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On The Local Metric Dimension of Line Graph of Special Graph

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ABSTRACT

Let G be a simple, nontrivial, and connected graph. $W = \{w_1, w_2, w_3, \dots, w_k\}$ is a representation of an ordered set of k distinct vertices in a nontrivial connected graph G. The metric code of a vertex v, where $v \in$ G, the ordered $r(v|W) = \big(d(v, w_1), d(v, w_2), \dots, d(v, w_k)\big)$ of k-vector is representations of v with respect to W, where $d(v, w_i)$ is the distance between the vertices v and w_i for $1 \leq i \leq k$. Furthermore, the set W is called a local resolving set of G if $r(u|W) \neq r(v|W)$ for every pair u,v of adjacent vertices of G. The local metric dimension ldim(G) is minimum cardinality of W. The local metric dimension exists for every nontrivial connected graph G. In this paper, we study the local metric dimension of line graph of special graphs , namely path, cycle, generalized star, and wheel. The line graph L(G) of a graph G has a vertex for each edge of G, and two vertices in L(G) are adjacent if and only if the corresponding edges in G have a vertex in common.

Keywords: metric dimension, local metric dimension number, line graph, resolving set.

INTRODUCTION

All graph in this paper are simple, nontrivial, and undirected, for more detail basic definition of graph, see [1]. In [2] define the distance $d(u, v)$ between two vertices u and v in a connected graph G is the length of a shortest path between these two vertices. Suppose that $W = \{w_1, w_2, w_3, \dots, w_k\}$ is an ordered set of vertices of a nontrivial connected graph G. The metric representation of v with respect to W is the k-vector $r(v|W) = \big(d(v, w_1), d(v, w_2), \dots, d(v, w_k)\big)$. Distance in graphs has also been used to distinguish all of the vertices of a graph. The set W is called a resolving set for G if distinct vertices of G have distinct representations with respect to W. The metric dimension dim(G) of G is the minimum cardinality of resolving set for G [3]. Furthermore, we consider those ordered sets W of vertices of G for which any two vertices of G having the same code with respect to W are not adjacent in G. If $r(u|W) \neq r(v|W)$ for every pair u, v of adjacent vertices of G, then W is called a local metric set of G. The minimum k for which G has a local metric k-set is the local metric dimension of G, which is denoted by ldim(G) [4].

In this paper, we study the local metric dimension number of line graph of special graphs, namely path, cycle, generalized star, and wheel graph. Line graphs are a special case of intersection graphs. The line graph L(G) of a graph G has a vertex for each edge of G, and two vertices in L(G) are adjacent if and only if the corresponding edges in G have a vertex in common. Thus, the line graph L(G) is the intersection graph corresponding to the endpoint sets of the edges

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of G. As for an example, Figure 1 shows a graph G and its line graph L(G) [5]. The results show that distinct vertices of each L(G), with G is a special graph, namely path, cycle, generalized star, and wheel graph have distinct representations with respect to W, and their local metric dimension attain a minimum number. Furthermore, [6] showed that on the metric dimension, the upper dimension and the resolving number of graphs, [7] showed the metric dimension of amalgamation of cycles, [8] studied the constant metric dimension of regular graphs, [2] obtained resolvability in graphs and the metric dimension of a graph. The last, [9] showed the Fault-tolerant metric and partition dimension of graph.

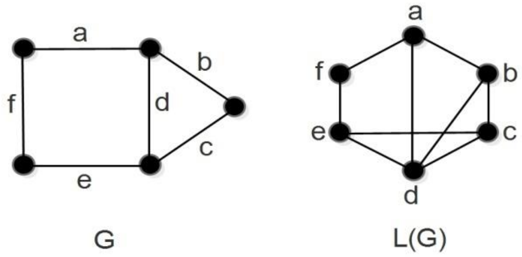

Figure 1. A graph and its line graph

RESULTS AND DISCUSSIONS

We have some observation to show the result of line graph of path, cycle, generalized star, and wheel graph. Thus, we have four main theorem about local metric dimension to discuss as follows.

Observation 1. *The order and the size of* $L(P_n)$ *are* $|V(L(P_n))| = n-1$ *and* $|E(L(P_n))| = n-2$, *respectively*.

Proof. Line graph of Path $L(P_n)$ is connected, simple, and undirected graph with vertex set $V(L(P_n)) = \{v_i; 1 \leq i \leq n-1\}$ and edge set $E(L(P_n)) = \{v_i v_{i+1}; 1 \leq i \leq n-2\}$. Thus, $|V(L(P_n))| = n-1$ and $|E(L(P_n))| = n-2$. See Figure 2 (a) and 2 (b) for illustration. ∎

Theorem 1. *For* $n \geq 2$, *the local metric dimension of line graph of path is* $ldim(L(P_n)) = 1$.

Proof. By observation 1, line graph of path $L(P_n)$ is isomorphic to P_{n-1}. Thus, the vertex set and the edge set of $L(P_n)$ are $V(L(P_n)) = \{v_i; 1 \leq i \leq n-1\}$ and $E(L(P_n)) = \{v_i v_{i+1}; 1 \leq j \leq n-2\}$, respectively. The number of vertices $|V(L(P_n))| = n-1$ and the size $|E(L(P_n))| = n-2$. By Theorem 3, $ldim(P_n) = 1$, thus $ldim(L(P_n)) = 1$. It concludes the proof. See Figure 2 (c) for illustration . ∎

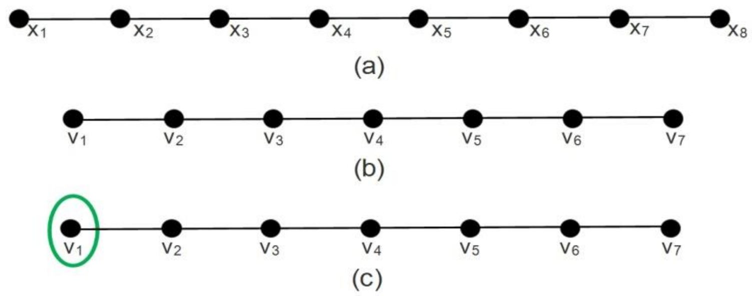

Figure 2. (a) P_8 , (b) $L\{P_8\}$, (c) Construction of local resolving set $W = \{v_1\}$

Observation 2. *The order and the size of* $L(C_n)$ *are* $|V(L(C_n))| = n$ *and* $|E(L(P_n))| = n$, *respectively*.

Proof. Line graph of cycle $L(C_n)$ is connected, simple, and undirected graph with vertex set $V(L(C_n)) = \{v_i; 1 \le i \le n\}$ and edge set $E(L(C_n)) = \{v_i v_{i+1}; 1 \le j \le n-1\} \cup \{v_n v_1;\}$. Thus, $|V(L(C_n))| = n$ and $|E(L(C_n))| = n$. See Figure 3 (a) and 3 (b) for illustration. ■

Theorem 2. *For $n \ge 3$, the local metric dimension of line graph of cycle is*

$$ldim(L(C_n)) = \begin{cases} 1, & for\ n\ even \\ 2, & for\ n\ odd \end{cases}$$

Proof. By observation 2, line graph of cycle $L(C_n)$ is isomorphic to C_n. Thus, the vertex set and the edge set of $L(C_n)$ are $V(L(C_n)) = \{v_i; 1 \le i \le n\}$ and $E(L(C_n)) = \{e_j; 1 \le j \le n\}$, respectively. The order of vertices $|V(L(C_n))| = n$ and the size $|E(L(C_n))| = n$. By Theorem 3,

$$ldim(C_n) = \begin{cases} 1, & \text{for } n \text{ even} \\ 2, & \text{for } n \text{ odd} \end{cases}$$

thus

$$ldim(L(C_n)) = \begin{cases} 1, & \text{for } n \text{ even} \\ 2, & \text{for } n \text{ odd} \end{cases}$$

It concludes the proof. See Figure 3 (c) for illustration. ■

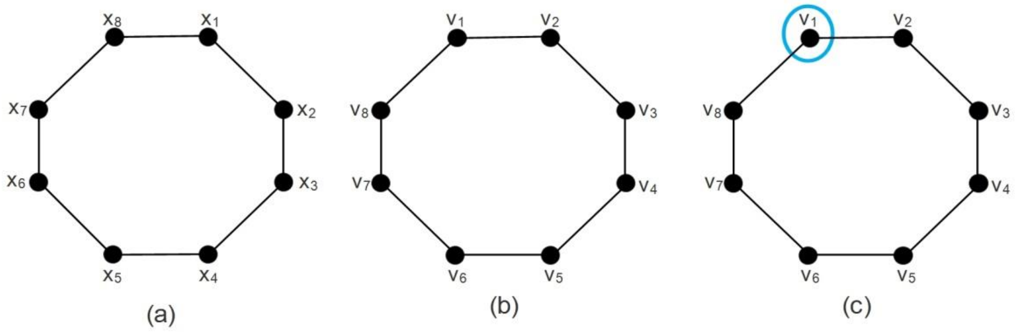

Figure 3. (a) C_8 , (b) $L\{C_8\}$, (c) Construction of local resolving set $W = \{v_1\}$

Observation 3. *The order and the size of $L(S_{n,m})$ are $|V(L(S_{n,m}))| = mn$ and $|E(L(S_{n,m}))| = mn + \frac{n(n-3)}{2}$, respectively.*

Proof. Line graph of generalized star $L(S_{n,m})$ is connected, simple, and undirected graph with vertex set $V(L(S_{n,m})) = \{x_{i,j}; 1 \le i \le n, 1 \le j \le m\}$ and edge set $E(L(S_{n,m})) = \{x_{i,j}x_{i,j+1}; 1 \le i \le n, 1 \le j \le m-1\} \cup \{x_{i,1}x_{t,1}; i \ne t, 1 \le i,t \le n\}$. Thus, $|V(L(S_{n,m}))| = mn$ and $|E(L(S_{n,m}))| = mn + \frac{n(n-3)}{2}$. See Figure 4 (a) and 4 (b) for illustration. ■

Theorem 3. For $n \ge 3$, the local metric dimension of line graph of generalized star is $dim(L(S_{n,m})) = n - 1$.

Proof. By observation 3, the order and the size of $L(S_{n,m})$ are $|V(L(S_{n,m}))| = mn$ and $|E(L(S_{n,m}))| = mn + \frac{n(n-3)}{2}$, respectively. Suppose the lower bound is $ldim(L(S_{n,m})) \ge n - 2$ with the resolving set $W' = \{x_{i,1}; 1 \le i \le n-2\}$. Thus two vertices of $V(L(S_{n,m})) - W'$ with respect to W' are

$$r(x_{n-1,1}|W') = r(x_{n,1}|W') = \left(\underbrace{1, \dots, 1}_{n-2\ times} \right)$$

It is easy to see that the line graph of generalized star has the same vertex representation respecting to W'. Thus, the lower bound is $ldim(L(S_{n,m})) \ge n - 1$. Now, we will show that $ldim(L(S_{n,m})) \le n - 1$ by determining the resolving set $W = \{x_{i,1}; 1 \le i \le n-1\}$ and the vertex representation of $V(L(S_{n,m})) - W$ respect to W, as follows

$$r(x_{n,j}|W) = \Big(\underbrace{j,\dots,j}_{n-1\ times}\Big); 1 \le j \le m$$

$$r(x_{i,j}|W) = \Big(\underbrace{j,\dots,j}_{i-1\ times}, j-1, \underbrace{j,\dots,j}_{n-i-1\ times}\Big); 1 \le i \le n-1, 2 \le j \le m$$

It easy to see that $\forall u, v \in V(L(S_{n,m})) - W$ have a different representation respect to W for every pair u, v of adjacent vertices in $L(S_{n,m})$. The cardinality of resolving set $L(S_{n,m})$ is $n-1$, thus $ldim(L(S_n)) \le n-1$. It concludes the proof. See Figure 4 (c). ■

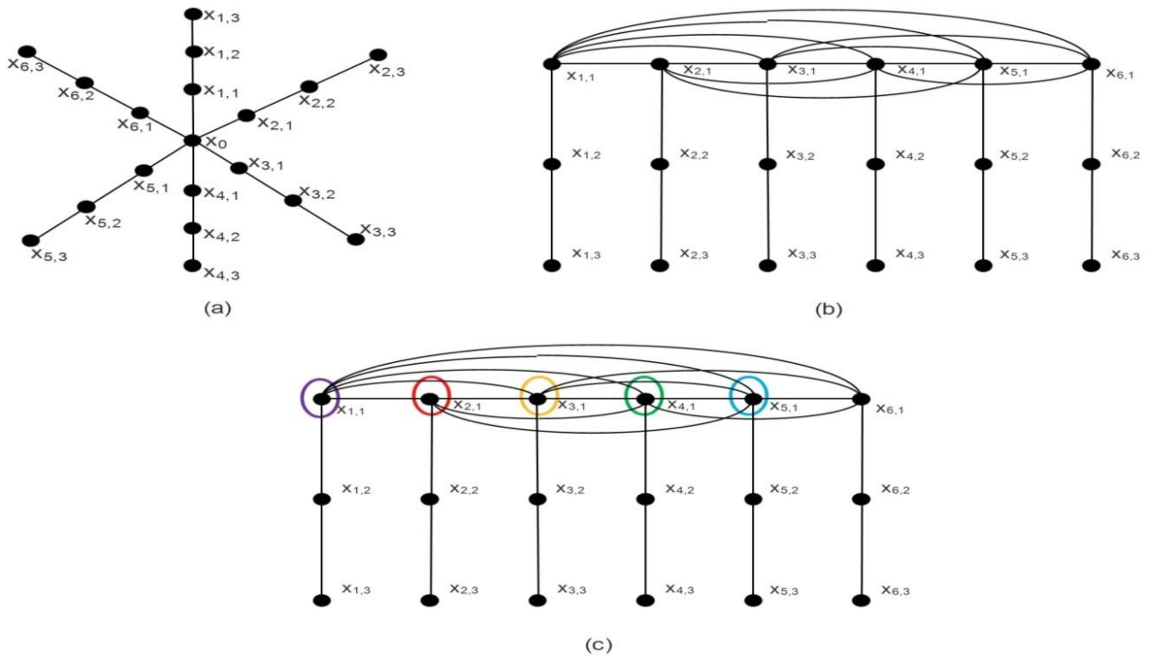

Figure 4. (a) $S_{6,3}$, (b) $L(S_{6,3})$, (c) Construction of local resolving set $W = \{x_{1,1}, x_{2,1}, x_{3,1}, x_{4,1}, x_{5,1}\}$

Observation 4. *The order and the size of* $L(W_n)$ *are* $|V(L(W_n))| = 2n$ and $|E(L(W_n))| = \frac{n(n+5)}{2}$, *respectively*.

Proof. Line graph of wheel $L\{W_n\}$ is connected, simple, and undirected graph with vertex set $V(L(W_n)) = \{x_{i,j}; 1 \le i \le n, 1 \le j \le 2\}$ and edge set $E\left(L(S_{n,m})\right) = \{x_{i,1}x_{i,2}; 1 \le i \le n\} \cup \{x_{i,1}x_{t,1}; i \ne t, 1 \le i, t \le n\} \cup \{x_{i,2}x_{i+1,2}; 1 \le i \le n-1\} \cup \{x_{i,2}x_{i+1,1}; 1 \le i \le n-1\} \cup \{x_{1,2}x_{n,2}\} \cup \{x_{1,1}x_{n,2}\}$. Thus, $|V(L(W_n))| = 2n$ and $|E(L(W_n))| = \frac{n(n+5)}{2}$. See Figure 5 (a) and 5 (b) for illustration. ■

Theorem 4. *For* $n \ge 3$, *the local metric dimension of line graph of wheel is* $LDim(L(W_n)) = n-1$.

Proof. By observation 4, the order and the size of $L(W_n)$ are $|V(L(W_n))| = 2n$ and $|E(L(W_n))| = \frac{n(n+5)}{2}$, respectively. Suppose the lower bound of $L(W_n)$ is $ldim(L(W_n)) \ge n-2$ with the resolving set $W' = \{x_{i,1}; 1 \le i \le n-2\}$. Thus two vertices of $V(L(W_n)) - W'$ with respect to W' are

$$r(x_{n-1,1}|W') = r(x_{n,1}|W') = \Big(\underbrace{1,\dots,1}_{n-2\ times}\Big)$$

It is easy to see that the line graph of wheel possess the same vertex representation respecting to W'. Thus the lower bound is $ldim(L\{W_n\}) \ge n-1$. Now, we will show that $ldim(L\{W_n\}) \le n-1$

by determining the resolving set $W = \{x_{i,1}; 1 \leq i \leq n-1\}$ and the vertex representation of $V(L\{W_n\}) - W$ respect to W, as follows.

$$W = \{x_{1,1}, x_{2,1}, x_{3,1}, x_{4,1}, x_{5,1}, x_{6,1}, x_{7,1}\}$$

$$r(x_{n,1}|W) = \left(\underbrace{1, \dots, 1}_{n-1\ times} \right)$$

$$r(x_{i,2}|W) = \left(\underbrace{2, \dots, 2}_{i-1\ times}, 1, \underbrace{2, \dots, 2}_{n-i-1\ times} \right); 2 \leq i \leq n-1$$

$$r(x_{1,2}|W) = \left(1,1, \underbrace{2 \dots, 2}_{n-3\ times} \right)$$

$$r(x_{n,2}|W) = \left(1, \underbrace{2, \dots, 2}_{n-2\ times} \right)$$

It easy to see that $\forall u, v \in V(L(W_n)) - W$ have a different representation respect to W for every pair u, v of adjacent vertices in $L(W_n)$. The cardinality of resolving set $L(W_n)$ is $n-1$, thus $ldim(L(W_n)) \leq n-1$. It concludes the proof. See Figure 5 (c) for illustration. ■

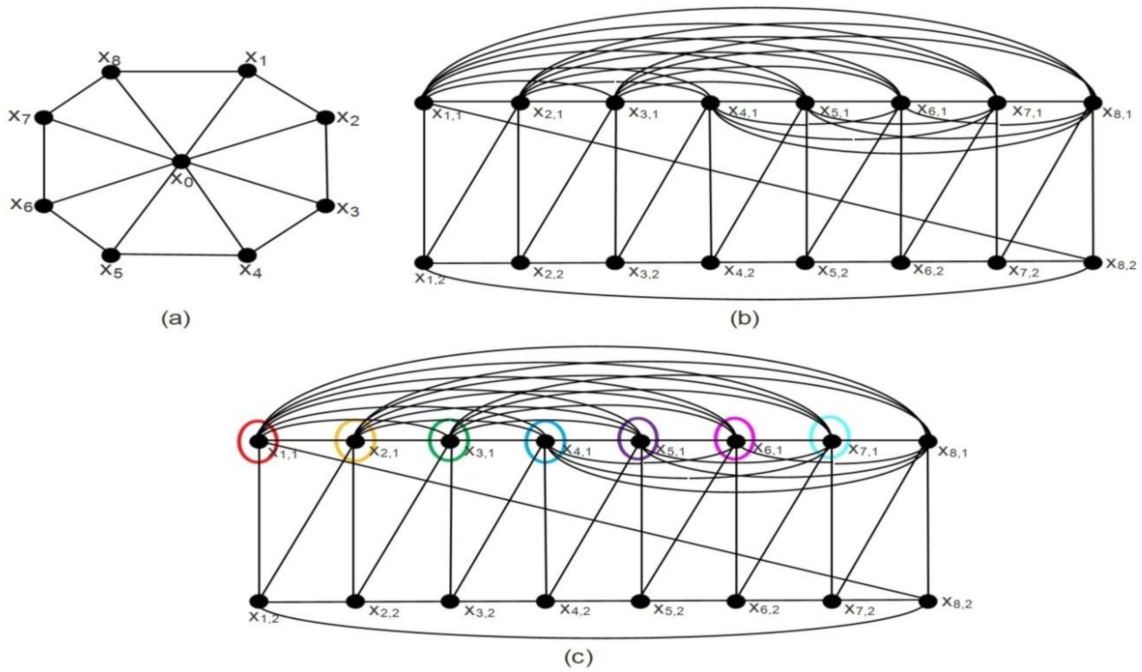

Figure 5. (a) W_8, (b) $L\{W_8\}$, (c) Construction of local resolving set

CONCLUSION

We have shown the local metric dimension number of line graph of special graphs, namely line graph of path, cycle, star, and wheel. The results show that the local metric dimension numbers attain the best lower bound. However we have not found the sharpest lower bound for any connected graph, therefore we proposed the following open problem.

Open Problem 1. *Let G be a connected graph, obtain the best lower bound of the local metric dimension of any graph G.*

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