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MATHEMATICAL REPRESENTATION ABILITY OF PROSPECTIVE MATHEMATICS TEACHERS IN SOLVING TRIGONOMETRIC PROBLEMS

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ABSTRACT

This study aims to investigate the proficiency of prospective mathematics teacher students in mathematical representation. To achieve this goal, we used a qualitative research design with a descriptive approach. The research subjects consisted of three undergraduate students from Mathematics Education department at a private university in Yogyakarta. Data were collected using a test for mathematical representation ability and interviews. The data were analysed descriptively using techniques by Miles and Huberman. It shows that the mathematical representation ability of prospective teacher students was in low category. This can be seen from the average achievement of indicators of mathematical representation ability (34.74%). The ability to create and use representations for collecting/organizing, recording, and communicating mathematical ideas is 33.33%. The ability to select, apply and translate various forms of representation mathematics to solve problems is 29.76%. And the ability to use multiple representations is 41.11%.

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INTRODUCTION

In today's fast-paced and ever-evolving technological landscape, there is a pressing need for experts who possess a strong aptitude for managing novel concepts, adaptability to change, resilience in the face of uncertainty, proficiency in maintaining order, and problem-solving skills. Maccini and Gagnon's (2002) argued that the ability to tackle mathematical and real-world challenges is commonly called Mathematical Power.

The National Council of Teachers of Mathematics (NCTM) (2010) revealed that there are at least five mathematical abilities that students need to have, namely problem solving, reasoning and proof, connection, communication, and representation. One of the mathematical abilities that have been mentioned is mathematical representation. Mathematical representation can be interpreted as the ability to state or express mathematical ideas or ideas in the form of pictures, graphs, tables, diagrams, mathematical equations, symbols, writing, or a combination of these forms (Mandur et al., 2016; Syafri, 2017). Furthermore, NCTM (2000) explained that the word 'representation' refers to processes and products. In other words, representation can be interpreted as capturing mathematical concepts or relationships in several forms.

Representational abilities are integral to the standard skills required for learning mathematics. The Ministry of National Education (now known as the Ministry of Education and Culture) and the National Council of Teachers of Mathematics (NCTM) are the key parties involved. According to the Ministry of National Education (Effendi, 2012), one of the fundamental objectives of teaching mathematics in primary and secondary schools is to equip students with problem-solving skills, including the ability to comprehend problems, develop mathematical models, and solve them effectively. Furthermore, students are expected to interpret and evaluate the solutions obtained from the mathematical models.

As previously explained, NCTM also incorporates representation skills into the standard abilities that students need to have. NCTM (2010) stated that schools, from pre-kindergarten to Grade 12, must aim so that all students can: (1) create and use representations to collect/organize, record, and communicate mathematical ideas; (2) select, apply and translate various forms of mathematical representations to solve problems; and (3) use various kinds of representations to model and interpret physical phenomena, social phenomena, and mathematical phenomena. These three skills are often referred to as indicators of mathematical representation ability and are commonly used to determine whether students already have good mathematical representation skills.

Mathematical representation skills facilitate problem-solving, express mathematical concepts, and enhance students' mathematical abilities (Sabirin, 2014; Noer & Gunowibowo, 2018). This poses a significant challenge for educators, including prospective teachers, who must be equipped with effective strategies to address students' difficulties in representing mathematical problems. Moreover, a teacher's limitations in conveying mathematical content can have a detrimental impact on students' learning outcomes, as students tend to emulate the approaches modeled by their teachers (Mandur et al., 2016).

Not only for students but the ability to represent mathematically is also crucial in the teaching process (Mainali, 2021; Fennel & Rowan, 2001). That is, a teacher or prospective teacher should also master the ability of mathematical representation. At least one group of researchers have examined the mathematical representation abilities of prospective mathematics teachers. Astuti (2017) found that some prospective mathematics teachers exhibit inadequate mathematical representation skills, as evidenced by unfulfilled representational ability indicators. The causes of this deficiency may be attributed to several factors, such as inadequate learning media, insufficient experience, and inadequate training.

Proficient mathematical representation skills are essential for prospective teachers to effectively guide their students in problem-solving. However, research suggests that many prospective mathematics teachers lack representation skills. Therefore, this study aims to investigate the mathematical representation ability of prospective mathematics teachers at a private university in Yogyakarta. From these research results, the mathematics education study program will be able to arrange learning and test items in accordance with the abilities of the prospective mathematics teacher.

Based on this explanation, the researchers were interested in describing the profile of the mathematical representation ability of prospective mathematics teachers. In addition, this research is expected to be the first step to solving problems related to students' mathematical representation abilities.

METHOD

This research was conducted using a qualitative method with a descriptive approach. The qualitative research method is also called the naturalistic research method because the research was carried out in natural conditions or settings (Sugiyono, 2018). A natural setting is used to find the authenticity of data from qualitative research (Bachri, 2010).

This research was conducted online on May 19, 2021. The subjects in this study were three students of the Mathematics Education Study Program, class of 2018, from a private university in Yogyakarta. The subjects were selected based on their mathematics learning outcomes:1 subject with high ability, one with moderate ability, and one with low ability. The subjects worked on five questions about trigonometry.

The main instrument in this research was the researchers because, in qualitative research, the researchers act as the main instrument in collecting and analyzing data (Fraenkel & Wallen, 2008; Merriam, 2009). Besides, in this study, the instruments were test questions and interview guidelines to assist researchers in data collection. The test items consist of five items to measure the subject's mathematical representation ability on trigonometry comparisons. Test items were made based on indicators of mathematical representation abilities issued by NCTM. The interview guide consisted of seven questions grouped into three aspects based on the indicators of mathematical representation previously described. The three aspects were creating and using representations for collecting/organizing, recording, and communicating mathematical ideas; selecting, applying, and translating various forms of representation mathematics to solve problems; using multiple representations to model and interpret physical phenomena, social phenomena, and mathematical phenomena. Interviews were used as a form of triangulation to verify the data from the mathematical representation ability test results. The supporting instruments used in this study were validated using the Focus Group Discussion (FGD).

The data collection process in this study produced two types of data: (1) test and (2) questionnaire data. The data were analyzed using qualitative data analysis techniques, according to Miles & Huberman. There are at least three stages carried out in analyzing qualitative data: data reduction, data display, and conclusion drawing/verification (Miles and Huberman in Sugiyono, 2018). Data reduction is summarizing, selecting, and focusing data on the main or essential aspects of data. During data reduction, researchers must also pay attention to any outliers or unpatterned data points (Sugiyono, 2018). Once data has been reduced, it must be effectively communicated to various stakeholders in a clear and understandable format. This can be achieved using different presentation formats, such as narrative text, network diagrams, charts, and other visual aids. After data reduction and presentation, the next step is drawing and verifying conclusions. The conclusions presented should address the research questions or hypotheses formulated earlier and provide insights or recommendations based on the findings. This verification process is essential to ensure the accuracy and reliability of the research results and to prevent erroneous conclusions that could undermine the research outcomes.

RESULT AND DISCUSSION

Result

As previously explained, in this study, five items were used to determine the mathematical ability of prospective mathematics teachers. Each item has been prepared based on indicators of mathematical representation ability according to NCTM. Besides, each item had also been prepared based on the question indicators to suit students' abilities regarding comparative trigonometry. Details of the item indicators used and the corresponding items can be observed in Table 1. Furthermore, Table 2 presents detailed descriptions of the questions used to measure mathematical representation ability.

Table 1. Details of Question Indicators

Question Indicators	Question Number
Determine the height of an object based on information about the angle of elevation and/or depression using trigonometry	1,4
Evaluate statement(s) about the angle of elevation with the help of trigonometry	2,3
Determine the length from a side of a triangle using trigonometry	5

Table 2 shows the question used to test the prospective mathematics teachers.

Table 2. Detailed Question Descriptions

No	Question
1	The angle of depression formed between the top of a tower and a point on the ground is 30°. Ten meters above that point, a drone was flying. If the elevation angle of the drone to the top of the tower is 45°, a. Make an illustration representing the case. b. Which shape do you find easier to understand to help determine the height of a tower? Illustration or verbal statement? Why? c. Determine the height of the tower!
2	Aldo and Syella argue about a statement. The statement is as follows: “A person looks at the top of a tower with an elevation angle of 30°. If the height of the tower is doubled, the elevation angle will also be doubled. Aldo agreed with this statement, but Syella did not. Create an illustration of the case. Next, indicate whose opinion is more appropriate and provide reasons!
3	Wika is observing the top of a building from a certain point. She believes that if the height of the building were to increase by 20% and the distance between her and the building was to increase by 20%, then the elevation angle would change. Investigate whether Wika's opinion is correct or not! If it is incorrect, make the correct statement regarding the problem!
4	Surya is at the top of a building with a height of 20 meters. He uses binoculars to see the hot air balloon flying higher than the height of the tower. The sun's elevation angle to the blimp is 60°, while the sun's depression angle to the blimp's shadow is 30° (assuming the blimp's image falls 'right' below the balloon). a. Sketch the situation and write the relationship between the height of the building, the height of the hot air balloon, and the distance from the building to the hot air balloon! b. This situation can be represented in different forms, namely series of words and sketches. In your opinion, which

	representation is easier to understand for determining the height of a hot air balloon from the ground? Provide reasoning!
	c. Determine the height of the hot air balloon from the ground!
5	It is known that triangle ABC has side lengths AC=b cm, BC=a cm, and a+b=12 cm. If angle A is 60° and angle B is 30°, determine the length of side AB!

Table 3 presents the research results of the mathematical communication skills of the three subjects.

Table 3. The Mathematical Representation Skills of Prospective mathematics teachers

Indicator of mathematical representation	Achievement
Create and use representations to collect/organize, record, and communicate mathematical ideas	33.33%
Select, apply, and translate various forms of representation mathematics to solve problems	29.76%
Use multiple representations to model and interpret physical phenomena, social phenomena, and mathematical phenomena	41.11%
Average	34.74%

It can be seen that the achievement of mathematical communication skills for each indicator is 33.33%, 29.76%, and 41.11%, with an average of 34.74%. This indicates that the ability of prospective mathematics teachers is lacking. The following section discusses the representation skills of each subject in more depth.

Discussion

This section discusses the ability of each subject to solve mathematical representation problems.

Problem 1

The following describes subject 1's answers based on Problem 1. The answer to Subject 1 is shown in Figure 1.

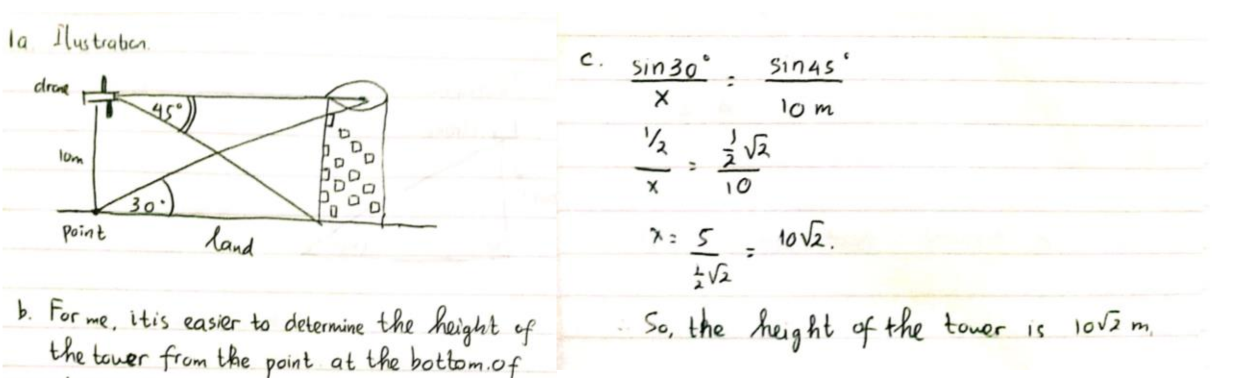


Figure 1. Subject 1's answer to Problem 1

Based on the test results, representation errors were found in the work of Subjects 1 and 3. In Subject 1's work, the 30^0 was described as the elevation angle from the ground level to the top of the tower. It was also reinforced by the results of interviews, showing similar results. Geometrically, this value is correct based on the relation of lines and angles. However, the meaning process by Subject 1 was inappropriate with the question. In addition, in subject 1's work, 45^0 was depicted from a line segment, with the tangent to the top of the tower and the base of the tower. It is incorrect because one of the line segments formed should touch the top of the tower, and the other line segments have the same height as the height of the drone parallel to the ground. In Part C, Subject 1 also provided incorrect solutions because the representations made by Subject 1 were incorrect.

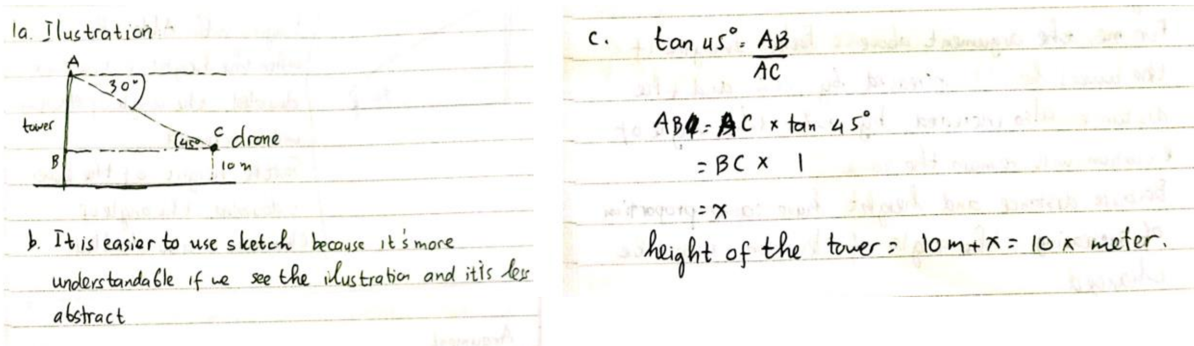


Figure 2. Subject 2's answer to Problem 1

Subject 2 could represent the situation in the questions correctly. In Part C, Subject 2 also provided the correct results. The errors found in Subject 2's work were algebraic operations errors, so they did not affect the representation.

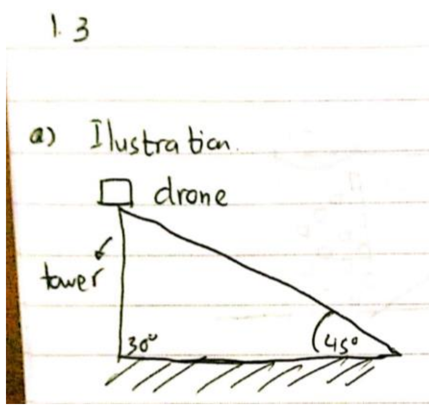


Figure 3. Subject 3's answer to Problem 1

Subject 3 correctly represented the elevation angle in the question. However, errors were found in the depiction of the angle of depression and the height of the drone. On the other hand, Subject 3 did not solve Problem 1c.

Problem 2

In Question 2, the three subjects gave similar answers. Figure 4 presents the answer of one of the subjects related to Question 2.

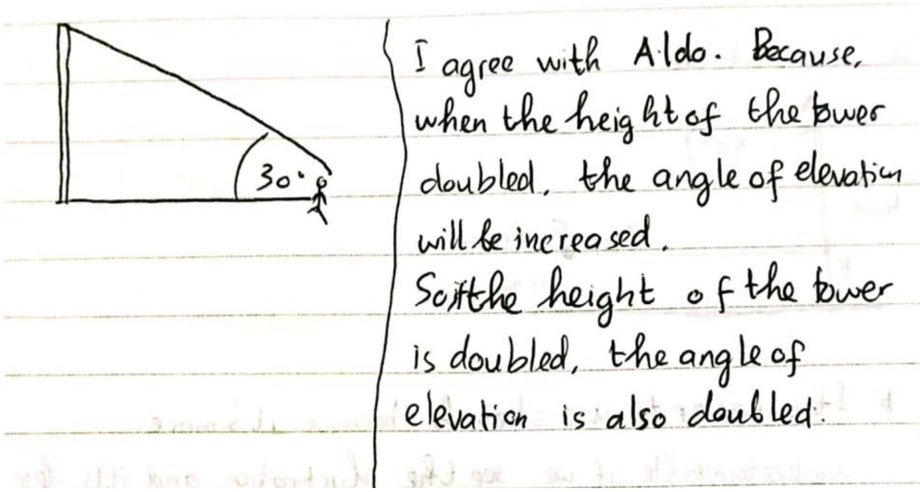


Figure 4. Subject 2's answer to Problem 2.

In general, both Subjects 1 and 3 gave similar answers to Subject 2. They thought if the elevation angle is doubled, the elevation angle formed is also doubled. The results of interviews also reinforced this statement.

They thought the elevation angle corresponds to the height of the building; if the height is doubled, the elevation angle is also doubled, and vice versa. According to Subject 2, when the height of the tower is doubled, the elevation angle will change because the person sees a tower with a higher height (tower). This statement is not entirely accurate. Subject 2's incorrect answer was due to the size of the angle formed. The error was due to oversimplification rather than a lack of understanding of the underlying principles. Look at Figure 5.

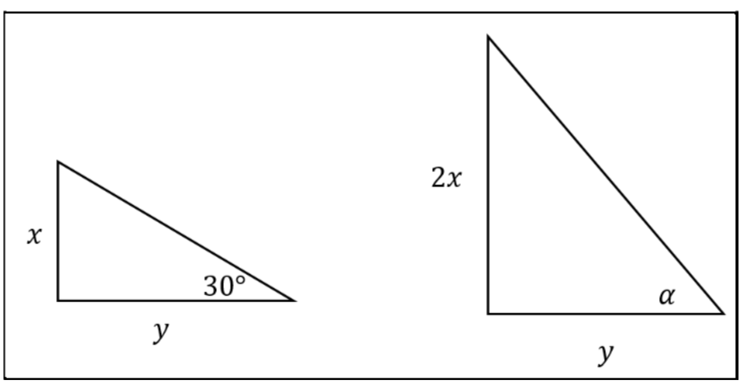


Figure 5. Sketch for Problem 2

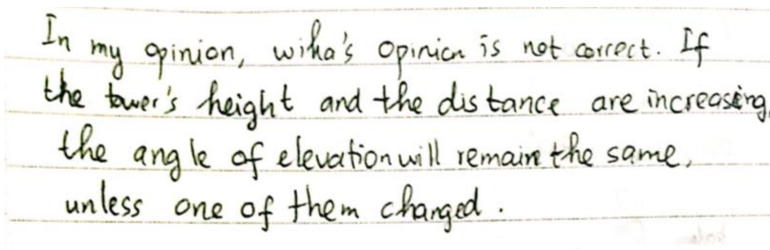
The image on the left is the 'initial' condition when Aldo and Syella saw the top of the tower, while the image on the right is the 'final' condition where the building's height has doubled. Using a trigonometric ratio at the "initial" condition, it is obtained $\tan 30^0 = \frac{1}{3}\sqrt{3} = \frac{x}{y}$.

Furthermore, trigonometric ratios can be applied to the final conditions generated $\tan 30^0 = \frac{1}{3}\sqrt{3} = \frac{x}{y}$. Note that $\tan \alpha = \frac{2}{3}\sqrt{3} \neq \sqrt{3} = \tan 60^0 = \tan(2 \times 30^0)$.

From this statement, it is clear that $\alpha \neq 60^0$, meaning that if the height of the tower is doubled, the elevation angle formed is not doubled, so Aldo's statement is incorrect.

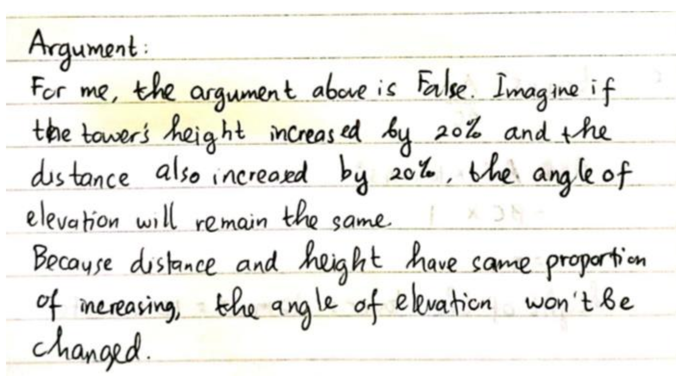
Problem 3

Like Question 2, the three subjects gave similar answers for Question 3. They believed that Wika's opinion regarding the figure was not quite right. However, the explanation given by each subject regarding this statement was different. Figures 6-8 display the subject's answer to Question 3.



In my opinion, wika's opinion is not correct. If the tower's height and the distance are increasing, the angle of elevation will remain the same, unless one of them changed.

Figure 6. Subject 1's answer to Problem 3



Argument:
For me, the argument above is False. Imagine if the tower's height increased by 20% and the distance also increased by 20%, the angle of elevation will remain the same.
Because distance and height have same proportion of increasing, the angle of elevation won't be changed.

Figure 7. Subject 2's answer to Problem 3

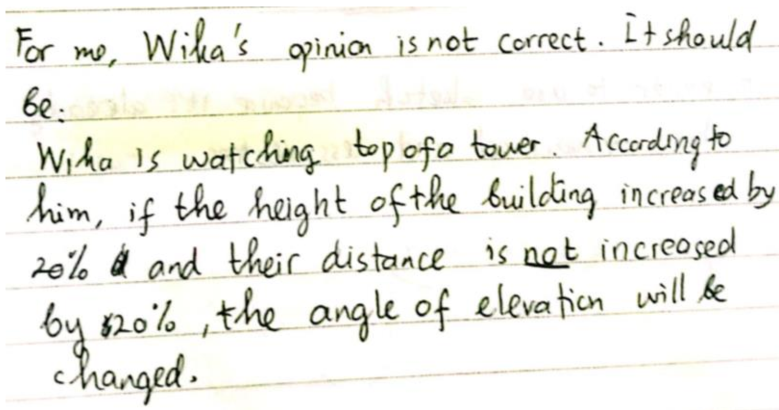

For me, Wika's opinion is not correct. It should be:
Wika is watching top of a tower. According to him, if the height of the building increased by 20% and their distance is not increased by 20%, the angle of elevation will be changed.

Figure 8. Subject 3's answer to Problem 3

Based on the test results, the three subjects presented the correct answers regarding Wika's statement. Their reasoning was also diverse. Subject 1 thought if the building height increases and the distance (building and observer) also increases, the elevation angle formed is constant. While Subject 1's reasoning is partially sound, it is not entirely accurate. The argument from Subject 1 only applies if the percentage change in building height and the observer's distance to the building are the same/constant. If the percentage increase in distance and building height is different, Subject 1's argument loses its validity.

Subjects 1 and 2 also had the same idea. Subject 2 thought that because height and distance have the same increase, the elevation formed will not change. The opinion given by Subject 2 was correct and more complete compared to Subject 1. However, a few statements were not in accordance with Subject 2's answer, namely, "... then it is the same as the initial position/previous position". This statement is incorrect because the initial and final conditions (after the distance and height of the building are increased by 20%) are different. Even so, the 'idea' of Subject 2 is correct regarding the problem.

Unlike the other two subjects, Subject 3 had slightly different answers compared to Subjects 1 and 2. According to Subject 3, if the height of the building increases by 20% and the distance from the building does not increase by the same percentage, the elevation angle will change. In addition, Subject 3 also thought that Wika's opinion was less relevant. Subject 3's idea was rather appropriate because it was in line with the idea of Subject 2, requiring the same increase in distance and height of the building.

Overall, the three subjects' answers regarding Wika's statement were correct. The reasons given by the three subjects also varied, but the main idea was similar, namely, related to the consistency of increasing the height and distance of buildings. The results of the subject's answers to the questions were also strengthened by the interview results. Even so, the three subjects had not provided a mathematical representation to support or strengthen their answers to Question 3.

Problem 4

In Question 4, only Subject 2 correctly represented the given situation. Subjects 1 and 3 could not represent the given situation correctly. Figure 9 displays the results of Subject 2's representation regarding Question 4.

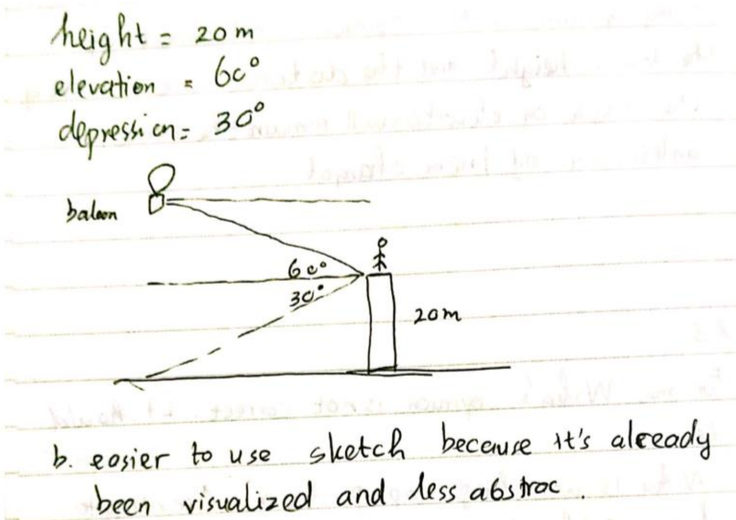


Figure 9. Subject 2’s answer to Problem 4

Subject 1 gave a slightly different answer from Subject 2, with the difference only in the delineation of the angle of depression. Subject 2 placed the angle of depression facing downwards and just below the elevation angle, while Subject 1 placed the angle of depression facing downwards and right next to the hot air balloon, as shown in Figure 10. Subject 1's answer was incorrect because, in the question, the angle of depression in question was from Surya to the balloon's shadow. In Subject 1's answer, the angle of depression formed from the blimp towards the sun. For other components, such as body height and elevation angle, Subject 1 successfully illustrated them correctly. Furthermore, the results of Subject 1's work can be observed in Figure 10.

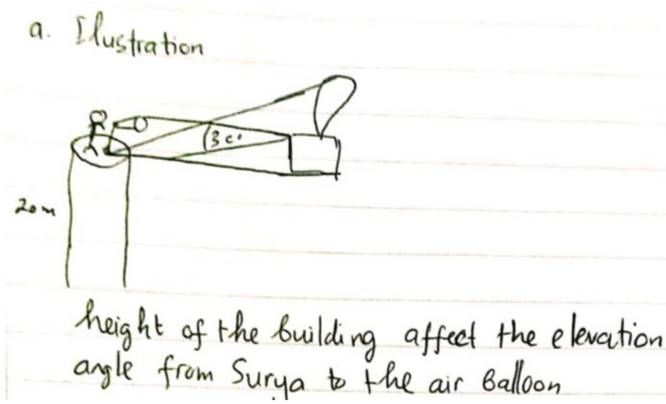


Figure 10. Subject 1's answer to Question 4

Test results shows that Subject 3 could not properly represent the given situation. Subject 3 described the angle of depression at the bottom of the observer. Besides, the hot air balloon was also described as being right above Surya, and the distance between the building and the balloon was depicted vertically so that it resembled the height of the balloon. Based on the test results, only 1 component was correctly represented by Subject 3, namely the height of the building. Furthermore, the results of Subject 3's work on Problem 4 can be observed in Figure 11.

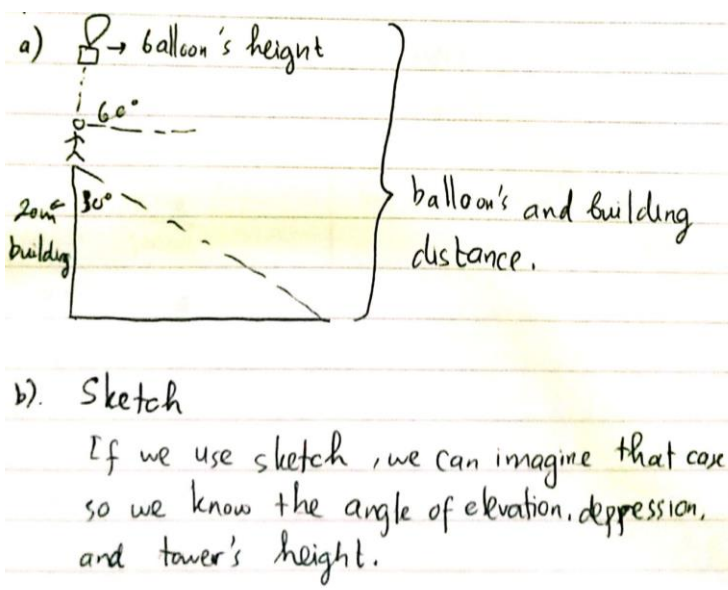


Figure 11. Subject 3's answer to Question 4

For Question 4, all subjects agreed that the easiest representation to use is a sketch or illustration. They believed that by using pictures, the information contained in the problem could be known clearly and quicker. In addition, the three subjects also did not answer the question in Part C regarding the height of the hot air balloon from the ground. Based on the interview results, Subjects 2 and 3 admitted they had not had time to work on these questions. Subject 1 was also found to have not worked on Part c's problem. When interviewed, Subject 1 revealed that the time to work on the problem was sufficient, but Subject 1 had no idea how to solve the problem. Similar to Subject 1, Subject 3 also had difficulty finding the idea of the problem.

Problem 5

In Question 5, only Subject 1 provided the correct answer even though Subject 1 did not illustrate the problem provided. Subjects 2 and 3 could represent the given problem or situation. However, Subjects 2 and 3 made a slight calculation error, so the final result was incorrect. In addition, the error was found in the value substitution process for b. In the third line, Subject 3 replaced b with 12 − b. As a result, the value $b = \frac{12 - 12\sqrt{3}}{1 - \sqrt{3}}$ was generated, which can be simplified to 12. However, Subject 3 did not continue solving Problem 3 because a = 0 was obtained. Subject 3 thought this result was impossible and was confused about doing it. Subject 2 made an error when substituting in solving procedure. Subject 2 reversed in writing the size of angles A and B at sin (A) and sin (B). In addition, the procedures and calculations carried out were correct. Furthermore, the results of Subjects 2 and 3's answers can be observed in Figures 12 and 13.

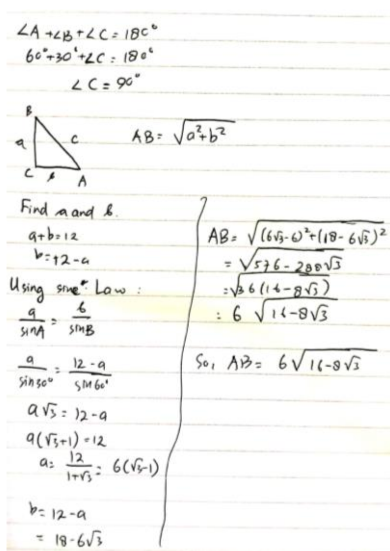


Figure 12. Subject 2’s answer to Question 5

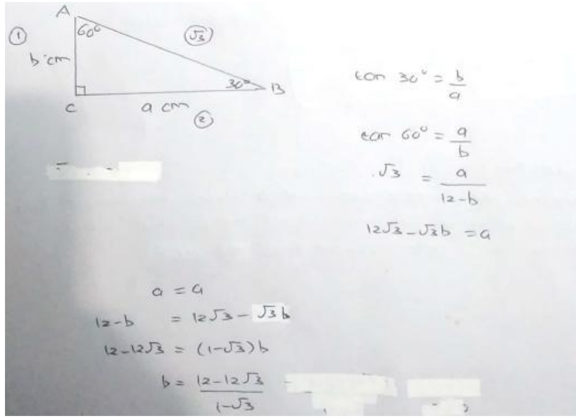


Figure 13. Subject 3's answer to Question 5

In order to enhance the comprehensiveness and depth of the research findings, the researchers recommend expanding the sample size for future studies. Furthermore, the researchers suggest that educators at all levels utilize the Problem-Based Learning (PBL) model to improve students' mathematical representation skills. By employing this approach, educators may be able to promote a more nuanced understanding of mathematical concepts and facilitate the development of more effective problem-solving strategies among students. This suggestion was also inspired by research conducted by Fitri, Munzir, and Duskri (2017). They suggested that PBL can be an alternative to learning in class because it presents an interactive learning environment. It only needs to be noted that starting problems in each math topic is not easy. In carrying out learning in class, the teacher is expected to provide opportunities for students to build their understanding of mathematics. Furthermore, Inayah (2018) said that the Quantum learning model could improve mathematical problem-solving and multiple representation’s ability. From this research, the mathematics education study program may use PBL or quantum teaching to improve mathematical representation ability.

CONCLUSION

Based on the findings of this study, it can be concluded that the mathematical representation skills of prospective mathematics teachers, particularly those in the 2018 cohort at Sanata Dharma University, are inadequate. This conclusion is drawn from an analysis of the mathematical representation indicators, revealing that students struggle with representing contextual problems using mathematical illustrations. In addition, students also have difficulty using representations to evaluate mathematical statements that are contextual. Each indicator of mathematical representation showed that the ability to create and use representations for collect/organize, record, and communicate mathematical ideas was 33.33%, the ability to select, apply and translate various forms of representation mathematics to solve problems was 29.76%, and the ability to use multiple representations to model and interpret physical phenomena, social phenomena, and mathematical phenomena was 41.11%.

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