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Student or Teacher? A Look at How Students Facilitate Public Sensemaking During Collaborative Groupwork

Michelle T. Lo
Stanford University

Jennifer L. Ruef
University of Oregon

As institutions strive to improve teaching and learning for all, educators must consider equitable instruction. This includes equitable distributions of authority and agency among students. The authors define distribution of authority as shared opportunities for decision-making in enacting mathematical tasks and agency as the power to carry out these self-made decisions. Equitable distributions of authority and agency can be enhanced in mathematics public sensemaking classrooms where students participate in discourse as active members of the classroom. In public sensemaking classrooms, students understand and acknowledge one another's ideas, present and revise arguments, and take risks by sharing ideas. This study investigates one group of students and how they positioned one another during mathematical groupwork in a public sensemaking classroom as well as how the positioning impacted the distribution of agency and authority. At the time of data collection, the students attended a school for grades 6–12 situated in an urban public school that focuses largely on preparing students for careers in the health sciences and other health-related fields. Data was collected as video footage and analyzed using a priori codes. Analyses indicate that one student replicates the role of teacher by redistributing authority and agency back to other students through self-removal. This is contrasted with other motivations for self-removal while doing mathematics in a team. The findings inform researchers and classroom teachers of potential metacognitive awareness of equity between students, positioning patterns that may occur during collaborative groupwork, and the effectiveness of public sensemaking classrooms on distributing authority and agency equitably during groupwork.

KEYWORDS: groupwork, authority, agency, mathematics education, public sensemaking

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MICHELLE T. LO is Master of Arts and teacher candidate at Stanford Teacher Education Program, Graduate School of Education, 485 Lasuen Mall, Stanford, CA 94305-3096; e-mail: mlo3@stanford.edu. Her research interests include student development of authority and agency in the classroom, teacher action research, mathematical student identities, and kinesthetic learning.

JENNIFER L. RUEF is Assistant Professor of Mathematics Education in the University of Oregon's Department of Education Studies, 5277 University of Oregon, Eugene, OR 97403-1215; email: jruef@uoregon.edu. Her research interests include how students identify themselves, or are identified by others, as being "good at math" as well as the ways in which students and teachers learn to make sense of mathematics and construct convincing and powerful arguments.

Introduction

Historically, particular groups of people, such as women, African Americans, Hispanics, Indigenous peoples, and Southeast Asians, have been marginalized and underrepresented in science, technology, engineering, and mathematics (STEM) fields. This restriction of access presents an equity problem and impoverishes STEM fields by limiting participation. The requirements of a 21st century workforce call for increased collaboration and problem solving (National Research Council, 2011). Thus, the ability to work productively with diverse groups is critical. As collaboration, problem solving, and diverse perspectives are required for future success, educators must shift their practices to ensure all students are gaining access to rigorous mathematics, especially those who have been traditionally excluded and marginalized (Burris et al., 2006, 2008; Darling-Hammond, 1995; Gamoran, 2009; Jett, 2019; Langer-Osuna, 2011; Oakes, 1990). Because mathematics is evident in the other STEM fields and beyond, an increasing number of mathematics educators are concerned with establishing equity in classrooms and implementing equitable practices to keep students, especially those who have been historically marginalized, interested in mathematics throughout their K–12 education (National Council of Teachers of Mathematics [NCTM], 2018). When students become active partners in creating learning opportunities for each other, the power of an equitable classroom manifests in a network of actors. No longer the sole province of the teacher, distributions of agency allow students to create opportunities for their colleagues to succeed. But how might teachers foster such distributions of equity among students? How might teachers and researchers recognize such agency when it occurs? Our study examines this problem space and expands knowledge of how students might co-construct equitable teaching and learning practices.

Our study was situated within an urban public school. The students and their teacher are thus part of a larger context that is often framed in terms of deficits (e.g., Hand, 2010; Horn, 2007; Jackson, 2010; Martin, 2012; Tate et al., 2018). This is evident in policies that punish schools and districts for low performance on high-stakes tests or impose impoverished forms of teaching and testing based on beliefs that urban students are less capable of engaging with challenging curriculum and pedagogy than their suburban peers (Cabana et al., 2014). Our study joins a growing body of research that frames historically minoritized and urban students in terms of assets (e.g., Dunleavy, 2015; Langer-Osuna, 2018; Leonard & Martin, 2013; Ruef, in press; Watanabe & Evans, 2015). This research is, at its roots, focused on equitable teaching and learning practices. We share an example of how one group of students mirrored their teacher's model for distributed authority and agency.

Distributions of authority and agency are core elements of teaching for equity. We define equity in teaching and learning mathematics in terms of learning environments that are inclusive and comprehensive. Inclusive learning environments provide students "a fair distribution of opportunities to learn" (Esmonde, 2009, p. 249) and comprehensive learning environments "open up interactional space for a broad range of competent ideas" (Dunleavy, 2015, p. 62). Effective mathematics classes should support students in becoming collaborative and critical problem solvers prepared to leverage productive change in the world. Gutiérrez (2009) framed equity in terms of *access, achievement, identity, and power.* Viewed through this lens, equitable mathematical teaching and learning practices enhance access to challenging curriculum and engaging learning environments. They support achievement in mathematics by widening the ways students can perform competence—when there are more ways to be "good at math," more students will be competent. This definition of competence draws from research on Complex Instruction (Cohen & Lotan, 2014; Horn, 2012; Ruef, in press). Seeing oneself as mathematically competent increases identification with mathematics, both individually and collectively. Power, in turn, refers to relational equity within the classroom or, more specifically, how students and teachers treat one another (Boaler & Staples, 2008). Power must also be considered beyond the classroom, and we must recognize how politics impact school policy and even how classroom achievements can impact policy (e.g., Gutstein, 2003). For the purposes of this paper however, we focus on equity within a classroom where teaching and learning mathematics are inclusive and comprehensive.

Although educators may generally agree that teaching and striving for equity are important, these practices are often difficult to enact. This article shares a success story that sheds light on how a teacher and her students co-constructed an equitable teaching and learning culture. Equitable distributions of authority and agency can be enhanced in mathematics classrooms where students participate in discourse as active members of the classroom. Thus, one way to investigate equity in action is through the degrees to which authority and agency are equitably distributed among members of a classroom. Authority is the amount of "given opportunities to be involved in decision-making," which may include "establishing priorities in task completion, method, or pace of learning" (Gresalfi & Cobb, 2006, p. 51). We define agency as the ability to carry out those self-made decisions in the classroom. These decisions are not necessarily restricted to a mathematical task but may extend to how students physically situate themselves in order to complete a task (Ruef, in press). In this paper, we focus primarily on authority and agency during the completion of a specific task. Note that authority and agency are not limited to the students in the classroom. Teachers are generally recognized as the primary holders of authority and agency and often must actively work to share these with students (Cobb et al., 2009; Cohen & Lotan, 2014; Horn, 2012; Ruef, in press).

Educators are increasingly utilizing collaborative learning through groupwork in mathematics classrooms (NCTM, 2018). Previously, transmission-based models of teaching and learning were frequently utilized in United States mathematics classrooms. In this form of learning, students often passively participate as learners by quietly taking notes or reproducing ways to solve problems (Ruef, 2013). Transmission-based learning may reinforce typical representations of what it means to be "good at math," which suggest that those who are able to absorb mathematical knowledge through lecture and notetaking, sitting still, being quiet, and doing well on exams are seen as being "good at math" (e.g., Boaler, 2002). Transmission-based instruction is problematic both in terms of learning mathematics and fostering equity.

In contrast, sensemaking-based instruction requires students to be active learners. Building a sensemaking culture in the classroom can unlock new ideas about what it means to be "good at math" for students, especially when students learn by participating in *public sensemaking* (Ruef, 2016, in press). Features of public sensemaking include striving to respect, acknowledge, and understand each other's perspectives; welcoming mistakes and productive struggle as aspects of learning; and taking risks by "sharing one's thinking; presenting, critiquing, revising, and refining arguments" (Ruef, 2016, p. v). These aspects of public sensemaking clearly contrast the common images of what it means to be "good at math" in transmission-based instruction.

Few classrooms land squarely at either end of the transmission-sensemaking continuum. Most contain elements of both ends of the spectrum with teachers and students working hard to create productive learning environments. The NCTM (2018) publication *Catalyzing Change* details the current general state of high school mathematics teaching and learning in the United States and offers a vision for its evolution. One of the equitable teaching practices suggested in *Catalyzing Change* is for teachers to "facilitate meaningful mathematical discourse" among students (p. 30). Productive groupwork (e.g., Complex Instruction) is one tool that brings forth meaningful discourse.

When students engage in groupwork, they have opportunities to collaborate. During this work, students are often tasked with collectively solving mathematical problems, sharing or double-checking solutions from assignments, or putting together presentations. In all three cases, students are asked to publicly communicate their ideas, questions, and solutions as they engage in public sensemaking. Groupwork allows teachers to delegate authority towards the students, empowering "students to argue, evaluate, and confirm the validity of their mathematical ideas" (Dunleavy, 2015, pp. 63–64). The result is a broader definition of what it means to be "good at math" in the classroom.

Conceptual Framework

Our work is situated within related work attending to student identity and learning opportunities. Both of these are informed by distributions of agency and authority and broader definitions of competence. To determine distributions of authority and agency, our conceptual framework draws from positioning theory (Esmonde, 2009;

van Langenhove & Harré, 1999), and it is framed by Esmonde's (2009) and DeJarnette & González's (2015) classifications of positions and actions in systems of negotiation. We extend this classification with the addition of the positions of contributor and remover.

Positioning Theory

This study framed the students' mathematical interactions in terms of *positioning*. Positioning theory utilizes speech and actions to identify someone's rights, obligations, and duties (van Langenhove & Harré, 1999). The researchers analyzed students' interactions during groupwork. Specifically, the current study centered on *interactive positioning*, which occurs when students position one another in relation to each other (Davies & Harré, 1999). Students can position themselves or be positioned by others. Additionally, positioning can be "intentional or unintentional, explicit or implicit" (Ruef, 2016, p. 11). This study examines the types of positioning that occurred in one public sensemaking classroom in terms of evident positions created during groupwork.

Esmonde (2009) described three types of positioning that may be present during groupwork: expert, novice, and facilitator. The expert is often deferred to mathematically and granted or ceded authority to decide whether an idea or work is correct. Novices often defer to experts, asking for and receiving help from others. Novices position themselves as less competent, but they may sometimes challenge the expert. Facilitators regulate group activity and participation from group members, and they actively elicit group members' contributions to joint problem solving and include them in discussions. In addition to Esmonde's (2009) three types of positioning during groupwork, we define two additional types of positions that may occur. The first is the *contributor*, who participates in groupwork non-mathematically but increases the group's productivity in some way. This might include keeping track of time or gathering necessary supplies. The second position we define is the *remover*. This position is evident when a student removes themselves from the task at hand, leaving other students to complete the task. The reasons a student is positioned as a remover may vary. For example, a student might remove themself from the task when home or family life present distractions from classwork. Although each type of positioning is common during groupwork, each one is not always present or obvious.

Ideally, in an equitable classroom, positions are impermanent as an equitable classroom culture would produce relatively frequent shifts in positioning. This is because each student brings a different level of prior knowledge or expertise to the task at hand. A student who recalls a strategy used in a similar problem may be viewed as an expert on one task and a novice on another where the student has less mathematical insight. Regardless of prior knowledge, any student may step into the role of facilitator, guiding the group's social interactions and mathematical work. Similarly, any student may play the role of contributor in moving the task forward through nonmathematical work. Finally, students might remove themselves from the task at hand, stepping into the role of remover. Some students may rarely change their role either by choice or because they are excluded by peers (Cohen & Lotan, 2014; Esmonde et al., 2009; Horn, 2012). This limits their opportunities to sensemake and is an indicator that authority and agency are not equitably distributed in the classroom.

System of Negotiation

To classify speech or actions produced by students, it is important to discuss how certain moves may map onto the five positions (i.e., expert, novice, facilitator, contributor, and remover). According to Halliday (1984), an interaction is an exchange that has two variables. The first variable is a commodity that is exchanged; this includes either information or goods and services. The second variable is the role of the speaker/actor, which involves giving or demanding a commodity. For this study, the role of the speaker/actor could also include rejecting or ignoring a commodity. This study employed a system of negotiation that includes five moves that a speaker/actor can employ during an interaction, all of which are combinations of rejecting, giving, or demanding information or services. Table 1 provides the five moves that may occur during an interaction and an example of what the move might look like in a mathematics classroom. The moves are divided into those made by primary (K1) and secondary (K2) knowers, primary (A1) and secondary (A2) actors (Berry, 1981), and primary deflectors (D1). In terms of authority and agency, note that K1 moves hold high levels of authority because primary knowers create opportunities for themselves to provide or make decisions on mathematical knowledge. A2 moves hold high levels of agency because secondary actors initiate decisions to be carried out.

Table 1
Types of Negotiation Moves and Examples

Negotiation Move	Definition	Example
Primary Knower (K1)	Provides mathematical information.	"Area is length times width."
Secondary Knower (K2)	Requests mathematical information.	"What's the formula for area of a rectangle?"
Primary Actor (A1)	Provides, or offers to provide, an action related to the activity of doing mathematics. Performs or offers to perform activity.	[reads the math problem aloud]
Secondary Actor (A2)	Requests or commands an action related to the activity of doing mathematics. Directs activity.	"Can you read the problem out loud?"
Primary Deflector (D1)	Deflects a request for information or knowledge or request to present information or knowledge.	"I'm not telling you the formula for area."

The study adapted and added to the mapping created by DeJarnette & González (2015) to relate the system of negotiation and the types of positioning that appear during groupwork (Figure 1). In addition to the mapping used by DeJarnette & González, we created two more mappings such that each negotiation move maps directly onto a position, creating a one-to-one mapping.

K1 moves are mapped onto the expert because the student providing information is also one that is most likely to be deferred to during mathematical groupwork. K2 moves, negotiations that request information, align with the position of novice. By definition, a novice holds less knowledge about the mathematical problem at hand and, as a result, needs more information to build understanding. Next, A1 moves are mapped onto the contributor because the contributor participates non-mathematically in such a way that moves the group's productivity forward. A2 moves are mapped onto the facilitator role because those requesting action are inherently prompting all students to participate in completing the mathematical task in some way. Finally, through performing D1 moves by rejecting, deflecting, or ignoring requests by other students, the remover excuses themself from participation in the groupwork.

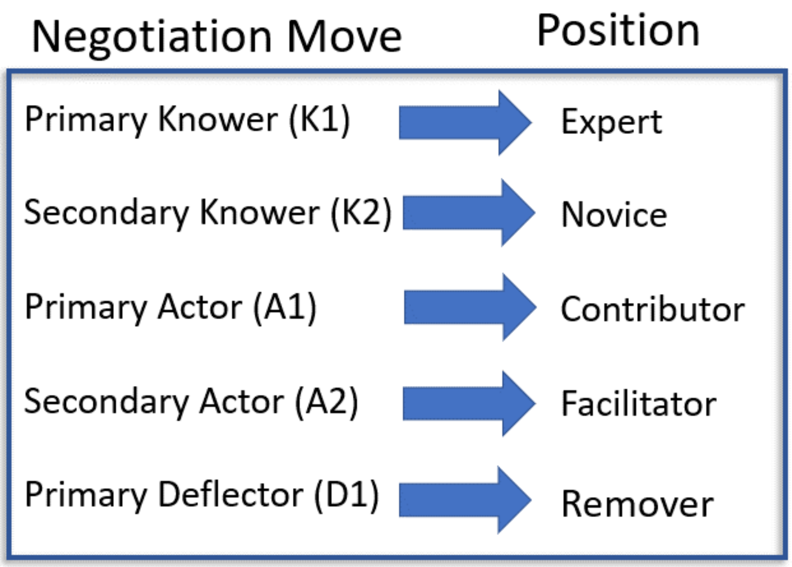

Figure 1. One-to-one mapping: Negotiation moves to position assigned to a student by self or other.

Research Questions

Prompted by the prevalent and growing use of groupwork in mathematics classrooms, this study examined the different ways a group of students positioned each other in a classroom that encouraged public sensemaking. We also considered how positioning affects the distribution of agency and authority among students in a group. These goals translate to the following research questions:

1. How do students negotiate positioning during mathematical groupwork in a public sensemaking classroom?
2. How does interactive positioning impact distributions of agency and authority among students?

The following study provided a snapshot of the ways students positioned one another as well as how agency and authority were distributed in a public sensemaking classroom during collaborative groupwork.

Methods

This embedded case study (Yin, 2017) draws from a larger study and body of data that took place across the 2015–2016 school year in three sixth-grade mathematics classes all taught by one teacher (Ruef, 2016). The larger study made use of presurveys, postsurveys, ad hoc exit tickets (i.e., short answer questions), participant observation collection of field notes and video footage, and formal and ad hoc interviews with all participants. Analysis from the larger study was ongoing and iterative, making use of both quantitative (i.e., linear regression analysis) and qualitative (i.e., a priori and emergent coding; analytic memoing) methods (Emerson et al., 2011).

Participants & Setting

The participants in this study were students who attended City School (a pseudonym) in California. City School is housed in a large urban area and focuses largely on preparing students for careers in the health sciences and other health-related fields. Additionally, City School educates students in grades 6–12, and many of its graduates meet the criteria to become first generation college students. Over 91% of City School's students qualified for free and reduced lunch at the time of data collection, and statistics from 2009 demonstrate that 74% of the students identified as Latinx/Hispanic, 11% as African-American, 11% as Asian, 2% as Filipinx, 1% as Native American, and 1% as White. The participants in this study were sixth graders taking a mathematics class taught by Ms. Isabella Mayen. Ms. Mayen was chosen for her focus on public sensemaking. All names are pseudonyms chosen or approved by the participants.

At the time of the study, Isabella Mayen was a second-year teacher of mathematics. She identifies as Latina, had attended a well-known teacher education program, and is well-versed in how to facilitate public sensemaking. The tables in her classroom were arranged in five or six groups of four, and the groups were known as "teams." In Ms. Mayen's classroom, working in a team implicitly meant that "everyone has equal status as a sensemaker, and including everyone in a team is an important social function" (Ruef, 2016, p. 58–59). The message of teamwork attempted to prevent opportunities for exclusion, thus promoting equity in learning mathematics. Team membership changed every two to three weeks and was randomly assigned (e.g., Cohen & Lotan, 2014; Horn, 2012). It is important to note that Ms. Mayen and her colleagues had control over the content and pacing of the curriculum they taught. They selected, composed, and adapted curriculum to best fit the learning trajectories of their students and prepare them for success in STEM-related studies and careers.

This study centered on one particular team in September of 2015, about one month into the academic year. The students in this team were Brooklyn, Kazaly, Flor, and Elena. At this point in the school year, Ms. Mayen was leading a three-day lesson on finding the areas of polygons. She tasked the teams with finding the area of an irregular trapezoid (Figure 2) accounting for the number of unit squares without using standard formulas.

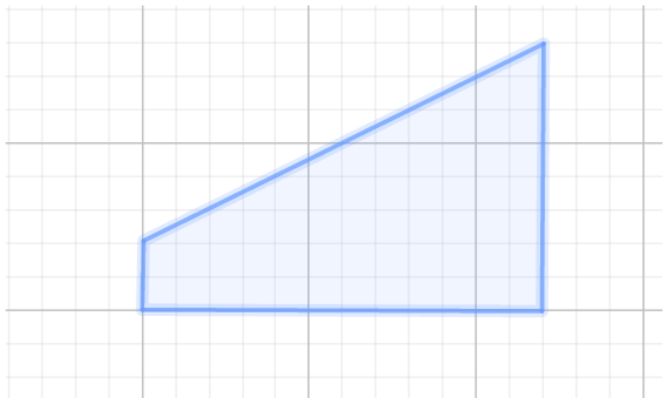

Figure 2. Irregular trapezoid with area of 60 square units.

Ms. Mayen's goal was to have the entire team agree on one solution and for representatives to present the team's method to the class. One or more students from each team were expected to present, and the teams decided who would represent them.

Data Sources

This study utilized preexisting classroom video footage of students at City School from 2015. The footage was originally recorded by Ruef. Video recording focused heavily on the first four weeks of the school year, including 13 of the first 18 days. Because Ruef collected the primary data, she was able to choose when and which groups to film during class. The accumulation of video footage was captured from the "perspectives and lenses" (Ruef, 2016, p. 34) of the original researcher. According to Ruef, she "strategically and purposively" picked moments where there was "strong evidence of positioning by both students and the teacher" (Ruef, 2016, p. 34). For the purposes of the current study, a 26-minute video was excerpted from the total video footage to be analyzed as a case study. This portion of the accumulated video footage from Ms. Mayen's class was selected for analysis because it displayed salient attributes of distributing agency and authority among students during mathematical groupwork.

Data Analysis

Analysis of the video footage was completed using MAXQDA 18.1.1, a qualitative data analysis software. The videos were first transcribed, then coded using the negotiation and positioning moves (see Table 1) as a priori codes. Table 2 provides an example of how student interactions during one continuous conversation were coded.

Table 2
Example of Interaction Using Negotiation Moves as Codes

Line	Speaker	Transcription	Code
1	Elena	Okay, let's get back to work.	A2
2	Flor	What are we getting back to do? We're finished!	D1
3	Brooklyn	¡No sé! (I don't know!)	K2
4	Kazaly	How do you got 66?	K2
5	Brooklyn	You add them all together and you get 66.	K1
6	Flor	You add them altogether once you get all these numbers right here.	K1
7	Brooklyn	And even if you - and if you count them one by one too, you still get 66.	K1
8	Flor	Yeah.	K1

Note that not all interactions were coded with the five moves if the interaction did not fit any of the defined codes. The unit of analysis was a talk turn. As it evolved, our analysis was discussed in regular research meetings with colleagues to invite "outside eyes" into the process. We discussed existing and emergent codes to determine face validity, asking if the codes mapped to what others observed in the data. We also considered additional codes suggested by colleagues. Given the focus of the research questions and the data we were analyzing, we were satisfied that we had reached saturation of observed phenomena in relation to both codes and the data set. The authors further discussed and refined the coding scheme. We then selected 20% of the video footage to calculate interrater agreement, resulting in a Cohen's Kappa value of 0.616. Satisfied with this level of interrater agreement, Lo calculated the percentages of individual negotiation moves out of the total number of negotiations (see Table 3).

Results

The percentages in Table 3 revealed several findings. On this team and for this task, Brooklyn was positioned as expert-facilitator-remover. Brooklyn accounted for 54 percent of the total number of K1 moves, 55% of the total number of A2 moves, and 47% of the total D1 moves. Flor was positioned as contributor because 47% of the total A1 moves belonged to her. There was no prominent novice because none of the students exhibited a high percentage of the total K2 moves.

Table 3
Percentage of Frequencies of Negotiation Moves Among Students

Negotiation Move	Elena	Brook-lyn	Kazaly	Flor	Total # of Negotia-tions
Primary Actor (A1)	2 (12%)	5 (29%)	2 (12%)	8 (47%)	17
Secondary Actor (A2)	9 (27%)	18 (55%)	3 (9%)	3 (9%)	33
Primary Knower (K1)	25 (17%)	81 (54%)	14 (9%)	31 (20%)	151
Secondary Knower (K2)	7 (19%)	13 (36%)	6 (17%)	10 (28%)	36
Primary Deflector (D1)	4 (21%)	9 (47%)	2 (11%)	4 (21%)	19

Distribution of Agency

Recall that removers can vary in their reasons for excluding themselves from groupwork. Although Brooklyn was positioned as the expert-facilitator of the group, her self-positioning as the remover provides evidence that she distributed agency back to the other team members. Table 4 shares an instance where all four students worked to decide which one of the two methods they had developed for finding the area of an irregular trapezoid to present to the class. Although Elena and Flor initially volunteered to present to the class before this discussion, both deflected possible opportunities to make decisions about the method they were to present. At one point, Elena removed herself from the role of presenter saying, “I don’t wanna go up ‘cause they ask so many questions!” Note that this decision to choose not to present is evidence for agency. The team was thus in flux regarding who would present their method to the class.

Table 4
Deciding Who Will Present to the Class

Line	Speaker	Transcription	Code
1	Elena	Which one should we do?	K2
2	Flor	Brooklyn, which one should we do?	K2
3	Brooklyn	You guys are going up there, so you guys decide,	D1
4	Brooklyn	but remember you still have to count the little ones	K1

In the transcript in Table 4, Elena attempted to broadly ask the team for help (Line 1), but Flor explicitly funneled the opportunity to make a decision to Brooklyn (Line 2) perhaps due to Brooklyn's established position as expert. However, Brooklyn redistributed agency back to Elena and Flor by reminding them they volunteered to present and pressing them to choose (Line 3). This move positioned Brooklyn as the remover. By not making the decision on which method to use, Brooklyn removed herself from the decision-making process and redistributed agency back to her teammates in the process. Brooklyn also retained her position as expert by reminding them to count "the little ones," or the partial square units (Line 4).

Distribution of Authority

In addition to distributing agency, Brooklyn also distributed authority to the other members of the group by positioning herself as remover in addition to expert-facilitator. By the end of the lesson, Brooklyn, Elena, Flor, and Kazaly had discussed a second new method on how to find the area of the trapezoid. Specifically, the team discussed the idea that the diagonal line through two square units divides the two squares into one square unit each on the top and bottom half of the rectangle. Table 5 makes it clear they had yet to confirm which agreed method to present to the class.

Table 5
Deciding Which Method to Present to the Class

Line	Speaker	Transcription	Code
1	Brooklyn	I think you guys should do this one – and then explain – remember it's half? Half of two is one?	K1
2	Elena	I don't know how to explain that.	K2
3	Elena	Or you should go.	A2
4	Brooklyn	I'm not going.	D1
5	Flor	I'll go up with you, Brooklyn.	A2
6	Brooklyn	I don't want to go.	D1
7	Elena	Everybody goes.	A2

Brooklyn exhibited traits of a participant in a public sensemaking classroom by acknowledging alternative ideas. However, Elena refrained from risk-taking and presenting the newer method (Line 2). Because she does not yet know how to explain it, it is highly probable that Elena rescinded her offer to present in front of the class in fear of embarrassment.

Next, Brooklyn turned down Elena's request for Brooklyn to go to the board to present the team's method (Line 4). By rejecting Elena's request, "Or you should go," Brooklyn attempted to redistribute authority back to her teammates, Elena and Flor, through a deflector (D1) move. Brooklyn increased the number of opportunities for Elena and Flor to act as experts when she rejected the invitation to present to the class. In response, Flor, who originally volunteered with Elena to present the team's method to the class, took up the new opportunity to present with Brooklyn (Line 5). Had Brooklyn readily accepted the invitation to present the newer method, the opportunity for Flor to participate in public sensemaking may have been diminished, overshadowed by Brooklyn's implicit authority through her position as expert. Finally, it is interesting to note that as Brooklyn redistributed authority to her teammates, Elena declared, "Everybody goes" (Line 7). This statement created an opportunity to present as a team and reinstated Elena as a presenter.

Discussion

This paper aimed to answer two questions. First, 1) How are students positioned during mathematical groupwork in a public sensemaking classroom? From this snapshot drawn from video footage of Brooklyn, Kazaly, Elena, and Flor, one combination of positioning that may occur in public sensemaking classrooms resulted in a clear expert, facilitator, contributor, and remover, but no clear novice. Identifying positions during groupwork can help determine what equity issues may be present in mathematics classrooms, especially when teachers are looking out for and circumventing shifts from temporary positioning to calcified positioning. In this case study, the lack of a clear novice did not raise any red flags. However, asking questions and requesting information to make sense of mathematics, a critical role of the novice position, is important in public sensemaking. In this instance, the teacher of the classroom, Ms. Mayen, would need to ensure that there is never a permanent absence of a novice, as that would indicate that the class has no need for mathematical sensemaking. On the other hand, in order to maintain equity in the classroom, Ms. Mayen would also need to look out for potential calcifications of positioning during groupwork.

Analysis of the video footage also aimed to answer the following question: 2) How does interactive positioning impact distributions of agency and authority? The two instances discussed above highlight the ways in which two students committed deflection (D1) moves and removed themselves from participation in presenting their strategy to the class for different reasons. Elena refused to present because she was apparently anxious about answering questions while at the board. Brooklyn, while positioned as an expert-facilitator, redistributed authority and agency to her teammates by "refusing to be the source of authority" (Ruef, in press) and so positioned herself as a remover. By taking this action and others like it, Brooklyn stepped into the role of their teacher, Ms. Mayen. Brooklyn exhibited the traits of a teacher of a public sensemaking classroom by encouraging risk-taking as she pressed Elena and Flor to present while removing herself from the discussion. This allowed others to make decisions while they were also attempting to make sense of the mathematics they were engaged with.

Teaching and learning become more equitable when students do not permanently position each other. Temporary positioning, fluid across activities, creates space for students to fluctuate between expert, facilitator, contributor, novice, and remover positions. Even though Brooklyn is positioned as the expert-facilitator on the team, she attempts to dissolve her position by allowing others to decide on a method to present and providing space for Elena and Flor to take risks, thus positioning herself as a remover. In other words, her self-positioning as remover limited her positioning as the expert-facilitator as temporary. She accomplished this through self-refusal and invitations to peers to participate in presenting the group's method.

Brooklyn's imitation of Ms. Mayen's role as a teacher surfaces several interesting points. First, as Brooklyn took up actions similar to those of a classroom teacher, there are new questions about whether students are metacognitively aware of equity issues in their own classrooms and teams. Dunleavy (2015) stated that "striving toward equity in mathematics education invokes constant, purposeful work from the teacher as she or he seeks to diminish differences in access, opportunities, and outcomes for students" (p. 64). Although teachers are typically the ones tasked with noticing and treating inequitable participation in mathematics classrooms, this case study reveals the possibility that students may also have the capacity to address the distribution of authority and agency by monitoring how many opportunities they have to participate in sensemaking. Teachers cannot attend to every student individually and simultaneously, but their bandwidth is extended and amplified when students foster equity within groups, as demonstrated by Brooklyn. Public sensemaking classrooms with established norms for students to take risks, acknowledge ideas, grapple with mistakes, and share their thinking can open pathways for students to maintain equitable distributions of authority and agency within their own teams.

Limitations and Next Steps

This case study is specific to a particular group engaged in a specific task at a fixed time and place. Group dynamics vary by task, participation structures, and participants. Thus, generalizing student positioning in the classroom is beyond the scope of this paper. Extending this research might include analyzing different groups of students in Ms. Mayen's classes to see what kinds of positioning are present and to map their fluidity. Student interactions are also more complicated than five types of negotiation moves. Future studies and expanded teaching and learning contexts may surface additional types of moves.

Conclusion

In summary, this paper aims to provide a window into how agency and authority were redistributed in terms of public sensemaking when students positioned one another during groupwork. It is heartening to present evidence of students from historically minoritized backgrounds taking up the work of equitable distributions of agency and authority in facilitating public sensemaking for themselves. This study shows, empirically, that historically marginalized students of color are capable not only of making sense of mathematics at a deeply conceptual level, but also in providing opportunities for equitable learning. It also dives deeply into the ways this group of four students positioned each other and challenged one another to present their thinking to the class. It was relatively early in the school year, and the norms for public revision of work were still under development (e.g., Jansen, 2020; Ruef, in press). Public sensemaking thus required bravery and persistence as the team did not know what questions their classmates would ask.

Teachers who facilitate public sensemaking develop lenses through which they notice various classroom phenomena. They notice risk-taking, productive discourse, and potential status issues. Teachers who notice, and praise, students who act as

Brooklyn did—creating opportunities for classmates to step into agentic roles by stepping back—can communicate new ways of being competent to their students. One can be good at math by deflecting if that deflection invites a colleague into agency and authority.

Teachers must occasionally support students who remove themselves from instructional activity for a variety of reasons. Although the reasons for "checking out" are often legitimate, removal has real consequences (e.g., Hand, 2010). When students do not participate, at best they deprive their group of an engaged thought partner in sensemaking. However, the researchers argue that removal can be a positive and supportive act.

Both Elena and Brooklyn deflected requests to present at the board but for different reasons. Elena's motivation appeared to stem from fear: what would happen if she were at the board and her colleagues asked her challenging questions? Brooklyn's motivation appeared to be sharing power: who would step up if she refused the role of presenter? The answer appears in the results; Brooklyn's self-removal created a vacuum. Her encouragement to Elena to present provided a push, and Elena stepped back into the role of presenter. Elena's temporary removal and her explanation for it allowed her colleagues opportunities to rebut her reasons and encourage her to take up the challenge.

This study supports equitable teaching and learning practices by expanding frameworks for positioning. This framework can be applied in a systematic manner in researching student and group interactions. Our findings support teachers in examining group and class dynamics for fluidity in status (i.e., expert, novice, facilitator, contributor, and remover). The increased presence of collaboration and problem solving happening in urban public sensemaking classrooms gives us hope for the future of diversity and success in STEM-related fields.

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