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RESISTANCE COEFFICIENT OF A PARTICLE MOVING IN THE PRESENCE OF A DEFORMED WALL

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In 1988 Falade and Brenner obtained the results which enables us to calculate the low Reynolds number hydrodynamic interaction only for a sphere moving in a fluid bounded by an arbitrary deformed wall. In this paper another combined analytical-numerical method for solving the same problem is presented for a wide class of bodies, shape of which can be described in separable coordinates (elipsoid, torus, spheroid, sphere). This method of calculation is based on the perturbation method, linearity of the Stokes equation and the collocation method, respectively. As an example, applicability of this method is tested for an axisymmetric translation of a sphere.

Key words: Stokes flow, viscous incompressible flow past a body, bounded flow

1. Introduction

In many industrial and biological processes the important class of hydrodynamic interaction problems is posed by the effect of nearby wall on the particle motion. In general, such problems are very complicated since non-planar wall and arbitrary shaped particles are involved. In order to construct a mathematical model of this physical phenomenon it is necessary to resort to a number of simplifications, based on its physical analysis. The investigation of this complicated problem can be started with the Stokes flow past a single rigid spherical particle, near a curved wall of the constant curvature radiuses $R1$, $R2$. For the plane wall $R1 = R2 = \infty$, for the cylindrical one $R1 = c1$, $R2 = \infty$, and in the case of spherical wall $R1 = R2 = c2$. Adopting of this approach has dated back to the works of Lorentz (1896) and Faxen (1924). The data and results on particle-wall interaction may be found in Happel and

Brenner (1967), Hasimoto and Sano (1980), Kim and Karilla (1991), Fuillebois (1989), Falade and Brenner (1988).

Approach to particle wall interactions depends on the ratio of two length scales: the particle size a and the distance between the particle and wall d (we denote it $k = d/a$). In spite of verification of the solution methods the following cases have been studied in earlier papers separately:

- Sphere far away from the wall $(k > 1)$ – mainly by means of the reflection technique

- Particles and wall near contact $(k \to 0)$ – using a combination of lubrication and numerical methods; matched asymptotic expansion was often applied too

- Moderate separation $(k \approx 1)$ – using various numerical schemes.

It is worth mentioning, that the results obtained in the papers cited above are devoted only to the situation when the wall bounding a flow has constant curvature radiuses and they prove that a strong dependence between the effect of a wall on the hydrodynamic force of a moving particle and a particle shape.

The first result for the flow past a sphere in the presence of an arbitrary curved wall $(R1 = R2 < \infty)$ have obtained by Falade and Brenner (1988). The authors examined translational and rotational motions of a spherical particle and obtained the results, which were valid within the entire range of the parameter k. They showed, that the wall curvature the Stokes force F and torque T depended lineary on the two scalar principal curvature coefficients $R1$, $R2$ of the wall at the foot of the shortest normal to the wall from the sphere centre.

To construct the method of solution the authors used spherical bipolar coordinates. It enables description of a sphere and a plane wall surfaces, respectively, by given values of a single variable. Moreover, they used perturbation method for velocity $\boldsymbol{v}$ and pressure p, which permited – after expanding velocity $\boldsymbol{v}$ in a Taylor series – to transfer curved wall boundary condition to the plane wall. Unfortunately, this method does not work for the flow past a non-spherical particle due to the properties of bipolar coordinates mentioned above. So, we need another solution scheme to solve this problem, if we want to know dependence between the effect of an arbitrary curved wall in the flow field and a shape of the moving particle.

The aim of this paper is to present a new numerical-analytical method which allows us to analyze the flow past a non-spherical particle, shape of which can be described in separable coordinates in the presence of arbitrary curved wall in the Stokes approach. It is based on the perturbation method and the concept, that each of the particular perturbation fields can be sought as a sum of the fundamental solutions: in the half space above a wall and outside the moving particle, in the space. The boundary conditions instead *on the curved wall are satisfied on a plane wall after* expanding the velocity v in a Taylor series. The essential difference between the approach given by Falade and Brenner (1988) and this one consists in the fact, that instead of the Stokes fundamental solution in bipolar coordinates we use a sum of the fundamental solutions: in the half space and in the separable coordinates. Next, we use the collocation method to determine the constants appearing in those formulas.

As an example of efficiency applicability of this method, the axisymmetrical, the translational flow problem past a sphere in the presence of the deformed wall $z = z_0[1 + \varepsilon \exp(br^2)]$, $b < 0$, is solved.

2. Formulation of the problem

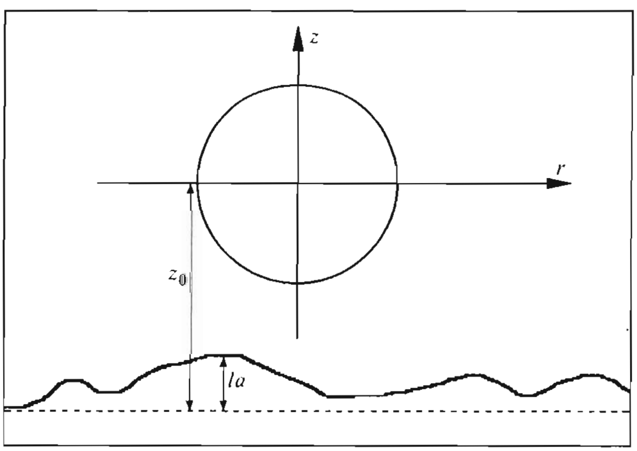

Fig. 1. Geometry of the flow

We consider the creeping motion of a solid particle in a stationary viscous fluid, which is bounded by a rigid, arbitrary curved wall, curvature of which slowly changes. At infinity the fluid is at rest. The particle moves at a constant translational velocity V. Hereinafter, a stands for the characteristic dimension of the particle, and d for the distance between particle and wall. By introducing the Cartesian coordinate system (x, y, z) origin of which is in the particle center the wall surface can be described by the function

$$z = fa(x, y) + z_0 \qquad f \in C^2$$

which satisfies the condition $|fa(x, y)| < la$, where z_0 and la are constants, $|z_0|$ represents the characteristic distance between the two parallel planes $z = 0$ and $z = z_0$ which is defined as the crossing the wall at the point at which the function $fa(x, y)$ attains the mimimum. For comparing shapes of different curved walls it is useful to consider the function f instead of fa, defined as $f(x, y) = fa(x, y)/la$. In such a case $|f(x, y)| \leq 1$. It has been assumed, that the parameter ε defined as the ratio $\varepsilon = la/z_0$ is small, $|\varepsilon| < 1$, moreover the parameter $\beta = la/a \ll 1$. Then the equation describing the wall reads $z = z_0\Big(1 + \varepsilon f(x, y)\Big)$. In order to find the solution to this problem the Stokes approach is applied. The range of small Reynolds numbers $\mathrm{Re} \leq 1$ is considered, $\mathrm{Re} = av/\nu$, where ν denotes the kinematic viscosity of the fluid.

The governing equations of the fluid motion are

$$\nu \Delta \boldsymbol{v} = \nabla p \qquad \nabla \cdot \boldsymbol{v} = 0 \tag{2.1}$$

where $\boldsymbol{v}$ and p stand for the velocity vector and the pressure, respectively.

Because of the connection of the coordinate system with the moving particle, the velocity $\boldsymbol{v}$ satisfies the following boundary conditions

$$\boldsymbol{v} = \begin{cases} -\boldsymbol{V} & \text{on the curved wall} \\ \mathbf{0} & \text{on the surface } S \text{ of the moving particle} \\ -\boldsymbol{V} & \text{at infinity} \end{cases} \tag{2.2}$$

3. Method of solution

The solution to the problem represented by Eqs (2.1) is sought in the form of perturbation expressions for the velocity $\boldsymbol{v}$ and pressure p. Assuming, that $|\varepsilon| < 1$ it is possible to expand velocity $\boldsymbol{v}$ and pressure p into the infinite series

$$\boldsymbol{v} = \sum_{i=0}^{\infty} \varepsilon^i \boldsymbol{v}_i \qquad p = \sum_{i=0}^{\infty} \varepsilon^i p_i \tag{3.1}$$

Substituting Eqs (3.1) into Eq $(2.1)_1$ and then equating terms with the same powers of ε, one finds that each perturbation field $(\boldsymbol{v}_i, p_i)$ satisfies the Stokes equations

$$\nu \Delta \boldsymbol{v}_i = \nabla p_i \qquad \nabla \cdot \boldsymbol{v}_i = 0 \tag{3.2}$$

and the boundary conditions

$$\begin{array}{lll} \boldsymbol{v}_i = \boldsymbol{0} & & i = 0,1,\dots \text{ on the body} \\ \boldsymbol{v}_0 = -\boldsymbol{V} & \boldsymbol{v}_i = \boldsymbol{0} & i = 1,2,\dots \text{ at infinity} \end{array} \tag{3.3}$$

Some troubles with fulfilling the boundary condition appears at the curved wall. Namely, after substituting Eq $(3.1)_1$ into Eq $(2.2)_1$ we have

$$\begin{aligned} -\boldsymbol{V} &= \boldsymbol{v}_0\Big(x, y, z_0(1 + \varepsilon f(x,y))\Big) + \varepsilon \boldsymbol{v}_1\Big(x, y, z_0(1 + \varepsilon f(x,y))\Big) + \\ &+ \varepsilon^2 \boldsymbol{v}_2\Big(x, y, z_0(1 + \varepsilon f(x,y))\Big) + \dots \end{aligned} \tag{3.4}$$

In this formula the perturbation parameter ε appears implicitly, in the first argument of the function, as well as explicitly, so that it is not directly possible to equate like powers to zero. This obstacle can overcome only by expanding the perturbation fields $\boldsymbol{v}_i$ in Taylor series to exhibit explicitly their dependence on ε. If we assume that $\boldsymbol{v}_i$, like $\boldsymbol{v}$, is analytic in its dependence upon z, we can expand it in a Taylor series about $z = z_0$. Considering only the linear terms at ε Eq (3.4) is

$$\begin{aligned} -\boldsymbol{V} &= \boldsymbol{v}_0(x,y,z_0) + (z - z_0)\frac{\partial \boldsymbol{v}_0}{\partial z}(x,y,z_0) + \varepsilon \boldsymbol{v}_1(x,y,z_0) + \\ &+ \varepsilon(z - z_0)\frac{\partial \boldsymbol{v}_1}{\partial z}(x,y,z_0) + \dots \end{aligned} \tag{3.5}$$

Since $(z - z_0) = \varepsilon f(x,y) z_0$, the boundary condition represented by Eq (3.4) can be rewriten as

$$\boldsymbol{v}_0 + \sum_{i=1}^{\infty} \varepsilon^i \Big[\boldsymbol{v}_i + \sum_{j=1}^{i} \frac{1}{j!} z_0^j f(x,y)^j \Big(\frac{\partial^j \boldsymbol{v}_{i-j}}{\partial z^j}\Big)\Big] = -\boldsymbol{V} \qquad \text{at } z = z_0 \tag{3.6}$$

Therefore, the boundary condition on the deformed wall, Eq $(2.2)_1$, can be satisfied with respect to any order in ε by requiring that various perturbation fields $\boldsymbol{v}_i$ satisfy the following boundary conditions on the flat wall $z = z_0$

$$\begin{aligned} &\boldsymbol{v}_0 = -\boldsymbol{V} \\ &\boldsymbol{v}_i = -\sum_{j=1}^{i} \frac{1}{j!} f(x,y)^j z_0^j \Big(\frac{\partial^j \boldsymbol{v}_{i-j}}{\partial z^j}\Big) \qquad \text{for } i = 1,2,\dots \end{aligned} \tag{3.7}$$

To solve the flow problem defined above, owing to the linearity of Stokes equation, we seek the solution $\boldsymbol{v}_i$ as a sum $\boldsymbol{v}_i = \boldsymbol{w}_i + \boldsymbol{u}_i$, where $\boldsymbol{u}_i$ is a general solution of the Stokes equation $(2.1)_1$ outside the moving body in the space in the separable coordinates in which the surface of the moving body can be described. Part $\boldsymbol{w}_i$ represents the general solutions in the half space $z \geq z_0$. They are regular in the flow field. In view of the boundary conditions we take $\boldsymbol{v}_0$ to be $\boldsymbol{v}_0 = \boldsymbol{u}_0 + \boldsymbol{w}_0 - \boldsymbol{V}$, where $\boldsymbol{V}$ is the velocity of the moving particle under consideration.

The boundary conditions on the wall (3.7) can be rewriten now in the equivalent form

$$
\begin{aligned}
&\boldsymbol{u}_0 = -\boldsymbol{w}_0 \\
&\boldsymbol{u}_i = -\boldsymbol{w}_i - \sum_{j=1}^{i} \frac{1}{j!} f(x,y)^j z_0^j \Big(\frac{\partial^j \boldsymbol{v}_{(i-j)}}{\partial z^j} \Big) \qquad \text{for} \quad i = 1,2,\dots
\end{aligned}
\tag{3.8}
$$

which enables us to express the unknown constants appearing in the global solution $\boldsymbol{w}_i$ by the constants (still unknown) from the global solution $\boldsymbol{u}_i$. They can be determined from the boundary condition to be satisfied by the velocity $\boldsymbol{v}$ on the moving body, Eq $(2.2)_2$

$$
\boldsymbol{u}_0 + \boldsymbol{w}_0 = -\boldsymbol{V} \qquad\qquad \boldsymbol{u}_i + \boldsymbol{w}_i = \boldsymbol{0} \qquad\qquad i = 1,2,\dots
$$

applying collocation technique and solving the derived set of equations. Then, the velocity field is known.

4. Solution for a sphere

As an example of applicability of this method we consider the flow field resulting from an axisymmetrical translation of a solid sphere in fluid. This problem was choosen because in this case the accuracy and convergence of the present method can be verified by comparison with the results presented by Falade and Brenner (1988).

Let the sphere move at the translation velocity $\boldsymbol{V}$ towards the wall. In the polar system of coordinates (r,z) with the origin at the sphere centre the surface of the bounding wall will be given by the equation

$$
z = z_0\Big(1 + \varepsilon \exp(br^2)\Big) \qquad\qquad b < 0 \qquad z_0 < 1
$$

We assume, that $|\varepsilon| < 1$, $\beta < 1$ and $\mathrm{Re} < 1$. The governing equations of the fluid motion are the Stokes equations (2.1) and the boundary conditions are given by Eqs (2.2). Due to the axisymmetric nature of the flow, we introduce the stream function $\Psi(r,z)$ satysfying Eq $(2.1)_2$, which in cylindrical coordinates is given by

$$v_r = \frac{1}{r}\frac{\delta\Psi}{\delta z} \qquad v_z = -\frac{1}{r}\frac{\delta\Psi}{\delta r} \tag{4.1}$$

The symbols v_r, v_z stand for the radial and axial velocities $\boldsymbol{v}$, respectively. Now, from Eq $(2.1)_1$, the stream function satisfies the equation

$$D^2(D^2\Psi) = 0 \tag{4.2}$$

where D^2 is the generalized axisymmetric Stokes operator

$$\frac{\delta^2}{\delta r^2} - \frac{1}{r}\frac{\delta}{\delta r} + \frac{\delta^2}{\delta z^2} \tag{4.3}$$

We look for the solution of Eq (4.2) in the form of perturbation expansion series and expand the stream function $\Psi(r,z)$

$$\Psi(r,z) = \sum_{i=0}^{\infty} \varepsilon^i \Psi_i(r,z) \tag{4.4}$$

Moreover we assume, that each stream function $\Psi_i(r,z)$ is composed of the two parts $\Psi_i = \Psi_{ui} + \Psi_{wi}$. The part Ψ_{ui} represents the infinite series containing all simply separable solutions of Eq (4.2) (in spherical coordinates) which are regular in the flow field, given by Happel and Brenner (1988) as

$$\Psi_{ui} = \sum_{n=2}^{\infty} (B_n^i \varrho^{-n+1} + D_n^i \varrho^{-n+3}) I_n(\zeta) \tag{4.5}$$

Here $\zeta = \cos\theta$ and $I_n(\zeta)$ is the Gegenbauer function of the first kind of order n and degree $-1/2$, r and θ are the spherical coordinates measured from the sphere centre. B_n^i and D_n^i are unknown constants which will be determined by satisfying the non-slip boundary conditions on the surface of the sphere in the presence of bounding wall.

The part Ψ_{wi} represents the integral of all the separable solutions of Eq (4.2) (in cylindrical coordinates) which produce finite velocities everywhere in the flow field, given by the Fourier-Bessel integral

$$\Psi_{wi} = \int_0^{\infty} [B^i(\alpha)e^{-\alpha z} + D^i(\alpha)\alpha z e^{-\alpha z}] J_1(\alpha r)\, d\alpha \tag{4.6}$$

Here $B^i(\alpha)$, $D^i(\alpha)$ are unknown functions of the variable α and J_1 is the Bessel function of the first kind of order one. The disturbances produced by the sphere along the wall can be completely reduced when the proper choice of the functions $B^i(\alpha)$, $D^i(\alpha)$ in Eq (4.6) is made.

We denote by u_{ri}, u_{zi}, w_{ri}, w_{zi} the velocity components derived from Eqs (4.1) after substituting into them the stream functions Ψ_{ui} and Ψ_{wi}, respectively. Thus the axial v_r and radial v_z components of velocity $\boldsymbol{v}$ of the fluid flow are

$$
\begin{aligned}
v_r &= \sum_{i=0}^{\infty} \varepsilon^i v_{ri} = \sum_{i=0}^{\infty} \varepsilon^i (u_{ri} + w_{ri}) \\
v_z &= \sum_{i=0}^{\infty} \varepsilon^i v_{zi} = \sum_{i=0}^{\infty} \varepsilon^i (u_{zi} + w_{zi})
\end{aligned}
\tag{4.7}
$$

Now substituting Eqs (4.5) and (4.6) into Eqs (4.1) and using Eqs (4.7) one obtains the formulas for the axial and radial components of velocities $\boldsymbol{v}_i$

$$
\begin{aligned}
v_{ri} &= \sum_{n=2}^{\infty} (B_n^i \mathcal{B}_{rn} + D_n^i \mathcal{D}_{rn}) + \int_0^{\infty} \mathcal{E}^i(\alpha, z)\alpha J_1(\alpha r)\, d\alpha = u_{ri} + w_{ri} \\
& \qquad\qquad i = 0, 1, 2, \ldots \\
v_{zi} &= \sum_{n=2}^{\infty} (B_n^i \mathcal{B}_{zn} + D_n^i \mathcal{D}_{zn}) + \int_0^{\infty} \mathcal{F}^i(\alpha, z)\alpha J_0(\alpha r)\, d\alpha = u_{zi} + w_{zi} \\
& \qquad\qquad i = 1, 2, \ldots \\
v_{z0} &= \sum_{n=2}^{\infty} (B_n^0 \mathcal{B}_{zn} + D_n^0 \mathcal{D}_{zn}) + \int_0^{\infty} \mathcal{F}^0(\alpha, z)\alpha J_0(\alpha r)\, d\alpha - V = u_{z0} + w_{z0} - V
\end{aligned}
\tag{4.8}
$$

The functions $\mathcal{B}_{rn}$, $\mathcal{D}_{rn}$, $\mathcal{B}_{zn}$, $\mathcal{D}_{zn}$, $\mathcal{E}^i$, $\mathcal{F}^i$ are listed in Appendix (Eqs (A.1)).

The boundary conditions which should be satisfied by the velocity $\boldsymbol{v}$ on the deformed wall are represented now by Eqs (3.8). In order to determine the velocity field to the first order in ε it is reduced to

$$
\begin{aligned}
w_{r0}(r, z_0) &= -u_{r0}(r, z_0) \\
w_{r1}(r, z_0) &= -e^{br^2} z_0 \frac{\delta}{\delta z} v_{r0}(r, z_0) - u_{r1}(r, z_0) \\
w_{z0}(r, z_0) &= -u_{z0}(r, z_0) \\
w_{z1}(r, z_0) &= -e^{br^2} z_0 \frac{\delta}{\delta z} v_{z0}(r, z_0) - u_{z1}(r, z_0)
\end{aligned}
\tag{4.9}
$$

Substituting Eqs (4.8) into Eqs (4.9) we see that Eqs $(4.9)_{1,3}$ can be easily inverted and integration can be performed using results of Hankel transforms. It is

$$\mathcal{E}^0(\alpha, z_0) = -\int_0^\infty t \sum_{n=1}^\infty [B_n^0 \mathcal{B}_{rn}(t, z_0) + D_n^0 \mathcal{D}_{rn}(t, z_0)] J_1(\alpha t)\, dt$$

$$\mathcal{F}^0(\alpha, z_0) = -\int_0^\infty t \sum_{n=1}^\infty [B_n^0 \mathcal{B}_{zn}(t, z_0) + D_n^0 \mathcal{D}_{zn}(t, z_0)] J_0(\alpha t)\, dt \tag{4.10}$$

These integrals can be calculated analytically. We have

$$\begin{aligned}
\mathcal{B}^*_{rn} &= -\int_0^\infty t\mathcal{B}_{rn}(t, z_0) J_1(\alpha t)\, dt = -\frac{1}{n!}\Big(\frac{\alpha|z_0|}{z_0}\Big)^{(n-1)} \mathrm{e}^{-\alpha|z_0|} \\
\mathcal{B}^*_{zn} &= -\int_0^\infty t\mathcal{B}_{zn}(t, z_0) J_0(\alpha t)\, dt = -\frac{\alpha^{(n-1)}}{n!}\Big(\frac{|z_0|}{z_0}\Big)^{n} \mathrm{e}^{-\alpha|z_0|} \\
\mathcal{D}^*_{rn} &= -\int_0^\infty t\mathcal{D}_{rn}(t, z_0) J_1(\alpha t)\, dt = \\
&= -\frac{1}{n!}\Big(\frac{\alpha|z_0|}{z_0}\Big)^{(n-3)} \mathrm{e}^{-\alpha|z_0|}[(2n-3)\alpha|z_0| - n(n-2)] \\
\mathcal{D}^*_{zn} &= -\int_0^\infty t\mathcal{D}_{zn}(t, z_0) J_0(\alpha t)\, dt = \\
&= -\frac{\alpha^{(n-3)}}{n!}\Big(\frac{\alpha|z_0|}{z_0}\Big)^{n} \mathrm{e}^{-\alpha|z_0|}\Big[(2n-3)\alpha|z_0| - (n-1)(n-3)\Big]
\end{aligned} \tag{4.11}$$

then w_r^0, w_z^0 can be now written as

$$\begin{aligned}
w_{r0} &= \sum_{n=2}^\infty B_n^0 \mathcal{W}\mathcal{B}_{rn} + D_n^0 \mathcal{W}\mathcal{D}_{rn} \\
w_{z0} &= \sum_{n=2}^\infty B_n^0 \mathcal{W}\mathcal{B}_{zn} + D_n^0 \mathcal{W}\mathcal{D}_{zn}
\end{aligned} \tag{4.12}$$

The functions $\mathcal{W}\mathcal{B}_{rn}$, $\mathcal{W}\mathcal{D}_{rn}$, $\mathcal{W}\mathcal{B}_{zn}$, $\mathcal{W}\mathcal{D}_{zn}$ are listed in Appendix (Eqs (A.2)).

Thus, substituting the above formulas into Eqs $(4.8)_{1,3}$ one obtains to zero order velocity fields v_{r0}, v_{z0} still in terms of unknown coefficients B_n^0, D_n^0

$$\begin{aligned} v_{r0} &= \sum_{n=2}^{\infty} B_n^0(\mathcal{B}_{rn} + \mathcal{W}\mathcal{B}_{rn}) + D_n^0(\mathcal{D}_{rn} + \mathcal{W}\mathcal{D}_{rn}) \\ v_{z0} &= \sum_{n=2}^{\infty} B_n^0(\mathcal{B}_{zn} + \mathcal{W}\mathcal{B}_{zn}) + D_n^0(\mathcal{D}_{zn} + \mathcal{W}\mathcal{D}_{zn}) - V \end{aligned} \tag{4.13}$$

In order to obtain a unique solution, we should apply the boundary conditions imposed on the moving particle (Eq $(2.2)_2$) to the velocities expressed by Eqs (4.13) at a finite number of discrete points on the sphere. Next – after truncating the infinite series which appears in Eqs (4.13) into the finite one – we solve the derived equations set with respect to B_n^0, D_n^0 and obtain the velocity field $\boldsymbol{v}_0$.

Then, beginning with the zero-order field, each higher-order field $\boldsymbol{v}_m$, $m > 0$ can be successively determined by satisfying the appropriate boundary conditions at the wall, and the surface of the particle. The wall boundary conditions to be satisfied by $\boldsymbol{v}_m$ on $z = z_0$, Eq (3.8), require prior calculation of $\boldsymbol{v}_{m-1}$ and its derivatives.

The algorithm for a computing higher order field $\boldsymbol{v}_m$ can be summarized as follows:

Stage (i) Compute lower-order field derivatives of $\boldsymbol{v}_k$, $k < m$ which appear in Eqs (3.8)

Stage (ii) Inverte equations, Eqs (3.8) to obtain $\mathcal{E}^m$ and $\mathcal{F}^m$ in terms of the unknown spherical coefficients B_n^m and D_n^m

Stage (iii) Solve the set of equations in the B_n^m, D_n^m unknown coefficients derived from Eqs $(4.8)_{1,2}$ after applying the results obtained at Stage (ii) and boundary conditions on the sphere. The collocation technique is applied.

After Stage (ii) the final formulas for the first-order velocity fields v_{r1}, v_{z1} are

$$\begin{aligned} v_{r1} &= \sum_{n=2}^{\infty} \Big[B_n^1(\mathcal{B}_{rn} + \mathcal{W}\mathcal{B}_{rn}) + D_n^1(\mathcal{D}_{rn} + \mathcal{W}\mathcal{D}_{rn}) + B_n^0\mathcal{W}\mathcal{B}\mathcal{R}_n + D_n^0\mathcal{W}\mathcal{D}\mathcal{R}_n\Big] \\ v_{z1} &= \sum_{n=2}^{\infty} \Big[B_n^1(\mathcal{B}_{zn} + \mathcal{W}\mathcal{B}_{zn}) + D_n^1(\mathcal{D}_{zn} + \mathcal{W}\mathcal{D}_{zn}) + B_n^0\mathcal{W}\mathcal{B}\mathcal{Z}_n + D_n^0\mathcal{W}\mathcal{D}\mathcal{Z}_n\Big] \end{aligned} \tag{4.14}$$

The coefficients B_n^0, D_n^0 are known from zero order solution $\boldsymbol{v}_0$, the coefficients B_n^1, D_n^1 are unknown yet and should be found at Stage (iii). The functions $\mathcal{WB}_{rn}$, $\mathcal{WD}_{rn}$, $\mathcal{WB}_{zn}$, $\mathcal{WD}_{zn}$ are the same as in Eqs (4.13) and $\mathcal{WBR}_n$, $\mathcal{WDR}_n$, $\mathcal{WBZ}_n$, $\mathcal{WDZ}_n$ are listed in Appendix (Eqs (A.3)).

5. Force

The non-dimensional hydrodynamic force exerted upon translating sphere is

$$f = \frac{\mathcal{F}}{6\pi\mu a} \tag{5.1}$$

where $\mathcal{F} = F_0 + \varepsilon F_1 + O(\varepsilon^2)$.

The force exerted by the fluid on the sphere takes the form (cf Falade and Brenner, 1988)

$$F_i = \mu\pi \int_S r^3 \frac{\delta}{\delta n}\left(\frac{D^2\Psi_i}{r^2}\right)\, ds \tag{5.2}$$

S denotes the particle surface. Performing the above integration one obtains the simple relation

$$F_i = 4D_2^i \mu\pi \tag{5.3}$$

In order to determine the force exerted by the fluid on the sphere to the first order in ε we first calculate the fields $\boldsymbol{v}_0$, p_0 and next $\boldsymbol{v}_1$, p_1. The present paper will be devoted to determination of the first order curvature effects only.

6. Numerical results and conclusions

The scheme of spacing the collocation points on the surface of the sphere is based on the paper by Ganatos et al. (1980) in which the corresponding problem of the sphere motion in the presence of a flat wall was considered. The calculations of the wall correction factor f in this study were performed using the set of points: 0°, 45°, 90°, 132°, 145°, 175°, 177°, 180°).

It is noteworthy owing to the nature of the problem that each boundary point represents a ring.

The time necessary for computing the force f for set of equations of the n collocation points in IBM PENTIUM 100 is about $4n \cdot 10^{-2}$ sec.

To study this algorithm there were made series of calculations for a sphere moving towards the wall $z = z_0(1+\varepsilon\exp(br^2)$, $b < 0$, $z_0 < 0$ for various parameters b and ε. In result, the magnitudes of non-dimensional hydrodynamic force f (Eq (5.1) were obtained acting on the translating sphere in dependence of the two parameters $dis = |z_0|/a$ and b. Parameter dis denotes the non-dimensional distance between the centre of a sphere and the wall $z = z_0$ and parameter b is associated with the shape of the wall.

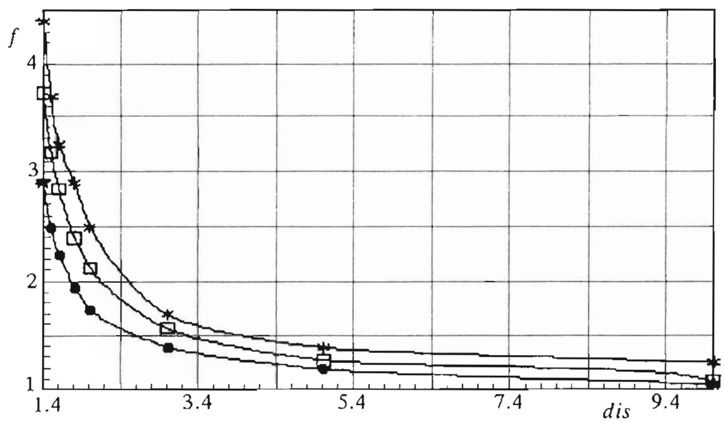

Fig. 2. Typical f (dis) for a moving sphere: (a) towards flat wall □, (b) towards spherical wall – results derived from Falade, and Brenner (1988) ⋆, (c) for deformed wall $z = z_0(1 - 0.2\exp(-0.1r^2)$, $z_0 < 0$, •

In the Fig.2 the presented results are compared with those given by Falade and Brenner (1988) for a spherical wall and by Ganatos et al. (1980) for a flat wall for the chosen values of parameters b and ε. As an example it was taken $b = -0.1$, $\varepsilon = -0.2$. To make this comparison, first, the local radius of curvature of the wall must be calculated and then the tabulated values from Falade and Brenner (1988) must be used. Because the non-dimensional force f was given by Ganatos et al. (1980) only for selected values of parameter dis, so the same values of dis were used in this test. The results of such computations are plotted in Fig.2 and the curves show the dependence of force f acting on a translating sphere on its position dis.

It can be observed that from a qualitative point of view, the results presented are similar to those obtained by Falade and Brenner (1988). The differences can be explained by the fact, that the results given by Falade and Brenner (1988) refer to the spherical wall with the same curvature radiuses at the foot of the sphere as the considered wall.

To study the effect of deformation of the wall and possible gains from the presented algorithm there make the calculations of non-dimensional force f for various values of the parameter $\beta = la/a$. Fig.3 presents the results obtained for $\beta = 0.2$, 0.25, 0.3 and $b = -0.01$.

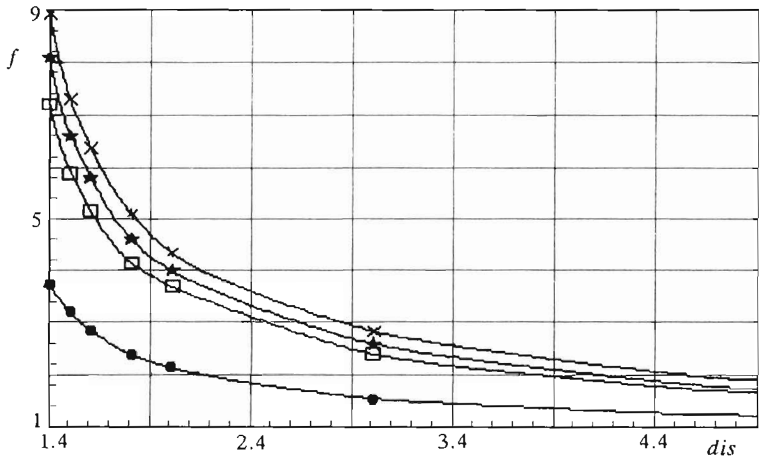

Fig. 3. Comparison between the results for various deformation of the wall: (a) $b = -0.01, \beta = 0.2$ □, (b) $b = -0.01, \beta = 0.25$ ⋆, (c) $-b = 0.01, \beta = 0,3$ ×, (d) $\beta = 0$ – flat wall •

Summing up, the following conclusions can be drawn:

- The numerical method presented in this paper enables one to determine the hydrodynamic interaction between an arbitrary deformed wall and a moving body, the shape of which can be described in the separable coordinates
- The area of an active interaction between a body and a wall is the most important factor which increases the drag.

Appendix

The functions $\mathcal{B}_{rn}$, $\mathcal{D}_{rn}$, $\mathcal{B}_{zn}$, $\mathcal{D}_{zn}$, $\mathcal{E}^i(\alpha,z)$, $\mathcal{F}^i(\alpha,z)$ which appear in Eqs (4.4) and (4.5) can be expressed as

$$\begin{aligned}
\mathcal{B}_{rn} &= \frac{n+1}{\sqrt{(r^2+z^2)^n}}\frac{1}{r}I_{n+1}\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)\\
\mathcal{D}_{rn} &= \frac{n+1}{\sqrt{(r^2+z^2)^{n-2}}}\frac{1}{r}I_{n+1}\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)+\\
&\quad -\frac{2}{\sqrt{(r^2+z^2)^{n-1}}}\frac{2}{r}I_n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)\\
\mathcal{B}_{zn} &= \frac{1}{\sqrt{(r^2+z^2)^{n+1}}}P_n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)\\
\mathcal{D}_{zn} &= \frac{2}{\sqrt{(r^2+z^2)^{n-1}}}I_n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)+\frac{1}{\sqrt{(r^2+z^2)^{n-1}}}P_n\Big(\frac{z}{\sqrt{r^2+z^2}}\Big)\\
\mathcal{E}^i(\alpha,z) &= (1-\sigma)\mathrm{e}^{-\sigma}\mathcal{E}^i(\alpha,z_0)+\sigma\mathrm{e}^{-\sigma}\mathcal{F}^i(\alpha,z_0)\\
\mathcal{F}^i(\alpha,z) &= -\sigma\mathrm{e}^{-\sigma}\mathcal{E}^i(\alpha,z_0)+(1+\sigma)\mathrm{e}^{-\sigma}\mathcal{F}^i(\alpha,z_0)
\end{aligned} \tag{A.1}$$

The functions, which appear in Eqs (4.12) and (4.13) are as follows

$$\begin{aligned}
\mathcal{WB}_{rn} &= -\int\limits_0^\infty[(1-\sigma)\mathrm{e}^{-\sigma}\mathcal{B}^*_{rn}+\sigma\mathrm{e}^{-\sigma}\mathcal{B}^*_{zn}\alpha]J_1(\alpha r)\,d\alpha\\
\mathcal{WB}_{zn} &= -\int\limits_0^\infty[-\sigma\mathrm{e}^{-\sigma}\mathcal{B}^*_{rn}+(\sigma+1)\mathrm{e}^{-\sigma}\mathcal{B}^*_{zn}\alpha]J_0(\alpha r)\,d\alpha\\
\mathcal{WD}_{rn} &= -\int\limits_0^\infty[(1-\sigma)\mathrm{e}^{-\sigma}\mathcal{D}^*_{rn}+\sigma\mathrm{e}^{-\sigma}\mathcal{D}^*_{zn}\alpha]J_1(\alpha r)\,d\alpha\\
\mathcal{WD}_{zn} &= -\int\limits_0^\infty[-\sigma\mathrm{e}^{-\sigma}\mathcal{D}^*_{rn}+(\sigma+1)\mathrm{e}^{-\sigma}\mathcal{D}^*_{zn}\alpha]J_0(\alpha r)\,d\alpha
\end{aligned} \tag{A.2}$$

The functions, which appear in Eqs (4.14) were obtained using the transformation and can be written as

$$\mathcal{WBR}_n = -\int\limits_0^\infty[(1-\sigma)\mathrm{e}^{-\sigma}\mathcal{BB}^*_{rn}+\sigma\mathrm{e}^{-\sigma}\mathcal{BB}^*_{zn}]\alpha J_1(\alpha r)\,d\alpha$$

$$\mathcal{WBZ}_n = -\int_0^\infty [-\sigma e^{-\sigma}\mathcal{BB}^*_{rn} + (\sigma+1)e^{-\sigma}\mathcal{BB}^*_{zn}\sigma]\alpha J_0(\alpha r)\, d\alpha$$

$$\mathcal{WDR}_n = -\int_0^\infty [(1-\sigma)e^{-\sigma}\mathcal{DD}^*_{rn} + \sigma e^{-\sigma}\mathcal{DD}^*_{zn}]\alpha J_1(\alpha r)\, d\alpha \qquad (A.3)$$

$$\mathcal{WDZ}_n = -\int_0^\infty [-\sigma e^{-\sigma}\mathcal{DD}^*_{rn} + (\sigma+1)e^{-\sigma}\mathcal{DD}^*_{zn}]\alpha J_0(\alpha r)\, d\alpha$$

where the symbols $\mathcal{BB}_{rn}$, $\mathcal{BB}_{zn}$, $\mathcal{DD}_{rn}$, $\mathcal{DD}_{zn}$ denote derivatives of the function given by Eqs (4.11) and (A.2) and can be written as

$$\mathcal{BB}_{rn} = \frac{\partial \mathcal{B}^*_{rn}}{\partial z} + \frac{\partial \mathcal{WB}_{rn}}{\partial z} \qquad \mathcal{DD}_{rn} = \frac{\partial \mathcal{D}^*_{rn}}{\partial z} + \frac{\partial \mathcal{WD}_{rn}}{\partial z}$$

$$\mathcal{BB}_{zn} = \frac{\partial \mathcal{B}^*_{zn}}{\partial z} + \frac{\partial \mathcal{WB}_{zn}}{\partial z} \qquad \mathcal{DD}_{zn} = \frac{\partial \mathcal{D}^*_{rn}}{\partial z} + \frac{\partial \mathcal{WD}_{zn}}{\partial z} \qquad (A.4)$$

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Współczynnik oporu cząsteczki poruszającej się w obecności ścianki

Streszczenie

W pracy została przedstawiona nowa analityczno-numeryczna metoda na wyznaczanie siły działającej na poruszającą się cząstkę w obecności ścianki o dowolnym kształcie, w przybliżeniu Stokesa. Opiera się ona na liniowości równań Stokesa, metodzie zaburzeń oraz metodzie kollokacji. Umożliwia wyznaczenie siły dla dowolnego ciała, którego kształt jest opisany we współrzędnych krzywoliniowych. Jako przykład obliczono siłę działającą w ruchu osiowosymetrycznym cząstki kulistej i porównano z wynikami otrzymanymi przez Fallade i Brenner (1988).

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