\boldsymbol{v}_\alpha$.

Taking drying problems into consideration it is convenient to formulate the field equations in terms of material co-ordinates of the solid matrix. Therefore, we reformulate the balance equations in such a way that all the thermodynamic functions are referred to the mass of dry body. Such an approach provides a rational way of constructing a more practical model of drying. Besides, choosing the porous matrix as a reference constituent allows for construction of a general function (moisture potential) being responsible for the moisture transport and the phase transition. Furthermore, the boundary is clearly defined and formulation of the boundary conditions creates no difficulties.

We rearrange the balance equations using functions referred to the dry body mass, namely, specific moisture content $X_\alpha = \rho_\alpha/\rho_s$, relative moisture flux $\boldsymbol{w}_\alpha = \rho_\alpha(\boldsymbol{v}_\alpha - \boldsymbol{v}_s)$, total internal energy per unit mass of the dry body $e = \sum_\alpha X_\alpha e_\alpha$, total stress tensor for the multi-phase medium $\mathbf{T} = \sum_\alpha \mathbf{T}_\alpha$, total heat flux $\boldsymbol{q} = \sum_\alpha \boldsymbol{q}_\alpha$, total heat supply per unit volume $\rho r = \sum_\alpha \rho_\alpha r_\alpha$, where $\rho = \sum_\alpha \rho_\alpha$ is the total mass density. The individual material derivative of the constituent α is replaced by the material derivative with convection velocity of the solid (denoted by a dot over the symbol)

$$
\phi'_\alpha = \dot{\phi}_\alpha + (\boldsymbol{v}_\alpha - \boldsymbol{v}_s) \operatorname{grad} \phi_\alpha
\tag{2.2}
$$

Neglecting the accelerations and kinetic energies as small in drying processes and ignoring the stress deviator for the moisture, we rearrange Eqs (2.1) as follows

$$
\begin{aligned}
&\rho_s \dot{X}_\alpha = -\operatorname{div} \boldsymbol{w}_\alpha + \widehat{\rho}_\alpha && \alpha = L, V, A \\
&\operatorname{div} \mathbf{T} + \rho \boldsymbol{g} \cong \mathbf{0} && \mathbf{T} = \mathbf{T}^\top \qquad (2.3)\\
&\rho_s \dot{e}_\alpha = \operatorname{tr}(\mathbf{T}\mathbf{D}_s) - \operatorname{div}\Big(\boldsymbol{q} + \sum_\alpha \boldsymbol{w}_\alpha h_\alpha\Big) + \rho r + \sum_\alpha \boldsymbol{w}_\alpha \cdot \boldsymbol{g}
\end{aligned}
$$

where $\mathbf{D}_s = [\operatorname{grad} \boldsymbol{v}_s + (\operatorname{grad} \boldsymbol{v}_s)^\top]/2$ is the strain rate tensor of the porous solid, $h_\alpha = e_\alpha - P_\alpha/\rho_\alpha$ is the enthalpy of the constituent α, and P_α is the partial pressure of the constituent α.

3. Caratheodory's principle. The second law of thermodynamics

Eq $(2.3)_3$ represents the energy balance for the medium contained in a unit volume appointed by the porous body in a current configuration, i.e., it is expressed in the space variables $(\boldsymbol{x}, t)$. Its form expressed in the material variables $(\boldsymbol{X}, t)$ (reference configuration) is

$$
\begin{aligned}
\dot{E} = {} & \operatorname{tr}(\mathbf{T}^R \dot{\mathbf{E}}_s^{(r)}) + \rho_s^R \sum_\alpha h_\alpha \dot{X}_\alpha + \operatorname{tr}(\mathbf{T}^R \dot{\mathbf{E}}_s^{(ir)}) - \operatorname{Div} \boldsymbol{q}^R + R + \\
& - \sum_\alpha \boldsymbol{w}_\alpha^R \operatorname{Grad}(h_\alpha + \mu_{grav}) - J \sum_\alpha \widehat{\rho}_\alpha h_\alpha
\end{aligned}
\qquad (3.1)
$$

where $E = \rho_s^R e$ and $\rho_s^R = \rho_s J$ are the internal energy and the porous body mass density, respectively, per unit volume of the reference configuration, $J = \det \mathbf{F}$ is the determinant of the deformation gradient $\mathbf{F} = \operatorname{Grad}_X \boldsymbol{x}$, $\dot{\mathbf{E}}_s^{(r)}$ and $\dot{\mathbf{E}}_s^{(ir)}$ are the reversible and irreversible strain rate tensors of the porous solid, $\mathbf{T}^R = J\mathbf{F}^{-1}\mathbf{T}(\mathbf{F}^{-1})^\top$ is the second Piola-Kirchoff stress tensor, $\boldsymbol{q}^R = J\boldsymbol{q}(\mathbf{F}^{-1})^\top$ and $\boldsymbol{w}_\alpha^R = J\boldsymbol{w}_\alpha(\mathbf{F}^{-1})^\top$ are the heat and moisture fluxes, both measured per unit area of the reference configuration, and $R = J\rho r$ is the external heat supply per unit volume of the reference configuration.

According to Caratheodory's principle there are adiabatically inaccessible states in the neighbourhood of every state, which means that for an adiabatic reversible process the deferential (Pfaffian)

$$\frac{D\Psi}{Dt} = \dot{E} - \operatorname{tr}(\mathbf{T}^R\dot{\mathbf{E}}_s^{(r)}) - \rho_s^R \sum_\alpha h_\alpha \dot{X}_\alpha \tag{3.2}$$

has to be equal to zero (see e.g. Guminski, 1974).

It can be shown (see e.g. Naphtali, 1966; Hutter, 1977) that the differential form in Eq (3.2) for a reversible process is integrable, which means that there exists an integrating factor $T(\theta) \geqslant 0$ which turns the differential form in Eq (3.2) into a potential, which consists of the sum of entropies of individual constituents

$$\frac{1}{T(\theta)}\Big[\dot{E} - \operatorname{tr}(\mathbf{T}^R\dot{\mathbf{E}}_s^{(r)}) - \rho_s^R \sum_\alpha h_\alpha \dot{X}_\alpha\Big] = \rho_s^R \sum_\alpha X_\alpha \dot{s}_\alpha \tag{3.3}$$

We express Eq (3.3) in terms of the free Helmholtz energy $F = E - ST$ with $S = \rho_s^R \sum_\alpha X_\alpha s_\alpha$ being the total entropy of the medium per unit volume of the reference configuration

$$\dot{F} = \operatorname{tr}(\mathbf{T}^R\dot{\mathbf{E}}_s^{(r)}) + \rho_s^R \sum_\alpha \mu_\alpha \dot{X}_\alpha - S\dot{T} \tag{3.4}$$

where $\mu_\alpha = h_\alpha - s_\alpha T$ is the free enthalpy per unit mass of the constituent α (chemical potential).

Combining Eq (3.3) with Eq (3.1) one obtains

$$T\dot{S} = \operatorname{tr}(\mathbf{T}^R\dot{\mathbf{E}}_s^{(ir)}) - \operatorname{Div}\Big(\boldsymbol{q}^R + \sum_\alpha \boldsymbol{w}_\alpha^R s_\alpha T\Big) + R - \sum_\alpha \boldsymbol{w}_\alpha^R \cdot \operatorname{Grad}\widehat{\mu}_\alpha - J\sum_\alpha \widehat{\rho}_\alpha \mu_\alpha \tag{3.5}$$

where $\widehat{\mu}_\alpha = \mu_\alpha + \mu_{grav}$.

The thermodynamic system is limited by the boundary of the porous solid. The heat flow appear due to conduction and mass transported through the boundary. We separate from Eq (3.5) the entropy exchanged by the thermodynamic system with the ambient medium and subtract it from the total entropy. The difference is called the entropy produced in the system during irreversible process. According to the second law of thermodynamics this part of entropy is always positive. Thus, we have

$$\operatorname{tr}(\mathbf{T}^R\dot{\mathbf{E}}_s^{(ir)}) - \sum_\alpha \boldsymbol{w}_\alpha^R \operatorname{Grad}\widehat{\mu}_\alpha - J\sum_\alpha \widehat{\rho}_\alpha \mu_\alpha - \frac{1}{T}\Big(\boldsymbol{q}^R + \sum_\alpha \boldsymbol{w}_\alpha^R s_\alpha T\Big) \cdot \operatorname{grad} T \geqslant 0 \tag{3.6}$$

This residual inequality expresses the amount of energy dissipated per unit time and per unit volume. From the mathematical point of view it is a constraint which helps to draw some conclusions concerning the form of thermodynamic fluxes.

4. Constitutive assumptions

One can find a family of admissible processes which satisfy the inequality (3.6). Taking into account another tensorial representation of the individual terms in this inequality and Curie's principle, we formulate the sufficient conditions for satisfying it

$$
\begin{aligned}
&\boldsymbol{w}_\alpha^R = -\Lambda_\alpha \operatorname{Grad} \widehat{\mu}_\alpha && \Lambda_\alpha \geqslant 0 \\
&\boldsymbol{q}^R + \sum_\alpha \boldsymbol{w}_\alpha^R s_\alpha T = -\Lambda_T \operatorname{Grad} T && \Lambda_T \geqslant 0 \\
&\widehat{\rho}_L = \frac{\omega}{T}(\mu_V - \mu_L) = -\widehat{\rho}_V && \omega \geqslant 0 \\
&\dot{\mathsf{E}}_s^{(ir)} = \mathsf{E}^{(4)}\mathsf{T}^R && \mathsf{E}^{(4)}\mathsf{T}^R \otimes \mathsf{T}^R \geqslant 0
\end{aligned}
\tag{4.1}
$$

Thus, the mass fluxes, Eq $(4.1)_1$, depend on the gradients of moisture potentials. The heat flux and heat transported by the mass flux, Eq $(4.1)_2$, are proportional to the temperature gradient. The rate of mass transformation due to the liquid-vapour phase transition, Eq $(4.1)_3$, is proportional to the difference between chemical potentials of liquid and vapour. Eq $(4.1)_4$ is similar to the Newton-Cauchy-Poisson law for viscous bodies or the law of plastic flow under an additional condition concerning yielding, (see Kowalski, 1996a).

We assume that the free energy, Eq (3.4) being a function of the thermodynamical state depends on strain tensor $\mathsf{E}_s^{(r)}$, moisture content X_α, and temperature T. Thus, the equations of state have the form

$$
\begin{aligned}
&\mathsf{T}^R = \frac{\partial F}{\partial \mathsf{E}_s^{(r)}} = \mathsf{T}^R(\mathsf{E}_s^{(r)}, \boldsymbol{X}_\alpha, T) && S = -\frac{\partial F}{\partial T} = S(\operatorname{tr} \mathsf{E}_s^{(r)}, \boldsymbol{X}_\alpha, T) \\
&\mu_\alpha = \frac{1}{\rho_s^R}\frac{\partial F}{\partial X_\alpha} = \mu_\alpha(\operatorname{tr} \mathsf{E}_s^{(r)}, \boldsymbol{X}_\alpha, T)
\end{aligned}
\tag{4.2}
$$

The explicit forms of the above functions can be found e.g. by, expanding them into a Taylor series and interpreting properly the respective coefficients.

The fundamental set of Eqs (2.3), (4.1) and (4.2), together with the initial and boundary conditions give the basis for formulation of boundary value problems.

5. Set governing of equations

Now, we shall apply the thermomechanical model of drying of capillary-porous materials presented above to solving some boundary value problems. After making the assumptions about preheating and constant drying rate periods, the complete system of differential equations describing deformations and distributions of temperature and moisture potential inside the dried body (see for instance Kowalski and Rybicki, 1994, 1996b; Kowalski et al., 1997; Rybicki, 1993) has the form

$$
\begin{aligned}
&M\nabla^2 u_i + \Big(M + A - \frac{\gamma_X^2}{c_X}\Big)u_{j,ji} = \Big(\gamma_\theta - \gamma_X\frac{c_\theta}{c_X}\Big)\theta_{,i} + \frac{\gamma_X \rho_s}{c_X}\mu_{,i} \\
&K_m\nabla^2\mu = \dot{\mu} + \frac{\gamma_X}{\rho_s}\dot{u}_{i,i} - \frac{c_\theta}{\rho_s}\dot{\theta} \qquad (5.1)\\
&K_T\nabla^2\theta = \dot{\theta} + K_u\dot{u}_{i,i} - K_\mu\dot{\mu}
\end{aligned}
$$

Here u_i, μ, θ denote the displacement vector of the porous solid, moisture potential, and temperature, respectively, and

$$
K_m = \frac{\Lambda_m c_X}{\rho^2} \qquad K_T = \frac{\Lambda_T}{c_v^*} \qquad K_\mu = \frac{T_r c_\theta \rho_s}{\gamma_\theta c_{X\theta} c_v^*}
$$

$$
\gamma_\theta = (2M + 3A)\alpha_\theta \qquad \gamma_X = (2M + 3A)\alpha_X
$$

$$
K_u = T_r\Big(1 - \frac{c_\theta \gamma_X}{c_X\gamma_\theta}\Big)c_v^* \qquad c_v^* = c_v + \frac{T_{rr} c_\theta^2}{\gamma_\theta c_X}
$$

where c_v is the specific heat of medium at a constant volume, c_θ and c_X are the coefficients of thermal capacity and moisture content capacity of the moisture potential, T_r is the reference temperature (absolute).

The system of equations represents: the deformations of the dried materials, Eq $(5.1)_1$; the moisture potential distribution , the gradient being responsible for the moisture transport, Eq $(5.1)_2$; and the temperature distribution, Eq $(5.1)_3$.

The moisture potential is considered here as a linear function of the moisture content X, temperature θ and volumetric strain ε

$$
\mu = \frac{1}{\rho_s}[c_\theta(\theta - \theta_r) - \gamma_\theta\varepsilon + c_X(X - X_r)] \tag{5.2}
$$

The internal stresses occur during drying due to non-uniform distributions of the moisture content and temperature. We use the following physical relation between stresses σ_{ij}, strains ε_{ij}, temperature θ and moisture content X

$$\sigma_{ij} = 2M\varepsilon_{ij} + [A\varepsilon - \gamma_\theta(\theta - \theta_r) - \gamma_X(X - X_r)]\delta_{ij} \tag{5.3}$$

where X_r is the reference moisture content.

During the constant drying rate period the temperature θ is equal to the wet-bulb thermometer and remains constant. In this period the system of governing equations is reduced to Eqs $(5.1)_{1,2}$. The differential equation for the temperature field is trivial in this case.

6. Formulation of the initial-boundary-value problem

We shall discuss the problem of convective drying of a prismatic bar, Fig.1. This is a 2D problem and all functions are assumed to be dependent on the co-ordinates x, y and time t.

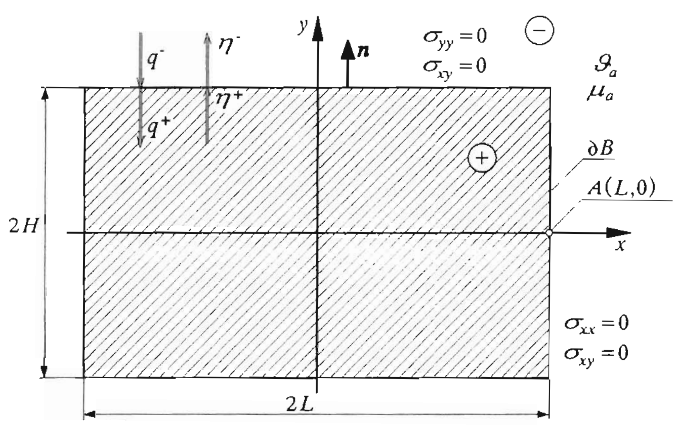

Fig. 1. Rectangular cross-section of the dried bar. Boundary conditions

For the 2D problem, the displacement of the porous solid in z-direction is assumed to be zero, and all other functions are assumed to be dependent on the co-ordinates x, y and time t, i.e. $u_x = u_x(x, y, t)$, $u_y = u_y(x, y, t)$, $\mu = \mu(x, y, t)$, $\theta = \theta(x, y, t)$.

We formulate the initial-boundary-value problem as follows: find the functions u_x, u_y, θ and μ which, within the rectangle $(-L, L) \times (-H, H)$ and for $t \in R^+$, satisfy the system of equations (5.1) under the following conditions (see Fig.1.):

— for the stresses

$$\sigma_{ij}\Big|_{\partial B} = 0 \tag{6.1}$$

— for the moisture exchange

$$\Lambda_m \boldsymbol{n} \cdot \operatorname{grad} \mu\Big|_{\partial B^+} = \pm\alpha_m\Big(\mu\Big|_{\partial B^-} - \mu_a\Big) \tag{6.2}$$

— for the heat exchange

$$\Lambda_T \boldsymbol{n} \cdot \operatorname{grad} \theta\Big|_{\partial B^+} = \pm\alpha_T\Big(\theta_a - \theta\Big|_{\partial B^-}\Big) \pm l\alpha_m\Big(\mu\Big|_{\partial B^-} - \mu_a\Big) \tag{6.3}$$

under the initial conditions: $\sigma_{ij}(x,y,0) = 0$, $\mu(x,y,0) = \mu_0$, $\theta(x,y,0) = \theta_0$. In the above conditions μ_a and θ_a denote the chemical potential and temperature of the drying medium (air) and l is the latent heat of evaporation.

7. Results

The method for solving the considered problem is based on the Galerkin discretization (finite elements) in space and on the difference approximation of time derivatives in the final system of ordinary differential equations (see e.g. Wait ant Mitchell, 1986; Rybicki, 1994; Musielak, 1991).

7.1. Drying of the bar with the rectangular cross-section

Fig.2 illustrates the distribution of moisture content in a quarter of the rectangular cross-section of the dried bar. It is seen that the value of X near the boundary is smaller than in the middle of the cross-section. This non-uniform distribution of X together with non-uniformity of the temperature distribution are the reasons the strains of the dried material arise and, as a consequence, the reasons for the internal stress generation. The stresses will be shown further on.

Fig.3 shows the evolution of the moisture potential μ and temperature θ along the x-axis. Because of the temperature elevation near the boundary it can happen that the moisture potential becomes a local extremum as it is shown in Fig.3 (dashed line). In such a case, there is a negative moisture potential gradient on the right-hand side of the maximum (solid line) and a positive one on its left-hand side (dashed line). It is the reason why the flow of moisture from the interior of the dried body to its boundary is blocked due to the thermodiffusional effect. The boundary layer is dried very quickly

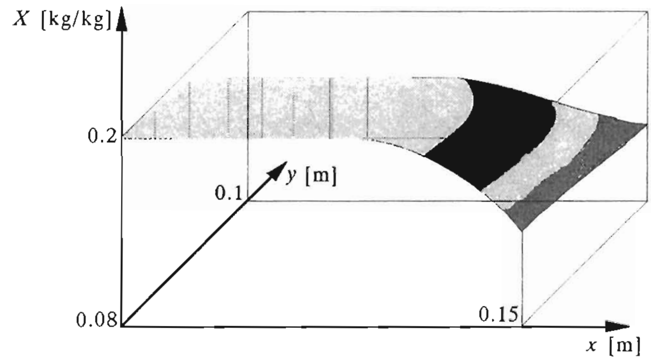

Fig. 2. Distribution of the moisture content after five hours of the process

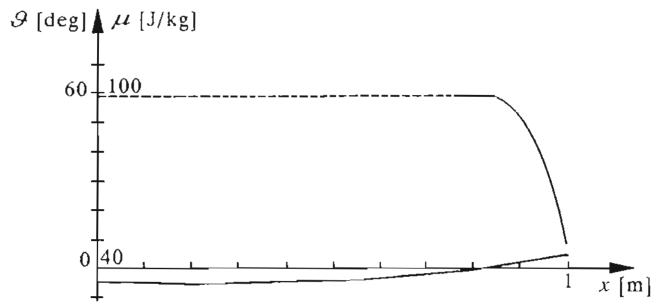

Fig. 3. Distribution of the moisture potential and temperature along x-axis at $t = 10\,\mathrm{min}$

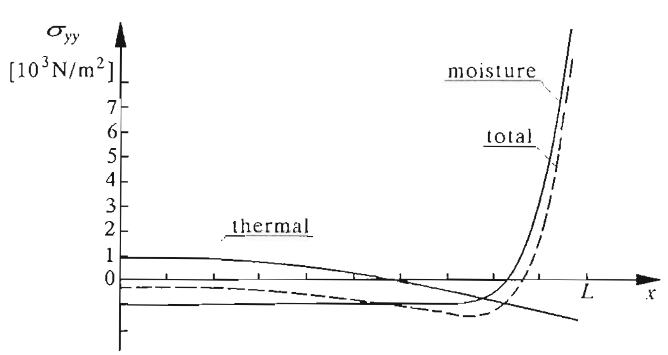

Fig. 4. Evolution of the drying induced stresses due to the temperature and the moisture content alteration along section $y = 0$, $0 < x < L$ after 7 min of the drying process

and shrinks rapidly, whereas the interior does not or even swells a bit. The shrinkage stresses which appear in the boundary layer at that moment can cause its cracking.

Fig.4 illustrates the distribution of thermal, shrinkage and total stresses along the x-axis. It can be seen that near the boundary the thermal stresses are negative and the shrinkage ones are positive. And, vice versa, in the interior the thermal stresses are positive and the shrinkage ones are negative. In both cases they reduce each other and the total stresses become smaller.

7.2. Drying induced stresses dependent on the shape of the dried body

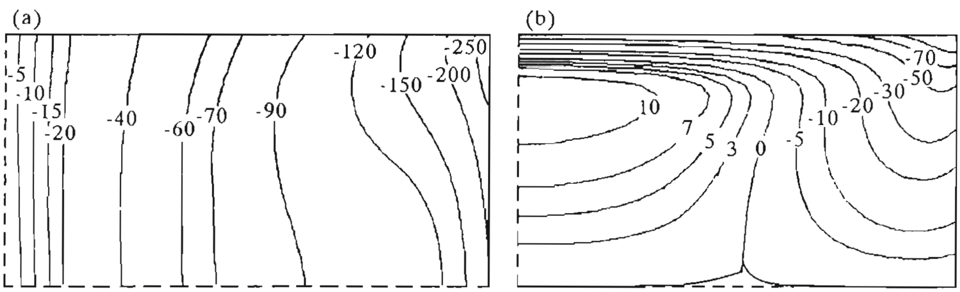

Fig. 5. Displacement contour lines (a) u_x (b) u_y $[10^{-7}\mathrm{m}]$ at the instant $t = 10\,\mathrm{min}$ in the the rectangular bar

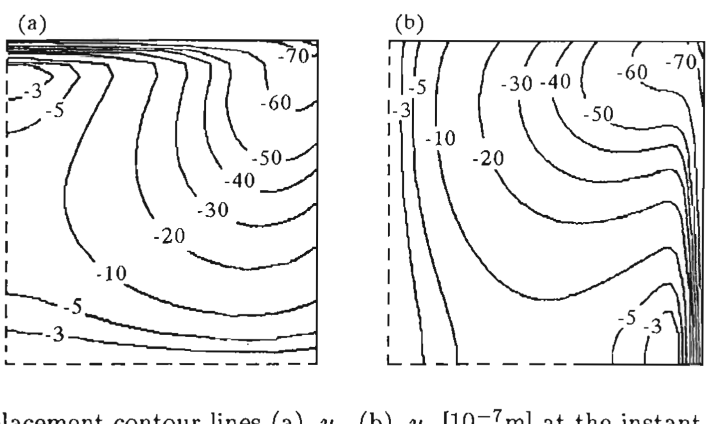

Fig. 6. Displacement contour lines (a) u_x (b) u_y $[10^{-7}\mathrm{m}]$ at the instant $t = 10\,\mathrm{min}$ in the square bar

The influence of the dried body shape on shrinkage strains is illustrated by the drying of two different bars: the first one having a rectangular cross-section $(-L, L) \times (-2L, 2L)$, and the second one having a square cross-section $(-L, L) \times (-L, L)$. Displacement in the x-direction at an instant is depicted in Fig.6a for the first bar and in the Fig.6a for the second one. Shrinkage in the x-direction appears at all points of the regions but is much grater near the edge where the moisture is intensively removed. The displacements u_x are qualitatively similar for both bars, unlike the displacements u_y. These displacements are shown in Fig.5b for the rectangular bar and in Fig.6b for the square one. Unexpectedly, in the internal region of rectangular bar there appear the positive displacements despite a decrease in moisture content in the whole rectangle. The shape of the dried body affects shrinkage on both sides of the rectangle, being much stronger,however, on the longer side. This asymmetrical shrinkage leads to some strange *tension force* appearing inside the body. This effect is observed only for a rectangular bar. No such effects can be observed for a square domain whose displacement u_y is shown in Fig.6b.

The drying of bars with square cross sections: (a) without any groove, (b) with circular grooves, (c) with rectangular grooves and (d) with triangular grooves (see Fig.7) is considered. It acknowledges the influence of the dried body shape on the drying induced stresses.

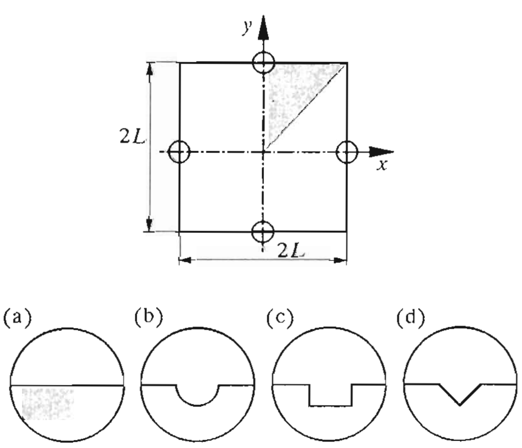

Fig. 7. Cross-section of the dried bar with marked grooves

Fig.8 shows the distributions of normal stresses σ_{xx} plotted on the half of quarter of the bars cross-sections. The tensile stresses appear near the bar boundary and the compressive ones in the internal part of cross-section. They attain their maximum values on the boundary. For all the grooves considered stresses reach zero at the border of the grooves, and they have maximum values inside the grooves. This phenomenon is especially dangerous in the

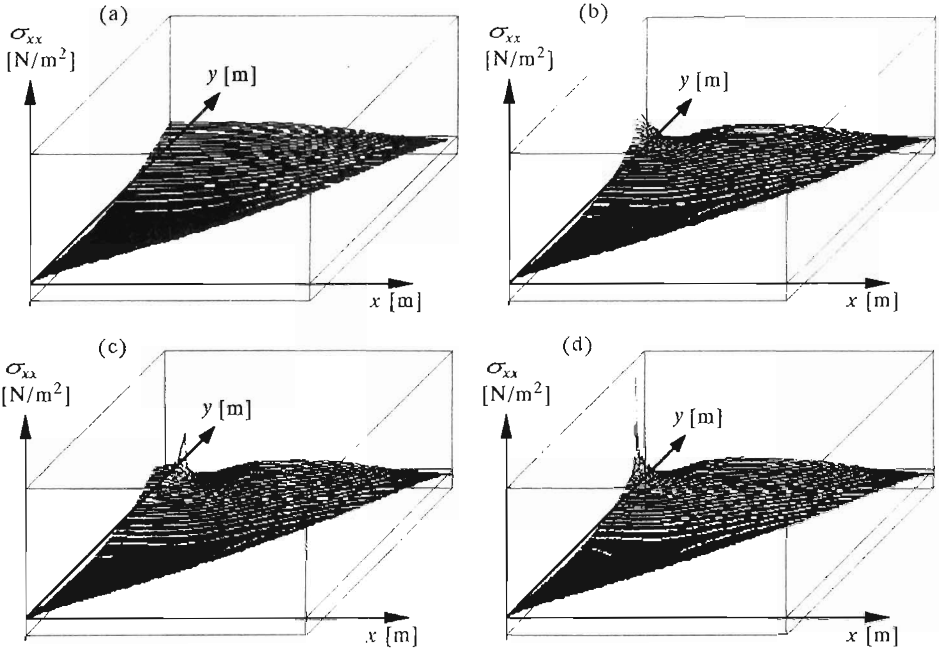

Fig. 8. Distributions of the normal stresses σ_{xx} after five hours of the drying process in the cross-section of the bar: (a) without grooves, (b) with circular grooves, (c) with rectangular grooves, (d) with triangular grooves

case of triangular-shaped groove. One to this groove in stresses increase even more than six times in comparison with a bar without grooves.

7.3. Moisture dependent mechanical properties

This numerical example is connected with the influence of the moisture content on materials constants. We assume here that the stress-strain-moisture relation (Eq (5.3)) is non-linear. The Young modulus E is assumed to be a linear function of the moisture content X, and as a consequence of it the constants M, A, γ_X and γ_θ (Eq (5.3)) are also linear functions of the moisture content.

In Fig.9 maximal values of the normal stresses σ_{xx} and σ_{yy} are shown. If we take the mechanical constants to be independent of the moisture content (averaged values), we obtain understated values of the maximum of tensional stresses. In the case considered the discrepancy for the σ_{yy} stresses is greater than 20%. The results obtained for the average Young modulus, that the tangential stresses on the longer side are greater than those on the shorter

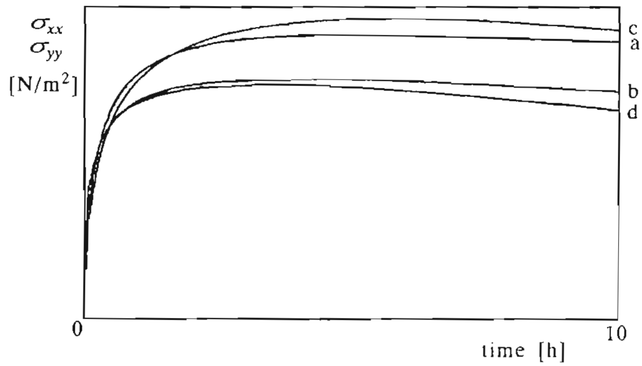

Fig. 9. Maximal normal stresses: (a) σ_{xx} for $E = E(X)$ (Eq $(2.3)_1$) at the point $(0,H)$, (b) σ_{xx} for $E = E_a$ (Eq (3.1)) at the point $(0,H)$, (c) σ_{yy} for $E = E(X)$ (Eq $(2.3)_1$) at the point $(L,0)$, (d) σ_{yy} for $E = E_a$ (Eq (3.1)) at the point $(L,0)$

one (see Fig.9b,d). But if we assume the mechanical constants to be linear functions of the moisture content we see that the tangential stresses on the shorter side are higher. It is due to the fact that this side has a smaller moisture content, so it is more rigid and the stresses are higher.

7.4. Optimal control of computer simulation of drying

The mathematical model of drying may be used to construction of a computer programme for simulation of the drying process. The simulated processes are carried out in such a way that the drying-induced stresses never exceed the stress limit.

The object of control in our considerations is a simulated drying process, and precisely, the mathematical model of this process given by Eqs (5.1) and the boundary conditions (6.1) to (6.3). We formulate our problem as follows: find such a programme of alteration of the drying parameters (temperature T_a and moisture content X_a) at which the drying rate is possibly maximal and the drying stresses never exceed the admissible stresses for a given dried material. Thus, one must observe the tension stresses at the boundary of the dried material, which are usually the largest. These stresses must never exceed the admissible value but their necessary reduction during drying process, when they tend to exceed the admissible stresses, ought to be carried out in such a way that they remain close to the admissible value. Otherwise, the rate of drying would reduced too much. This is the principle of maximum drying rate (see also Leitman, 1966).

Fig.10 presents a controlled drying process from the beginning to the 11th

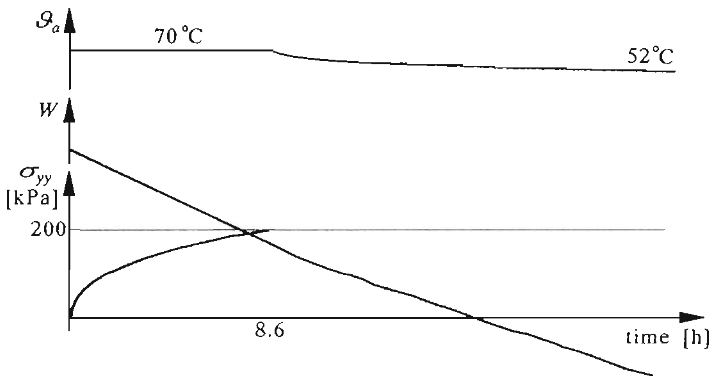

Fig. 10. Optimal drying process

hour through a suitable alteration of the temperature T_a. The upper part of the figure presents the moisture content alteration W and the lower part illustrates the evolution of the stresses at the point $(L, 0)$ for the following programme: from the beginning to 8.6 hours of the drying process the drying parameters were $T_a = 70°\mathrm{C}$ and $Y_a = 0.035$, since that time the humidity was kept constant and the temperature of the drying medium was altered by the computer automatically in such a way that the stresses have never exceeded the permissible stress $\sigma_{yy} = 200\,\mathrm{kPa}$. Moreover, it is seen that the stress σ_{yy} is nearly constant and equal to $200\,\mathrm{kPa}$ through the whole course of drying. One expects that the drying rate in this case is the optimal one.

8. Short review of authors' works concerning drying

Analysis of the mechanical behaviour of porous materials during drying has been carried out by the authors for about ten years. Within the framework of this studies two thermomechanical models have been developed: the first one based on the thermoelasticity of fluid-saturated porous media, see Kowalski (1987), (1990), and the second one based on the continuum mixture theory, see Boer and Kowalski (1995). The models were extended onto permanent deformations (Kowalski, 1996a) and the boundary conditions for them were widely discussed by Kowalski and Rybicki (1995a), Kowalski and Strumiłło (1997). At the same time a number of numerical examples were solved. First, the influence of the deformation of the dried body upon heat and moisture diffusion in its inside was examined, Kowalski and Musielak (1988). Then, the problem of shrinkage strains and stresses was studied (see Kowalski, 1996b; Kowalski et al., 1992; Kowalski and Rybicki, 1994, 1996b; Rybicki, 1993, 1994) based on the linearised model (Kowalski, 1987, 1990). 1D and 2D initial-boundary valued problems, including various phenomena, were analysed. The second model of drying (Boer and Kowalski, 1995; Kowalski, 1994) including gas phase and evaporation inside the dried body was applied by Felcyn et al. (1996), Kowalski et al. (1996a,c), Musielak (1991), (1996) to examine the drying-induced strains and stresses. In these works the equations contained nonlinear terms describing phase transitions. Then, other nonlinear phenomena and their influence on stress generation were investigated: the nonlinear elasticity for ceramics (Kowalski and Musielak, 1995) and the orthotropic diffusion with the orthotropic nonlinear elasticity for wood (Kowalski and Musielak, 1998). The response of dried materials to drying conditions was analysed by Kowalski et al. (1997), Kowalski and Rybicki (1995b). The results of these papers were applied to the problem of optimal control of drying (Kowalski and Rybicki, 1996a, 1998).

Except for the construction of the drying models based on the phenomenological approach and the numerical studies connected with the computer simulated drying processes the authors carried out also some experimental investigation concerning both the drying processes (Felcyn et al., 1996) and wetting processes (Kowal et al., 1992; Kowal and Kowalski, 1995; Kowalski and Kowal, 1998). Three review papers concerning the movement of the phase transition zone by drying (Kowalski and Musielak, 1989), the damage models with the view to their applications to dried materials (Kowalski and Musielak, 1995) and the acoustic emission as a method for the diagnosis of the dried materials damage (Kowalski and Musielak, 1992) were written as well.

Main results of the aforementioned research have been published in the manuscript (Kowalski et al., 1996b).

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Procesy suszenia w kontekście teorii zawilżonych materiałów porowatych

Streszczenie

W pracy przedstawiono termomechaniczny model suszenia zawilżonych materiałów kapilarno porowatych. Zaprezentowano równania podstawowe, wyprowadzone na bazie równań bilansu i termodynamiki procesów nieodwracalnych. Rozwiązanie tych równań pozwala na analizę ewolucji rozkładu zawartości wilgoci, opis deformacji suszonego materiału i analizę naprężeń suszarniczych. Odkształcenia i naprężenia są analizowane na przykładzie pryzmatycznego pręta suszonego konwekcyjnie (dwuwymiarowy problem początkowo-brzegowy). Dyskutowany jest wpływ różnych warunków suszenia i kształtu przekroju suszonego pręta na odkształcenia i naprężenia przy zastosowaniu liniowego i nieliniowego modelu.

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