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A KIND OF UNIFICATION IN CONTINUUM MECHANICS[1]

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The aim of this paper is to propose a kind of unification within the framework of continuum mechanics theories, by means of which the inelastic behaviour of solids is modelled. In this approach deformation (kinematics) of the continuum and its substructure is described in terms of a structure of the Finsler bundle. This new theoretical background, being physically justified, is used for formulation of an alternative continuum description of a solid behaviour. The additive decomposition of the total deformation gradient is defined with no additional assumptions like: intermediate stress-free configuration, yield rule, hardening and/or softening laws. This leads to new strain measures which are both anisotropic and internal-variable-dependent. The rate-independent example of a solid deformation is included to show that this approach requires no extra theories for description of the residual state, softening and hardening phenomena.

Key words: Finsler geometry, continuum with microstructure, inelastic deformation

1. Introduction

The classical approach to modelling of mechanical behaviour of solids introduces *ad hoc* conditions such as intermediate configuration, yield rule, hardening and/or softening laws, etc., which are not satisfactory for physical reasons, however, obviously useful in certain cases. Evolution equations are generally used to describe a feature of a solid which is unaware that something is postulated about it. The idea of distinguishing between loading and unloading processes is also rather misleading since the response of any solid is in agreement with its internal structure, but not with our ability to analyse such physical processes.

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The assumption that the plastic strain does not influence the elastic response (cf Dafalias, 1987; Pęcherski, 1992) accepted commonly is in opposition to experimental results (Korbel, 1992; Wack and Tourabi, 1992; Young, 1962). It turns out that an inelastic deformation of solids, even within the range of engineering stresses, is irreversible from the beginning, and such materials never behave exactly in an elastic manner. The hysteresis phenomenon, recognized as the dominating phenomenon in solid behaviour, also never exists alone and it is generally accompanied by other material phenomena like: hardening, softening, viscosity, relaxation effects, localization of shear bands, etc.

Despite a very detailed mathematical formulation and treatment of constitutive equations, many problems make their use difficult if one considers the physical behaviour of the material like the onset of strain localization, the stability of plastic flow at the micro-structural scale (Asaro, 1979; Asaro and Rice, 1977; Campbell, 1967; Kocks et al., 1979). These obstacles result from the assumed differences between kinematics of the continuum and kinematics of the idealized underlying microstructures. Departures from the idealization of underlying structure of the material are discussed by Kocks and Mecking (1981).

These inconsistencies of classical theories of inelasticity can be eliminated, or at least reduced, within the framework of Finslerian methodology motivated by Saczuk's (1996a,b) approach rather than the geometrization of a continuum proposed by Bilby et al. (1955), Kondo (1952, 1955, 1963). It stems also from the following observation. The hysteresis effect, a leading phenomenon of the inelastic behaviour, is consequence of the anisotropic character of micro-structural mechanisms of the deformation process. This fundamental evidence implies that the geometric nature of inelastic phenomenon has a non-Riemannian character. On the other hand, anisotropy and hysteresis loops are modelled easily within the geometry with an anisotropic metric, i.e. with the aid of metric which depends both on the position and direction. The anisotropic character of such a geometry is represented properly by the concept of indicatrix used to model a characteristic feature of the solid. A type of such geometry is known in the literature as the Finsler geometry (Rund, 1959; Matsumoto, 1986).

Notation which will be used in this paper is slightly different form the one used in continuum mechanics (Truesdell and Noll, 1965). The coordinates in the reference configuration we will denote by small letters. In the actual configuration we will denote them by capital letters. Here we opt for the notation, which is to a certain degree opposite to the convention used in classical continuum mechanics, but which is widely used in the differential geometry (Rund, 1959; Matsumoto, 1986).

2. Alternative decomposition of kinematics

Nearly all formulations of plasticity that have been developed series of assumptions are accepted at the starting point. Two of them are of fundamental importance for classical approaches (cf Dafalias, 1987; Pęcherski, 1992):

1. The distinction between the kinematics of the continuum and its underlying substructure,
2. The multiplicative decomposition of kinematics.

The continual description of solid behaviour based on these assumptions (Fig.1) involves some inconsistencies in comparison with the experimental results (cf Korbel, 1992; Wack and Tourabi, 1992) sketched in Section 1.

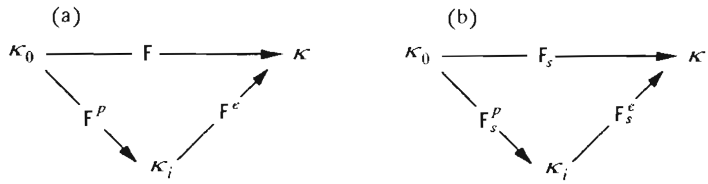

Fig. 1. Kinematics of a continuum (a) and a substructure (b)

In Fig.1 $\mathsf{F}(\mathsf{F}_s)$ are the position-dependent deformation gradients composed multiplicatively from elastic $\mathsf{F}^e(\mathsf{F}^e_s)$ and plastic $\mathsf{F}^p(\mathsf{F}^p_s)$ parts of the continuum (a substructure), respectively. κ_0, κ_i and κ denote the reference, intermediate and actual configurations of the body, respectively.

The inelastic description of solid proposed here utilizes kinematics defined by means of a structure of the Finsler bundle, without using the notion of intermediate configuration (Fig.2). Our main assumption is then reduced to the following:

The kinematics of the continuum and its substructure are described by means of a structure of Finsler bundle.

Logical implication of this assumption consists in the additive decomposition of kinematics (Section 4). The decomposition in Fig.2, in contrast to that shown in Fig.1, is based on the exact geometric background and does not demand any additional assumptions.

In Fig.2 $\widehat{\mathsf{F}}$ is the position-direction-dependent deformation gradient composed additively by horizontal F^h and vertical F^v parts. In the limit transition

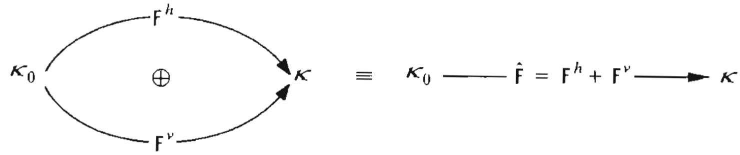

Fig. 2. Kinematics of a generalized continuum

of the internal state F^h reduces to the deformation gradient known from the classical continuum mechanics, while F^v vanishes. κ_0 and κ denote the reference and actual configurations of the body, respectively. The symbol $\oplus$ stands for the direct sum.

3. Comments on experimental results

In this section we only sketch some background for this topic. We present a general scheme of our methodology without its detailed explanation. Korbel's theory (cf Korbel, 1992) of a hierarchy of solid responses on the applied force system showed that there was an obvious intercorrelation between the micro-events of plastic deformation and global mechanical properties (Korbel and Berveiller, 1991). The research discussed by Korbel (1992) revealed different, even of a catastrophic nature, mechanisms of the metal deformation like shear bands, Lüders and Portevin-Le Chatelier bands originate in the same basic mechanism. This mechanism depends on the spatial-temporal organization of dislocation ensembles, avalanche-like movement of dislocation substructures (Basinski and Jackson, 1965) and formation of the micro-shear bands of non-crystallographic orientations (Korbel and Pęcherski, 1997). On the other hand, direct observation of the deformation substructure in solids proved that the dislocation distributions were very complex. Thus, it cannot be surprising that a proper continuous theory describing a deformation process should be in agreement with these facts. This progress is to some degree hindered by specific drawbacks of the experimental techniques (Basinski and Basinski, 1979).

In recent years considerable progress has been made towards modelling of the polycrystalline material response using approach (cf Asaro, 1979). However, there is still a gap in our complete understanding of the relationship between the response of single crystals and their relation to the polycrystalline behaviour (cf Zaoui, 1986). Even though the literature contains accounts of many types of measurements of mechanical properties of solids, direct comparisons are difficult to make and often inconclusive due to quite different experimental conditions imposed (Nabarro et al., 1964). The necessity for changing the present orientation to the description of inelastic behaviour of solids within the continuum mechanics seems to be evident (cf Section 5).

Our analysis which follows is free from the "artificial" assumptions and is consistent with the nature of inelastic behaviour of solids observed in experiments. The present methodology can be illustrated by the scheme presented in Fig.3.

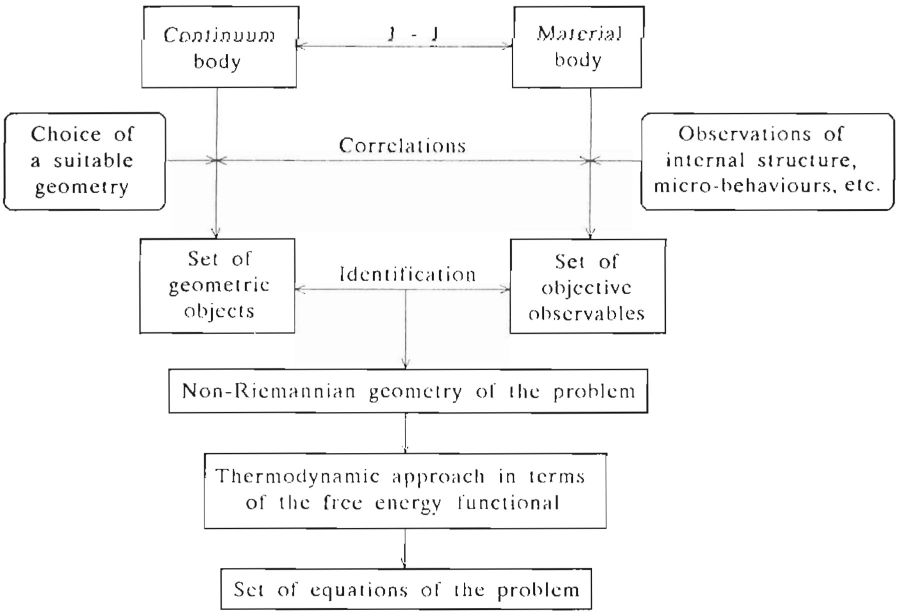

Fig. 3. Scheme of the proposed methodology

The level of correlations and suitable identifications between geometric and physical counterparts is of fundamental importance in the above scheme. As an illustration one can take into account the morphological features of shear bands in metals at different levels of observation. At the micro-level (say, one visible in the electron microscope) they are formed by complicated heterogeneous substructures, but at the macro-level (light microscope) they can be observed as the single slip lines (Korbel, 1992). If this level of description is made correct then the kinematics allows one to predict the correct deformation path of the solid under a given load system. In practice this step may be realized properly on the mathematical level.

4. Kinematics

We propose a continuous model of inelastic behaviour of solids from the Finslerian point of view (Saczuk, 1996a,b). We consider an interacting generalized (microstructural) continuum (a body $\mathcal{B}$), described by the Finsler bundle structure in static equilibrium within the following assumptions:

- Material body (a continuum) $\mathcal{B}$ is assumed to be a 3-dimensional Finsler bundle F^3 whose points will be called line-elements (Rund, 1959)
- Motion of the body $\mathcal{B}$ is defined by mapping

$$\chi \ : \ \mathcal{B} \times R \to E^3 \times E^3 \times R \qquad (\boldsymbol{x}, \boldsymbol{y}, t) \mapsto \widehat{\boldsymbol{X}} = \chi(\boldsymbol{x}, \boldsymbol{y}, t)$$

  where E denotes the Euclidean space and R is a real number space
- A time-space of events is the product: $E^3 \times E^3 \times R$
- The body $\mathcal{B}$ is subject to external and internal force fields
- Laws of evolution of the body $\mathcal{B}$ result from a variational principle for the first order functional describing its motion.

We will consider kinematic aspects of this formulation, while the details of the static (or dynamic) ones the reader can find in the paper by Saczuk (1996a,b).

In the classical approach to continuum mechanics (Truesdell and Noll, 1965), by a body $\mathcal{B}_c$ we understand a pair $(\mathcal{B}_c, \chi)$, where $\mathcal{B}_c$ is both an oriented connected 3-dimensional manifold and a measure space whose any element x is called a (material) particle, and χ is a diffeomorphism of $\mathcal{B}_c$ into the 3-dimensional euclidean space E^3. A family of such diffeomorphisms is called a family of configurations of the body. The fact that $\mathcal{B}_c$ is a measure space means that it is endowed with a non-negative scalar measure called the mass distribution of the body.

If $\kappa : \mathcal{B}_c \to E^3$ is a (reference) configuration of $\mathcal{B}_c$ then κ is characterized by three smooth functions x^i (the coordinate functions of κ) such that $\kappa(P) = (x^1(P), \dots, x^3(P))$, $P \in \mathcal{B}_c$. If ϕ is any other (current) configuration of $\mathcal{B}_c$ then the deformation from κ to ϕ, i.e. $\phi \circ \kappa^{-1} : \kappa(\mathcal{B}_c) \to \phi(\mathcal{B}_c)$; $x^i \mapsto \phi \circ \kappa^{-1}(x^i)$, $\forall x^i \in \kappa(\mathcal{B}_c)$ is assumed to be the diffeomorphism. In the local coordinate system we have

$$X^i = \chi^i(\boldsymbol{x}) \qquad \text{or} \qquad \boldsymbol{X} = \chi(\boldsymbol{x}) \tag{4.1}$$

where $\chi \equiv \phi \circ \kappa^{-1} : E^3 \to E^3$. The classical deformation gradient $\mathsf{F} \in \mathcal{L}(E^3)$ is then defined to be the tangent map of $\chi : \mathsf{F} = T\chi$. By $\mathcal{L}(E^3)$ we denote the set of linear mappings of E^3 into itself.

The geometric relation (4.1) within the Finsler formalism can be defined analogously

$$\widehat{\boldsymbol{X}} = \widehat{\chi}(\boldsymbol{x}, \boldsymbol{y}) \tag{4.2}$$

where the diffeomorphism $\widehat{\chi} : E^6 \supset F^3 \to F^3 \subset E^6$ is a deformation of the body $\mathcal{B}$. The line-element $(\boldsymbol{x}, \boldsymbol{y})$ = (a position vector, a direction vector) can be identified with an oriented particle of the body $\mathcal{B}$. For our purpose it is enough to consider the direction vector $\boldsymbol{y}$ as the micro-displacement (cf Woźniak, 1968), or deviation of the mean displacement (cf Kondo, 1955), at the micro-level.

To introduce the concept of a deformation gradient in the generalized continuum we start from the Finsler space with the Cartan connection (cf Rund, 1959). First we define the direct sum of covariant derivatives ∇^h and ∇^v as the following composition

$$\nabla^h + \nabla^v = [1,1] \begin{bmatrix} \nabla^h & \mathbf{0} \\ \mathbf{0} & \nabla^v \end{bmatrix} \begin{bmatrix} \mathbf{1} \\ \mathbf{1} \end{bmatrix} \tag{4.3}$$

where $\mathbf{1}$ is the identity tensor on $\mathcal{B}$. The operators ∇^h and ∇^v are defined as follows

$$\nabla^h(\cdot) = \nabla^h_{\delta_i}(\cdot) \otimes dx^i \tag{4.4}$$

where (cf Matsumoto, 1986)

$$\nabla^h_{\delta_i}\delta_j = \Gamma^{\star k}_{ij}\delta_k \qquad \delta_i = \partial_i - N^k_i\dot{\partial}_k \qquad \partial_i = \frac{\partial}{\partial x^i} \qquad \dot{\partial}_i = \frac{\partial}{\partial y^i}$$

and

$$\nabla^v(\cdot) = \nabla^v_{\dot{\partial}_i}(\cdot) \otimes Dl^i \tag{4.5}$$

where

$$\nabla^v_{\dot{\partial}_i}\dot{\partial}_j = C^k_{ij}\dot{\partial}_k \qquad Dl^i = dl^i + N^i_k dx^k \qquad l^i = \frac{y^i}{L}$$

Here L is the fundamental function, N^i_k, C^k_{ij} and $\Gamma^{\star k}_{ij}$ are connection coefficients of the Finsler space. One should remember here that the Finsler space has a structure of the principal bundle over the tangent bundle of the base manifold (see Matsumoto, 1986).

From the geometric point of view the fundamental function L through the equation $L = 1$, which defines the equation of indicatrix, can be identified with the yield surface (cf Saczuk, 1996b). On the other hand, this function as a distance function (Rund, 1959) is used to define the length (metric) $ds = L(\boldsymbol{x}, d\boldsymbol{x})$ of a line-element $(\boldsymbol{x}, d\boldsymbol{x})$ with the origin $\boldsymbol{x}$ and, next, components of the metric tensor g_{ij} from

$$ds = L(\boldsymbol{x}, d\boldsymbol{x}) = \sqrt{g_{ij}(\boldsymbol{x}, d\boldsymbol{x})\, dx^i dx^j}$$

see (4.12). In turn, the metric tensor g_{ij} is applied to definition of connection coefficients (see Eqs (4.13)÷(4.16)). From the fact that $\det[\dot{\partial}_i \dot{\partial}_j L] = 0$ (the function $L(\boldsymbol{x}, \boldsymbol{y})$ is, by definition, homogeneous of degree one with respect to $\boldsymbol{y}$) the physical sense of L is attributed to $L^2 = W$ and it is identified by us with the strain energy density of dislocations and induced by the deformation process (see Section 5).

The map $\widehat{X} \mapsto (\nabla^h + \nabla^v)(\widehat{X})$ written as

$$\widehat{\mathsf{F}} = \mathsf{F}^h + \mathsf{F}^v \tag{4.6}$$

defines $\widehat{\mathsf{F}} \in \mathcal{L}(E^3) \oplus \mathcal{L}(E^3)$ as the deformation gradient of $\mathcal{B}$. Its vertical $\mathsf{F}^v \in \mathcal{L}(E^3)$ and horizontal $\mathsf{F}^h \in \mathcal{L}(E^3)$ parts are, respectively, equal to (Saczuk, 1996a)

$$\begin{aligned} \mathsf{F}^v &= \nabla^v \widehat{X} = {}_vX^i_k \bar{\partial}_i \otimes Dl^k \\ \mathsf{F}^h &= \nabla^h \widehat{X} = {}_hX^i_k \bar{\partial}_i \otimes dx^k \end{aligned} \tag{4.7}$$

where $\bar{\partial}_i$ is the unit vector in the current configuration ϕ and $\otimes$ denotes the tensor product.

The additive decomposition (4.6), opposite to the multiplicative one used in classical plasticity (cf Lee, 1969), is the purely kinematic concept and has no counterparts in the literature on continuum mechanics.

In the case of convected coordinate system (cf Pietraszkiewicz and Badur, 1983) the definitions (4.7) can be written as

$$\mathsf{F}^v = \dot{\bar{\partial}}_i \otimes Dy^i \qquad\qquad \mathsf{F}^h = \bar{\delta}_i \otimes dx^i \tag{4.8}$$

where $\bar{\delta}_i = \bar{\partial}_i - N^k_i \dot{\bar{\partial}}_k$ and $\dot{\bar{\partial}}_i$ constitute base vectors of $\mathcal{B}$ in the actual configuration ϕ. Relations inverse to Eq (4.8) have the following forms

$$(\mathsf{F}^v)^{-1} = \dot{\partial}_i \otimes D\bar{y}^i \qquad\qquad (\mathsf{F}^h)^{-1} = \delta_i \otimes d\bar{x}^i \tag{4.9}$$

After introducing an additional intermediate basis one can obtain from Eqs (4.8) and (4.9) a series of valuable relations. This standard problem is here omitted; the reader should consult the paper of Pietraszkiewicz and Badur (1983).

In classical continuum mechanics $(\mathsf{F}^v \equiv \mathbf{0})$ we restrict ourselves with F^h to the well-known form of deformation gradient

$$\mathsf{F} = \bar{\partial}_i \otimes dx^i$$

We shall denote further the horizontal and vertical components of any tensor T by ${}_hT^{i\cdots}_{j\ldots}$ and ${}_vT^{i\cdots}_{j\ldots}$, respectively. The h-derivative and v-derivative of the position-direction-dependent vector $\widehat{\boldsymbol{X}} = \widehat{\boldsymbol{X}}(\boldsymbol{x},\boldsymbol{y})$ are defined as follows (Matsumoto, 1986; Rund, 1959)

$$(\mathsf{F}^h)^i_k \equiv {}_hX^i_k = \partial_k \widehat{X}^i - \dot{\partial}_l \widehat{X}^i \dot{\partial}_k G^l + \Gamma^{\star i}_{lk} \widehat{X}^l \tag{4.10}$$

$$(\mathsf{F}^v)^i_k \equiv {}_vX^i_k = L\dot{\partial}_k \widehat{X}^i + A^i_{lk} \widehat{X}^l \tag{4.11}$$

where the remaining unknowns in Eqs (4.10), (4.11) are defined by means of the components of the metric tensor

$$g_{ij}(\boldsymbol{x},\boldsymbol{y}) = \frac{1}{2}\frac{\partial^2 L^2(\boldsymbol{x},\boldsymbol{y})}{\partial y^i \partial y^j} \tag{4.12}$$

as follows

$$\Gamma^\star_{ijk} = \Gamma_{ijk} - C_{jkl}\frac{\partial G^l}{\partial y^i} = \gamma_{ijk} - C_{kjl}\frac{\partial G^l}{\partial y^i} - C_{ijl}\frac{\partial G^l}{\partial y^k} + C_{ikl}\frac{\partial G^l}{\partial y^j} \tag{4.13}$$

$$\Gamma^\star_{ijk} = g_{jl}\Gamma^{\star l}_{ik} \qquad \Gamma_{ijk} = g_{jl}\Gamma^l_{ik} \qquad 2G^l = \gamma^l_{jk} y^j y^k \tag{4.14}$$

$$\begin{aligned} N^l_k &= \dot{\partial}_k G^l = \frac{\partial G^l}{\partial y^k} = \Gamma^l_{jk} y^j = \Gamma^{\star l}_{jk} y^j \\ \gamma_{ijk} &= \frac{1}{2}\Big(\frac{\partial g_{ij}}{\partial x^k} + \frac{\partial g_{jk}}{\partial x^i} - \frac{\partial g_{ki}}{\partial x^j}\Big) \end{aligned} \tag{4.15}$$

$$\begin{aligned} C_{ijk} &= \frac{1}{2}\frac{\partial g_{ij}}{\partial y^k} \qquad & C_{ijk} y^k = C_{ijk} y^j = C_{ijk} y^i = 0 \\ C_{ijk} &= g_{jl} C^l_{ik} \qquad & A^i_{jk} = L C^i_{jk} \end{aligned} \tag{4.16}$$

Assume that the deformation $\widehat{\chi}$ is an orientation-preserving diffeomorphism, i.e.

$$\widehat{J} = \det \widehat{\mathsf{F}} > 0 \tag{4.17}$$

or

$$\widehat{J} = \det \begin{bmatrix} \mathbf{F}^h & \mathbf{0} \\ \mathbf{0} & \mathbf{F}^v \end{bmatrix} > 0$$

i.e. $\widehat{J} = J^h J^v > 0$ with $J^h = \det \mathbf{F}^h$ and $J^v = \det \mathbf{F}^v$. For the mappings which have continuous derivatives, this is the necessary and sufficient condition for invertibility. Since $\widehat{\mathbf{F}}$ is invertible, one can use the polar decomposition (see Chevalley, 1946) to decompose $\widehat{\mathbf{F}}$ into

$$\widehat{\mathbf{F}} = \widehat{\mathbf{R}}\widehat{\mathbf{U}} \tag{4.18}$$

or

$$\widehat{\mathbf{F}} = \begin{bmatrix} \mathbf{R}^h & \mathbf{0} \\ \mathbf{0} & \mathbf{R}^v \end{bmatrix} \begin{bmatrix} \mathbf{U}^h & \mathbf{0} \\ 0 & \mathbf{U}^v \end{bmatrix}$$

i.e.

$$\mathbf{F}^h = \mathbf{R}^h\mathbf{U}^h \qquad \mathbf{F}^v = \mathbf{R}^v\mathbf{U}^v \tag{4.19}$$

where $\widehat{\mathbf{U}}$ or $\mathbf{U}^h$ and $\mathbf{U}^v$ are the positive definite tensors and since $\widehat{J} > 0$, $\widehat{\mathbf{R}}$ or $\mathbf{R}^h$ and $\mathbf{R}^v$ are the proper orthogonal tensors.

For finite deformation various classical strain measures coaxial with the Lagrangian triad have been introduced in the literature (Truesdell and Noll, 1965; Ogden, 1984). They are called by Hill (1968) the material strain measures. This class of strain measures requires that the strain should vanish at no deformation, should be positive and increasing when the corresponding fibres are extended, and should reduce to the usual linear measure when linearized. These conditions are satisfied when the function f defining the strain measure $\widetilde{E} = f(\widetilde{\lambda})$ ($\widetilde{\lambda}$ is the principal value of $\widetilde{E}$) has the properties

$$f(1) = 0 \qquad f'(1) = 1 \qquad f'(\widetilde{\lambda}) > 0 \tag{4.20}$$

The subclass of strain measures which complies with these requirements is (cf Hill, 1968; Ogden, 1984)

$$\widetilde{E}_{(\alpha)} = \frac{1}{2m}\left(\widetilde{\lambda}_{(\alpha)}^{2m} - 1\right) \tag{4.21}$$

Various commonly used strain measures result for special values of m. For instance, for $m = 1$ we obtain the so-called Lagrange strain tensor. In our case of the generalized deformation gradient $\widehat{\mathbf{F}}$, Eq (4.6), such an alternative Lagrangian strain tensor can be put into the form

$$\widehat{\mathbf{E}} = \sum_{\alpha=1}^{6}\left(\widetilde{\lambda}_{(\alpha)}^{2} - 1\right)\boldsymbol{n}^{(\alpha)} \otimes \boldsymbol{n}^{(\alpha)} \tag{4.22}$$

where $\boldsymbol{n}^{(\alpha)}$ is the principal direction of the generalized stretch tensor $\widehat{\mathbf{U}}$, or

$$\widehat{\mathbf{E}} = \frac{1}{2}(\widehat{\mathbf{C}} - \mathbf{1}) \tag{4.23}$$

where $\widehat{\mathbf{C}} = \widehat{\mathbf{F}}^{\mathsf{T}}\widehat{\mathbf{F}}$ is the right Cauchy-Green deformation tensor. In the representation of the direct sum, Eq (4.23) is equivalent to

$$\widehat{\mathbf{E}} = \begin{bmatrix} \mathbf{E}^h & \mathbf{0} \\ \mathbf{0} & \mathbf{E}^v \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \mathbf{C}^h - \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}^v - \mathbf{1} \end{bmatrix}$$

The other cases of $\widetilde{E}_{(\alpha)}$ and the time rates of $\widetilde{E}_{(\alpha)}$ can be defined after Mehrabadi and Nemat-Nasser (1987), Truesdell and Toupin (1960).

Using Eqs (4.10) and (4.11) the horizontal and vertical parts of the Cauchy-Green strain tensor are then equal to

$$\begin{aligned}\mathbf{C}^h = \Big(&\partial_h X^i \partial_l X^j + \dot{\partial}_m X^j \dot{\partial}_l G^m \dot{\partial}_k X^i \dot{\partial}_h G^k + \Gamma^{\star j}_{nl} \Gamma^{\star i}_{kh} X^n X^k + \\ &-\partial_{(l} X^{(j} \dot{\partial}_{|k|} X^{i)} \dot{\partial}_{h)} G^k - \dot{\partial}_m X^{(j} \dot{\partial}_{(l} G^{|m|} \Gamma^{\star i)}_{|k|h)} X^k + \partial_{(l} X^{(j} \Gamma^{\star i)}_{|k|h)} X^k\Big) \bar{g}_{ij}\, dx^l \otimes dx^h\end{aligned} \tag{4.24}$$

$$\mathbf{C}^v = L^2 \Big(\dot{\partial}_h X^i \dot{\partial}_l X^j + \dot{\partial}_{(l} X^{(j} C^{i)}_{|k|h)} X^k + C^i_{kh} C^j_{ml} X^k X^m\Big) \bar{g}_{ij}\, Dl^l \otimes Dl^h \tag{4.25}$$

respectively, where $(\cdot)$ means the symmetric part with respect to the enclosed indices, the sign $|\cdot|$ around the index is used to exclude it from the symmetrization operation and $\bar{g}_{ij}$ are components of the metric tensor in the actual configuration ϕ. The interrelated pair of measures Eqs (4.24) and (4.25), of any deformation process is defined in the invariant way.

In the case of classical continuum, i.e. when $\boldsymbol{y} \equiv \mathbf{0}$

$$\mathbf{C}^h = \Big(\partial_h X^i \partial_l X^j + \Gamma^{\star j}_{nl} \Gamma^{\star i}_{kh} X^n X^k\Big) \bar{g}_{ij}\, dx^l \otimes dx^h \tag{4.26}$$

$$\mathbf{C}^v \equiv \mathbf{0} \tag{4.27}$$

where $\boldsymbol{X}$ and $\mathbf{g}$ are functions of $\boldsymbol{x}$. The case $\boldsymbol{y} = \boldsymbol{y}_r$ with $\boldsymbol{y}_r$ being a residual or imperfection vector leads to non-singular $\mathbf{C}^h$ and $\mathbf{C}^v$. To specify the connection coefficients C^i_{jk}, N^i_j and $\Gamma^{\star i}_{jk}$ we first have to estimate the local internal (dislocation) structure of the solid under consideration defining its fundamental function L or its metric tensor $\mathbf{g}$.

5. The rate-independent case

The aim of this Section is to present the unified concept (Saczuk, 1996a) of modelling the residual states, hardening and softening phenomena in solids. We show how this approach makes it possible to describe the above properties and phenomena in the solid deformed under various conditions and how it can be used for controlling the solid behaviour through a change of the deformation path.

5.1. Residual state

The deformation process of the solid below the yield stress is treated within the framework of continuum mechanics as the fully elastic regime, and is not subjected to a detailed microstructural analysis. On the other hand, the experimental observations show clearly that for stresses below the yield stress dislocations move back, when the applied stress is released (Marukawa, 1967). Brown and Ekvall (1962) have shown that plastic flow can take place in single iron crystals at stresses well below the upper yield stress. The stress relaxation technique used by Shaw and Sargent (1964) to study the phenomenon of plastic strain in the pre-yield (the microstrain) region have shown that the movement of free dislocations at any given stress level, below the upper yield stress, is a function of the loading rate. Extensive studies of changes in a substructure of crystals during unloading were conducted by Young (1962), Crump and Young (1968). They have shown that considerable rearrangement of dislocations occurs during unloading and significant differences exist between the dislocation configurations, which appear during loading and unloading processes, respectively.

5.2. Hardening process

The problem of hardening in solids is sufficiently well documented in experimental studies presented, among others, by Schmid and Boas (1935), Nabarro et al. (1964), Basinski and Basinski (1979).

The real nature of the hardening process is still not completely understood. The reason for this is a very large number of parameters like: temperature and strain rate, slip system, applied stress, crystal orientation, purity, initial imperfection, crystal size and shape which influence the deformation of any crystalline material (Basinski and Basinski, 1979). It seems that there are at least several hardening mechanisms. In general, an increase in hardeness is connected with an increased difficulty of dislocation movement. The work hardening effect is, following Taylor (1934), a consequence of the strain system induced by dislocations distributed throughout the lattice and generated in the process of plastic flow. The mechanisms governing the strain-hardening and strain-softening in face-centred cubic metals are discussed by Seeger et al. (1957). The role played by the forest dislocations (dislocations intersecting the active glide plane) in the mechanisms of strain hardening is analysed by Basinski and Basinski (1964).

The theories of work hardening, which are based on the concepts of continuum mechanics, include isotropic and/or kinematic hardening. To predict more complex hardening behaviour one can use combined hardening rules (Wilimas and Svensson, 1971; Baltov and Sawczuk, 1965) to describe translation and distortion of the yield surface. The two-surface (Philips and Sierakowski, 1965) or multisurface (Mróz, 1967) theories of hardening have led to many propositions of modelling the cases of multiaxial loading. Some of these theories were verified by experimental results (Lu and Mohamed, 1987).

5.3. Rationale

Our rate-independent constitutive modelling has intimate connection with the classical one. Therefore, for the moment, we shall deal with the classical methodology. Various rate-independent models of plastic behaviour of one-phase (poly)crystals, proposed in the literature by e.g. Bishop and Hill (1951), Hill (1965), Zaoui (1986), are based mainly upon average relations and/or localization procedures. Typical models, connected with the Taylor (1938) model, are directed into the accurate description of interaction between a crystal and neighbouring grains. The extension of the Taylor model proposed by Lin (1957) assumes uniformity of the total strain, while the plastic strain differs from one grain to another. Modifications of this approach were given by Eshelby (1957) and Kröner (1961) for polycrystals. The more general form of interaction of the whole crystal with neighbouring grains is proposed by Hill (1965), where the difference between stresses produced by the interaction of the matrix $\mathbf{T}_m$ and the grain $\mathbf{T}_g$ is expressed by the appropriate difference of strains $\mathbf{E}_m - \mathbf{E}_g$ as follows

$$\mathbf{T}_m - \mathbf{T}_g = \mathbf{L}^\star \cdot (\mathbf{E}_m - \mathbf{E}_g) \tag{5.1}$$

where the "overall" constraint tensor $\mathbf{L}^*$ is responsible for interactions between grains (cf Zaoui, 1986; Korbel and Pęcherski, 1997). The above relation does not explain how to calculate $\mathbf{E}_g$, $\mathbf{E}_m$ and $\mathbf{L}^*$ and, moreover, their interrelation with microstructure of any crystal is not clear.

According to our model an inelastic flow of a solid takes place from the very beginning of deformation process in accord with evolution of its internal substructures. Therefore, both the yield condition and evolution law are in general superfluous. The response of the solid depends on the history of its deformation and the applied external fields. The well-known material effects of a solid deformation like hardening (Fig.6) and softening (Fig.5) do not need here extra theories. The calculations made here provide qualitative rather than quantitative insight into these problems.

5.4. The model proposed

A phenomenological study of plastic flow in crystalline solids is based on information about the current distribution of dislocations and internal stresses. A description of plastic flow under the applied stress demands suitable information on the influence of stresses opposing dislocation motion, known as the internal back stresses. For a given applied stress only certain dislocations in the regions with low back stresses are free to move and contribute to the plastic flow (Gasca-Neri and Nix, 1974). Hence, the concept of the effective stress, defined as a difference between the applied stress and the back stress, is of fundamental importance for the modelling of the deformation process. This stress acting on dislocations does some work during their motion. This situation can be clarified as follows. Suppose that a dislocation line glides on a slip plane containing point defects. Its movement may be completely stopped by obstacles if the stress is above a certain level. In this situation the dislocation segment between the obstacles is displaced to the positive curvature position and the back stress is generated by its bowing, cutting or climbing. In order to produce substantial macroscopic strain, a large number of dislocations must be in motion. That takes place when a sufficient stress applied to a solid causes spontaneous nucleation of dislocations. The observed multiplication of dislocations is caused by the Frank-Read sources and/or the multiple cross-glide mechanism. To explain the above facts we introduce the physical model as follows.

We assume that each particle of our body is in a structure-dependent potential well formed by the stresses around neighbouring particles. We assume that molecular motions are frozen at the center of each particle. Under external agencies a particle moves and meets local obstacles. In consequence of interactions between neighbouring particles and defects produced as a result of deformation, the motion of the particle is in general hindered. To consider such a motion we introduce the f-system containing a particle and an obstacle, which interacts with its surrounding. Along any path connecting *configurations of the particle the internal and effective stresses act* on our f-system. The first stress component compensates the influence of neighbouring particles and other sources of the internal stress outside the f-system. The second one acts on the particle inside the f-system and does work during its motion. Deformation of the body is possible only if the applied stress is raised to the level that is proportional to the strain associated both with internal and effective stresses above a certain level.

In the sequel, the internal part of deformation will be modelled by the vertical strain tensor, while, the effective one by the horizontal strain tensor. This identification is obvious for the following reasons. First, the vertical part of deformation is responsible for the internal state space in which the act of slip takes place. Second, the horizontal one describes an average result of internal interactions and external conditions.

5.5. Algorithm

An algorithm, which includes the relation (5.1), can be realized within the Finslerian methodology in the following steps.

1) <u>Calculate the structure-dependent energy distribution in the solid</u>

In general, we have no sufficient knowledge of the energy distribution in the solid, and no idea of how it changes as the solid deforms. The stored energy associated with dislocations is very often calculated from the (anisotropic) elasticity theory (Nabarro, 1967; Hirth and Lothe, 1968). The general conclusion is that this energy is proportional to the square of the Burgers vector of dislocation. Therefore, we can only proceed by introducing simplifying assumptions consistent with known general properties of this distribution.

For the purpose of presentation, we assume the position-direction-dependent functional W being, by definition, an anharmonic approximation of interactions between dislocations, in the form

$$W(\boldsymbol{x},\boldsymbol{y}) \equiv L^2(\boldsymbol{x},\boldsymbol{y}) = g_{ij}(\boldsymbol{x},\boldsymbol{y})y^iy^j = \\ = \alpha(\boldsymbol{x},\boldsymbol{y})(y^1)^2 + \beta(\boldsymbol{x},\boldsymbol{y})(y^2)^2 + \gamma(\boldsymbol{x},\boldsymbol{y})(y^3)^2 \tag{5.2}$$

with the coefficients α, β and γ homogeneous of degree zero with respect to $\boldsymbol{y}$. For simplicity we assume

$$\alpha(\boldsymbol{x},\boldsymbol{y}) = \frac{2x^1y^1}{l_y^2} \qquad \beta(\boldsymbol{x},\boldsymbol{y}) = 2x^2 \qquad \gamma(\boldsymbol{x},\boldsymbol{y}) = 2x^3$$

where $l_y = \sqrt{(y^1)^2 + (y^2)^2}$. According to (4.12) we have

$$g_{ij}(\boldsymbol{x},\boldsymbol{y}) := \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} \qquad g^{ij}(\boldsymbol{x},\boldsymbol{y}) := \begin{bmatrix} \alpha^{-1} & 0 & 0 \\ 0 & \beta^{-1} & 0 \\ 0 & 0 & \gamma^{-1} \end{bmatrix} \tag{5.3}$$

The metric tensor (5.3) gives rise to the non-vanishing connection coefficients. Hence, the coefficients of vertical connection C^i_{jk}, Eqs (4.16), are defined by the relations

$$C^1_{11} = \frac{1}{2}\Big(\frac{1}{y^1} - \frac{y^1}{l_y^2}\Big) \qquad C^1_{12} = -\frac{1}{2}\frac{y^2}{l_y^3}$$

the coefficients of the connection $N^i_j = \dot{\partial}_j G^i$, Eqs (4.15), by

$$\dot{\partial}_1 G^1 = \frac{y^1}{2x^1} \qquad \dot{\partial}_2 G^2 = \frac{y^2}{2x^2} \qquad \dot{\partial}_3 G^3 = \frac{y^3}{2x^3}$$

and finally, the non-vanishing coefficients of the horizontal connection $\Gamma^{\star i}_{jk}$, Eq (4.13), by

$$\Gamma^{\star 1}_{11} == -\frac{1}{4x^1}\Big(1 - \frac{(y^1)^2}{l_y^2}\Big) - \frac{1}{8x^2}\Big[\frac{x^1}{x^2}\frac{(y^1)^2(y^2)^3}{l_y^5} - \Big(\frac{1}{y^1} - \frac{y^1}{l_y^2}\Big)\frac{(y^2)^3}{l_y^2}\Big]$$

$$\Gamma^{\star 1}_{12} = \frac{1}{4x^2}\frac{(y^2)^2}{l_y^2} \qquad \Gamma^{\star 2}_{11} = -\frac{x^1}{4(x^2)^2}\frac{y^1(y^2)^2}{l_y^3}$$

$$\Gamma^{\star 2}_{22} = \frac{1}{2x^2} \qquad \Gamma^{\star 3}_{33} = \frac{1}{2x^3}$$

2) Calculate the deformation measures

Having defined geometry of the internal state one can then obtain, for a given position-microposition vector $\boldsymbol{X}(\boldsymbol{x},\boldsymbol{y})$, the h- and v-components of the deformation gradient tensor and the strain tensor according to Eqs (4.10), (4.11) and (4.24), (4.25), respectively.

3) Formulate the strain-stress relation

An analogue of Eq (5.1) within our approach is reduced to the following natural form

$$\hat{\mathsf{T}} = \hat{\mathcal{C}} \cdot \hat{\mathsf{E}} \tag{5.4}$$

where $\hat{\mathsf{T}} \in \mathcal{C}(\mathcal{L}(E^3) \oplus \mathcal{L}(E^3))$ ($\mathcal{C}(\cdot)$ is used here to denote the set of continuous mappings on $(\cdot)$) is the generalized stress tensor and the fourth-order isothermal material tensor $\hat{\mathcal{C}} \in \mathcal{L}(\mathcal{L}(E^3) \oplus \mathcal{L}(E^3))$ is calculated according to (cf Brugger, 1964)

$$\hat{\mathcal{C}} = \rho_0 \frac{\partial^2 \mathcal{F}}{\partial \hat{\mathsf{E}} \partial \hat{\mathsf{E}}} \tag{5.5}$$

from the Helmholtz free energy $\mathcal{F} = \mathcal{F}(\hat{\mathsf{E}}, \theta)$, with ρ_0 being the mass density in κ and θ denoting temperature. In terms of horizontal and vertical terminology Eq (5.4) can be rewritten as follows

$$\mathsf{T}^h + \mathsf{T}^v = {}_h\mathcal{C} \cdot \mathsf{E}^h + {}_v\mathcal{C} \cdot \mathsf{E}^v \tag{5.6}$$

where the material tensors ${}_h\mathcal{C}$ and ${}_v\mathcal{C}$ are defined in the same manner as $\hat{\mathcal{C}}$.

According to definition of ∇^h, Eq (4.4), one can decompose ${}_h\mathcal{C}$ into the following parts

$${}_h\mathcal{C} = {}_x\mathcal{C} - {}_y\mathcal{C} \frac{\partial \mathsf{G}}{\partial \mathsf{E}^h} \tag{5.7}$$

with ${}_x\mathcal{C}$ and ${}_y\mathcal{C}$ being functions of $\mathsf{E}^x = \mathsf{E}^h(\boldsymbol{x})$ and $\mathsf{E}^y = \mathsf{E}^h(\boldsymbol{y})$, respectively. It should be emphasized here that the last step have been made in the direction of simplification of the physical sense of ${}_h\mathcal{C}$ only.

For the purpose of presentation we assume for Eq (5.4) or (5.6) the linear relation between strains and stresses (Hooke's law). Then the elastic stiffness tensor for isotropic material ${}_x\mathcal{C}$ takes the well-known form

$${}_x\mathcal{C} := \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}$$

where λ and μ are the Lame constants. The second part of ${}_h\boldsymbol{C}$ in Eq (5.7) can then be expressed in the following form

$$
{}_y\boldsymbol{C}\frac{\partial \mathsf{G}}{\partial \mathsf{E}^h} := \begin{bmatrix} \mathsf{A}_1 & 0 \\ 0 & \mathsf{A}_2 \end{bmatrix}
$$

where

$$
\mathsf{A}_1 = \begin{bmatrix} (\lambda+2\mu)\frac{y^1}{2x^1} & \frac{\lambda}{4}\left(\frac{y^1}{x^1}+\frac{y^2}{x^2}\right) & \frac{\lambda}{4}\left(\frac{y^1}{x^1}+\frac{y^3}{x^3}\right) \\ \frac{\lambda}{4}\left(\frac{y^1}{x^1}+\frac{y^2}{x^2}\right) & (\lambda+2\mu)\frac{y^2}{2x^2} & \frac{\lambda}{4}\left(\frac{y^2}{x^2}+\frac{y^3}{x^3}\right) \\ \frac{\lambda}{4}\left(\frac{y^1}{x^1}+\frac{y^3}{x^3}\right) & \frac{\lambda}{4}\left(\frac{y^2}{x^2}+\frac{y^3}{x^3}\right) & (\lambda+2\mu)\frac{y^3}{2x^3} \end{bmatrix}
$$

$$
\mathsf{A}_2 = \begin{bmatrix} \frac{\mu}{4}\left(\frac{y^2}{x^1}+\frac{y^3}{x^3}\right) & 0 & 0 \\ 0 & \frac{\mu}{4}\left(\frac{y^1}{x^1}+\frac{y^3}{x^3}\right) & 0 \\ 0 & 0 & \frac{\mu}{4}\left(\frac{y^2}{x^2}+\frac{y^1}{x^1}\right) \end{bmatrix}
$$

4) Final step

For the assumed function of motion (displacement field) which satisfies the equilibrium equations (30) in Saczuk (1996a), after neglecting the body forces, one can calculate strains (point 2) and stresses (point 3).

For simplicity, let us assume a displacement field which is linear both with respect $\boldsymbol{x}$ and $\boldsymbol{y}$. To this end we assume that the position vector $\widehat{\boldsymbol{X}}$ in the actual configuration is expressed by

$$
\begin{aligned}
\widehat{X}^1(\boldsymbol{x},\boldsymbol{y},t) &= a(t)x^1 + y^1 + \gamma(t)y^2 \\
\widehat{X}^2(\boldsymbol{x},\boldsymbol{y},t) &= x^2 + y^2 \\
\widehat{X}^3(\boldsymbol{x},\boldsymbol{y},t) &= x^3 + y^3
\end{aligned}
\tag{5.8}
$$

Here (x^1, x^2, x^3) are the components of the position vector $\boldsymbol{x}$, and (y^1, y^2, y^3) are the components of the micro-displacement vector $\boldsymbol{y}$ in any slip system. The motion (5.8) is superposition of the simple macroextension along the x^1-axis and the simple microshear in the (y^1, y^2)-plane.

5.6. Numerical results

Mechanical behaviour of the solid under the given loading is most compactly described by means of a set of stress-strain curves. For this reason, our numerical calculations will be restricted to such curves.

5.6.1. Residual state

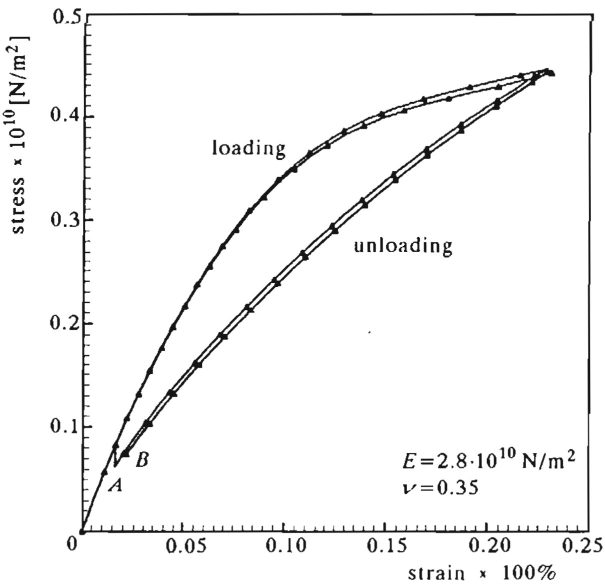

Fig. 4. Cyclic loading-unloading curves for showing the residual state effects

To describe the plastic deformation of material, a conventional approach uses generally the yield surface to distinguish between loading and unloading. In our case the differences between loading and unloading processes are due to the irreversible character of dislocation motion at the microlevel. The situation depicted in Fig.4 was realized by changing the distance travelled by dislocations backwards when the applied stress is removed. The residual state (point A in the first cycle, point B in the second cycle) is the limit point of the unloading process under the condition of recoverable initial macrostate. From Fig.4 the following features can be seen:

(a) Loading and unloading curves are nonlinear and concave downward

(b) Cycles of loading-unloading curves show open hysteresis loops. An amount of openess of the hysteresis loop is a measure of the residual state resulting from inelastic flow under loading-unloading cycles.

Our numerical results are in close agreement with the experimental results of Lukáš and Klesnil (1965).

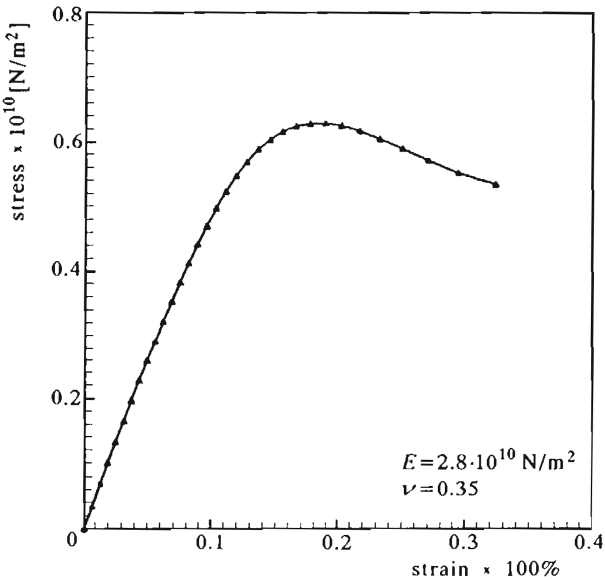

Fig. 5. Strain-stress relation for a tension test

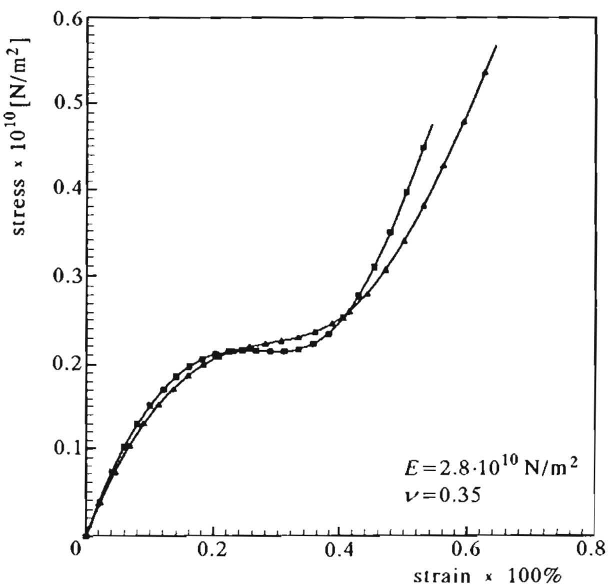

Fig. 6. Strain-stress relations for different simple shear tests

5.6.2. Hardening/softening behaviour

The effect of the change of deformation path on the mode of inelastic flow is presented in Fig.5 and Fig.6. The results were obtained under different load systems while our numerical calculations were recorded by means of changing the direction of slip with respect to the direction of dislocation glide. The program of change of the deformation path was set up. The curve in Fig.5 is the result of tension in the x^1-direction, and the curves in Fig.6 are consequence of the simple shear in the x^1x^2-plane. The strain and stress values are calculated from the intensities of the deviatoric strain tensor and deviatoric stress tensor (Urbanowski, 1965), respectively.

It is seen that the mechanical properties of a solid may be profoundly altered by the change of deformation path. They depend not only on the history of deformation, but also on the stress system used to study these properties (Basinski and Jackson, 1965). They clearly show a practical possibility of non-monotonic behaviour of strain-stress relation induction. Such a possibility results from the instability of a metal substructure after changing the of loading path (Korbel and Martin, 1988). Fig.5 is in close agreement with the experimental data of Korbel and Martin (1988).

As a conclusion one may state that the presented approach is thus capable of describing residual state, hardening and softening phenomena of a solid behaviour at the same foundation.

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O pewnej unifikacji w mechanice kontinuum

Streszczenie

Celem pracy jest zaproponowanie pewnej unifikacji w ramach teorii mechaniki kontinuum, która modeluje niesprężyste zachowanie się ciał stałych. W tym podejściu, deformacja (kinematyka) kontinuum i jego substruktura opisane są za pomocą struktury wiązki Finslera. Nowe teoretyczne podstawy, fizycznie uzasadnione, zostały wykorzystane do sformułowania alternatywnego kontynualnego opisu zachowania się ciała stałego. Addytywny rozkład gradientu deformacji został zdefiniowany bez wprowadzania dodatkowych założeń takich jak pośrednia beznaprężeniowa konfiguracja, reguła płynięcia, prawa wzmocnienia i/lub osłabienia. Prowadzi to do nowych miar deformacji, które są zarówno anizotropowe, jak i zależne od zmiennych wewnętrznych. Przytoczony przypadek deformacyjnej teorii zachowania się ciała stałego został zamieszczony celem pokazania, że w tym podejściu stan residualny oraz zjawiska osłabienia i umocnienia nie wymagają do opisu dodatkowych teorii.

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