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OPTIMIZATION OF MONOSYMMETRIC I-BEAMS SUBJECT TO LATERAL BUCKLING CONSTRAINTS

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In this paper a transversally loaded monosymmetric I-beam is optimized. The beam consists of a number of uniform segments and their optimal dimensions are sought for in order to maximize the lateral buckling load with the total volume of the beam material kept fixed. Various types of support are allowed for and the beams of such supports are subjected to either constant external bending moment or varying bending moment that occurs when both concentrated and uniformly distributed loads are applied. The analysis is carried out by using numerical integration of the set of differential equations describing the critical state of the beam under consideration. Both the influence of monosymmetry on the design and the effect of loading location are discussed.

Key words: monosymmetric I-beam, lateral buckling, optimization

1. Introductory remarks

The majority of papers dealing with rods optimized against instability is devoted to design of columns. Many results of non-prismatic elements having solid or thin-walled cross-sections have been obtained for various support conditions and load cases. On the other hand it is known that thin walled rods are very often used also as beams and in such a case they are exposed to lateral buckling. There has not been much attention in the literature paid to optimization of non-uniform beams with respect to that type of instability. Usually when optimizing elements like that only the form of cross-section is designed and the beam remains prismatic. For prismatic elements instead of designing the cross-section the optimal location of rigid braces can also be determined so as to maximize the elastic buckling load (Ang et al., 1993). As far as non-uniform beams are concerned most of the literature on lateral buckling deals with the analysis only. Lateral buckling of non-prismatic I-beams has been considered by Trahair and Kitipornchai (1971) (stepped beams), Kitipornchai and Trahair (1972) (beam taper free to take an arbitrary shape), Anderson and Trahair (1972) (monosymmetric cross-section). There has been little research on optimization of non-uniform beam resistance to the lateral buckling, limited however to design of narrow rectangular beams with various types of beam taper (e.g. Popelar, 1976, 1977; Wang et al., 1986). For I-beams the approximate optimal solutions obtained on the assumption of negligible influence of warping rigidity are also given by Popelar (1976, 1977).

Only Wang et al. (1990) presented optimal solutions of linearly tapered I-section beams. The Rayleigh-Timoshenko energy approach was used in the instability analysis to derive the Rayleigh quotient and then for optimal design that buckling capacity was maximized subject to a volume constraint. The three design cases were considered so that either the degree of monosymmetry for prismatic beam was found or flange or web optimal linear tapers were determined.

The stepped doubly-symmetric I-beams were optimized by Bochenek (1993) where several solutions for various types of support conditions and load cases were obtained. The analysis has been carried out with the use of numerical integration of the set of first order differential equations describing the critical state of the beam under consideration. The lateral buckling load was maximized with respect to both the constant volume requirement and inequality constraints, imposed on maximum permissible stress and local instability of both flange and web.

The present paper aims to complement the results of that paper by solving the design problem for monosymmetry of I-section allowed for. Similarily as in Bochenek (1993) the stepped beams are considered, and only the steps in the flange width are permitted. For beams with I-section steps in the web depth lead to the physical breaks in the flanges, whereas steps in the web thickness hardly affect the critical load. The assumption of the cross-section monosymmetry opens more possibilities for the design problem formulation. For example, only one of the two flanges can be designed and, what is probably more important, both flanges can be optimized independently. That significantly improves the efficiency of the design.

Various types of support are allowed for but simply supported beams and cantilevers are considered in details. Beams of such supports are subjected to either constant external bending moment or varying bending moment that occurs when both concentrated or uniformly distributed loads are applied. The effect of loading location (top flange loading, load application point coinciding with the shear centre, bottom flange loading) is also discussed.

The analysis is carried out by using numerical integration of the set of first order differential equations describing the critical state of the beam under consideration. The following assumptions are accepted in the analysis: the cross-section retains its original shape (no local buckling), deformation in the plane of applied loads is small.

2. Lateral buckling of I-beams

An I-section beam being bent in the plane of symmetry yz is considered. Since the flexural rigidity corresponding to the bending in that plane is significantly larger then the one for bending in the perpendicular plane xz and additionaly torsional rigidity is small then lateral buckling of the beam may occur. The state equations describing critical state of the beam can be presented in the following form (e.g. Vlasow, 1959; Timoshenko and Gere, 1962; Trahair and Woolcock, 1973)

$$
\begin{aligned}
&(EI_y\kappa_y)' = T_x + M_x\rho - M_z\kappa_x \\
&\Big[(GI_s + M_x\beta_x)\rho - (EI_\omega\rho')'\Big]' = -M_x\kappa_y + M_y\kappa_x + M_z
\end{aligned}
\tag{2.1}
$$

where

M_x, M_y	–	bending moments
M_z	–	torsional moment
T_x	–	shear force
κ_x, κ_y	–	curvature components
ρ	–	twist
EI_x, EI_y	–	flexural rigidities
GI_s	–	torsional rigidity
EI_ω	–	warping rigidity

whereas β_x defined as

$$
\beta_x = \frac{1}{I_x}\Big(\int_A x^2y\,dA + \int_A y^3\,dA\Big) - 2y_0
\tag{2.2}
$$

it is the monosymetry property (for a doubly symmetric cross-section it equals to zero). In Eq (2.2) y_0 is the coordinate of the shear centre and x, y stand for the axes with the origin at the centroid. Eqs (2.1) can easily be transformed to a convenient form of the set of first order differential equations

$$
\begin{aligned}
& u' = \theta && \theta' = -\frac{M_y}{EI_y} \\
& M_y' = T_x + M_x\rho && \phi' = \rho \\
& \rho' = -\frac{B}{EI_\omega} && M_z' = -M_x\frac{M_y}{EI_y} - Qa\phi \\
& B' = (-GI_s + M_x\beta_x)\rho + M_z &&
\end{aligned}
\tag{2.3}
$$

where the deflection u, the angles of rotation θ, ϕ have been introduced and the following definitions of the moment of free torsion M_s, the warping moment M_ω and the bimoment B functions have been adopted

$$
\begin{aligned}
& M_z = M_s + M_\omega && M_s = (-GI_s + M_x\beta_x)\phi' \\
& M_\omega = B' && B = -EI_\omega\phi''
\end{aligned}
\tag{2.4}
$$

The expression $Qa\phi$ stands for an additional torsional moment resulting from application of loading at the distance a above or below the shear centre. The terms representing bending in the plane of major rigidity have been omitted in Eqs (2.3) because of their negligible influence on the design (Bochenek, 1993).

The boundary conditions in the form dependent on the type of beam ends support are now added. Cantilevers and supported beams are considered and for the latter case we distinguish the support conditions in the plane of minor rigidity (simply supported or clamped) and in the plane of major rigidity (only simply supported). In what follows we get:

— cantilever

$$
u(0) = \theta(0) = \phi(0) = \rho(0) = M_y(0) = M_z(L) + Pa\phi(0) = B(L) = 0 \tag{2.5}
$$

— both ends simply supported

$$
u(0) = M_y(0) = \phi(0) = B(0) = u(L) = \phi(L) = B(L) = 0 \tag{2.6}
$$

— both ends clamped

$$
u(0) = \theta(0) = \phi(0) = \rho(0) = u(L) = \theta(L) = \phi(L) = 0 \tag{2.7}
$$

3. The optimal design problem

A straight monosymmetric I-beam of upper flange width D_{10}, lower flange width D_{20}, flange thickness T_1, web depth H, web thickness T_2 and length L is considered. This beam is referred to as the reference (prismatic) one and its material of given volume V_0 is redistributed during the optimization process.

The optimal design problem is formulated in the following way. The distribution of the flange widths D_{1i}, D_{2i} of n segments, beam has been divided into, are sought for so as to maximize the lateral buckling load subject to the volume constraint.

If the general buckling load F is introduced then the function

$$F = F(D_{11}, D_{21}, D_{12}, D_{22}, ...D_{1n}, D_{2n}) \tag{3.1}$$

is maximized with respect to the equality constraint representing the constant volume requirement

$$V = \sum_{i=1}^{n}\Big((D_{1i} + D_{2i})T_1 + HT_2\Big)L_i = V_0 \tag{3.2}$$

and the following flanges width geometrical bounds

$$D_{1i}^{min} \leq D_{1i} \leq D_{1i}^{max} \qquad D_{2i}^{min} \leq D_{2i} \leq D_{2i}^{max} \qquad i = 1, 2, ..., n \tag{3.3}$$

The above mathematical programming formulation can easily be extended by imposing additional constraints that, for example, take care of maximum permissible stress in prebuckling state, local instability of the flanges or local instability of the web (Bochenek, 1993).

Since the cross-section is monosymmetric the optimization problem can be formulated in the following four variants:

1. Proportional change of upper and lower flange dimensions, i.e.:
   $D_{1i}/D_{2i} = D_{10}/D_{20} = \text{const}$

2. Design of the upper flange, lower flange prismatic, i.e.:
   D_{1i} – variable, $D_{2i} = D_{20} = \text{const}$

3. Design of the lower flange, upper flange prismatic, i.e.:
   $D_{1i} = D_{10} = \text{const}$, D_{2i} – variable

4. Independent design of both flanges, i.e.:
   D_{1i}, D_{2i} – variable.

The quantities D_{10}, D_{20} stand for the width of upper and lower flange of the reference, prismatic beam. It is worth noting that the above four design cases can be applied to the optimization of both doubly symmetric and monosymmetric reference beams.

4. Results and discussion

The design problem formulated in Section 3 has been solved for various combinations of geometrical parameters, loading and beam support types. In particular, the following beam types were considered:

- simply supported ends – point loading in the middle of the beam
- simply supported ends – uniform loading
- cantilever – end point loading
- cantilever – uniform loading.

Table 1

	design case 1	design case 2	design case 3	design case 4
supported ends point load lower flange loaded	69.8	36.3	87.8	96.6
supported ends point load upper flange loaded	51.3	20.5	43.7	66.0
supported ends uniform loading upper flange loaded	36.6	13.4	40.3	57.8
cantilever point load lower flange loaded	30.4	49.9	37.4	90.1

Some of the results obtained for the doubly symmetric reference I-beam are now presented. Table 1 shows values of the design effectiveness parameter ε, defined as

$$\varepsilon = \frac{F_{opt} - F_{prism}}{F_{prism}} 100\%$$

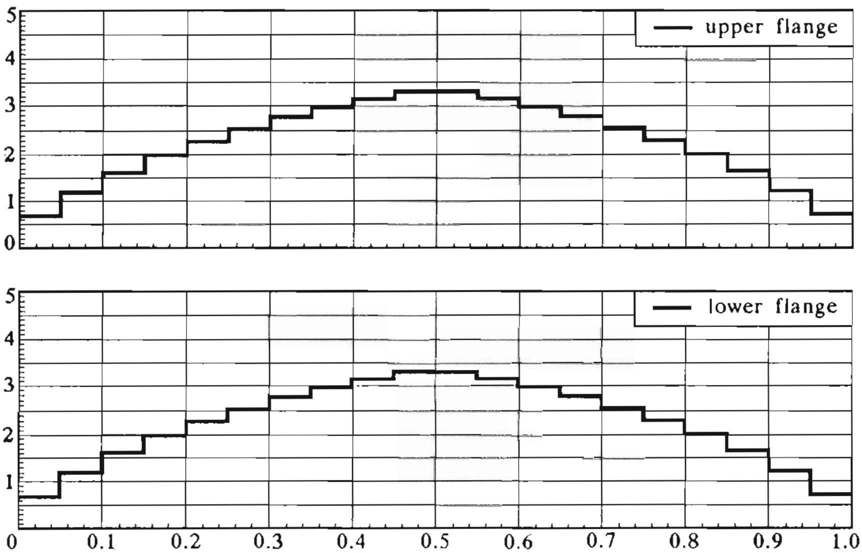

Fig. 1. Simply supported beam, point load, lower flange loaded, design case 1, $\varepsilon = 69.8$

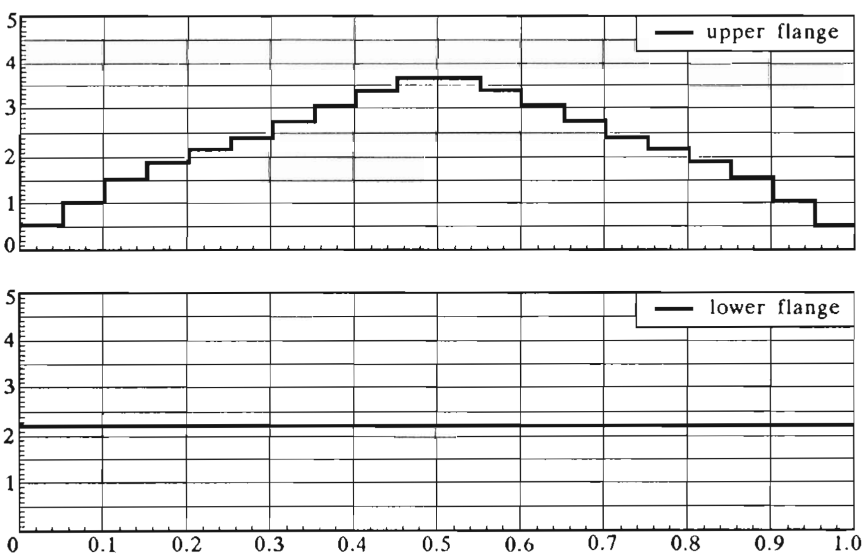

Fig. 2. Simply supported beam, point load, lower flange loaded, design case 2, $\varepsilon = 36.3$

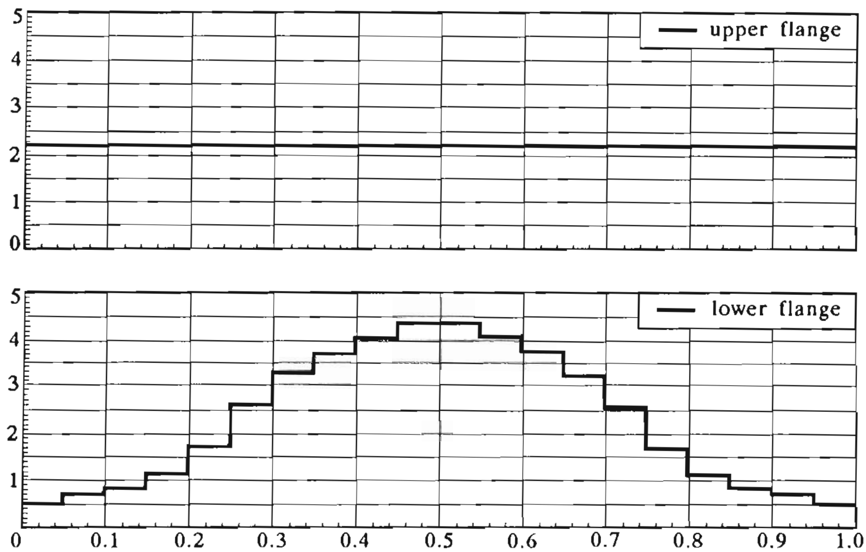

Fig. 3. Simply supported beam, point load, lower flange loaded, design case 3, $\varepsilon = 87.8$

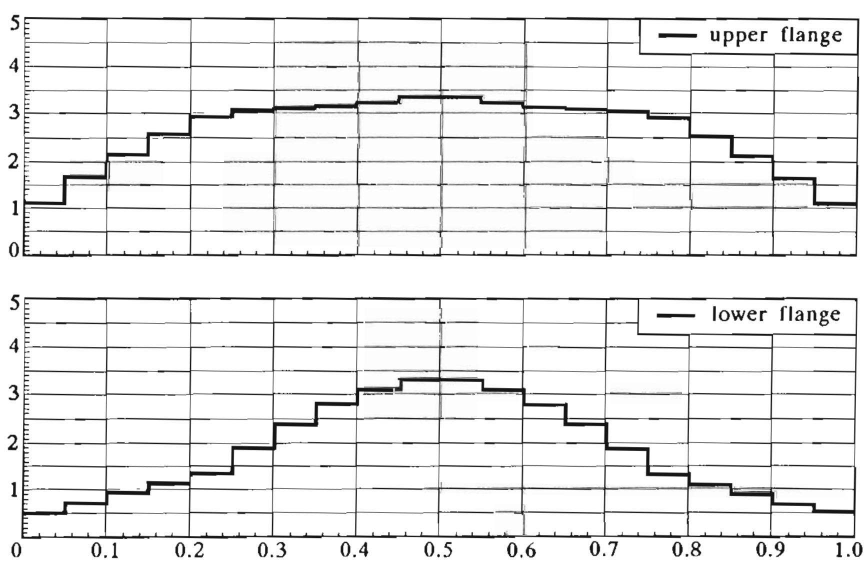

Fig. 4. Simply supported beam, point load, lower flange loaded, design case 4, $\varepsilon = 96.6$

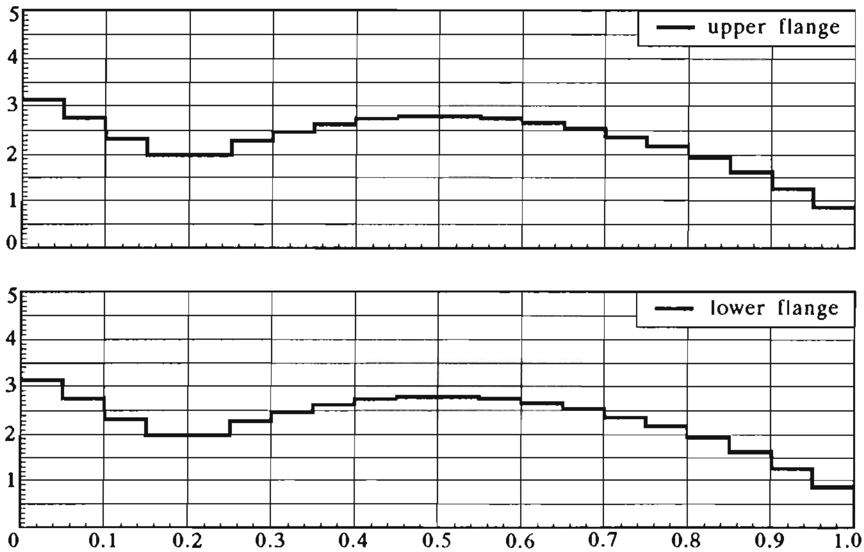

Fig. 5. Cantilever beam, point load, lower flange loaded, design case 1, $\varepsilon = 30.4$

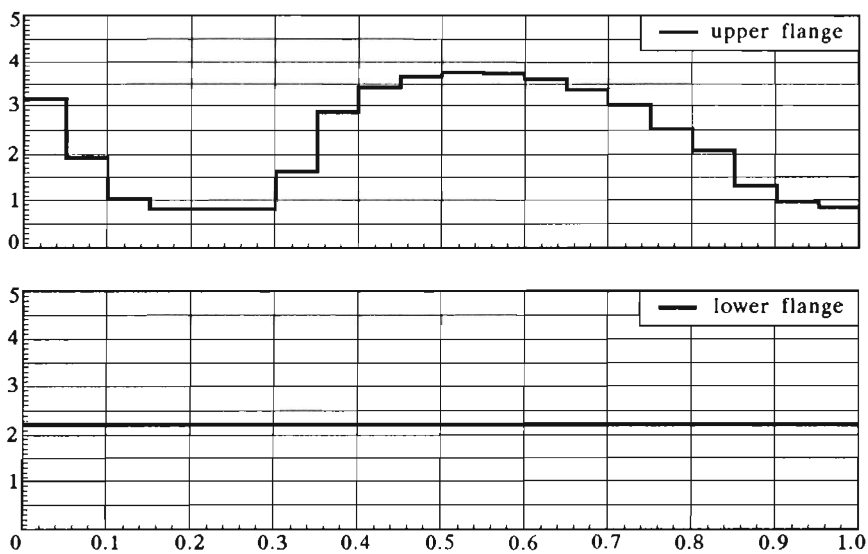

Fig. 6. Cantilever beam, point load, lower flange loaded, design case 2, $\varepsilon = 49.9$

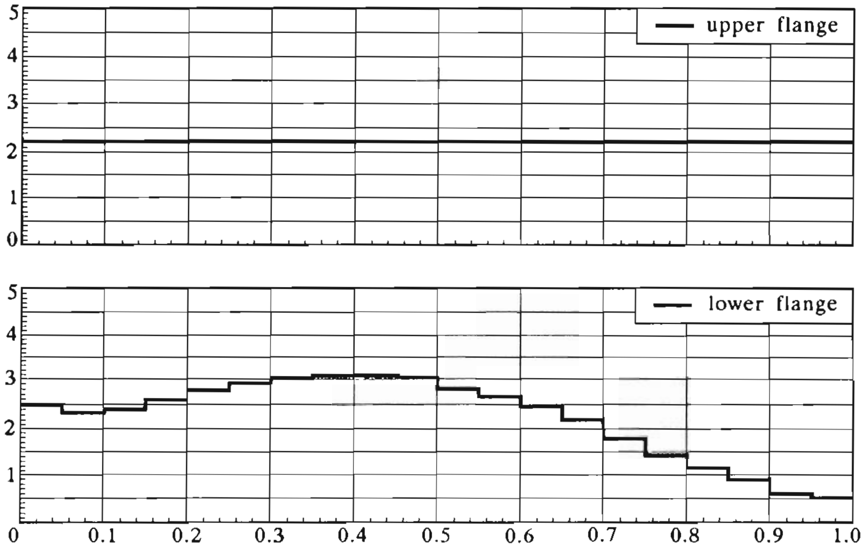

Fig. 7. Cantilever beam, point load, lower flange loaded, design case 3, $\varepsilon = 37.4$

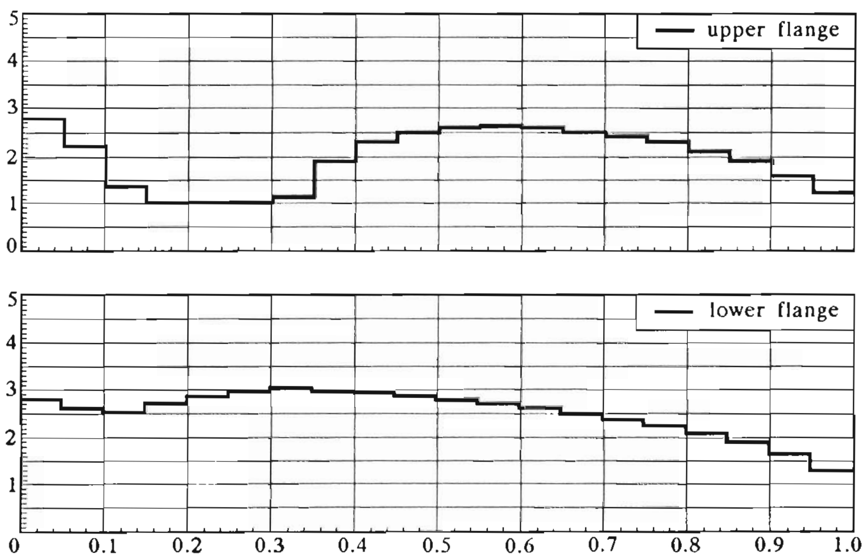

Fig. 8. Cantilever beam, point load, lower flange loaded, design case 4, $\varepsilon = 90.1$

that have been found for chosen support and load cases and various formulations of the optimization problem. F_{prism} and F_{opt} are the critical load values evaluated for reference prismatic and optimal beam, respectively.

It can be seen from the presented results that the design is more effective if the flange that is in tension is optimized. It is the lower one for supported beams and the upper one for cantilevers. For supported beams better results are obtained in the case of load applied to the lower flange. The values of the effectiveness parameter ε can be even two times greater than in the case of upper flange loading. The design of upper flange for cantilevers is usually more effective, however, in the case of load applied to the upper flange the result can be different. In such a case the effect of monosymmetry (critical load is greater when the tension flange is smaller) and the effect of position of the load above the shear center (critical load is greater when the upper flange is larger) act as the opposite ones. In Fig.1 $\div$ Fig.8 the optimal distribution of upper and lower flange width for selected beam types is shown. A simply supported beam (Fig.1 $\div$ Fig.4) and a cantilever (Fig.5 $\div$ Fig.8) have been chosen and for both of them the load has been applied to the lower flange.

The analysis of the obtained results has led to the following conclusions. The design of the flanges of I-beam significantly increases the value of the lateral buckling load, even up to 100%. That improvement is much more significant for monosymmetry of the cross-section allowed for and it is evident especially for independent design of both upper and lower flanges. It has been also shown that in some cases optimization of only one flange gives better results than the proportional design of both flanges.

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Optymalne kształtowanie narażonych na zwichrzenie monosymetrycznych belek dwuteowych

Streszczenie

W pracy przeprowadzono analizę zwichrzenia przegubowo podpartych oraz wspornikowych belek o monosymetrycznym przekroju dwuteowym. Sformułowano zadanie optymalizacji parametrycznej polegające na doborze wymiarów przekroju poszczególnych segmentów, na które podzielono belkę, tak aby dla ustalonej objętości materiału belki obciążenie krytyczne było maksymalne. Ze względów praktycznych zrezygnowano z dowolnej zmienności wymiarów belki koncentrując uwagę na belkach składających się z segmentów o ustalonych wymiarach. Dla każdego segmentu przyjęto jednakową wysokość środnika oraz grubości środnika i pólek, pozostawiając jako zmienne kształtowania szerokości pólek. Dopuszczenie monosymetrii przekroju pozwoliło na większą swobodę w formułowaniu zadania optymalizacji, w szczególności umożliwiło niezależne kształtowanie zmienności wymiarów górnej i dolnej pólki. W rezultacie zwiększyła się efektywność optymalizacji i możliwe było znaczne podniesienie wartości obciążenia krytycznego w porównaniu z belką o przekroju o dwóch osiach symetrii.

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