MECHANIKA TEORETYCZNA
I STOSOWANA
Journal of Theoretical
and Applied Mechanics
2, **34**, 1996

EXPERIMENTAL SETUP FOR TESTS UNDER BIAXIAL TENSION OF ANISOTROPIC CRUCIFORM SPECIMEN

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Testing method for suitable measurement of the mechanical properties of materials is considered. Mutually perpendicular biaxial tension forces applied to the edges of a flat cruciform specimen produce in its thinnest part in-plane biaxial stresses. The problem of homogeneity of stresses is discussed in terms of the specimen shape design. The laminar specimen structure is necessary for testing the directional properties of materials with the induced anisotropy such as the fibre reinforced plastics usually fabricated in a sheet form or the rolled sheet-metals. The uniformity condition of the stress-strain field allowing the strain field measurement on the plane surface by means of the mechanical extensometer the idea of which is here pointed out. This paper highlights the problems arising in measuring, monitoring and control of strains in cruciform specimen. The testing machine and gripping system for inelastic tests at room and elevated temperatures are also described.

List of symbols

δ_{ij} – Kronecker delta
F_{ij} – deformation gradient
ε_{ij} – strain tensor components
γ_{ij} – shear tensor components
$u_{i,j}$ – partial derivatives of the displacement vector $\boldsymbol{u}$
$\boldsymbol{u}$ – displacement vector
$d\boldsymbol{s}$ – linear element vector

x_i – coordinates of the Lagrangian formulation
a_i – coordinates of the Eulerian formulation
Δa_i – length of the undeformed fibre
Δx_i – length of the deformed fibre
J – Jacobian

1. Introduction

The laboratory testing of material mechanical properties for yield, ultimate, fracture, fatigue or creep under multiaxial loading conditions is usually carried out establishing biaxial and triaxial stress states as follows:

- Stressing of thin-walled tubes by combinations of applied tension, torsion, internal or external pressure
- Orthogonal loading of flat cruciform specimens.

The flat cruciform specimen allows a complete range of the in-plane stress state by altering the levels and phases of the loads to be applied to the specimen arms. The experimental technique of the cruciform specimen is useful for materials fabricated in a sheet form such as rolled sheet-metals or fibre reinforced materials subjected to the static or cyclic biaxial loading tested at low, medium or high strain rate including studies of crack propagation.

By means of the biaxial tension specimen shape, the different experimental techniques have been reviewed (cf Waniewski et al. (1993); Demmerle and Boehler (1993)) imposing the following requirements:

- The stress and strain distribution over the gauge area of the specimen should be as homogeneous as possible
- It should be possible to determine properly the developed stress state, i.e., the magnitudes of the stresses and the orientation of the stress principal directions
- Initial yielding should occur in the central gauge area of the specimen
- For any orientation Θ between the stress principal directions and the privileged axes of the initial orthotropy, either a constant loading ratio or variable loading ratios in a wide range during the test should be allowed.

By means of this technique, the cross-shaped specimen of uniform as well as of reduced thickness of the central measurement section with slotted arms is mentioned in the literature (cf Waniewski et al. (1993); Demmerle and Boehler (1993)).

The stress-strain analysis of a flat cruciform specimen has been carried out on the Cray X-MP computer of the Research Centre in Juelich, F.R.G. The PERMAS finite element codes have been used to get the first insight into the stress distribution of the model designed with a thinned down square centre (cf Betten et al. (1990)). Further investigations on the model with a thinned central circular test section have been undertaken in the elastic and inelastic ranges by applying the ABAQUS finite element codes (cf Waniewski et al. (1993)).

The strain field developed into the test section of specimen is evaluated at room and elevated temperatures using the mechanical extensometer, the idea of which is based upon a system of double bending flexible units allowing simultaneous and independent measurement of two orthogonal plane displacements at each of three points located onto the working section of the specimen (cf Waniewski (1990)).

The testing machine has been specially designed and built to enable the use of dead weights and levers for steady loading to be reliably maintained for the long periods of time encountered in creep testing.

2. Cruciform specimen geometry

In experimental procedures for material behaviour under combined loadings the shape design is a major problem, since of the central gauge area must reveal the maximum and uniform stress-strain state in the specimen. For that reason the stress-strain analyses in elastic and inelastic ranges, respectively, has been undertaken assuming a uniform in-plane stress field over as much of the thinned down circular centre as possible using the finite element code ABAQUS (cf Waniewski et al. (1993)).

The laminar specimen structure illustrated in Fig.1 is necessary for testing the directional properties of materials with the induced anisotropy such as the fibre reinforced plastics usually fabricated in a sheet form or the rolled sheet-metals. In this case, the middle layer of the laminar structure possesses the central measurement area, which must have the maximum strain in the specimen, to deform without disturbance in the material structure caused by the mechanical treatment along the varied thickness. Except the middle layer,

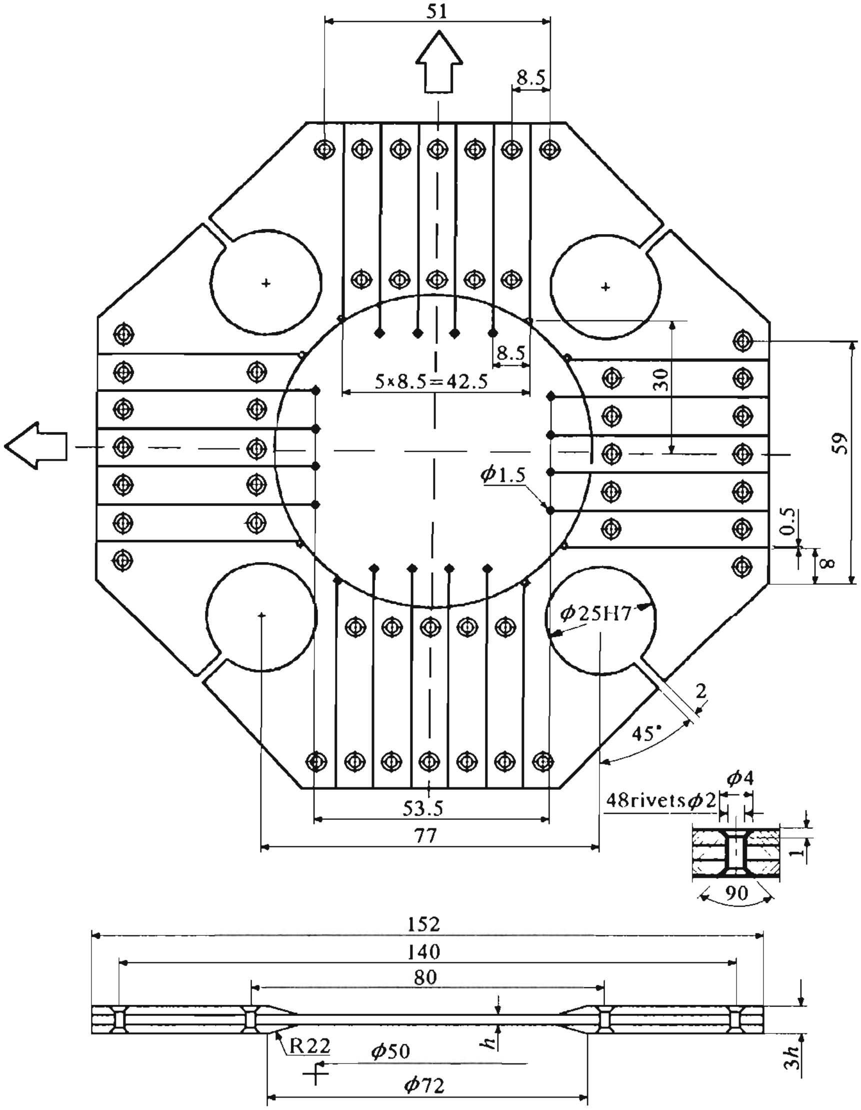

Fig. 1. Biaxial tensile flat cruciform specimen (all dimensions in mm)

the cross-shaped body consists of two outer layers with thickened four arms of the middle layer, each with the centrally placed circular hole surrounded by the filled region to reinforce specimen structure. The three layers of laminar body are of the same thickness h, strongly joined with each other by riveting and by bounding appropriate surfaces of the laminar structure. The specimen presented in Fig.1 is modified (cf Waniewski et al. (1993)) by trapezoidally-shaped arms with four holes needed for the appropriate mechanical treatment of the final laminar cruciform structure.

A test specimen illustrated in Fig.1 consists of a series of limbs as a result of slotting the material from the edge of the centre section to the outer surface. A series of seven slots on each thickened arm prevents from lateral constraint on the central test section when deformation occurs in the plane perpendicular to the slots.

3. Testing machine

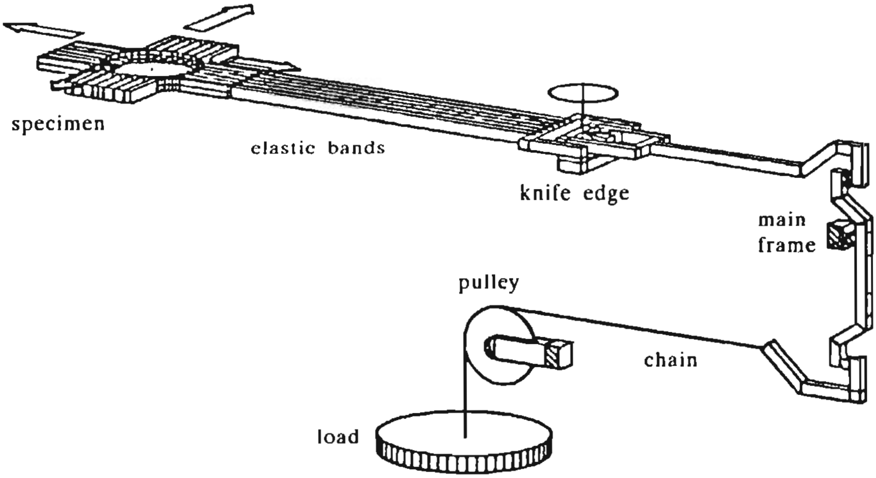

Fig. 2. Scheme of apparatus

In the classical testing procedure for tensile tests, the ends of the specimen are rigidly clamped and subjected to a translation parallel to the axial directions, without rotation. In order to avoid non-homogeneous stress and strain fields development during deformation of anisotropic materials in result of the off-axes tension test the new testing device has been proposed ensuring a uniform normal stress distribution on the ends of specimen (cf Demmerle and Boehler (1993); Betten et al. (1990)). Rigid universal grip heads are linked independently to each limb by individual clamps into its end allowing limb deformation without constraint. Flexible elastic band supplies tension force to each limb ensuring its free displacement, Fig.2. Each arm of cruciform is loaded independently but to maintain equilibrium the loads in opposite arms must be equal. A system of knife-edges located in two perpendicular surfaces minimizes the undesirable effect of specimen bending, Fig.2. This bending is also reduced by the addition of counterbalance weights on each lever arm as well on each horizontal pull rod due to self weight. The dead-weight loading of lever beams leads to a loading magnification factor of 17:1. The furnace was designed to heat up the specimen to a maximum of 600°C. A spiral sheathed element is located below the centre of the specimen.

4. Biaxial strain extensometer

To control a biaxial tension test most essential information can be established by monitoring both force and strain. Continuous force monitoring is made from load-cells set in series with the loading arms. However, continuous strain monitoring is not so straightforward, particularly in the tests at elevated temperatures. The simplest way is based on application of high temperature electrical resistance strain gauges bonded directly to the specimen centre in form of a gauge rosette but this solution can be too expensive to use. Moreover, the level of inelastic strain may be high enough to cause gauge debonding. On the other hand, there are many excellent commercially available extensometers for uniaxial strain measurement at room and elevated temperatures but they tend to be not easily adapted to the biaxial use.

The device, as an example of strain measurement on the plane surface at elevated temperatures, is presented by Morisson (1986) to provide the information about the strain/stress fields during the steady- and rupture-states of creep under biaxial forces condition. An extensometer incorporates a high temperature Linear Voltage Displacement Transducer (LVDT) and has been used successfully in tests up to 250°C. For temperatures above this the LVDT has been mounted outside the furnace and the measurement was extended from the specimen surface by a rod/tube assembly.

Three types of extensometers are described by Smith and Pascoe (1985) which have been used for testing high strain fatigue in steels, fatigue crack pro-

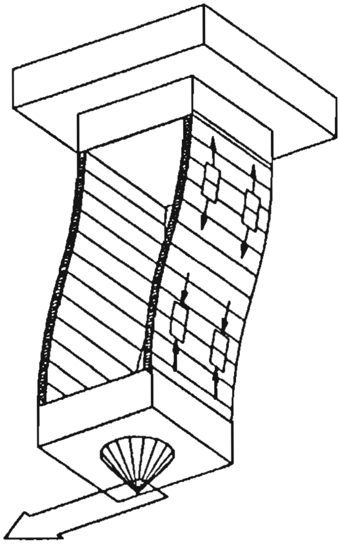

Fig. 3. Flexible deformable double bending unit

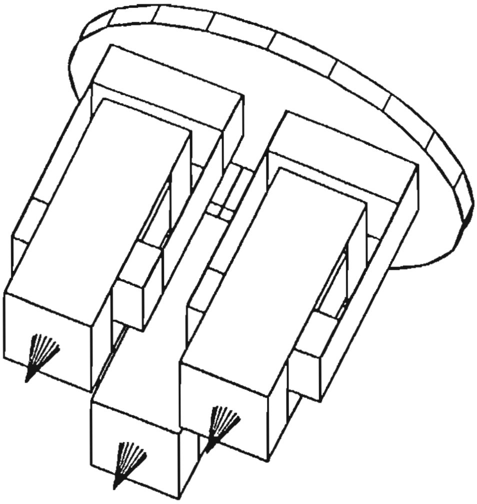

Fig. 4. Scheme of extensometer

pagation in steels and fatigue-fracture of glass reinforced polyester composite based on bending of a beam and on deformation of a thin ring, respectively.

The process of uniform deformation of a measurement zone is taken into account by means of the recently designed mechanical extensometer, the idea of which is based upon a system of double bending flexible units allowing simultaneous and independent measurement of two orthogonal plane displacements at each of three points of the working section (cf Waniewski (1990)). A specimen deformation during the test is transformed onto two double bending thin beams symmetrically combined in one unit presented in Fig.3. The unit dimensions allowing four strain gauges to be cemented on one of two beams so forming a full bridge circuit when upper and lower part of its surface is extended and compressed, respectively, Fig.3. Each of three extensometer legs consists of two identical double bending units turned with respect to each other through an angle of 90°. The material and dimensions of the beam working section are chosen in that way, allowing double bending only in one direction, therefore the combination of two flexible units causes the resultant displacement to be found by display of its two orthogonal components, Fig.4. Three extensometer legs are located on horizontally positioned flat surface via small conical indentations held in position by means of the elastic bands.

During the short time intervals the strain bridge outputs are amplified, and fed via A/D converters into a computer which stimulates both measurement device and data processing.

5. Strain measurement

In the undeformed body two adjacent material points A_0 and B_0 define the end points of a linear element vector $d\boldsymbol{s}_0$, Fig.5. During the deformation the linear element $d\boldsymbol{s}$ connecting the same material points varies in length and direction. Hence, A_0 undergoes the displacement $\boldsymbol{u}$ and moves to A, while B_0 experiences a slightly different displacement $\boldsymbol{u} + d\boldsymbol{u}$ when moving to B and the relation between the notations $d\boldsymbol{s}_0$ and $d\boldsymbol{s}$ is defined by the deformation gradient F_{ij}, Fig.5. The state of body in motion can be considered by the choice between two formulations. In the Lagrangian formulation the coordinates are attached to the individual particles which are moved and deformed with the material, $x_i = x_i(a_p, t)$ for $i, p = 1, 2, 3$. In the Eulerian formulation a fixed, rigid coordinate system is used and let the particle move in it, $a_i = a_i(x_p, t)$. The formulations mentioned above coincide for small motions of a solid body.

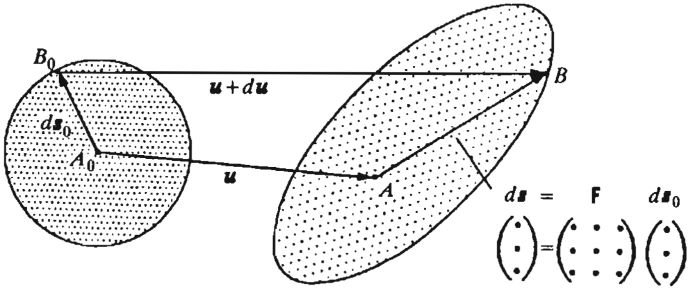

Fig. 5. Definition of deformation gradient F_{ij} and displacement vector u_i

Now, a thin plane sheet of material is considered in two Cartesian coordinate systems, each consisting of two coordinates x_i and a_i, $i = 1,2$, in the plane and a third coordinate x_3 and a_3 normal to it, respectively. It has been assumed that material properties are not changing along the sheet thickness and, therefore, the symmetric strain tensor $\varepsilon_{ij} = \varepsilon_{ji}$ has the following form

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{11} & \gamma_{12}/2 & 0 \\ & \varepsilon_{22} & 0 \\ & & \varepsilon_{33} \end{bmatrix} \tag{5.1}$$

and can be decomposed into strain $(\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33})$ and shear $(\gamma_{12}, 0, 0)$ of the deformable material. Considering the system of two infinitesimal line elements extended and rotated in the plane, the strain field is evaluated basing on the displacement vectors measured at the end points of these fibres. Hence, the kinematic relation has a general form

$$\varepsilon_{ij} = \frac{u_{i,j} + u_{j,i}}{2} = \frac{F_{ki}F_{kj} - \delta_{ij}}{2} \tag{5.2}$$

assuming the displacement sufficiently small (the quadratic terms in the displacement are therefore negligible). The general form of Eq (5.2) is independent of the forces acting and of the elastic or inelastic character of the deformation and results from the geometry of motion which leads from the undeformed to the deformed position.

Geometric interpretation of the displacement vectors and their partial derivatives of two equal infinitesimal fibres attached at the origin of the initial Cartesian coordinate system is shown in Fig.6, assuming that one of these fibres lies along the axis but the second one is turned about this axis through an angle of $\pi/3$.

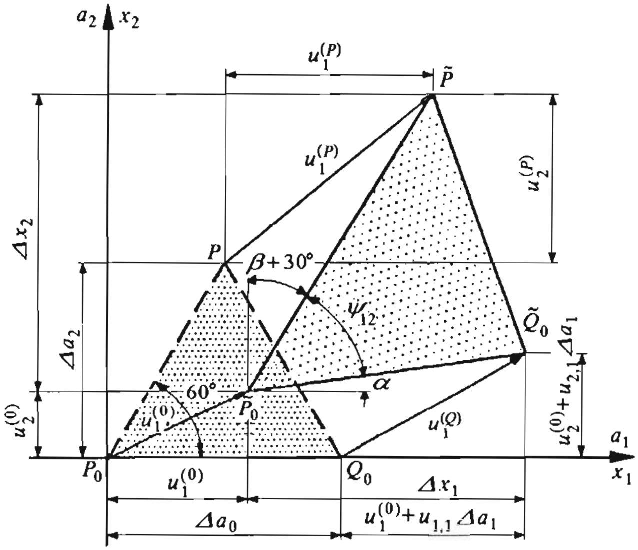

Fig. 6. Section of plane surface with two infinitesimal fibres before and during deformation

During deformation, the fibre defined by the end points P_0, Q_0 and the length Δa_0 is extended and rotated to the end points $\tilde{P}_0$, $\tilde{Q}_0$ related to its length equal to Δx_i. The point Q_0 undergoes projection relative to the point P_0 is then represented as follows

$$u_1^{(Q)} = u_1^{(0)} + u_{1,1}\Delta a_1 \qquad\qquad u_2^{(Q)} = u_2^{(0)} + u_{2,1}\Delta a_1 \tag{5.3}$$

The second fibre defined by the end points P_0, P and the length Δa_i is influenced during deformation by displacements at both end points $\tilde{P}$ and $\tilde{Q}$ due to the off-axis position of fibre relative to the initial configuration. The projection is made at point P can be found applying the Taylor series expansion (cf Betten (1993))

$$u_i^{(P)} = u_i^{(0)} + u_{i,j}\Delta a_j + \ldots \tag{5.4}$$

limited to the first order term by assuming sufficiently small deformation. Then, by virtue of Eq (5.4), the plane displacement components have the forms

$$\begin{aligned} u_1^{(P)} &= u_1^{(0)} + u_{1,1}\Delta a_1 + u_{1,2}\Delta a_2 \\ u_2^{(P)} &= u_2^{(0)} + u_{2,1}\Delta a_1 + u_{2,2}\Delta a_2 \end{aligned} \tag{5.5}$$

By solving Eqs (5.3) and (5.5) in terms of the partial derivative displacements, the fibres extension and rotation can be found

$$
\begin{aligned}
u_{1,1} &= \frac{u_1^{(Q)} - u_1^{(0)}}{\Delta a_1} & \qquad u_{2,2} &= \frac{2}{\sqrt{3}}\frac{u_2^{(P)} - u_1^{(Q)}}{\Delta a_1}\\
u_{1,2} &= \frac{2}{\sqrt{3}}\frac{u_1^{(P)} - u_1^{(Q)}}{\Delta a_1} & \qquad u_{2,1} &= \frac{u_2^{(Q)} - u_2^{(0)}}{\Delta a_1}
\end{aligned}
\tag{5.6}
$$

respectively, as a function of the known values of the displacement vectors to be measured at the ends points $\widetilde{P}_0$, $\widetilde{Q}_0$ and $\widetilde{P}$.

The partial derivative of displacement defined as $u_{3,3}$, has been derived assuming incompressibility of the material for which the Jacobian determinant must be equal to 1

$$
J = \det(F_{ij}) = \det(u_{i,j} + \delta_{ij}) \equiv 1 \tag{5.7}
$$

then, considering the first order terms only, the following relation has been obtained

$$
u_{3,3} = -(u_{1,1} + u_{2,2}) \tag{5.8}
$$

In this section, the procedure of the strain field determination has been presented by the kinematic relation in Eq (5.2), based on the partial derivatives of displacements shown in Eq (5.6) and (5.8), identified by means of displacement vector field, the value of which is known at three points $\widetilde{P}_0$, $\widetilde{Q}_0$ and $\widetilde{P}$, Fig.6.

As may be seen from Fig.6, the shear component $\gamma_{12} = \pi/3 - \Psi_{12} = \alpha + \beta$ can be decomposed into the angles α and β which denote the angles of rotation of respective linear elements, where

$$
\begin{aligned}
\tan\alpha &= \frac{u_{2,1}}{1 + u_{1,1}}\\
\tan\Big(\beta + \frac{\pi}{6}\Big) &= \frac{\frac{\sqrt{3}}{3} + u_{1,2}}{1 + \frac{2\sqrt{3}}{3}u_{2,1} + u_{2,2}}
\end{aligned}
\tag{5.9}
$$

This discussion can be extended onto the problem of strain field determination by means of the extensometer presented at previous section, only if the strain field is uniform in the range of small deformations. In this case, the kinematic relation, presented in the form of Eq (5.2), can be calculated basing on the displacement field (i.e., $u_i^{(0)}$, $u_i^{(P)}$ and $u_i^{(Q)}$ for $i = 1,2$) which is to be measured by the system of double elastic bending units, substituting the value of Δa_i for references basis of the extensometer and assuring sufficiently short time interval between two measurements to cause the sufficiently small gradient of deformation.

6. Conclusions

Direct loading of a flat cruciform specimen was considered to be a useful method of investigation into the directional properties of such materials as fibre reinforced plastics usually fabricated in the sheet form or the rolled sheet-metals.

The efforts have been concentrated on the appropriate specimen shape selection to ensure the uniform stress/strain field occurence into its circular test zone without disturbance in the material structure caused by the mechanical treatment along the varying thickness of the specimen. For that reason, the laminar specimen structure has been assumed with the thickened four arms to reinforce the cross-shaped body.

An analysis of the off-axes tests for which the principal directions of the applied loads do not coincide with the privileged axes of the material symmetries proved that a strongly non-homogeneous stress and strain field develops when the classical testing procedure consisting of rigidly clamped heads has been applied. Here, the new testing device has been proposed for which the load is applied through the elastic bands to the rigid universal grip heads linked independently to each limb by individual clamps allowing limb deformation without constraint.

Assuming the uniform strain field in the measurement zone, the new mechanical extensometer has been applied basing on the concept of double bending flexible units. The strain measurement procedure has been proposed allowing identification of strain components as well as the rotation angle of strain principal axes.

To verify the operational condition of the extensometer the creep tests are now in operation at room temperature on the cruciform specimen with the thinned down measurement zone cut out from the large bulk of the electrolytic copper. The preliminary creep tests were carried out for three different ratios of biaxial loading at room temperature to verify the strain measurement procedure by means of the new mechanical extensometer with indication of the strain gauges bonded to the measurement area of the specimen.

The special calibration device has been used to find the linear master curves (displacement/electrical signal) for each of six measurement units located along the plane axes of a specimen. Then, two perpendicular forces acting along axis of a specimen were detected by the load cells while the axial (0°), transversal (90°) and medial (45°) strain components were identified and compared by the use of three foil strain gauges.

Acknowledgements

The support of the State Committee for Scientific Research (KBN) under contract No. 301 54 91 01 is gratefully acknowledged.

References

1. BETTEN J., 1993, *Kontinuumsmechanik Elasto-, Plasto- und Kriechmechanik*, Springer-Verlag, Berlin, Heidelberg
2. BETTEN J., BREITBACH G., WANIEWSKI M., 1990, Multiaxial Anisotropic Creep Behaviour of Rolled Sheet-Metals, *Z. Angew. Math. Mech., (ZAMM)*, **70**, 371-379
3. DEMMERLE S., BOEHLER J.P., 1993, Optimal Design of Biaxial Tensile Cruciform Specimens, *J. Mech. Phys. Solids*, **21**, 143-181
4. MORISSON C.J., 1986, Biaxial Testing using Cruciform Specimens, *Techniques for Multiaxial Creep Testing*, Gooch D.J. and How T.M. edit., Elsevier Applied Science, London, New York, 111-126
5. SMITH E.W., PASCOE K.J., 1985, Practical Strain Extensometry for Biaxial Cruciform Specimens – Part 2, *Strain, J. British Soc. Strain Measur.*, **21**, 175-181
6. WANIEWSKI M., 1990, Strain Field Evaluation into Creep Process of Biaxial Tension Cruciform Specimen by Means of Mechanical Extensometer, *Proceedings of the 9th International Conference on Experimental Mechanics*, Lyngby, Denmark, 627-636
7. WANIEWSKI M., WOLF U., BREITBACH G., 1993, Stress-Strain Analysis of a Biaxial Tension Cruciform Specimen for Tests under Creep Conditions, *Strain, J. British Soc. Strain Measur.*, **29**, 15-24

Stanowisko badawcze do dwuosiowego rozciągania anizotropowych próbek krzyżowych

Streszczenie

Zaproponowano pomiar własności mechanicznych materiałów konstrukcyjnych przy pomocy metody opartej na wzajemnie prostopadłym dwuosiowym rozciąganiu płaskiej próbki krzyżowej. Dyskutowano problem jednorodności pola naprężenia poprzez zmianę kształtu próbki. Zaproponowana struktura warstwowa próbki jest konieczna w przypadku badania ukierunkowanych własności materiałów z wprowadzoną anizotropią, takich jak blachy lub kompozyty. Warunek jednorodności pól naprężenia i odkształcenia pozwala na pomiar deformacji w części pomiarowej próbki przy pomocy mechanicznego ekstensometru, którego konstrukcja jest tutaj przedstawiona. Praca uwypukla problemy związane z identyfikacją pola odkształcenia w części pomiarowej próbki. Przedstawiono również konstrukcję maszyny wytrzymałościowej z systemem obciążającym płaską próbkę krzyżową w zakresie pokojowej i podwyższonej temperatury.

Manuscript received October 18, 1995; accepted for print November 29, 1995