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BEHAVIOUR OF ACCELERATION WAVES IN A VISCOELASTIC POROUS MEDIUM WITH A STRUCTURE. Part II – LINEAR VISCOELASTIC FLUID-SATURATED POROUS SOLID

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The influence of a medium structure on the propagation of acceleration wave in a fluid-saturated porous medium has been studied. The medium is composed of a viscoelastic solid skeleton and compressible viscous fluid. It has been found that the acoustic tensor is non-symmetric in the considered case. The medium structure influences viscoelastic attenuation of the longitudinal waves only but does not influence attenuation of the rotational waves.

Key words: fluid-saturated porous solid, acceleration wave, viscoelastic solid

1. Introduction

The aim of this paper is to study the influence of the medium structure on the propagation of acceleration wave in the fluid-saturated porous medium with a viscoelastic solid skeleton. In the first part of the paper, the following propagation conditions of the acceleration wave have been obtained

$$\begin{aligned} \Big[\mathrm{div}\mathbf{T}^s\Big] + (1-\kappa)\Big[\mathrm{div}\mathbf{T}^f\Big] &= u^2[\bar{\rho}^s + (1-\kappa)\bar{\rho}^f]\boldsymbol{a}^s \\ \Big[\mathrm{div}\mathbf{T}^f\Big] &= \frac{1}{\kappa}\bar{\rho}^f u^2[\boldsymbol{a}^f - (1-\kappa)\boldsymbol{a}^s] \end{aligned} \tag{1.1}$$

where

$\mathbf{T}^s, \mathbf{T}^f$	–	stress tensors in the solid skeleton and fluid
$\bar{\rho}^s, \bar{\rho}^f$	–	partial densities of the constituents
$\boldsymbol{a}^s, \boldsymbol{a}^f$	–	amplitudes of the acceleration wave
u	–	speed of displacement of the acceleration wave
κ	–	parameter connected with the medium structure (cf the first part of the paper).

The derived growth equations are of the form

$$\Big[\frac{D(\mathrm{div}\mathbf{T}^s)}{D^s t}\Big] + (1-\kappa)\Big[\frac{D(\mathrm{div}\mathbf{T}^f)}{D^s t}\Big] + \Big[\frac{D\pi}{D^s t}\Big] = u^2[\bar{\rho}^s + (1-\kappa)\bar{\rho}^f]\Big(\boldsymbol{c}^s + 2\frac{\delta \boldsymbol{a}^s}{\delta t}\Big) + \\ +u^3\Big\{\frac{1-\kappa}{2\kappa}\bar{\rho}^f(\boldsymbol{a}^f - \boldsymbol{a}^s)[(\boldsymbol{a}^f - \boldsymbol{a}^s)n] + \bar{\rho}^s\boldsymbol{a}^s(\boldsymbol{a}^s\boldsymbol{n})\Big\} \tag{1.2}$$

$$\kappa\Big[\frac{D(\mathrm{div}\mathbf{T}^f)}{D^f t}\Big] - \Big[\frac{D\pi}{D^f t}\Big] = \bar{\rho}^f u^2\Big(\boldsymbol{c}^f + 2\frac{\delta \boldsymbol{a}^f}{\delta t}\Big) - (1-\kappa)\bar{\rho}^f u^2\Big(\boldsymbol{c}^s + 2\frac{\delta \boldsymbol{a}^s}{\delta t}\Big) + \\ +\bar{\rho}^f u^3\Big\{-\frac{5(1-\kappa)}{2\kappa}(\boldsymbol{a}^f - \boldsymbol{a}^s)[(\boldsymbol{a}^f - \boldsymbol{a}^s)n] + \boldsymbol{a}^f(\boldsymbol{a}^f\boldsymbol{n})\Big\} \tag{1.3}$$

The considered medium is described by the linear physical relations proposed by Biot (1956a), in the form

$$\begin{aligned} \mathbf{T} &= 2\widetilde{N}\mathbf{E}^s + (\widetilde{A}\mathrm{tr}\mathbf{E}^s + \widetilde{Q}\mathrm{tr}\mathbf{E}^f)\mathbf{1} \\ \mathbf{T}^f &= (\widetilde{Q}\mathrm{tr}\mathbf{E}^s + \widetilde{R}\mathrm{tr}\mathbf{E}^f)\mathbf{1} \end{aligned} \tag{1.4}$$

where $\widetilde{A}$, $\widetilde{N}$, $\widetilde{Q}$ are the integral operators

$$\begin{aligned} \widetilde{A} &= A(0) - \int\limits_0^t A'(t-\tau)\ldots\,d\tau \\ \widetilde{N} &= N(0) - \int\limits_0^t N'(t-\tau)\ldots\,d\tau \\ \widetilde{Q} &= Q(0) - \int\limits_0^t Q'(t-\tau)\ldots\,d\tau \end{aligned} \tag{1.5}$$

however $\widetilde{R} = R(0) = R$ =const; $A(0)$, $N(0)$, $Q(0)$ are the initial values of the stress-relaxation functions and $A'(t)$, $N'(t)$, $Q'(t)$ are the relaxation kernels.

These kernels are limited functions of the class C^1 in the interval $(0,\infty)$. E^s and E^f are the linear strain tensors

$$\mathsf{E}^s = 0.5[\mathrm{grad}\boldsymbol{U}^s + (\mathrm{grad}\boldsymbol{U}^s)^\top]$$
$$\mathsf{E}^f = 0.5[\mathrm{grad}\boldsymbol{U}^f + (\mathrm{grad}\boldsymbol{U}^f)^\top] \tag{1.6}$$

$\boldsymbol{U}^s$ and $\boldsymbol{U}^f$ are the displacement vectors of the solid skeleton and fluid, respectively, and T denotes transposition. Velocities of both the constituents expressed by the displacement vectors are of the form

$$\boldsymbol{v}^s = \frac{D\boldsymbol{U}^s}{D^s t} \qquad \boldsymbol{v}^f = \frac{D\boldsymbol{U}^f}{D^f t} \tag{1.7}$$

2. Propagation conditions

Now we substitute Eqs (1.6) into the physical relations (1.4) and then into Eqs (1.1). The terms $\Big[\mathrm{div}\mathsf{T}^s\Big]$ and $\Big[\mathrm{div}\mathsf{T}^f\Big]$ lead to the jumps of type, e.g., $\Big[\widetilde{N}\mathrm{div}\ \mathrm{grad}\boldsymbol{U}^s\Big]$ having the following index notation form

$$\Big[\widetilde{N}U^s_{i,jj}\Big] = \Big[N(0)U^s_{i,jj}\Big] - \Big[\int\limits_0^t N'(t-\tau)U^s_{i,jj}(x_k,\tau)\,d\tau\Big] \tag{2.1}$$

$N'(t)$ is a limited function of the class C^1 in the interval $(0,\infty)$ and $U^s_{i,jj}$ is a limited and continuous function everywhere except for a possible discontinuity jump at the discontinuity surface. Thus, by virtue of the theorem proved by Fisher and Gurtin (1965), the integral on the right-hand side of Eq (2.1) is a continuous function and hence

$$\Big[\int\limits_0^t N'(t-\tau)U^s_{i,jj}(x_k,\tau)\,d\tau\Big] = 0 \tag{2.2}$$

We therefore obtain

$$\Big[\widetilde{N}U^s_{i,jj}\Big] = N(0)\Big[U^s_{i,jj}\Big] \tag{2.3}$$

Using the relations (1.7), the definitions of amplitudes of the wave and applying the geometrical and kinematical compatibility conditions, we arrive at the propagation condition

$$(\mathsf{A} - \mathsf{D}u^2)\boldsymbol{a} = \boldsymbol{0} \tag{2.4}$$

In the case of propagation of one-dimensional acceleration wave along the x_1-axis, the matrix representation $\widehat{\mathbf{A}}$ of the acoustic tensor $\mathbf{A}$ is

$$\widehat{\mathbf{A}} = \begin{bmatrix} P(0)+(1-\kappa)Q(0) & 0 & 0 & Q(0)+(1-\kappa)R(0) & 0 & 0 \\ 0 & N(0) & 0 & 0 & 0 & 0 \\ 0 & 0 & N(0) & 0 & 0 & 0 \\ Q(0) & 0 & 0 & R(0) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{2.5}$$

The tensor of densities is represented by the matrix

$$\widehat{\mathbf{D}} = \begin{bmatrix} \xi\bar{\rho}^s & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi\bar{\rho}^s & 0 & 0 & 0 & 0 \\ 0 & 0 & \xi\bar{\rho}^s & 0 & 0 & 0 \\ (1-1/\kappa)\bar{\rho}^f & 0 & 0 & \bar{\rho}^f/\kappa & 0 & 0 \\ 0 & (1-1/\kappa)\bar{\rho}^f & 0 & 0 & \bar{\rho}^f/\kappa & 0 \\ 0 & 0 & (1-1/\kappa)\bar{\rho}^f & 0 & 0 & \bar{\rho}^f/\kappa \end{bmatrix} \tag{2.6}$$

where

$$\xi = 1 + (1-\kappa)\frac{\bar{\rho}^f}{\bar{\rho}^s} \tag{2.7}$$

and the vector $\boldsymbol{a}$ is represented by the matrix of amplitudes

$$\widehat{\boldsymbol{a}} = \Big[a_1^s, a_2^s, a_3^s, a_1^f, a_2^f, a_3^f\Big]^\top \tag{2.8}$$

$\boldsymbol{a}^s(a_1^s, a_2^s, a_3^s)$ and $\boldsymbol{a}^f(a_1^f, a_2^f, a_3^f)$ denote the vectors of amplitudes in the solid skeleton and in the fluid, respectively.

a_1^s, a_1^f are the amplitudes of longitudinal wave in the solid skeleton and fluid, respectively; a_2^s, a_3^s and a_2^f, a_3^f are the amplitudes of rotational waves in the solid skeleton and fluid, respectively.

In the case of two-parameter description of a porous medium structure the acoustic tensor (2.5) is non-symmetric. If this structure is described by one parameter f_v then $\kappa = 1$ and the acoustic tensor (2.5) is symmetrical. The acoustic tensor is formally the same as in the paper by Dzięcielak (1995). However viscoelastic properties of the solid skeleton cause that the material constants of a medium are replaced by the initial values of stress-relaxation function. Therefore the propagation conditions of acceleration waves in the fluid-saturated porous viscoelastic solid with a structure are formally identical as in the case of elastic solid skeleton (cf Dzięcielak (1995)). The propagation condition (2.4) leads to the propagation conditions of two orthogonal rotational waves and two longitudinal waves. A detailed discussion of the medium structure influence on the speeds of propagation of these waves is presented in section 6 of this paper.

3. Growth equations

More information about behaviour of the acceleration wave in the linear fluid-saturated porous medium will be obtained from the growth equations (1.2), (1.3). The force $\boldsymbol{\pi}$ of internal interaction appears in these equations, thus the constitutive relation for $\boldsymbol{\pi}$ is required. Following Biot (1956a), we assume this constitutive relation in the form of Darcy's law

$$\boldsymbol{\pi} = b(\boldsymbol{v}^f - \boldsymbol{v}^s) \tag{3.1}$$

The coefficient b is related to Darcy's coefficient of permeability k by $b = \mu f_v^2/k$, where μ is the fluid viscosity.

To obtain the growth equations describing the changes of amplitudes of the wave in the considered fluid-saturated porous medium we substitute the constitutive relations (1.4) and (3.1) into Eqs (1.2) and (1.3) using the Eqs (1.5), (1.6) and (1.7). Applying the aforementioned Gurtin and Fisher theorems and the known geometrical and kinematical conditions of compatibility we arrive at the growth equations of the form

$$\begin{aligned}
&2[\bar{\rho}^s + (1-\kappa)]u^2\frac{\delta \boldsymbol{a}^s}{\delta t} - bu^2(\boldsymbol{a}^f - \boldsymbol{a}^s) + [A'(0) + N'(0) + (1-\kappa)Q'(0)](\boldsymbol{a}^s\boldsymbol{n})\boldsymbol{n} + \\
&+N'(0)\boldsymbol{a}^s + [Q'(0) + (1-\kappa)R'(0)](\boldsymbol{a}^s\boldsymbol{n})\boldsymbol{n} + \bar{\rho}^s\boldsymbol{a}^s(\boldsymbol{a}^s\boldsymbol{n}) + \\
&+\frac{1-\kappa}{\kappa}\bar{\rho}^f(\boldsymbol{a}^f - \boldsymbol{a}^s)[(\boldsymbol{a}^f - \boldsymbol{a}^s)\boldsymbol{n}] = [A(0) + N(0) + (1-\kappa)Q(0)](\boldsymbol{c}^s\boldsymbol{n})\boldsymbol{n} + \\
&+N(0)\boldsymbol{c}^s + [Q(0) + (1-\kappa)R(0)](\boldsymbol{c}^f\boldsymbol{n})\boldsymbol{n} - u^2[\bar{\rho}^s + (1-\kappa)\bar{\rho}^f]\boldsymbol{c}^s
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
&2\bar{\rho}^f u^2\Big[\frac{\delta \boldsymbol{a}^f}{\delta t} - (1-\kappa)\frac{\delta \boldsymbol{a}^s}{\delta t}\Big] + bu^2(\boldsymbol{a}^f - \boldsymbol{a}^s) + \kappa Q'(0)(\boldsymbol{a}^s\boldsymbol{n})\boldsymbol{n} + \\
&+\kappa R'(0)(\boldsymbol{a}^f\boldsymbol{n})\boldsymbol{n} + \bar{\rho}^f u^3\Big\{\boldsymbol{a}^f(\boldsymbol{a}^f\boldsymbol{n}) - \frac{5(1-\kappa)}{2\kappa}(\boldsymbol{a}^f - \boldsymbol{a}^s)[(\boldsymbol{a}^f - \boldsymbol{a}^s)\boldsymbol{n}]\Big\} = \\
&= \kappa Q(0)(\boldsymbol{c}^s\boldsymbol{n})\boldsymbol{n} + \kappa R(0)(\boldsymbol{c}^f\boldsymbol{n})\boldsymbol{n} - \bar{\rho}^f u^2\boldsymbol{c}^f + (1-\kappa)\bar{\rho}^s u^2\boldsymbol{c}^s
\end{aligned} \tag{3.3}$$

We confine our study of the acceleration wave behaviour to the one-dimensional plane wave. Additionally, we assume the induced discontinuities $\boldsymbol{c}^s$ and $\boldsymbol{c}^f$ to be right proper vectors of the acoustic tensor A (cf Chen (1976)). In this case the right terms of Eqs (3.2) and (3.3) vanish. The propagation condition (2.4) allows us to introduce the following vectors of amplitudes

$$\boldsymbol{a}^s = \alpha\check{\boldsymbol{a}}^s \qquad \boldsymbol{a}^f = \alpha\check{\boldsymbol{a}}^f \tag{3.4}$$

where:

- α – scalar parameter, which in general can vary with space and time
- $\check{\boldsymbol{a}}^s, \check{\boldsymbol{a}}^f$ – vectors collinear with $\boldsymbol{a}^s$ and $\boldsymbol{a}^f$.

Now we form a scalar product of Eq (3.2) and $\boldsymbol{a}^s$ and then Eq (3.3) and $\boldsymbol{a}^f$, summing then these equations; which gives us the growth equation

$$2C_r u^2\frac{\delta\alpha}{\delta t} + [bu^2(\check{\boldsymbol{a}}^s - \check{\boldsymbol{a}}^f)(\check{\boldsymbol{a}}^s - \check{\boldsymbol{a}}^f) + C_1'(0)]\alpha + C_2 u^3\alpha^2 = 0 \tag{3.5}$$

where

$$C_r = [\bar{\rho}^s + (1-\kappa)\bar{\rho}^f]\check{\boldsymbol{a}}^s\check{\boldsymbol{a}}^s + \bar{\rho}^f[\check{\boldsymbol{a}}^f\check{\boldsymbol{a}}^f - (1-\kappa)\check{\boldsymbol{a}}^s\check{\boldsymbol{a}}^f] \tag{3.6}$$

$$\begin{aligned} C_1'(0) = {} & [A'(0) + N'(0) + (1-\kappa)Q'(0)](\check{\boldsymbol{a}}^s\boldsymbol{n})^2 + \\ & + N'(0)(\check{\boldsymbol{a}}^s\check{\boldsymbol{a}}^s) + (1+\kappa)Q'(0)(\check{\boldsymbol{a}}^f\boldsymbol{n})(\check{\boldsymbol{a}}^s\boldsymbol{n}) \end{aligned} \tag{3.7}$$

$$\begin{aligned} C_2 = {} & \frac{1-\kappa}{\kappa}\bar{\rho}^f\Big[(\check{\boldsymbol{a}}^f - \check{\boldsymbol{a}}^s)\Big(\check{\boldsymbol{a}}^s - \frac{5}{2}\check{\boldsymbol{a}}^f\Big)\Big][(\check{\boldsymbol{a}}^f - \check{\boldsymbol{a}}^s)\boldsymbol{n}] + \\ & + \bar{\rho}^f(\check{\boldsymbol{a}}^f\check{\boldsymbol{a}}^f)(\check{\boldsymbol{a}}^f\boldsymbol{n}) + \bar{\rho}^s(\check{\boldsymbol{a}}^s\check{\boldsymbol{a}}^s)(\check{\boldsymbol{a}}^s\boldsymbol{n}) \end{aligned} \tag{3.8}$$

Eq (3.5) allows us to calculate the parameter α that describes the changes of amplitudes of the acceleration wave in the considered medium.

In the case of plane wave propagating in the positive direction of x_1-axis the solution of the growth equation (3.5) is

$$\alpha = \alpha_0\frac{A\exp\Big[-\frac{A}{2C_r u}(x_1 - x_0)\Big]}{A + \alpha_0 C_2 u^2\Big\{1 - \exp\Big[-\frac{A}{2C_r u}(x_1 - x_0)\Big]\Big\}} \tag{3.9}$$

where

$$A = b(\check{a}_i^s - \check{a}_i^f)(\check{a}_i^s - \check{a}_i^f) + C_1'(0) \tag{3.10}$$

and $\alpha_0 = \alpha(x_0)$ is the initial amplitude being the value of α at a distance $x_0 < x_1$.

The term $C_1'(0)$ represents viscoelastic properties of the porous solid skeleton in the solution (3.9) affecting additional attenuation of the amplitude. The influence of viscoelastic properties of the medium on attenuation of the wave depends on the structure of medium as it is seen in Eq (3.7). This formula indicates that the structure of medium influences only attenuation of the longitudinal waves but does not influence attenuation of the rotational wave. It is worth to note that the term C_2 disappears for the rotational waves.

The solution (3.9) implies that according to the value and sign of the initial amplitude α_0, the amplitude α of longitudinal wave is decreasing and becomes arbitrarily small or is increasing and becomes infinite within a finite distance x_1. The last conclusion, of course, suggests the formation of a shock (cf Chen (1976)); it is always $A > 0$, thus the condition $\alpha_0 C_2 < 0$ must be satisfied. The shock is formed at the distance

$$x_1 = x_0 + \frac{2C_r u}{A} \ln \frac{\alpha_0 C_2 u}{A + \alpha_0 C_2 u} \tag{3.11}$$

The formula (3.11) leads to the additional condition

$$A + \alpha_0 C_2 u < 0 \tag{3.12}$$

In the case of elastic fluid-saturated porous solid the solution (3.9) reduces to the results presented by Dzięcielak (1995), where one can find a detailed analysis of the influence of medium structure on the behavior of acceleration wave. If the medium structure is described by one parameter only we obtain the results presented in of Dzięcielak (1980).

4. Results and discussion

Now, to discuss the influence of medium structure on the speeds of displacement and amplitudes of acceleration waves we restrict our considerations to the linear fluid-saturated porous medium with an elastic solid skeleton. In this case the initial values of stress-relaxation functions become material constants of the medium: $A(0) = A$, $N(0) = N$, $Q(0) = Q$, and the relaxation kernels $A'(t)$, $N'(t)$ and $Q'(t)$ are equal to zero.

The propagation condition (2.4) leads to the propagation conditions of two orthogonal rotational waves and a longitudinal wave. In the considered case the amplitudes of rotational waves in the fluid are equal to zero. From the propagation conditions of rotational waves we obtain the speed of displacement of these waves

$$u_R = \sqrt{\frac{N}{\bar{\rho}^s + (1-\kappa)\bar{\rho}^f}} \tag{4.1}$$

The dimensionless speed of displacement may be written in the form

$$\widetilde{u}_R(\kappa) = \frac{u_R(\kappa)}{u_R(1)} = \sqrt{1 + (1-\kappa)\frac{\bar{\rho}^f}{\bar{\rho}^s}} \tag{4.2}$$

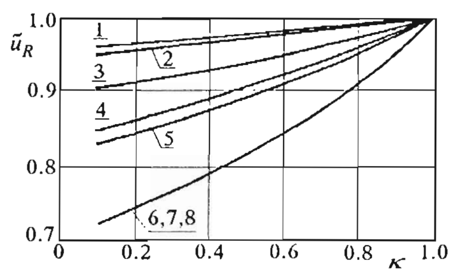

Fig. 1. Dimensionless speeds of displacement $\widetilde{u}_R$ of the rotational wave as functions of the structure parameter κ for various ratios of partial densities

It results from Eq (4.2) that the dimensionless speed of displacement of the rotational wave depends on the parameter κ that describes the medium structure and on the ratio of partial densities of both the medium constituents. This speed is always less then in the case of one-parameter description. In Fig.1, the dependence of the dimensionless speed of displacement of the rotational wave on the structure parameter κ for various media is presented. The curves in Fig.1 indicate that for increasing ratio of partial densities of the fluid and solid skeleton, the speed of displacement of rotational wave decreases. The influence of a medium structure on the speed of displacement of the rotational wave is insignificant for fluid-saturated rocks and soils (curve 1 and 2, $\bar{\rho}^f/\bar{\rho}^s \approx 0.1$) and is becoming considerable for media with greater ratios of partial densities ($\bar{\rho}^f/\bar{\rho}^s \approx 0.3 \div 1.0$).

A necessary condition for non-vanishing amplitudes in the propagation conditions of longitudinal wave leads to the biquadratic equation of the form

$$u_c^4 \frac{[\gamma_1 + (1-\kappa)]\gamma_2}{\kappa} - u_c^2 \Big\{ \frac{[\beta_{11} + (1-\kappa)(2\beta_{12} + \beta_{22})]\gamma_2}{\kappa} + \gamma_1\beta_{22} \Big\} + \\ + \beta_{11}\beta_{22} - \beta_{12}^2 = 0 \tag{4.3}$$

where the dimensionless densities and material constants are defined by

$$\gamma_1 = \frac{\bar{\rho}^s}{\rho} \qquad \gamma_2 = \frac{\bar{\rho}^f}{\rho} \qquad \rho = \bar{\rho}^s + \bar{\rho}^f \qquad \beta_{11} = \frac{P}{H}$$
$$\beta_{12} = \frac{Q}{H} \qquad \beta_{22} = \frac{R}{H} \qquad H = P + 2Q + R \qquad P = A + 2N \tag{4.4}$$

and satisfy the relations

$$\gamma_1 + \gamma_2 = 1 \qquad \beta_{11} + 2\beta_{12} + \beta_{22} = 1 \tag{4.5}$$

The dimensionless speed of displacement $u_c = u/V_c$ is related to the characteristic velocity $V_c = \sqrt{H/\rho}$. For our further considerations it is useful to introduce the ratio

$$\widetilde{u}_c(\kappa) = \frac{u_c(\kappa)}{u_c(1)} \tag{4.6}$$

that informs us about the relation between dimensionless speeds of displacement $u_c(\kappa)$ resulting from two-parameter and one parameter descriptions, respectively.

Numerical calculations have been made for dimensionless material constants presented in Table 1.

Table 1. Dimensionless material constants

	β_{11}	β_{12}	β_{22}	γ_1	γ_2	$\bar{\rho}^f/\bar{\rho}^s$
1	0.904	0.041	0.013	0.919	0.081	0.088
2	0.846	0.063	0.027	0.900	0.100	0.111
3	0.610	0.043	0.305	0.800	0.200	0.250
4	0.610	0.043	0.305	0.700	0.300	0.430
5	0.610	0.043	0.305	0.666	0.333	0.500
6	0.610	0.043	0.305	0.500	0.500	1.000
7	0.740	0.037	0.185	0.500	0.500	1.000
8	0.500	0.000	0.500	0.500	0.500	1.000

Data in the first row concern the water-saturated Berea's sandstone and were calculated on the base of data given by Yew and Jogi (1978); data in the second row describe mechanical properties of the the oil-saturated sandstone (cf Fatt (1958)); data in lines $3 \div 8$ have been taken from Biot (1956b).

The results of numerical calculations are presented in Fig.2 and Fig.3. The curves in Fig.2 show the change of displacement speed of the longitudinal wave of the first kind (fast wave) for different parameters κ and for various media (the curve number of corresponds to the number of a medium in Table 1). Fig.2 indicates that material constants and the parameter $\bar{\rho}^f/\bar{\rho}^s$ have fundamental meaning for the position of these curves. The influence of medium structure

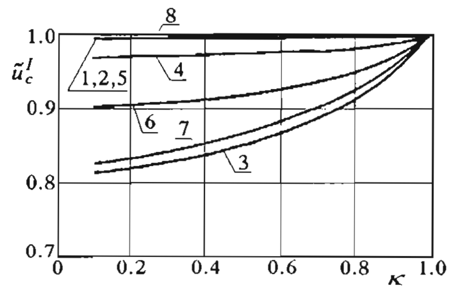

Fig. 2. Dimensionless speeds of displacement $\tilde{u}_c^I$ of the longitudinal wave of the first kind (fast wave) versus the structure parameter κ for various media

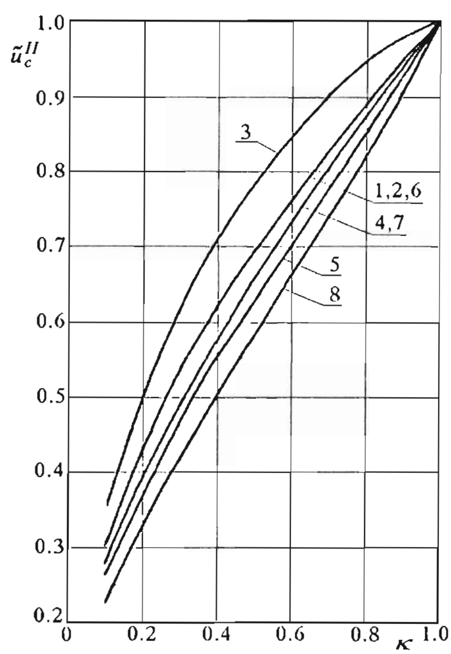

Fig. 3. Dimensionless speeds of displacement $\tilde{u}_c^{II}$ of the longitudinal wave of the second kind (slow wave) versus the structure parameter κ for various media

on speeds of displacement of the longitudinal wave of the first kind is less than 20% and for some media (1,2,8) can be omitted from the practical point of view. The curves in Fig.3 indicate that the medium structure influences strongly the speed of displacement of the longitudinal wave of the second kind (slow wave). This conclusion holds true for all media with the mechanical properties presented in Table 1.

Let us pass to the study of influence of the medium structure on amplitudes of the longitudinal and rotational waves. Numerical calculations were made taking the data given by Fatt (1959) for oil-saturated sandstone:

$$\begin{array}{lll} A = 4419\ \mathrm{MN/m^2} & N = 2755\ \mathrm{MN/m^2} & Q = 743\ \mathrm{MN/m^2} \\ R = 326\ \mathrm{MN/m^2} & \bar{\rho}^s = 2600\ \mathrm{kg/m^3} & \bar{\rho}^f = 820\ \mathrm{kg/m^3} \\ f = 0.26 & \mu = 2 \cdot 10^{-3}\ \mathrm{Ns/m^2} & k = 7.9 \cdot 10^{-13}\ \mathrm{m^2} \end{array}$$

The relative amplitudes of: longitudinal wave of the first kind (fast wave) α_r^I, longitudinal wave of the second kind (slow wave) α_r^{II} and the rotational wave α_r^R

$$\alpha_r^m(\kappa) = \frac{\alpha^m(\kappa)}{\alpha^m(1)} \qquad m = I, II, R \tag{4.7}$$

have been calculated. These three amplitudes are related to the values of respective amplitudes for $\kappa = 1$ (one-parameter description). The dimensionless amplitudes α^I/α_0, α^{II}/α_0, α^R/α_0 related to the initial value α_0 are determined as well. The results of these computations are presented in Fig.4. The influence of the medium structure on the wave amplitudes with increasing distance from a source of disturbance is shown in Table 2.

Table 2. The influence of the medium structure on the wave amplitudes with the distance from a source of disturbation

$x_1 - x_0$ [m]	α_r^I	α_r^{II}	α_r^R
10^{-8}	0.999998	1.00000000	1.000723
10^{-7}	0.999984	1.00000002	1.007356
10^{-6}	0.999844	1.00000024	1.076045
10^{-5}	0.998437	1.00000239	1.081213

The wave of the first kind is not very sensitive to the change of structure parameter κ, which is seen in Fig.4a,b. The amplitude of this wave decreases with the increasing parameter κ and reaches the minimum at $\kappa = 1$. This wave is weakly attenuated like in the case of one-parameter description ($\kappa = 1$). The medium structure influences strongly the amplitude of the wave of the second kind, see Fig.4c,d. The amplitude of this wave increases with the increasing parameter κ and its greatest value is reached at $\kappa = 1$. This

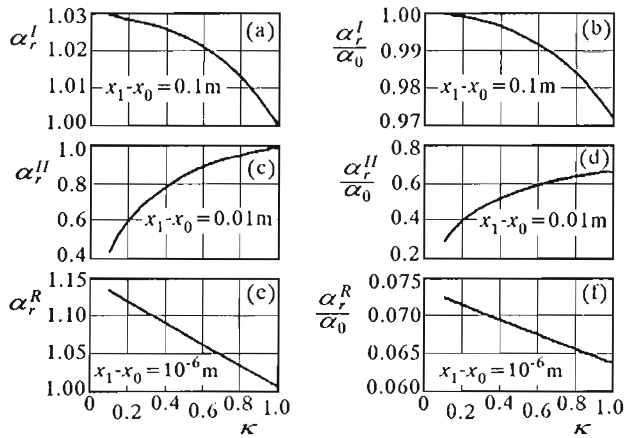

Fig. 4. Relative amplitudes α_r^m and the dimensionless amplitudes α^m/α_0 of the waves as functions of the structural parameter κ

wave is highly attenuated. The amplitude of the rotational wave is greater than in the case one-parameter description. This amplitude reaches minimum at $\kappa = 1$, see Fig.4e,f. The influence of considered description of a medium structure becomes stronger with the increasing distance from a source of disturbance. This is seen particularly in the case of longitudinal wave of the second kind and in the case of rotational wave.

5. Conclusions

Basing on the results obtained we can draw the following conclusions:

- The acoustic tensor is non-symmetric if the influence of a medium structure is taken into consideration.
- Description of a medium structure by two parameters causes that the speeds of displacement depend on a relative motion of both the constituents; this influence is neglegible in such media as fluid-saturated rocks or soils.
- A medium structure influences strongly the speed of displacement of the longitudinal wave of the second kind. The rotational wave and the

longitudinal wave of the first kind are less sensitive to the medium structure. For the longitudinal wave of the first kind the influence of medium structure is so weak that it can be omitted in some cases.

- The speeds of displacement are always smaller than in the case of structure description by one parameter (volumetric porosity) only.
- A medium structure influences the attenuation of the longitudinal waves due to viscoelastic properties of the solid skeleton but does not influence the attenuation of the rotational waves.
- A medium structure influences slightly the amplitude of longitudinal wave of the first kind; this amplitude is greater than in the case of one-parameter description. This influence is considerable for the longitudinal wave of the second kind (the amplitude is less) and the rotational wave (the amplitude is greater).
- The influence of the medium structure becomes stronger with the increasing distance from a source of disturbance.

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References

1. BIOT M.A., 1956a, Theory of Deformation of a Porous Viscoelastic Anisotropic Solid, *J. Appl. Phys.*, **27**, 459-467
2. BIOT M.A., 1956b, Theory of Propagation of Elastic Waves in a Fluid-Saturated Porous Solid, *J. Acoust. Soc. Am.*, **28**, 171-191
3. CHEN P.J., 1976, *Selected Topics in Wave Propagation*, Noordhoff International Publishing, Leyden
4. DZIĘCIELAK R., 1980, Propagation of Acceleration Waves in a Fluid-Saturated Porous Viscoelastic Solid, *Studia Geotech. et Mech.*, **2**, 13-23
5. DZIĘCIELAK R., 1995, The Influence of a Porous Medium Structure on the Behavior of Acceleration Waves, *Proceedigs of the IUTAM Symposium on Nonlinear Waves in Solids*, Victoria, Canada 1993, ASME Book No AMR 137, 386-391
6. DZIĘCIELAK R., 1996, Behaviour of Acceleration Waves in a Viscoelastic Porous Medium with a Structure. Part I Propagation Conditions and Growth Equations, *J. of Theoretical and Appled Mechanics*, **34**, 3, 521-531

7. FATT I., 1959, The Biot-Willis Elastic Coefficients for Sandstone, *J. Appl. Mech.*, **26**, 296-297

8. FISHER G.M.C., GURTIN M.E., 1965, Wave Propagation in the Linear Theory of Viscoelasticity, *Quart. Appl. Math.*, **23**, 257-263

9. YEW C.H., JOGI P.N., 1978, Determination of Biot's Parameters for Sandstone, Part 1: Static Tests, *Experimental Mechanics*, **18**, 167-172

Zachowanie się fali przyspieszenia w lepkosprężystym ośrodku porowatym ze strukturą. Część II – Liniowo lepkosprężysty ośrodek porowaty nasycony cieczą

Streszczenie

W pracy bada się wpływ stuktury ośrodka porowatego nasyconego cieczą na propagację fali przyspieszenia. Ośrodek składa się z lepkosprężystego szkieletu i cieczy lepkiej ściśliwej.

W rozważanym ośrodku tensor akustyczny jest niesymetryczny. Struktura ośrodka wpływa na tłumienie lepkosprężyste fali podłużnej, natomiast nie wpływa na tłumienie fali poprzecznej.

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