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On the study of Rainbow Antimagic Coloring of Special Graphs

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ABSTRACT

Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. The bijective function $f: V(G) \rightarrow \{1,2,\ldots,|V(G)|\}$ is said to be a labeling of graph where $w(xy) = f(x) + f(y)$ is the associated weight for edge $xy \in E(G)$. If every edge has different weight, the function f is called an edge antimagic vertex labeling. A path P in the vertex-labeled graph G, with every two edges $xy, x'y' \in E(P)$ satisfies $w(xy) \neq w(x'y')$ is said to be a rainbow path. The function f is called a rainbow antimagic labeling of G, if for every two vertices $x, y \in V(G)$, there exists a rainbow $x - y$ path. Graph G admits the rainbow antimagic coloring, if we assign each edge xy with the color of the edge weight $w(xy)$. The smallest number of colors induced from all edge weights of edge antimagic vertex labeling is called a rainbow antimagic connection number of G, denoted by $rac(G)$. In this paper, we study rainbow antimagic connection numbers of octopus graph O_n, sandat graph St_n, sun flower graph Sf_n, volcano graph V_n and semi jahangir graph J_n.

Keywords: antimagic labeling; rainbow coloring; rainbow antimagic connection number; special graphs

INTRODUCTION

The definition of graph used in this paper follows from Chartrand and Zhang [9]. In the latest days, graph theory has many applications, one of them is graph coloring. The application of graph coloring can be found in many area, such as data mining, image segmentation, clustering, image capturing, networking. Chartrand, *et al.* [10] extended the graph coloring concept into a rainbow coloring of graph. Let $c: E(G) \rightarrow \{1,2,\ldots,k\}, k \in \mathbb{N}$ be the edge coloring of a connected graph where the two adjacent edges may have the same color. If for every two vertices $x, y \in V(G)$, there exists a rainbow $x - y$ path, if no two edges of the $x - y$ path are the same color, then the path is called a rainbow path. A coloring of graph G is said to be rainbow connection, if for every two vertices $x, y \in V(G)$ have a rainbow $x - y$ path.

The edge colored G which every two different vertices have a rainbow connection is called rainbow coloring of graph, see [10]. Some results in regards to the concept of rainbow coloring of graphs can been found by Nabila, *et al* [21] and Ma, *et al.* [19]. Some other type of rainbow coloring are rainbow vertex coloring and rainbow total coloring. Some relevant results of rainbow vertex coloring can be found in Lie. H, *et al.* [15],

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Bustan *et al.* [8] and Li. X *et al.* [17], while some results of total rainbow coloring can be found results in Lie. H *et al.* [16] and Ma. Y *et al.*[20].

Furthermore, the other concepts in graph theory is graph labeling, one of the concept of graph labeling is an antimagic labeling of graph G, defined by Hartsfield and Ringel [13]. Baca *et al.* has found some antimagic labeling results in [4], [5], [6]. Moreover, some results on antimagic labeling have been contributed by Dafik $et\ al.$ in [11]. In addition, the research on antimagic labeling can also be found in several papers [2], [22], [25].

Arumugam *et al.* [3], defined a new concept by combining graph coloring and graph labeling. The bijective function $f : E(G) \rightarrow \{1, 2, \ldots, |E(G)|\}$, the vertex weight of the vertex x is $w(x) = \sum_{xy \in E(x)} f(xy)$ and $E(x)$ is the set of edges incident to x for every $x \in V(G)$. If for every two adjacent vertices $x, y \in V(G), w(x) \neq w(y)$, then the bijective function f is called a local antimagic labeling. So, each local antimagic label is a vertex coloring in G with vertex x colored with $w(x)$. Based on the definition of Arumugam [3], Dafik $et\ al.$ [12] defined the combination of the concepts of antimagic labeling and rainbow coloring into a new concept called rainbow antimagic coloring.

In this study, we will study the combination of rainbow coloring and antimagic labeling, and it tends to the new notion, namely a rainbow antimagic coloring. The lower bound of the rainbow antimagic connection number has been determined in Septory *et al.* stated in the following lemma.

Lemma 1. *Let G be any connected graph. Let $rc(G)$ and $\Delta(G)$ be the rainbow connection number of G and the maximum degree of G, $rac(G) \geq max\ \{rc(G), \Delta(G)\}$.*

While Dafik *et al.* also characterised the existence of rainbow $u - v$ path of any graph of $diam(G) \leq 2$ in the following theorem.

Theorem 1. Let G be a connected graph of diameter $diam(G) \leq 2$. Let f be any bijective function from $V(G)$ to the set $\{1,2, \ldots, |V(G)|\}$, there exists a rainbow $x - y$ path.

Some other results in regards on this notion can be read on [1], [7], [12], [14], [23] and [24]. In this paper, we will study the rainbow antimagic connection number of octopus graph O_n, sandat graph St_n, sun flower graph Sf_n, volcano graph V_n and semi jahangir graph SJ_n.

METHOD

To determine the number of rainbow antimagic coloring of graph, we use the following steps:

1. For any graph G, identify the set of vertices $V(G)$ and set of edges $E(G)$.
2. Analyze the lower bound of rainbow antimagic connection number (rac) based on Lemma: $rac(G) \geq \max\{rc(G), \Delta(G)\}$.
3. Label the vertices of the graph G with the function: $V(G) \rightarrow \{1,2,3, \ldots, |V(G)|\}$.
4. Determine the edge weight based on the sum of vertex label which incident with the edge. To calculate edge weight we give the function, $w(uv) = f(u) + f(v)$ for $u, v \in V(G)$.
5. Verify that every two vertex in the graph G have rainbow paths. If not, repeat the step 3.
6. Determine the upper bound of $rac(G)$ from the number of different edge weight.
7. The exact value of rainbow antimagic connection number can be determined if lower

bound is the same with upper bound of rainbow antimagic connection number.

RESULTS AND DISCUSSION

In this section, we will show our new results on those graph above stated in a theorem. We start to write the theorem, provide the cardinality of the graph, obtain lower and upper bound, establish the rainbow antimagic connection number and show the existence of rainbow path for any to vertices and finally conclude the proof.

Theorem 2. *For* $n \geq 3$, $rac(O_n) = 2n$.

Proof. The octopus graph O_n is a graph with vertex set $V(O_n) = \{x\} \cup \{y_i, z_i, 1 \leq j \leq n\}$, and edge set $E(O_n) = \{xy_i, xz_i, 1 \leq i \leq n\} \cup \{y_i y_{i+1}, 1 \leq i \leq n-1\}$. The cardinality of vertex set is $|V(O_n)| = 2n + 1$ and the cardinality of edge set is $|E(O_n)| = 3n - 1$. Based on definition of octopus graph, the graph O_n has maximum degree of $\Delta(O_n) = 2n$.

To prove the rainbow antimagic connection number of O_n, the first step is to determine the lower bound of $rac(O_n)$. Based on **Lemma 1.** we have $rac(O_n) \geq \Delta(O_n)$. Since, the labels of the vertices with the bijection $f: V(O_n) \to \{1,2, \dots, |V(O_n)|\}$, we have $f(u) \neq f(v)$ for every vertex $u, v \in V(G)$. It implies for each edge $ux, vx \in E(G)$, $w(ux) \neq w(vx)$. Thus $rac(O_n) \geq 2n$.

The second step is to determine the upper bound of $rac(O_n)$. Define the vertex labeling $f: V(O_n) \to \{1,2,\dots,2n+1\}$ as follows.

$$f(x) = 1$$
$$f(y_i) = \begin{cases} \frac{3+i}{2} & , \text{ for } i \text{ is odd} \\ \frac{3+n+i}{2} & , \text{ for } i \text{ is even}, n \text{ is odd} \\ \frac{2+n+i}{2} & , \text{ for } i \text{ is even}, n \text{ is even} \end{cases}$$
$$f(z_i) = n + i + 1 \quad , \text{ for } 1 \leq i \leq n$$

The edge weight f can be expressed as

$$w(xz_i) = 2 + n + i \quad , \text{ for } 1 \leq i \leq n$$
$$w(xy_i) = \begin{cases} \frac{5+i}{2} & , \text{ for } i \text{ is odd} \\ \frac{5+n+i}{2} & , \text{ for } i \text{ is even}, n \text{ is odd} \\ \frac{4+n+i}{2} & , \text{ for } i \text{ is even}, n \text{ is even} \end{cases}$$
$$w(y_i y_{i+1}) = \begin{cases} \frac{7+2i+n}{2} & , \text{ for } 1 \leq i \leq n, n \text{ is odd} \\ \frac{6+2i+n}{2} & , \text{ for } 1 \leq i \leq n, n \text{ is even} \end{cases}$$

The next step is to count the number of different edge weights inducing the rainbow antimagic coloring on the graph O_n. The edge weights are included in the sets $w(xy_i) = \{3,4,5, \dots, n+2\}$ and $w(xz_i) = \{n+3, n+4, n+5, \dots, 2n+2\}$. The number of distinct colors of $w(xy_i) \cup w(xz_i)$ is $2n$. To prove this number, we use the formula of an arithmetic sequence formula. The following is an illustration of determining the number of distinct colors.

$$U_s = a + (s-1)d$$
$$2n + 2 = 3 + (s-1)1$$
$$2n + 2 = 3 + s - 1$$
$$s = 2n$$

It implies that the edge weight $f : V(O_n) \rightarrow \{1,2,\ldots,2n+1\}$ induces a rainbow antimagic coloring of $2n$ colors. Therefore $rac\ (O_n) \leq 2n$. Combining two bounds, we have the exact value of $rac\ (O_n) = 2n$. The last is to show the existence of the rainbow $x - y$ path of O_n. According to the **Theorem 2**, since $diam(O_n) = 2$, for every two vertices of the $x, y \in V(G)$ there is a rainbow $x - y$ path. It completes the proof.

The illustration of a rainbow antimagic coloring of octopus graph O_n can be seen in Figure 1.

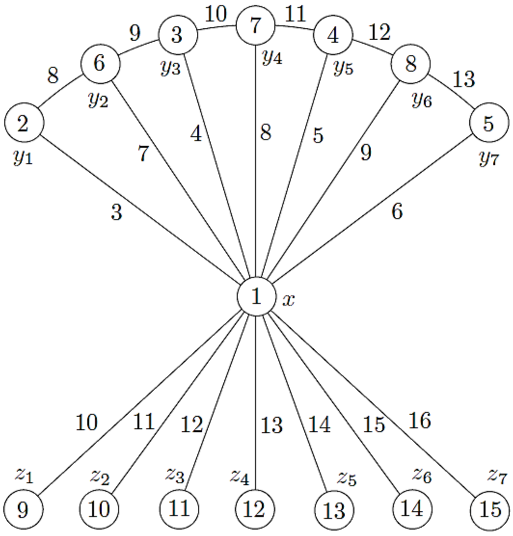

Figure 1. The illustration of rainbow antimagic coloring of octopus graph O_7

Theorem 3. *For* $n \geq 3, rac(St_n) = 3n$.

$Proof$. The sandat graph St_n is a graph with vertex set $V(\ St_n\ ) = \{a\} \cup \{\ x_i, y_i, z_i, 1 \leq i \leq n\}$ and edge set $E(St_n) = \{ax_i, ay_i, az_{i,} x_i y_i, y_i z_i\ 1 \leq\ i \leq\ n\}$. The cardinality of vertex set is $|V(St_n)| = 3n + 1$ and the cardinality of edge set is $|E(St_n)| = 5n$. Based on definition of sandat graph, the graph St_n has maximum degree of $\Delta\ (St_n)\ = 3n$.

To prove the rainbow antimagic connection number of St_n , the first step is to determine the lower bound of $rac(St_n)$. Based on **Lemma 1.** we have $rac(St_n) \geq \Delta(St_n)$. Since, the labels of the vertices with the bijection $f: V(St_n) \rightarrow \{1,2, \ldots, |V(St_n)|\}$, we have $f(u) \neq f(v)$ for every vertex $u, v \in V\ (G)$. It implies for each edge $ux, vx \in E\ (G), w\ (ux) \neq w\ (vx)$. Thus $rac\ (St_n) \geq 3n$.

The second step is to determine the upper bound of $rac(St_n)$. Define the vertex labeling $f\ :\ V(St_n) \rightarrow \{1,2,\ldots,3n+1\ \}$ as follows.

$$f(a) = 2$$
$$f(x_i) = 3n + 3 - 2i \quad , \quad \text{for } 1 \leq i \leq n$$
$$f(y_i) = \begin{cases} 1 & , \quad \text{for } i = n \\ i+1 & , \quad \text{for } \leq i \leq n \end{cases}$$
$$f(z_i) = 3n + 2 - 2i \quad , \quad \text{for } 1 \leq i \leq n$$

The edge weight f can be expressed as

$$w(ax_i) = 3n + 5 - 2i \quad , \quad \text{for } 1 \leq i \leq n$$

$$w(ay_i) = \begin{cases} 3 & , \text{ for } i = 1 \\ i+3 & , \text{ for } 2 \le i \le n \end{cases}$$
$$w(az_i) = 3n + 4 - 2i \quad , \text{ for } 1 \le i \le n$$
$$w(x_i y_i) = \begin{cases} 3n+2 & , \text{ for } i = 1 \\ 3n+4-i & , \text{ for } 2 \le i \le n \end{cases}$$
$$w(y_i z_i) = \begin{cases} 3n+1 & , \text{ for } i = 1 \\ 3n+3-i & , \text{ for } 2 \le i \le n \end{cases}$$

The next step is to count the number of different edge weights inducing the rainbow antimagic coloring on the graph St_n. The edge weights are included in the sets $w(ax_i) \cup w(ay_i) \cup w(az_i) \cup w(x_iy_i) \cup w(y_iz_i) = \{5,6,7,\ldots,3n+3\}$. The number of distinct colors of $w(ax_i) \cup w(ay_i) \cup w(az_i) \cup w(x_iy_i) \cup w(y_iz_i)$ is $3n$. Based on edge weights the number of edge wights is determined in the same way in **Theorem 2**.

It implies that the edge weight $f : V(St_n) \to \{1,2,\ldots,3n+1\}$ induces a rainbow antimagic coloring of $3n$ colors. Therefore $rac\ (St_n) \le 3n$. Combining two bounds, we have the exact value of $rac\ (St_n) = 3n$. The last is to show the existence of the rainbow $x-y$ path of St_n. According to the **Theorem 1**, since $diam(St_n) = 2$, for every two vertices of the $x, y \in V(G)$ there is a rainbow $x-y$ path. It completes the proof.

The illustration of a rainbow antimagic coloring of sandat graph St_n can be seen in Figure 2.

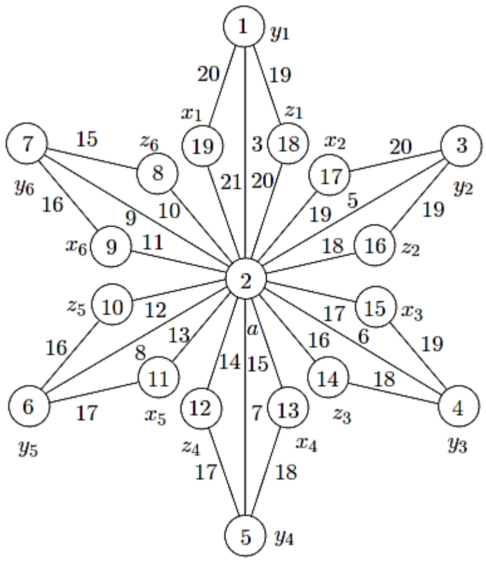

Figure 2. The illustration of rainbow antimagic coloring of sandat graph St_6.

Theorem 4. *For* $n \ge 4, rac(Sf_n) = 3n$.

Proof. The sunflower graph Sf_n is a graph with vertex set $V(Sf_n) = \{c\} \cup \{x_i, y_i, z_i, 1 \le i \le n\}$ and edge set $E(Sf_n) = \{cx_i, cy_i, cz_i, y_iz_i, z_iz_{i+1}, 1 \le i \le n\}$. The cardinality of vertex set is $|V(Sf_n)| = 3n+1$ and the cardinality of edge set is $|E(Sf_n)| = 5n$. Based on definition of sunflower graph, the graph Sf_n has maximum degree of $\Delta\ (Sf_n) = 3n$.

To prove the rainbow antimagic connection number of Sf_n, the first step is to determine the lower bound of $rac(Sf_n)$. Based on **Lemma 1.** we have $rac(Sf_n) \ge \Delta(Sf_n)$. Since, the labels of the vertices with the bijection $f: V(Sf_n) \to \{1,2,\ldots,|V(Sf_n)|\}$, we have

$f(u) \neq f(v)$ for every vertex $u, v \in V(G)$. It implies for each edge $ux, vx \in E(G), w(ux) \neq w(vx)$. Thus $rac(Sf_n) \geq 3n$.

The second step is to determine the upper bound of $rac(Sf_n)$. Define the vertex labeling $f : V(Sf_n) \to \{1,2,\dots,3n+1\}$ as follows.

$$
\begin{aligned}
f(c) &= 1 \\
f(x_i) &= 2n + i + 1 \quad , \quad \text{for } 1 \leq i \leq n \\
f(y_i) &= 2n - i + 2 \quad , \quad \text{for } 1 \leq i \leq n \\
f(z_i) &= i + 1 \quad , \quad \text{for } 1 \leq i \leq n
\end{aligned}
$$

The edge weight f can be expressed as

$$
\begin{aligned}
w(cx_i) &= 2n + i + 2 \quad , \quad \text{for } 1 \leq i \leq n \\
w(cy_i) &= 2n - i + 3 \quad , \quad \text{for } 1 \leq i \leq n \\
w(cz_i) &= i + 2 \quad , \quad \text{for } 1 \leq i \leq n \\
w(y_i z_i) &= 2n+3 \quad , \quad \text{for } 1 \leq i \leq n \\
w(z_i z_{i+1}) &= \begin{cases} 2i + 3 & , \quad \text{for } 1 \leq i \leq n-1 \\ n + 3 & , \quad \text{for } i = n \end{cases}
\end{aligned}
$$

The next step is to count the number of different edge weights inducing the rainbow antimagic coloring on the graph Sf_n. The edge weights are included in the sets $w(cx_i) \cup w(cy_i) \cup w(cz_i) = \{3,4,5,\dots,3n+2\}, w(y_i z_i) = \{2n+3\}$ and $w(z_i z_{i+1}) = \{n+3\} \cup \{5,7,9,\dots 2n+1\}$. The number of distinct colors of $w(cx_i) \cup w(cy_i) \cup w(cz_i) \cup w(y_i z_i) \cup w(z_i z_{i+1})$ is $3n$. Based on edge weights the number of edge wights is determined in the same way in **Theorem 2**.

It implies that the edge weight $f : V(Sf_n) \to \{1,2,\dots,3n+1\}$ induces a rainbow antimagic coloring of $3n$ colors. Therefore $rac(Sf_n) \leq 3n$. Combining two bounds, we have the exact value of $rac(Sf_n) = 3n$. The last is to show the existence of the rainbow $x - y$ path of Sf_n. According to the **Theorem 1**, since $diam(Sf_n) = 2$, for every two vertices of the $x, y \in V(G)$ there is a rainbow $x - y$ path. It completes the proof.

The illustration of a rainbow antimagic coloring of sunflower graph Sf_n can be seen in Figure 3.

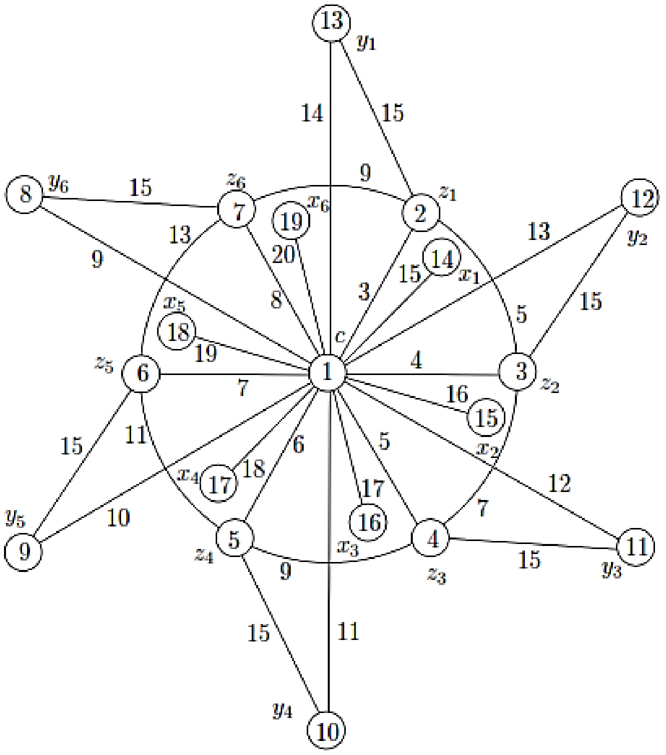

Figure 3. The illustration of rainbow antimagic coloring of sunflower graph Sf_6.

Theorem 5. *For* $n \geq 3, rac(V_n) = n + 2$.

Proof. The volcano V_n is a graph with vertex set $V(V_n) = \{x_1, x_2, x_3\} \cup \{y_i, 1 \leq i \leq n\}$ and edge set $E(V_n) = \{x_1x_2, x_2x_3, x_3x_1\} \cup \{x_iy_i, 1 \leq i \leq n\}$. The cardinality of vertex set is $|V(V_n)| = n + 3$ and the cardinality of edge set is $|E(V_n)| = n + 3$. Based on definition of volcano graph, the graph V_n has maximum degree of $\Delta(V_n) = n + 2$.

To prove the rainbow antimagic connection number of V_n , the first step is to determine the lower bound of $rac(V_n)$. Based on **Lemma 1.** we have $rac(V_n) \geq \Delta(V_n)$. Since, the labels of the vertices with the bijection $f: V(V_n) \to \{1,2, \dots, |V(V_n)|\}$, we have $f(u) \neq f(v)$ for every vertex $u, v \in V(G)$. It implies for each edge $ux, vx \in E(G), w(ux) \neq w(vx)$. Thus $rac(V_n) \geq n + 2$.

The second step is to determine the upper bound of $rac(V_n)$. Define the vertex labeling $f : V(V_n) \to \{1,2, \dots, n+3\}$ as follows.

$$f(x_1) = 1$$
$$f(x_2) = 2$$
$$f(x_3) = 3$$
$$f(y_i) = i + 3$$

The edge weight f can be expressed as

$$w(x_1x_2) = 3$$
$$w(x_2x_3) = 5$$
$$w(x_1x_3) = 4$$
$$w(x_iy_i) = i + 4$$

The next step is to count the number of different edge weights inducing the rainbow antimagic coloring on the graph V_n. The edge weights are included in the sets

$w(x_1x_2) \cup w(x_2x_3) \cup w(x_1x_3) = \{3,4,5\}$ and $w(x_iy_i) = \{5,6,7,\dots,n+4\}$. The number of distinct colors of $w(x_1x_2) \cup w(x_2x_3) \cup w(x_1x_3) \cup w(x_iy_i)$ is $n+2$. Based on edge weights the number of edge wights is determined in the same way in **Theorem 2**.

It implies that the edge weight $f : V(V_n) \to \{1,2,\dots,n+3\}$ induces a rainbow antimagic coloring of $n+2$ colors. Therefore $rac\ (V_n) \leq n+2$. Combining two bounds, we have the exact value of $rac\ (V_n) = n+2$. The last is to show the existence of the rainbow $x-y$ path of V_n. According to the **Theorem 1**, since $diam(V_n) = 2$, for every two vertices of the $x, y \in V(G)$ there is a rainbow $x-y$ path. It completes the proof.

The illustration of a rainbow antimagic coloring of volcano graph V_n can be seen in Figure 4.

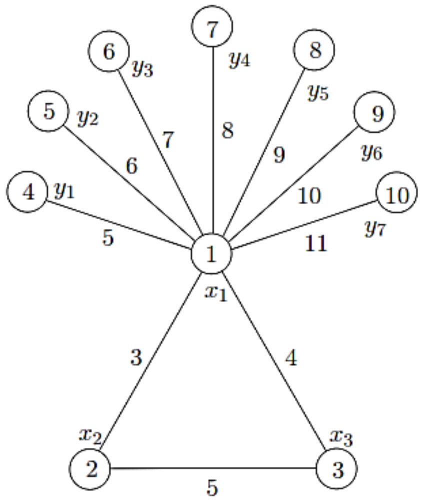

Figure 4. The illustration of rainbow antimagic coloring of volcano graph V_7.

Theorem 6. *For* $n \geq 3, rac(SJ_n) = n$.

Proof. The semi jahangir graph SJ_n is a graph with vertex set $V(SJ_n) = \{a\} \cup \{x_i, 1 \leq i \leq n\} \cup \{y_i, 1 \leq i \leq n-1\}$ and edge set $(SJ_n) = \{ax_i, 1 \leq i \leq n\} \cup \{x_iy_i, y_ix_{i+1}, 1 \leq i \leq n-1\}$. The cardinality of vertex set is $|V(SJ_n)| = 2n$ and the cardinality of edge set is $|E(SJ_n)| = 3n-2$. Based on definition of semi jahangir graph, the graph SJ_n has maximum degree of $\Delta\ (SJ_n) = n$.

To prove the rainbow antimagic connection number of SJ_n , the first step is to determine the lower bound of $rac(SJ_n)$. Based on **Lemma 1.** we have $rac(SJ_n) \geq \Delta(SJ_n)$. Since, the labels of the vertices with the bijection $f: V(SJ_n) \to \{1,2,\dots,|V(SJ_n)|\}$, we have $f(u) \neq f(v)$ for every vertex $u, v \in V\ (G)$. It implies for each edge $ux, vx \in E\ (G), w\ (ux) \neq w\ (vx)$. Thus $rac\ (SJ_n) \geq n$.

The second step is to determine the upper bound of $rac(SJ_n)$. Define the vertex labeling $f : V(SJ_n) \to \{1,2,\dots,2n\}$ as follows.

$$f(a) = \begin{cases} n & , \quad \text{for } n \text{ is odd} \\ n+1 & , \quad \text{for } n \text{ is even} \end{cases}$$

$$f(x_i) = \begin{cases} 2 & , \quad \text{for } i = 1 \\ 2i+2 & , \quad \text{for } 2 \leq i \leq n-1 \\ 4 & , \quad \text{for } i = n \end{cases}$$

$$f(y_i) = \begin{cases} 2n-2i+1 & , \quad \text{for } 1 \leq i \leq \left\lceil \frac{n}{2} \right\rceil - 1 \\ 2n-2i-3 & , \quad \text{for } \left\lceil \frac{n}{2} \right\rceil \leq i \leq n-2 \end{cases}$$

$$f(y_{n-1}) = \begin{cases} i-1 & , \text{ for } n \text{ is odd} \\ i & , \text{ for } n \text{ is even} \end{cases}$$

The edge weight f can be expressed as

$$w(ax_i) = \begin{cases} n+2 & , \text{ for } n \text{ is odd} \\ n+3 & , \text{ for } n \text{ is even} \end{cases}$$

$$w(ax_i) = \begin{cases} n+2i+2 & , \text{ for } n \text{ is odd}, 2 \le i \le n-1 \\ n+2i+3 & , \text{ for } n \text{ is even}, 2 \le i \le n-1 \end{cases}$$

$$w(ax_n) = \begin{cases} n+4 & , \text{ for } n \text{ is odd} \\ n+5 & , \text{ for } n \text{ is even} \end{cases}$$

$$w(x_iy_i) = \begin{cases} 2n+1 & , \text{ for } i = 1 \\ 2n+3 & , \text{ for } 2 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1 \\ 2n-1 & , \text{ for } \left\lceil \frac{n}{2} \right\rceil \le i \le n-2 \end{cases}$$

$$w(ax_{n-1}y_{n-1}) = \begin{cases} 3n-2 & , \text{ for } n \text{ is odd} \\ 3n-1 & , \text{ for } n \text{ is even} \end{cases}$$

$$w(y_ix_{i+1}) = \begin{cases} 2n+5 & , \text{ for } 2 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1 \\ 2n+1 & , \text{ for } \left\lceil \frac{n}{2} \right\rceil \le i \le n-2 \\ 2n-5 & , \text{ for } i = n-1 \end{cases}$$

The next step is to count the number of different edge weights inducing the rainbow antimagic coloring on the graph SJ_n. The edge weights are included in the sets $w(x_iy_i) \cup w(y_ix_{i+1}) = \{2n+1, 2n+3, 2n+5\}$ and $w(ax_i) = \{n+3, n+4, n+5, \ldots, 3n+1\}$. The number of distinct colors of $w(x_iy_i) \cup w(y_ix_{i+1}) \cup w(ax_i)$ is n. Based on edge weights the number of edge wights is determined in the same way in **Theorem 2**.

It implies that the edge weight $f : V(SJ_n) \rightarrow \{1,2,\ldots,3n-2\}$ induces a rainbow antimagic coloring of n colors. Therefore $rac\,(SJ_n) \le n$. Combining two bounds, we have the exact value of $rac\,(SJ_n) = n$. The last is to show the existence of the rainbow $x-y$ path of SJ_n. Suppose we take any $x, y \in V(SJ_n)$, there are two possibilities for x, y, namely: $x, y \in V(SJ_n)$ where $d(x,y) \le 2$ or $x, y \in V(SJ_n)$ where $d(x,y) \ge 3$. Suppose $x, y \in V(SJ_n)$ where $d(x,y) \le 2$, based on **Theorem 1**, we must have the rainbow $x-y$ path. For $x, y \in V(SJ_n)$ where $d(x,y) \ge 3$, we have two case: First case for path $x_i - y_j$ we use the path x_i, a, x_j, y_j or x_i, a, x_{j+1}, y_j. Second case for path $y_i - y_j$ we use the path y_i, x_i, a, x_j, y_j or $y_i, x_i, a, x_{j+1}, y_j$ or $y_i, x_{i+1}, a, x_j, y_j$ or $y_i, x_{i+1}, a, x_{j+1}, y_j$. Thus, for $x, y \in V(SJ_n)$ there is a rainbow $x-y$ path. It completes the proof.The illustration of a rainbow antimagic coloring of semi jahangir graph SJ_n can be seen in Figure 5.

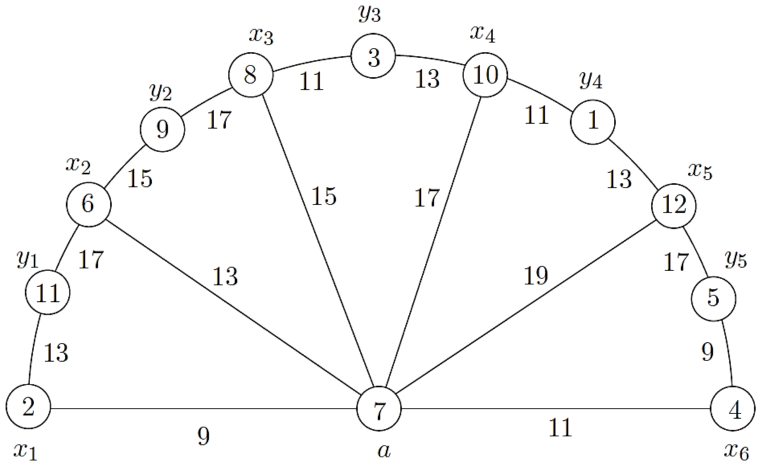

Figure 5. The illustration of rainbow antimagic coloring of semi jahangir graph SJ_6.

CONCLUDING REMARKS

Based on these results, the authors get the results of the rainbow antimagic connection number on several graphs. The authors finds the exact value of the octopus graph O_n, sandat graph St_n, sunflower graph Sf_n, volcano graph V_n and semi jahangir graph SJ_n.

Based on the results of this study, this study raises an open problem. Determine the exact value of the rainbow antimagic connection number of operation of graphs.

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