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MODEL OF STEERED TYRED WHEEL, THREE-DIMENSIONAL PROBLEM

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The paper presents a tyred wheel model. It describes an interaction between tyre and even as well as uneven road surface, in steady state or transient state. The influence of kingpin axis positioning as well as elastic properties of tyre and steering system is taken into account. The model results from a compromise between the following requirements: accuracy of phenomenon description, limitations of formal clearness, length of the computer program code, calculation time and feasibility of experimental verification of car movement simulation model. This tyred wheel model has been used effectively in simulation of biaxial vehicles movement and dynamics.

Key words: wehicle motion simulation, tyre model

1. Introduction

A model of tyred wheel is one of the main elements of simulation model of car's movement. It determines the qualitative and quantitative correctness of description of forces and moments generated within a zone of contact of the wheels with the road. Correct representation of the influence of three-dimensional positioning of the kingpin axis around which the steered wheel turns and to which elastic properties of the steering system and aligning moment of the wheels are reduced to also has significant importance in modelling.

Below, some works are listed, in which the reduction of forces existing in the contact of the wheel and road to the system fixed to axis of the kingpin which in effect gave a possibility of defining aligning moments of the steered wheels is presented. The simple calculation methods were employed, using only

geometric relations (cf Cronin, 1981; Lugner, 1981; Mitschke, 1977; Nordmark, 1984; Parraga, 1971; Studziński, 1980) or complex algorithms based on vector calculus (cf Berkefeld, 1983; Hiller and Woermie, 1985; Knapczyk and Kuranowski, 1986, 1987; Mitschke, 1977). Wicher (1995) considers the forces of inertia, which appear during transient states of the steering system movements, whereas other works include description of distortions of the elastic rubber-and-steel elements (silentblocks) (cf Berkefeld, 1983; Knapczyk and Kuranowski, 1986, 1987) as well as description of multi-arm suspension systems (cf Hiller and Woermie, 1985).

Much attention has been devoted to the interaction of vehicle with road through a such flexible element as the tyred wheel. Three-dimensional, elastic properties of the pneumatic tyre were described in terms of a matrix transforming the coupled linear and angular distortions of the tyre to the forces and moments (cf Bielecki, 1976; Bielecki and Maryniak, 1978). A model of string or ring with the elastic support (cf Pacejka, 1981; Zegelaar et al., 1994) were used or complex FEM models were applied (cf Böhm, 1979, 1989, 1993; Springer et al., 1989; Watanabe, 1984; Willumeit and Böhm, 1995) reflecting elastic properties in the range of frequencies up to 1000 Hz (cf Böhm, 1979, 1989, 1993) both in stationary and transient states (cf Springer et al., 1989; Willumeit and Böhm, 1995).

Bernard et al. (1981), Captain et al. (1979), Khachaturov et al. (1976), Davis (1975), Dunn and Olatunbosun (1989), Jackowski and Prochowski (1994), Lozia (1985), (1988), Lomako and Veremeev (1993) concentrated mainly on correct simulation of the elastic-damping forces (or elastic only) generation acting in radial direction in the plane of wheel. These models enable one to describe the forces resulting from the contact of the wheel with an uneven, rigid road surface with the lengths of irregularities comparable with those of the footprint. Lozia (1988) gave the description of radial and tangential (circumferential) forces for the above mentioned irregularities.

Largest number of publications has been devoted to description of forces and moments in the contact zone between wheel and road. Jędrzejczak (1980), Lozia (1985), Maryniak (1976), Mtschke (1977) presented the review of earlier efforts. In addition to description of the results of experimental studies of tyres (cf Baciński, 1969; Bakker et al., 1987; Bobrowski and Prochowski, 1989; Clark, 1981; Pacejka and Bakker, 1993; Sakai, 1981, 1982; Takahashi and Pacejka, 1988), numerous theoretical works were made, aimed at formulation of analytical description of tangential longitudinal (circumferential) force and the lateral one as well as the aligning moment of tyre (cf Dugoff et al., 1970; Fortunkov, 1980; Gim and Nikravesh, 1990, 1991; Maalej et al., 1989;

Mastinu and Fainello, 1992; Pacejka, 1972; Pacejka and Sharp, 1991; Palkovics and El-Gindy, 1993; Palkovics et al., 1994; Sakai, 1981, 1982; Sharp and El-Nashar, 1986, 1993). Currently most frequently used for this purpose are the semi-empirical models (cf Bakker et al., 1987; Dugoff et al., 1970;, Fancher and Bareket, 1993; Maalej et al., 1989; Pacejka and Bakker, 1993) which consist of approximations of experimental results. Despite a very close relation with a specific object and road surface, they are being used more and more frequently and so become a kind of standard in the simulation studies (HSRI-UMTRI model – see Dugoff et al., 1970, Fancher and Bareket, 1993; and "Magic Formula" – see Bakker et al., 1987, Pacejka and Bakker, 1993).

The most recent generation of models describing forces and moments in the tyre footprint are "Neuro- Tyre"-type models (cf Palkovics and El-Gindy, 1993; Palkovics et al., 1994), an application of the neural network method.

The models of transient states of interaction between tyre and road (cf Apetaur, 1993; Goraj, 1982; Mitschke, 1977; Palkovics et al., 1994; Takahashi and Pacejka, 1988; Willumeit and Böhm, 1995) are gaining more and more importance. Their applications allow for simulation of dynamic phenomena in the case of uneven road surfaces as well as better description of the "shimmy" – type phenomenon.

Below, the idea of a model of steered tyred wheel which enables description of many of the above mentioned phenomena is presented. It results from a compromise between the following requirements: accuracy of the phenomenon description, limitations of formal complexity, length of the computer code, calculation time and feasibility of experimental verification of the car movement simulation model. This tyred wheel model has been used effectively in simulation of biaxial vehicles movement (cf Lozia, 1993; Lozia and Stegienka, 1996).

2. Primary assumptions

The model of tyred wheel is an element of car movement simulation model (Fig.1). It should enable us to describe tyre footprint forces and elastic-damping properties of the front and rear wheels (steered and non-steered) and make allowance for geometric and kinematic properties of the steering system as well as elastic properties of the steering knuckle guidance system. It has been assumed that the road surface is uneven and undeformable. The road profile may result from real road measurements or realization of the random

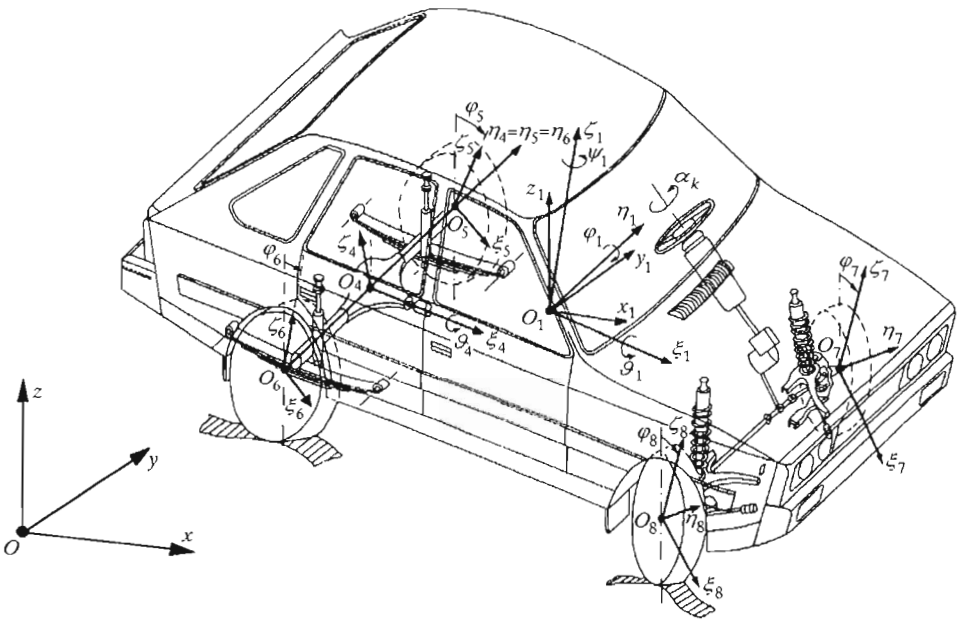

Fig. 1. Structure of the biaxial car model with the coordinate systems adopted

process (according to ISO/TC 108/253 recommendations) described using the bicubic spline interpolation. In the region of contact geometrical properties of the road are represented by the plane π_8 tangential to the three-dimensional road profile (Fig.2).

Since we examine the surface irregularities longer at least several times than the largest dimension of the contact zone. Shorter irregularities are being averaged within the footprint area. We analyze the case in which (Fig.2) the tyred wheel interacts with the road plane π_8 and we know: camber angle φ_{c8} related to the normal vector of this surface, normal reaction $\boldsymbol{N}_8$, velocity of the wheel center, longitudinal (circumferential) slip and lateral slip angle. In such a case it is always possible to use one of the known, experimentally verified models of interaction between the tyred wheel and flat road surface. Among the geometric properties of suspension only those influencing strongly the properties of tyred wheel model have been described. The relations representing the model are demonstrated on the example of front, right-hand side wheel.

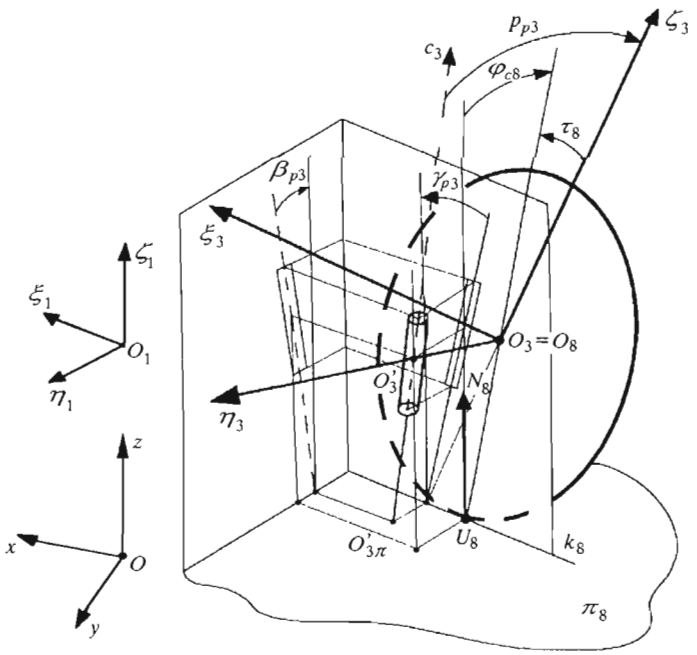

Fig. 2. Position of the kingpin axis $O'_3O'_{3\pi}$ in the body-fixed system $O_1\xi_1\eta_1$ (angles β_{p3}, γ_{p3}) and of the tyred wheel relative to the plane π_8 tangential to the road surface at point of contact U_8 (angle φ_{c8}). Here the plane π_8 is horizontal ($\varphi_{c8} = \varphi_{c3}$), and the steering angle of the wheel is equal to zero. Elastic deformations of the steering system have not been shown. Right-hand side, front wheel

3. Coordinate systems used for description of the vehicle movement

Vehicle motion is described in the inertial reference system $Oxyz$ connected with the road (Fig.1). In the center of the vehicle's body mass O_1 there has been attached a movable system $O_1x_1y_1z_1$ with the axes parallel to respective axes of the inertial system. The system $O_1\xi_1\eta_1\zeta_1$ is rigidly connected with the body. The angles: ψ_1 (yaw), φ_1 (pitch), ϑ_1 (roll) describe the body rotation about a fixed point O_1. In the middle of the rear axle O_4 (dependent suspension) there has been attached the system $O_4\xi_4\eta_4\zeta_4$ rigidly connected with the axle. Together with the wheels rotate the systems attached in the centers of the wheels. These are denoted by $O\xi\eta\zeta$ with indices 7,8 and 5,6 defining location of the wheels in the vehicle. This model of the vehicle has 14 degrees of freedom (Fig.1: x_{O_1}, y_{O_1}, z_{O_1}, ψ_1, φ_1, ϑ_1, ζ_{1O_4}, ϑ_4, $\zeta_{1O'_2}$, $\zeta_{1O'_3}$,

φ_5, φ_6, φ_7, φ_8, where O_2' and O_3' are the points of intersection of kingpins with the axes of wheels rotation).

4. Geometric properties of the suspension and geometric-elastic properties of steering column, steering gear and steering knuckle system

Three-dimensional location of the kingpin axis is described (Fig.2) in the body-fixed system $O_1\xi_1\eta_1\zeta_1$ by angles: kingpin inclination β_{p3} and castor angle γ_{p3}. Apart from this the wheel plane is the angle of p_{p3} from the kingpin and angle of φ_{p3} from the vertical and φ_{c8} from the normal to the contact plane.

For the independent suspension the angles β_{p3} and γ_{p3} are functions of its deflection

$$\beta_{p3} = \beta_{p3}(\zeta_{1O_3'}) \qquad \gamma_{p3} = \gamma_{p3}(\zeta_{1O_3'}) \tag{4.1}$$

For a dependent suspension these angles are also functions of deflection of the other wheel

$$\beta_{p3} = \beta_{p3}(\zeta_{1O_3'}, \zeta_{1O_2'}) \qquad \gamma_{p3} = \gamma_{p3}(\zeta_{1O_3'}, \zeta_{1O_2'}) \tag{4.2}$$

Driver is acting upon the vehicle by changing the steering wheel angle α_k. The arm of the steering gear is rotated by angle α_w which is a function of the angle α_k, steering gear ratio i_k and aligning moments R_{2c2}, R_{3c3} of the left and right road wheel

$$\alpha_w = \alpha_w(\alpha_k, i_k, R_{2c2}, R_{3c3}) \tag{4.3}$$

Therefore, there has been taken into account the flexibility of steering column and gear.

The steering angle of the wheel about the axis of the kingpin α_{p3} is a function of α_k, suspension deflections of both wheels $\zeta_{1O_3'}$ and $\zeta_{1O_2'}$, wheel toe-in angle α_{z3} and elastic deformation of the steering-linkage system which is a function of aligning moments R_{2c2} and R_{3c3}

$$\alpha_{p3} = \alpha_{p3}(\alpha_k, \zeta_{1O_3'}, \zeta_{1O_2'}, \alpha_{z3}, R_{2c2}, R_{3c3}) \tag{4.4}$$

The functions which appear in Eqs (4.1) $\div$ (4.4) are defined experimentally or result from the analysis of geometric and kinematic properties of suspension.

The wheel plane relative velocities (observed in the system $O_1\xi_1\eta_1\zeta_1$) resulting from Eqs (4.1) $\div$ (4.4) are neglected because they are many orders lower than those resulting from the considered degrees of freedom (Fig.1: x_{O_1}, y_{O_1}, z_{O_1}, ψ_1, φ_1, ϑ_1, ζ_{1O_4}, ϑ_4, $\zeta_{1O_2'}$, $\zeta_{1O_3'}$, φ_5, φ_6, φ_7, φ_8).

5. Elastic properties of the steering knuckle guidance system

In the majority of constructions the steering knuckle guidance system consists of: upper and lower A-arms with joints seated flexibly using rubber-and-steel elements (silentblocks), transverse link and McPherson strut with flexibly seated joints (McPherson suspension), leaf spring working as a guide, and rigid axle flexibly seated. The vector

$$\boldsymbol{w}_{k3} = \mathrm{col}[\gamma_3, \beta_3, \alpha_3] \tag{5.1}$$

defines the angular position of the steering knuckle in the body-fixed $O_1\xi_1\eta_1\zeta_1$ system. Elastic deformations of the guidance system are taken into account since the elements of this vector are sums of the values resulting from geometric properties of the suspension and steering system (Eqs (4.1) $\div$ (4.4)) as well as angular deformations of the steering knuckle guidance system caused by the resultant moment $\boldsymbol{R}_3$

$$\boldsymbol{w}_{k3} = \boldsymbol{w}_{p3} + \mathsf{D}_3\boldsymbol{R}_3 \tag{5.2}$$

where D – matrix of angular flexibility of the steering knuckle guidance system and

$$\boldsymbol{w}_{p3} = \mathrm{col}[\gamma_{p3}, \beta_{p3}, \alpha_{p3}] \qquad \boldsymbol{R}_3 = \mathrm{col}[R_{3a3}, R_{3b3}, R_{3c3}] \tag{5.3}$$

$\boldsymbol{R}_3$ is the resultant moment of the system of forces acting upon the tyred wheel, reduced to the point O_3' and the components defined in the $O_3'a_3b_3c_3$ system, where O_3' is the point of intersection of steering knuckle kingpin axis $O_3'c_3$ and the axis of rotation of the wheel $O_3'\eta_3$, whereas $O_3'a_3$ is parallel to the plane of wheel.

Because of high rigidity of the steering knuckle, changes in the angle p_{p3} caused by the system of forces acting on the wheel have been neglected.

6. Elastic properties of the steering system

Elasticity of the steering column and steering gear has been taken into

account in Eq (4.3), elasticity of the steering linkage – in Eq (4.4), and elasticity of the steering knuckle guidance system in Eq (5.2), respectively. Such a method of making allowance for the elastic properties of the steering system seems to be troublesome but nevertheless it allows us to apply the model described about in a simple way to defining flexible properties of steered and as well as non-steered wheels guidance system.

7. Model of elastic-damping properties of the tyred wheel

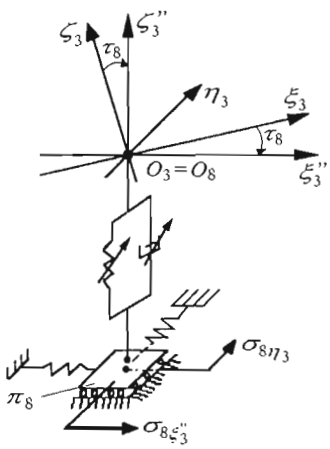

Fig. 3. Model of elastic-damping properties of tyre along the radial direction $O_3\zeta_3''$ and elastic properties in the longitudinal $\sigma_{8\xi_3''}$ and lateral $\sigma_{8\eta_3}$ directions. Right-hand side, front wheel

Tyre elastic-damping properties in a radial direction and only elastic properties in longitudinal and lateral directions have been taken into consideration (Fig.3). Generally, the elastic-damping characteristics are non-linear and they are being approximated by polynomials. Adoption - for the radial properties - of the "point-contact tyre model" (cf Captain et al., 1979; Khachaturev et al., 1976; Lozia, 1985) results from the assumption that the area of contact with uneven road surface is a sector of the plane π_8 tangential to the road at the center of footprint. Irregularities of the road are being averaged in this area (heights and inclinations in two perpendicular directions x and y) which makes the applied model similar to the "fixed-footprint tyre model" (cf Captain et al., 1979; Khachaturev et al., 1976; Lozia, 1985). Such a procedure

is justified by the fact that irregularities of the road shorter then the length of footprint are the source of oscillation of pneumatic tyre forces with the frequencies higher by several orders than the vehicle model natural frequencies. They are not significant for the analysis of the vehicle's movement.

8. Model of tangential properties of the tyred wheel

In accordance with the assumptions adopted here, any model of tangential (shear) properties of the pneumatic tyre can be applied – which can generally be noted analytically in the following way (without the indices identifying the wheel)

$$\begin{aligned} \mu_w &= \frac{B_w}{N} = \mu_w(v, s_w, \delta, N, \varphi_c, \dot{s}_w, \dot{\delta}, \dot{N}, \dot{\varphi}_c) \\ \mu_p &= \frac{B_p}{N} = \mu_p(v, s_w, \delta, N, \varphi_c, \dot{s}_w, \dot{\delta}, \dot{N}, \dot{\varphi}_c) \\ R_n &= R_n(v, s_w, \delta, N, \varphi_c, \dot{s}_w, \dot{\delta}, \dot{N}, \dot{\varphi}_c) \end{aligned} \tag{8.1}$$

where

μ_w – coefficient of longitudinal adhesion

μ_p – coefficient of lateral adhesion

R_n – aligning moment in the direction normal to the area of contact, reduced to the center of footprint or the center of wheel

B_w – tangential longitudinal force, in the plane of footprint

B_p – tangential lateral force, in the plane of footprint

N – normal reaction of the road

v – projection of the center of wheel velocity on the plane of footprint

s_w – relative longitudinal (circumferential) slip

δ – angle of lateral slip

φ_c – angle of wheel inclination relative to the normal to the plane of contact.

$\dot{s}_w, \dot{\delta}, \dot{N}, \dot{\varphi}_c$ – time derivatives of $s_w, \delta, N, \varphi_c$.

Taking into consideration the influence of $\dot{s}_w$, $\dot{\delta}$, $\dot{N}$, $\dot{\varphi}_c$ on the magnitudes values of forces and moment acting in the footprint enables us to describe transient states during the tyred wheel movement.

9. Systems of coordinates used for transformation from the system $O_1\xi_1\eta_1\zeta_1$ fixed to the vehicle body to the system $O_8\xi_8\eta_8\zeta_8$ connected with wheel

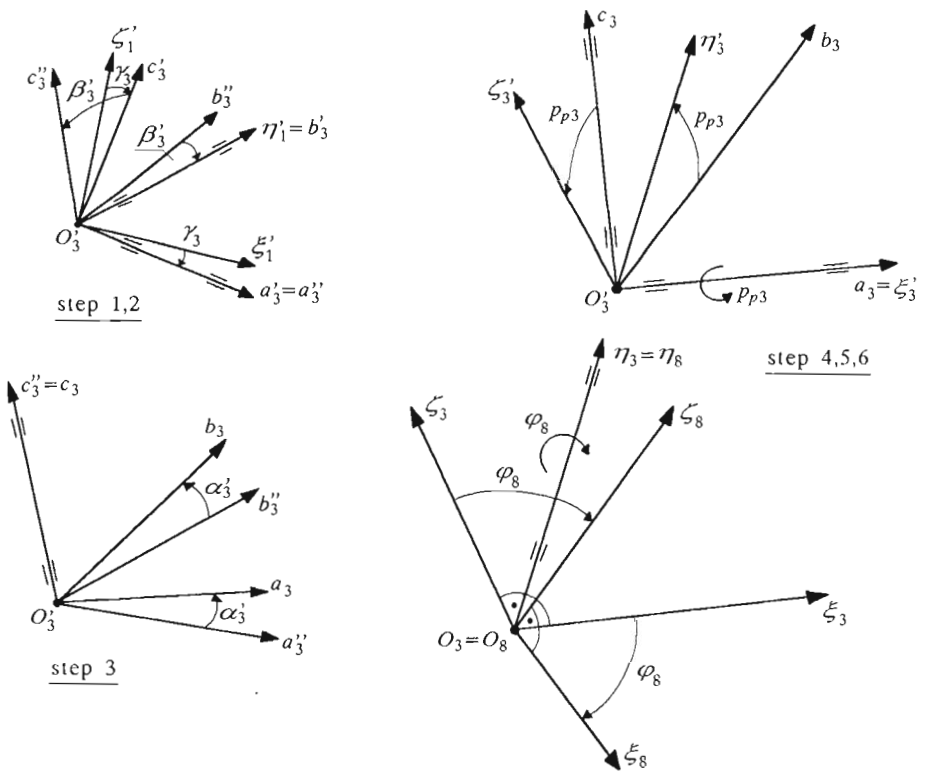

Fig. 4. Coordinate systems used for transformation from the body-fixed system $O_1\xi_1\eta_1\zeta_1$ to the system $O_8\xi_8\eta_8\zeta_8$ fixed to the right-hand side front road wheel

At the point O_3' of intersection of the kingpin axis with the wheel axis of rotation the system $O_3'\xi_1'\eta_1'\zeta_1'$ has been attached (Fig.4), with the axes parallel to the axes of the system $O_1\xi_1\eta_1\zeta_1$, respectively. Rotation has been executed about axis $O_3'\eta_1'$ by the castor angle γ_3.

Next, the resulting system $O_3'a_3'b_3'c_3'$ has been rotated by the angle β_3' which is the projection of castor angle β_3 onto the plane $O_3'b_3'c_3'$

$$\beta_3' = -\arcsin \frac{-\cos\gamma_3}{\sqrt{\cos^2\gamma_3 + \cot^2\beta_3}} \tag{9.1}$$

The system $O_3'a_3''b_3''c_3''$ was obtained. $O_3'c_3''$ is the axis of kingpin.

The third rotation is about axis $O_3'c_3''$ by angle α_3'. This creates the system $O_3'a_3b_3c_3$. The axis $O_3'c_3$ coincides with the axis $O_3'c_3''$ – it is the axis of kingpin. The angle α_3' should ensure position of the axis $O_3'a_3$ in the plane parallel to the wheel plane

$$\alpha_3' = \alpha_3 + \arcsin \frac{1}{\sqrt{\left(\frac{\cos\beta_3' \cos\varphi_{p3} + \sin\beta_3' \cos\gamma_3 \sin\varphi_{p3}}{\sin\gamma_3 \sin\varphi_{p3}}\right)^2 + 1}} \tag{9.2}$$

The second element of this relation results from the necessity for compensation of the component of the wheel turn which is the consequence of earlier rotation by the angle γ_3.

The fourth turn takes place around axis $O_3'a_3$ by the angle p_{p3}. The plane $O_3'\xi_3'\zeta_3'$ of the system $O_3'\xi_3'\eta_3'\zeta_3'$ created as a result of this turn is parallel to the plane of wheel.

Translation by the vector

$$\boldsymbol{s}_3 = \mathrm{col}[\xi_{3O_3}', \eta_{3O_3}', \zeta_{3O_3}'] \tag{9.3}$$

enables one to pass to the system $O_3\xi_3\eta_3\zeta_3$ fixed at the center of wheel $O_3 = O_8$.

Rotation by φ_8 (angle of rotation of the wheel) enables one to pass to the system $O_8\xi_8\eta_8\zeta_8$ fixed to the wheel rim.

Knowing the coordinates of the center of mass of the body O_1 in the inertial system $Oxyz$, knowing the transformation matrix of the body rotation about fixed point (angles $\psi_1, \varphi_1, \vartheta_1$), as well as the matrix defining the above described transformations one can calculate the coordinates of any point of the wheel in the inertial system $Oxyz$ and in system fixed to the vehicle body $O_1\xi_1\eta_1\zeta_1$.

10. Forces and moments appearing in the footprint

In order to calculate the forces transmitted by the tyred wheel we have to define the coordinates of the point of contact U_8 and the coordinates of location and components of velocity of this point relative to the center of wheel O_3. This is the reason why calculations are made within the moving systems linked to the vehicle body. Additionally, because the irregularities of the road surface are defined in the inertial reference system $Oxyz$, the calculations also have to be made in that system. Fig.5 shows the resultant

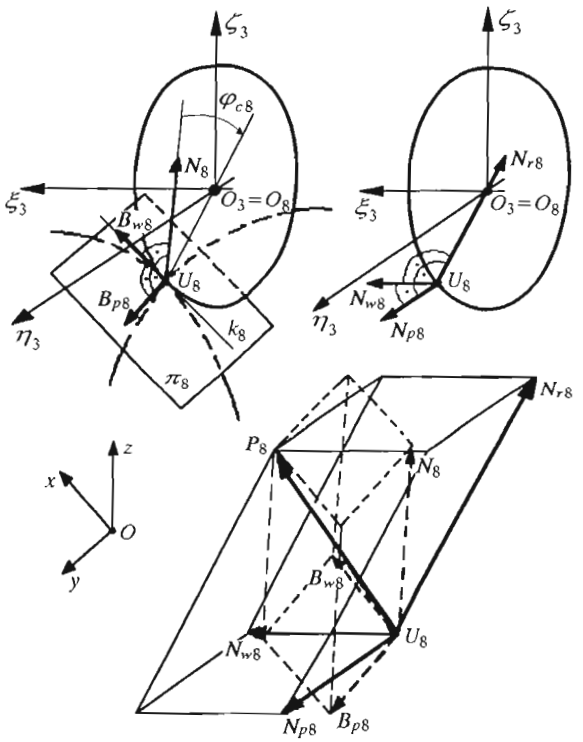

Fig. 5. Two equivalent force systems at the contact point U_8 between the tyred wheel and road

vector of contact forces $\boldsymbol{P}_8$. This is the resultant for the two systems of three mutually perpendicular vectors:

- System of three forces related to the road $\boldsymbol{N}_8$, $\boldsymbol{B}_{w8}$, $\boldsymbol{B}_{p8}$ (the reaction normal to the road profile and two tangential reactions located in the plane of contact π_8: the longitudinal one, at the same time located in the plane parallel to the plane of the wheel and lateral, perpendicular to k_8, the line of intersection of the road plane π_8 and the plane of wheel); these forces are defined using the tyre tangential (shear) properties model Eqs (8.1).

- System of forces related to the wheel $\boldsymbol{N}_{r8}$, $\boldsymbol{N}_{w8}$, $\boldsymbol{N}_{p8}$ (the radial reaction and longitudinal reaction – perpendicular to the former – located in the plane parallel to the plane of wheel and the lateral reaction, perpendicular to the plane of wheel); these forces are defined using a model of elastic-damping properties of tyre.

The algorithm of calculations of modules and components of versors for these six forces is as follows.

Basing on the current values of coordinates of the body center of mass $x_{O_1}, y_{O_1}, z_{O_1}$, the angles defining body rotation about the fixed point $O_1(\psi_1, \varphi_1, \vartheta_1)$, the coordinate $\zeta_{1O_3'}$ of relative movement of point O_3' as well as the angles γ_3, β_3, α_3, p_{p3}, coordinates and velocities of the center of wheel O_3 are defined in the inertial reference system $Oxyz$.

The point T_3 (not shown in Fig.5) is taken as the first approximation of the wheel and road contact point U_8. This point is the intersection point of the road surface and a straight line, the edge of intersection of the wheel plane and a vertical plane, perpendicular to the wheel plane and crossing its center. In order to simplify and speed-up the calculations, we look for a point lying on this line at the distance equal to the length of dynamic radius approximated by the value from the previous step of simulation. Next, using similar approximation of magnitudes of forces $\boldsymbol{N}_{r8}$ and $\boldsymbol{N}_{p8}$ we calculate the longitudinal and lateral dislocations of the center of footprint σ_{8w}, σ_{8p} which are then transformed to the inertial reference system $(\sigma_{8x}, \sigma_{8y}, \sigma_{8z})$

$$\boldsymbol{\sigma} = \mathrm{col}[\sigma_{8x}, \sigma_{8y}, \sigma_{8z}] \tag{10.1}$$

The coordinates of the point of contact U_8 are being calculated as

$$x_{U8} = x_{T3} + \sigma_{8x} \qquad y_{U8} = y_{T3} + \sigma_{8y} \tag{10.2}$$

The height of profile z_{U8} and the angles of inclination of the plane of contact π_8 in directions x and y: ε_{8x}, ε_{8y} are being calculated basing on description of the road profile in the following form

$$z_{U8} = z_U(x_{U8}, y_{U8}) \qquad \varepsilon_{8x} = \varepsilon_x(x_{U8}, y_{U8}) \qquad \varepsilon_{8y} = \varepsilon_y(x_{U8}, y_{U8}) \tag{10.3}$$

The length r_8 of projection of the vector $\boldsymbol{U_8O_8}$ onto the wheel plane $O_3\xi_3\zeta_3$ is being treated as dynamic radius of the pneumatic tyre. This projection enables us to define the versor $\boldsymbol{n}_{r8}$ of the radial force $\boldsymbol{N}_{r8}$ and the angle of rotation τ_8 of the system $O_3\xi_3\eta_3\zeta_3$ about the axis $O_3\eta_3$ in such a way that in thus created system $O_3\xi_3''\eta_3''\zeta_3''$ (Fig.3) the axis $O_3\zeta_3''$ has the direction of force $\boldsymbol{N}_{r8}$ and $O_3\xi_3''\zeta_3''$ is the wheel plane. Knowing the projections of the center of wheel velocity onto the plane Oxy and the inclinations of road surface ε_{8x}, ε_{8y} we may calculate the velocity dz_{U8}/dt and the vertical component of relative velocity dU_8O_8/dt, and next (knowing components of versor $\boldsymbol{n}_{r8}$) the velocity of radial deflection dr_8/dt of the tyre. The module of radial force is a function defined in the course of experimental studies

$$N_{r8} = N_r\left(r_8, \sigma_{8w}, \sigma_{8p}, \frac{dr_8}{dt}\right) \tag{10.4}$$

The next step is to calculate the geometric and kinematic parameters of tangential interaction with the road: φ_{c8}, s_{w8}, s_{p8}, v_8. They are functions of generalized coordinates and parameters of the model. Using the tyre tangential (shear) properties model Eqs (8.1) we calculate the values of coefficients of adhesion μ_{w8}, μ_{p8} as well as the aligning moment R_{n8}. The module of road normal reaction N_8 is being approximated here using the value from the previous step of simulation.

As we see in the above, for the six vectors shown in Fig.5 we known:

- module of radial force $\boldsymbol{N}_{r8}$
- versors of vectors $\boldsymbol{N}_{w8}$, $\boldsymbol{N}_{p8}$, $\boldsymbol{N}_{r8}$ (respective versors of the axes of system $O_3\xi_3''\eta_3''\zeta_3''$)
- versor of the force $\boldsymbol{B}_{w8}$ (versor of the edge k_8)
- versor of the force $\boldsymbol{B}_{p8}$ (vector product of normal vector versor of the contact plane π_8 and versor of the edge k_8)
- versor of the force $\boldsymbol{N}_8$ (normal vector versor of the contact plane π_8).

Unknown are the modules of vectors $\boldsymbol{N}_{w8}$, $\boldsymbol{N}_{p8}$, $\boldsymbol{B}_{w8}$, $\boldsymbol{B}_{p8}$, $\boldsymbol{N}_8$. The last three bear the relations (also see Eqs (8.1))

$$B_{w8} = \mu_{w8} N_8 \qquad B_{p8} = \mu_{p8} N_8 \tag{10.5}$$

The three remaining unknowns: N_{w8}, N_{p8}, N_8 are being found as solutions of the system of equations created as the result of projecting the footprint forces onto axes of the system $O_3\xi_3''\eta_3''\zeta_3''$

$$\begin{aligned} N_{w8} &= N_8 n_{8\xi_3''} + B_{w8} b_{w8\xi_3''} + B_{p8} b_{p8\xi_3''} \\ N_{p8} &= N_8 n_{8\eta_3''} + B_{p8} b_{p8\eta_3''} \\ N_{r8} &= N_8 n_{8\zeta_3''} + B_{w8} b_{w8\zeta_3''} + B_{p8} b_{p8\zeta_3''} \end{aligned} \tag{10.6}$$

where n and b with indices 8, ξ_3'', η_3'', ζ_3'' represents the components of versors of respective vectors on the axes indicated by the indices.

Next, we calculate the values B_{w8}, B_{p8}, Eqs (10.5), the resultant vector of forces in the footprint $\boldsymbol{P}_8$ and the vector of aligning moment $\boldsymbol{R}_{n8}$ relative to the center of footprint.

The system of forces in the footprint is reduced to the center of wheel O_3 and to the center of kingpin O_3'. In the latter case we take into consideration the fact that the component of the moment on the axis of rotation is not transferred as whole but only its part dependent on the value of angular deceleration and the moment of inertia of the wheel and the external torque (driving or breaking). So, we calculate the vector $\boldsymbol{R}_3$ (see Eq (5.3)).

11. Results of sample calculations

Applicability of the model shall be demonstrated on the example of tyre 165R13D124 and passenger car FSO Polonez. The geometric and elastic parameters of the tyre were adopted basing on static test stand studies. Geometrical parameters of the suspension and steering system were defined basing on the design drawings and laboratory measurements. Other values were estimated using specifications of tyres and car of similar class (cf Clark, 1981; Dugoff et al., 1970; Mitschke, 1977; Pacejka, 1972, 1981; Pacejka and Sharp, 1991; Sakai, 1981, 1982).

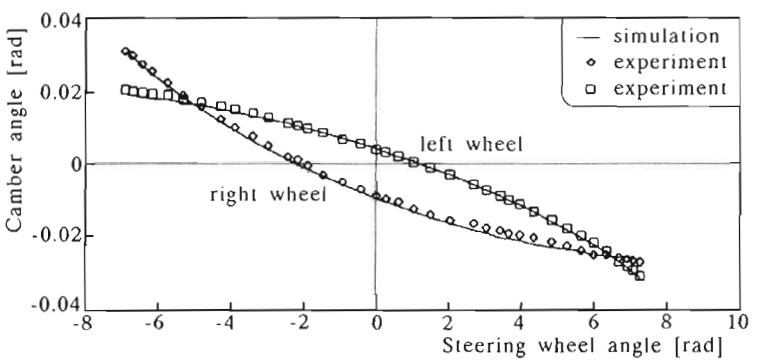

Fig. 6. Changes of the wheel camber angle φ_{p3} during turn which is the result of the rotation of steering wheel α_k. The wheels transmit only the normal reaction of horizontal fundation. Passenger car FSO Polonez of the type

In order to verify geometric properties of the model the simulation has been performed of the phenomena occurring during turning of the wheels caused by a quasi-static application of the steering wheel angle. Wheels do not rotate, instead they just sit on the so-called "turn-tables" which only transmit the normal reactions of the foundation. Vertical position of the centers of wheels is calculated by iteration in such a way as to maintain with a given accuracy 1 N a constant magnitude of 3000 N of normal reaction. The parameters of the car model correspond the specimen of FSO Polonez under study. Fig.6 shows the results of simulation and the results of experimental tests of the wheel camber angle as a function of the angle of steering wheel rotation. A very good consistency , both qualitative and quantitative, between the simulation and experiment results is visible.

It was very hard to obtain the data describing tangential (shear) properties of the tyre. That is why a relatively simple in use the HSRI-UMTRI model

(cf Dugoff et al., 1970; Fancher and Bareket, 1993) has been applied. Specifications of similar tyres have been used (cf Clark, 1981; Dugoff et al., 1970; Mitschke, 1977; Pacejka, 1972, 1981; Pacejka and Sharp, 1991; Sakai, 1981, 1982).

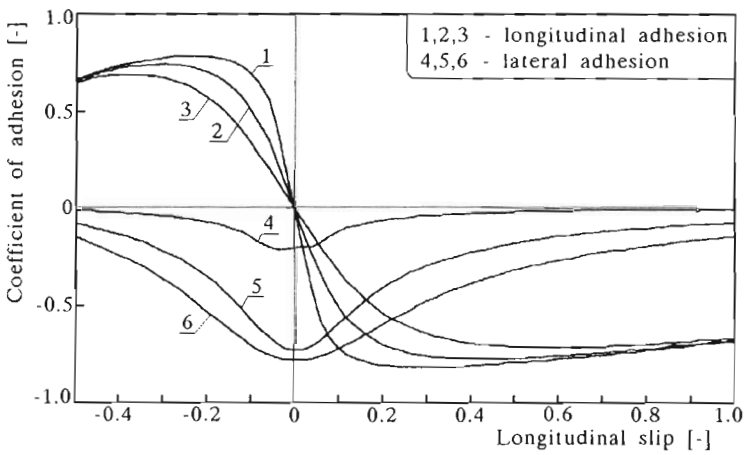

Fig. 7. Characteristics of the coefficients of longitudinal adhesion μ_w (lines 1, 2, 3) and lateral adhesion μ_p (lines 4, 5, 6) versus the relative longitudinal slip s_w (negative for drive, positive for breaking) for constant values of the lateral slip angle δ (zero for 1 and 4, 0.15 rad for 2 and 5, 0.3 rad for 3 and 6). Road surface: dry asphalt-concrete. Wheel cambar angle $\varphi_{p3} = 0.1$ rad, normal reaction of road $N_8 = 3000$ N. Velocity $V = 10$ m/s. Results of simulation

Fig.7 shows the characteristics of coefficient of longitudinal and lateral adhesion on dry surface for constant values of lateral slip angle and the camber angle of 0.1 rad value.

The model of steered tyred wheel presented here has been put into use as an element of the simulation model of movement of biaxial passenger car FSO Polonez (cf Lozia, 1993; Lozia and Stegienka, 1996). Fig.8 and Fig.9 show sample results of simulation of a single lane change maneuver. The lane width was equal to 3.5 m. We used the time history of steering wheel angle, recorded during an experiment. Fig.8 presents the comparison between lateral accelerations of the vehicle: resulting from measurements (E) and simulation (S), respectively. Fig.9 shows the projection of trajectory of the center of mass of the vehicle body O_1 onto the plane of road. Bold, solid lines define the boundaries of the first and second traffic lane. Both drawings prove correct functioning of the tyred wheel model. This applies to both qualitative as well as quantitative consistency.

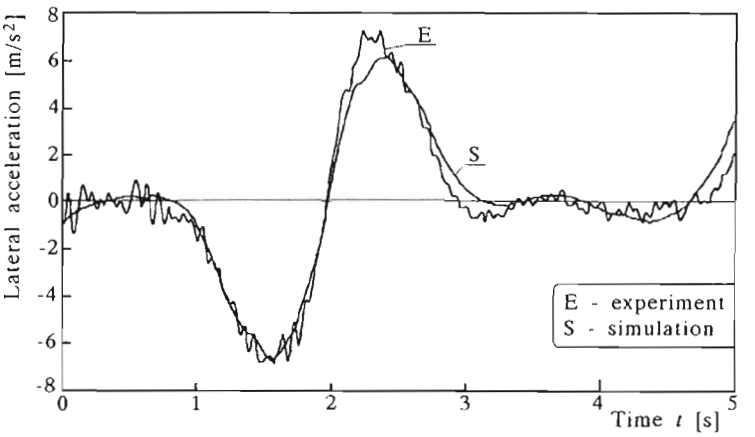

Fig. 8. Time histories of lateral acceleration during a single lane change maneuver. Experiment and simulation. Passenger car of the FSO Polonez type, $V = 19.1$ m/s

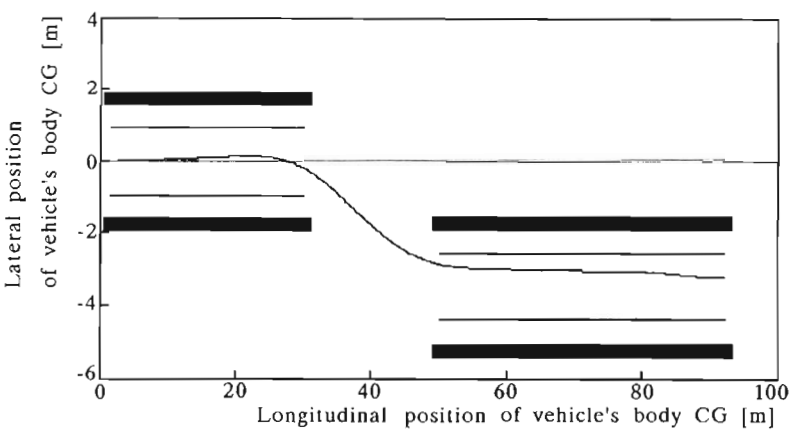

Fig. 9. Projection of trajectory of the center of body mass O_1 of the FSO Polonez type car onto the plane of road during a single lane change maneuver, $V = 19.1$ m/s. Results of simulation. Bold, solid lines define the boundaries of first and second traffic lanes

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Model kierowanego koła ogumionego, zagadnienie przestrzenne

Streszczenie

Artykuł przedstawia model kierowanego koła ogumionego, umożliwiający opis oddziaływania z równą i nierówną nawierzchnią drogi, w stanach ustalonych i nieustalonych. Uwzględniono charakterystyczne dla koła kierowanego, przestrzenne usytuowanie osi sworznia zwrotnicy (osi zataczania) oraz własności sprężyste opony i układu kierowniczego. Model stanowi kompromis między wymaganiami: dokładności opisu zjawisk, ograniczenia formalnej złożoności, długości kodu programu, czasu obliczeń i możliwością eksperymentalnej weryfikacji modelu symulacyjnego ruchu samochodu. Opis modelu uzupełniono przykładowymi wynikami obliczeń symulacyjnych oraz weryfikacji eksperymentalnej. Model ten jest od kilku lat z powodzeniem stosowany w symulacji ruchu pojazdów dwuosiowych.

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