

Fault-tolerant Gossip Graphs Based on Wheel Graphs

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Abstract

The gossip problem (telephone problem) is an information dissemination problem where each of n nodes of a communication network has a unique piece of information that must be transmitted to all the other nodes using two-way communications (telephone calls) between the pairs of nodes. Upon a call between the given two nodes, they exchange the whole information known to them at that moment. In this paper, we investigate the k -fault-tolerant gossip problem, which is a generalization of the gossip problem, where at most k arbitrary faults of calls are allowed. The problem is to find the minimal number of calls $\tau(n, k)$ needed to guarantee the spread of whole information. We constructed a k -fault-tolerant gossip scheme (sequences of calls) to find the upper bounds of $\tau(n, k)$, which improves the previously known results for some particular small values of n and k .

Keywords: Gossip, Information dissemination, Fault-tolerant gossiping.

1. Introduction

Gossiping is one of the basic problems of information dissemination in communication networks. The gossip problem (also known as a telephone problem) is attributed to A. Boyd (see e.g. [4] for review), although to the best knowledge of the reviewers, it was first formulated by R. Chesters and S. Silverman (Univ. of Witwatersrand, unpublished, 1970). Consider a set of n person (nodes) each of which initially knows some unique piece of information that is unknown to the others, and they can make a sequence of telephone calls to spread the information. During a call between the given two nodes, they exchange the whole information known to them at that moment. The problem is to find the sequence of calls with a minimum length (minimal gossip scheme), by which all the nodes will know all pieces of information (complete gossiping). It has been shown in numerous works [1]–[4] that the minimal number of calls is $2n - 4$ when $n \geq 4$ and 1, 3 for $n = 2, 3$, respectively. Since then many variations of gossip problem have been introduced and investigated (see e.g. [5]–[10], [12]–[17]).

One of the natural generalizations of this problem is the k -fault-tolerant gossip problem, which assumes that some of the calls in the call sequence can fail (do not take place) [13]–[16]. The nodes cannot change the sequence of their future calls depending on the current failed calls. Here the aim is to find a minimal gossip scheme, which guarantees the full exchange of the information in the case of at most k arbitrary fails, regardless of which the calls failed. The gossip schemes, which provide k -fault-tolerance, are called k -fault-tolerant

gossip schemes. Denote the minimal number of calls in the k -fault-tolerant minimal gossip scheme by $\tau(n, k)$.

Berman and Hawrylycz [14] obtained the lower and upper bounds for $\tau(n, k)$:

$$\left\lceil \left(\frac{k+4}{2} \right) (n-1) \right\rceil - 2 \lceil \sqrt{n} \rceil + 1 \leq \tau(n, k) \leq \left\lfloor \left(k + \frac{3}{2} \right) (n-1) \right\rfloor, \quad (1)$$

for $k \leq n-2$, and

$$\left\lfloor \left(\frac{k+3}{2} \right) (n-1) \right\rfloor - 2 \lceil \sqrt{n} \rceil \leq \tau(n, k) \leq \left\lfloor \left(k + \frac{3}{2} \right) (n-1) \right\rfloor, \quad (2)$$

for $k \geq n-2$.

Haddad, Roy and Schaffer [15] proved afterwards that

$$\tau(n, k) \leq \left(\frac{k}{2} + 2p \right) \left(n - 1 + \frac{n-1}{2^p-1} + 2^p \right), \quad (3)$$

where p is any integer between 1 and $\log_2 n$ inclusive. By choosing p appropriately, this result improves the upper bounds obtained by Berman and Hawrylycz for almost all k . Particularly, by choosing $p = \left\lfloor \frac{\log_2 n}{2} \right\rfloor$, the following bound is obtained: $\tau(n, k) \leq \frac{nk}{2} + O(k\sqrt{n} + n \log_2 n)$.

For the special case $n = 2^p$ for some integer p , Haddad, Roy, and Schaffer [15] also showed that

$$\tau(n, k) \leq \min \left\{ \left(\left\lceil \frac{k+1}{\log_2 n} \right\rceil + 1 \right) \frac{n \log_2 n}{2}, \left(\left\lfloor \frac{k+1}{\log_2 n} \right\rfloor + 1 \right) \frac{n \log_2 n}{2} + ((k+1) \bmod \log_2 n) (2n-4) \right\}. \quad (4)$$

Thus, $\tau(n, k) \leq \frac{nk}{2} + O(n \log_2 n)$, when n is a power of 2.

Later on, Hou and Shigeno [13] showed that

$$\left\lfloor \frac{n(k+2)}{2} \right\rfloor \leq \tau(n, k) \leq \frac{n(n-1)}{2} + \left\lfloor \frac{nk}{2} \right\rfloor. \quad (5)$$

So, it holds that $\frac{nk}{2} + \Omega(n) \leq \tau(n, k) \leq \frac{nk}{2} + O(n^2)$. These bounds improve the previous bounds for small n and sufficiently large k .

Recently, Hasunuma and Nagamochi [16] showed that

$$\tau(n, k) \leq \begin{cases} \frac{n \log_2 n}{2} + \frac{nk}{2}, & \text{if } n \text{ is a power of } 2 \\ 2n \lfloor \log_2 n \rfloor + n \lceil \frac{k-1}{2} \rceil, & \text{otherwise,} \end{cases} \quad (6)$$

and

$$\left\lfloor \frac{3n-5}{2} \right\rfloor + \left\lfloor \frac{1}{2} \left(nk + \left\lfloor \frac{n+1}{2} \right\rfloor - \lfloor \log_2 n \rfloor \right) \right\rfloor \leq \tau(n, k). \quad (7)$$

From their results, it holds that $\tau(n, k) \leq \frac{nk}{2} + O(n \log_2 n)$. Particularly, their upper bound improves the upper bound by Hou and Shigeno for all $n \geq 13$. They also improve the upper bound by Haddad *et al.* by showing that the factor $(k/2 + 2p)$ in their upper bound can be replaced with a smaller factor $(k/2 + p)$:

$$\tau(n, k) \leq \left(\frac{k}{2} + p \right) \left(n - 1 + \frac{n-1}{2^p-1} + 2^p \right), \quad (8)$$

where p is any integer between 1 and $\log_2 n$.

In this paper, we construct a k -fault-tolerant gossip scheme based on Wheel graphs, which improves the previously known results on the upper bound for the number of calls in case of small n and k . The obtained expressions for general n and k are (see Theorem 2) as follows:

$$\tau(n, k) \leq \frac{2}{3} nk + O(n) \quad (9)$$

2. Preliminaries

A gossip scheme (a sequence of calls between n nodes) can be represented by an undirected edge-labeled graph $G = (V, E)$ with $|V(G)| = n$ vertices. The vertices and edges of G represent correspondingly the nodes and the calls between the pairs of nodes of a gossip scheme. Such graphs may have multiple edges, but not self loops. An edge-labeling of G is a mapping $t_G : E(G) \rightarrow \mathbb{Z}^+$. The label $t_G(e)$ of the given edge $e \in E(G)$ represents the moment of time, when the corresponding call occurs.

A sequence $P = (v_0, e_1, v_1, e_2, v_2, \dots, e_k, v_k)$ with vertices $v_i \in V(G)$ for $0 \leq i \leq k$ and edges $e_i \in E(G)$ for $1 \leq i \leq k$ is called a walk of length k from a vertex v_0 to a vertex v_k in G , if each edge e_i joins two vertices v_{i-1} and v_i for $1 \leq i \leq k$. A walk, in which all the vertices are distinct is called a path. If $t_G(e_i) < t_G(e_j)$ for $1 \leq i < j \leq k$, then P is an ascending path from v_0 to v_k in G . Given two vertices u and v , if there is an ascending path from u to v , then v receives the information of u . Note that two different edges can have the same label. Since we consider only (strictly) the ascending paths, then such edges (i.e. calls) are independent, which means that the edges with the same label can be reordered arbitrarily but for any $t_1 < t_2$ all the edges with the label t_1 are ordered before any of the edges with the label t_2 .

Definition 1: *The communication between two vertices of G is called k -failure safe if an ascending path between them remains, even if arbitrary k edges of G are deleted (the corresponding calls fail). The graph G is called a k -fault-tolerant gossip graph if the communication between all the pairs of its vertices is k -failure safe.*

From the Menger theorem [22] it follows that a k -fault-tolerant gossip graph is a graph whose edges are labeled in such a way that there are at least $k + 1$ edge-disjoint ascending paths between two arbitrary vertices. A 0-fault-tolerant gossip graph is simply called a gossip graph.

To describe the construction of k -fault-tolerant gossip graphs (schemes), we use some important definitions and propositions given in the works [15], [16]. In order to simplify the discussion for edge-disjoint paths, we often omit the vertices (or edges) in the description of a path if there is no confusion.

Definition 2: *Let $P = (e_1, e_2, \dots, e_k)$ be a path with edges $e_i \in E(G)$ for $1 \leq i \leq k$ in a labeled graph G . If P is divided into $s+1$ subpaths $P^{(1)} = (e_1, \dots, e_{p_1})$, $P^{(2)} = (e_{p_1+1}, \dots, e_{p_2})$, \dots , $P^{(s+1)} = (e_{p_s+1}, \dots, e_k)$, then we write $P = P^{(1)} \odot P^{(2)} \odot \dots \odot P^{(s+1)}$, where \odot is the concatenation operation on two paths for which the last vertex of one path is the first vertex of the other. If $P = P^{(1)} \odot P^{(2)} \odot \dots \odot P^{(s+1)}$ such that $P^{(j)}$ is an ascending path for $1 \leq j \leq s+1$ and $P^{(j)} \odot P^{(j+1)}$ is not an ascending path for $1 \leq j \leq s$, then P is an s -folded ascending path in G . For an s -folded ascending path P , the folded number of P is defined to be s .*

Definition 3: Consider two graphs $G_1 = (V, E_1)$ and $G_2 = (V, E_2)$ with the same set of vertices V and labeled edge sets E_1 and E_2 , respectively. The edge sum of these graphs is the graph $G_1 + G_2 = G = (V, E)$ with $E = E_1 \cup E_2$, whose edges $e \in E$ are labeled by the following rules:

$$t_G(e) = \begin{cases} t_{G_1}(e), & \text{if } e \in E_1, \\ t_{G_2}(e) + \max_{e' \in E_1} t_{G_1}(e'), & \text{if } e \in E_2. \end{cases} \quad (10)$$

The edge sum $G_1 + G_2 + \dots + G_h$ of h identical graphs ($G_1 = G_2 = \dots = G_h \equiv G$) is denoted by hG . Each set $E(G_i)$ in hG is denoted by $E_i(hG)$, i.e., $E(hG) = \bigcup_{1 \leq i \leq h} E_i(hG)$. Note that the labels of the edges in $E_i(hG)$ are greater than the corresponding edges in $E(G)$ by $(i-1) \times \max_{e \in E(G)} t_G(e)$. Given a subset of edges $A \subseteq E(G)$, denote its copy in the set $E_i(hG)$ by A_i . By this analogy, a path P in G as a subset of $E(G)$ has a copy in $E_i(hG)$, which we denote by P_i .

Let $P = P^{(1)} \odot P^{(2)} \odot \dots \odot P^{(s+1)}$ be an s -folded ascending path from a vertex u to a vertex v in G , where $P^{(j)}$ is an ascending subpath for $1 \leq j \leq s+1$. Then, P_i is also an s -folded ascending path and $P_i = P_i^{(1)} \odot P_i^{(2)} \odot \dots \odot P_i^{(s+1)}$ for $1 \leq i \leq h$. Now consider the path $P(k) = P_k^{(1)} \odot P_{k+1}^{(2)} \odot \dots \odot P_{k+s}^{(s+1)}$ in hG . Then, $P(k)$ is an ascending path from u to v for $1 \leq k \leq h-s$ such that $P(k)$ and $P(k')$ are edge-disjoint if $k \neq k'$. Thus, based on P , we can construct $(h-s)$ edge-disjoint ascending paths from u to v in hG . Similarly, based on another s -folded ascending path P' from u to v , we can construct $(h-s)$ edge-disjoint ascending paths $P'(k)$ from u to v for $1 \leq k \leq h-s$. If P and P' are edge-disjoint, then all the paths $P(1), \dots, P(h-s)$ and $P'(1), \dots, P'(h-s)$ are pairwise edge-disjoint by construction. Therefore, the following lemma holds (see [15, 16]).

Lemma 1: Let u and v be vertices in a labeled graph G . If there are p edge-disjoint s -folded ascending paths from u to v in G , then there are $p(h-s)$ edge-disjoint ascending paths from u to v in hG for any integer $h \geq s$.

From this lemma, if there are p edge-disjoint s -folded ascending paths from u to v in G , then there are $k+1$ edge-disjoint ascending paths from u to v in $\left(s + \lceil \frac{k+1}{p} \rceil\right) G$.

Thus, the following corollary is obtained (see [15]).

Corollary 1: Let G be a graph with n vertices and m edges. If there are p edge-disjoint s -folded ascending paths between every pair of vertices in a labeled graph G , then $\tau(n, k) \leq \left(s + \lceil \frac{k+1}{p} \rceil\right) m$.

In order to improve this estimation of the upper bound, a stronger proposition is formulated and proved in [16].

Theorem 1: Let G be a labeled graph with n vertices. Suppose that

- $E(G)$ can be decomposed into l subsets $F^{(0)}, F^{(1)}, \dots, F^{(l-1)}$ such that for any two edges $e \in F^{(i)}$ and $e' \in F^{(j)}$, $t_G(e) < t_G(e')$ if $i < j$,
- for any two vertices u and v , there are p edge-disjoint paths from u to v such that the sum of their folded numbers is at most q , and the last edges of r_i paths are in $F^{(i)}$ for $0 \leq i \leq l-1$.

Then, the minimal number of edges in a k -fault-tolerant gossip graph is bounded

$$\tau(n, k) \leq \xi(n, k), \quad (11)$$

with $\xi(n, k)$ defined by the expression

$$\xi(n, k) = \sum_{0 \leq i \leq w} |F^{(i \bmod l)}|, \quad (12)$$

where w is an integer satisfying

$$\sum_{0 \leq i \leq w} r_{i \bmod l} \geq k + q + 1. \quad (13)$$

During the proof, the graph $\tilde{G} = hG + G'$ with $h = \lfloor \frac{w}{l} \rfloor$ and $G' = (V, \cup_{0 \leq i \leq w-hl} F^{(i)})$ is constructed, and showed that it is a k -fault-tolerant gossip graph. The number of edges of this graph is $|E(\tilde{G})| = \sum_{0 \leq i \leq w} |F^{(i \bmod l)}|$.

In the next section we construct a labeled Wheel graph and apply Theorem 1 to improve the known estimations of the upper bounds for $\tau(n, k)$.

3. Fault-tolerant Gossip Graphs Based on Wheel Graph

Consider a wheel graph $G = (V, E)$ with an odd number of vertices n , whose vertices and edges are labeled by the following rules: the label of the central vertex is u , the remaining vertices (which are located on the circle) are labeled consequently $v_1, v'_1, v_2, v'_2, \dots, v_k, v'_k$, where $n = 2k + 1$. Since the periodic boundary conditions are assumed, we identify the vertices $v_{i \pm k} \equiv v_i$ and $v'_{i \pm k} \equiv v'_i$ for $i = 1, 2, \dots, k$. The set of edges consists of three subsets

$$E(G) = F^{(0)} \cup F^{(1)} \cup F^{(2)} \quad (14)$$

with

$$F^{(0)} = \{(v_i, v'_i) : t_G((v_i, v'_i)) = 1, i = 1, 2, \dots, k\}, \quad (15)$$

$$F^{(1a)} = \{(v'_i, u) : t_G((v'_i, u)) = 2, i = 1, 2, \dots, k\}, \quad (16)$$

$$F^{(1b)} = \{(v_i, u) : t_G((v_i, u)) = 3, i = 1, 2, \dots, k\}, \quad (17)$$

$$F^{(1)} = F^{(1a)} \cup F^{(1b)}, \quad (18)$$

$$F^{(2)} = \{(v'_i, v_{i+1}) : t_G((v'_i, v_{i+1})) = 4, i = 1, 2, \dots, k\}. \quad (19)$$

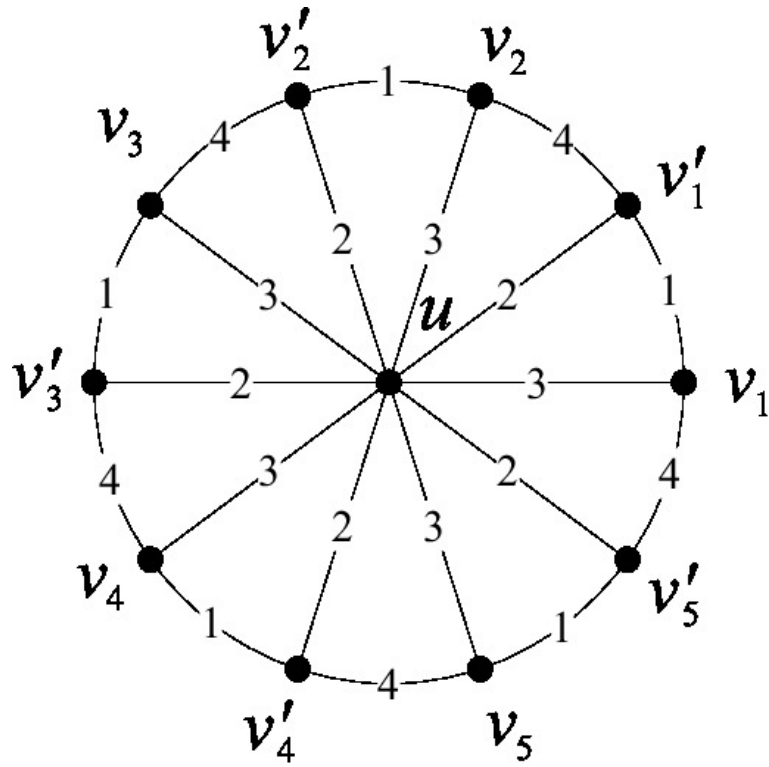


Fig. 1. Wheel graph for odd n (here $n=11$).

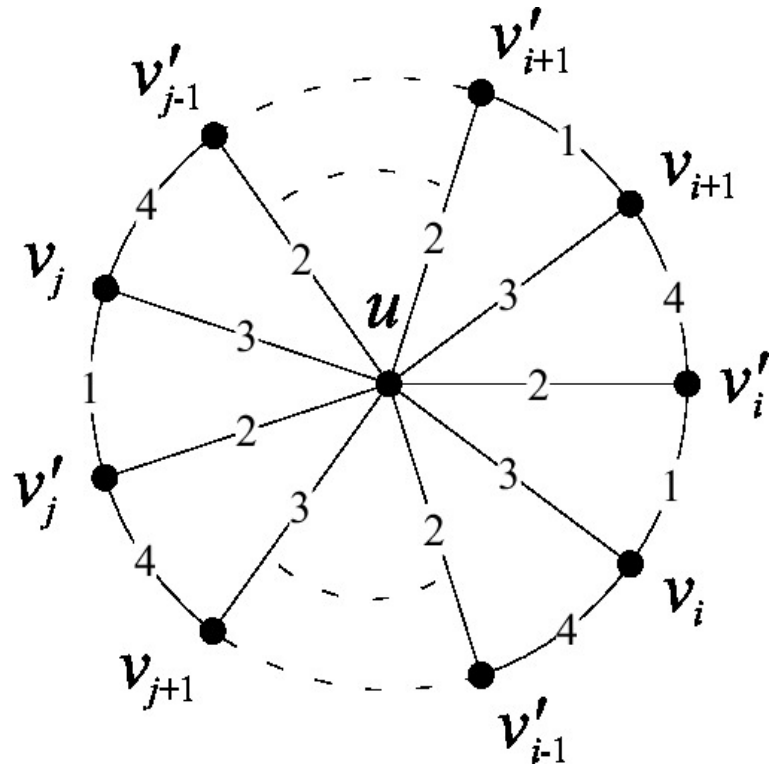


Fig. 2. The illustration of two arbitrary fixed vertices in the wheel graph.

Fig. 1 illustrates the wheel graph G for $n = 11$ vertices. Then we apply Theorem 1 to this graph. For all pairs of vertices in G , we first construct 3 edge-disjoint folded paths from the first vertex to the second one. For $i, j = 1, 2, \dots, k$ and $j \neq i, i - 1, i + 1, i + 2$ (see Fig. 2 for illustration) we have

- from v_i to u

$$\begin{aligned} & - v_i \xrightarrow{3} u \\ & - v_i \xrightarrow{1} v'_i \xrightarrow{2} u \\ & - v_i \xrightarrow{4} v'_{i-1} \xrightarrow{2} u \end{aligned}$$

- from u to v'_i

$$\begin{aligned} & - u \xrightarrow{2} v'_i \\ & - u \xrightarrow{3} v_i \xrightarrow{1} v'_i \\ & - u \xrightarrow{3} v_{i+1} \xrightarrow{4} v'_i \end{aligned}$$

- from v_i to v_j

$$\begin{aligned} & - v_i \xrightarrow{3} u \xrightarrow{2} v'_{j-1} \xrightarrow{4} v_j \\ & - v_i \xrightarrow{1} v'_i \xrightarrow{2} u \xrightarrow{3} v_{j+1} \xrightarrow{4} v'_j \xrightarrow{1} v_j \\ & - v_i \xrightarrow{4} v'_{i-1} \xrightarrow{2} u \xrightarrow{3} v_j \end{aligned}$$

- from v_i to v'_j

$$\begin{aligned} & - v_i \xrightarrow{3} u \xrightarrow{2} v'_j \\ & - v_i \xrightarrow{1} v'_i \xrightarrow{2} u \xrightarrow{3} v_j \xrightarrow{1} v'_j \\ & - v_i \xrightarrow{4} v'_{i-1} \xrightarrow{2} u \xrightarrow{3} v_{j+1} \xrightarrow{4} v'_j \end{aligned}$$

- from v'_i to v_j

$$\begin{aligned} & - v'_i \xrightarrow{2} u \xrightarrow{3} v_{j+1} \xrightarrow{4} v'_j \xrightarrow{1} v_j \\ & - v'_i \xrightarrow{1} v_i \xrightarrow{3} u \xrightarrow{2} v'_{j-1} \xrightarrow{4} v_j \\ & - v'_i \xrightarrow{4} v_{i+1} \xrightarrow{1} v'_{i+1} \xrightarrow{2} u \xrightarrow{3} v_j \end{aligned}$$

- from v'_i to v'_j

$$\begin{aligned} & - v'_i \xrightarrow{2} u \xrightarrow{3} v_j \xrightarrow{1} v'_j \\ & - v'_i \xrightarrow{1} v_i \xrightarrow{3} u \xrightarrow{2} v'_j \\ & - v'_i \xrightarrow{4} v_{i+1} \xrightarrow{1} v'_{i+1} \xrightarrow{2} u \xrightarrow{3} v_{j+1} \xrightarrow{4} v'_j \end{aligned}$$

The edge-disjoint folded paths for $j = i, i - 1, i + 1, i + 2$ are shorter; they have less or equal folded numbers and are easier to construct. Therefore, they are not presented here in order to avoid an artificial growth of the text. Finally, we have

$$|F^{(0)}| = (n - 1)/2, \quad |F^{(1)}| = n - 1, \quad |F^{(2)}| = (n - 1)/2, \quad (20)$$

$$p = 3, \quad r_0 = r_1 = r_2 = 1, \quad q = 3, \quad (21)$$

from which we obtain $w \geq k + 3$ and

$$\sum_{i=0}^{k+3} |F^{(i \bmod 3)}| = \begin{cases} \frac{2}{3}(n-1)k + \frac{5}{2}(n-1), & \text{if } (k \bmod 3) = 0 \\ \frac{2}{3}(n-1)(k-1) + \frac{7}{2}(n-1), & \text{if } (k \bmod 3) = 1 \\ \frac{2}{3}(n-1)(k-2) + 4(n-1), & \text{if } (k \bmod 3) = 2 \end{cases} \quad (22)$$

For even n , we modify the wheel graph by adding a new vertex u' and transforming the edge set to the following expression

$$E(G) = F^{(0)} \cup F^{(1)} \cup F^{(2)} \quad (23)$$

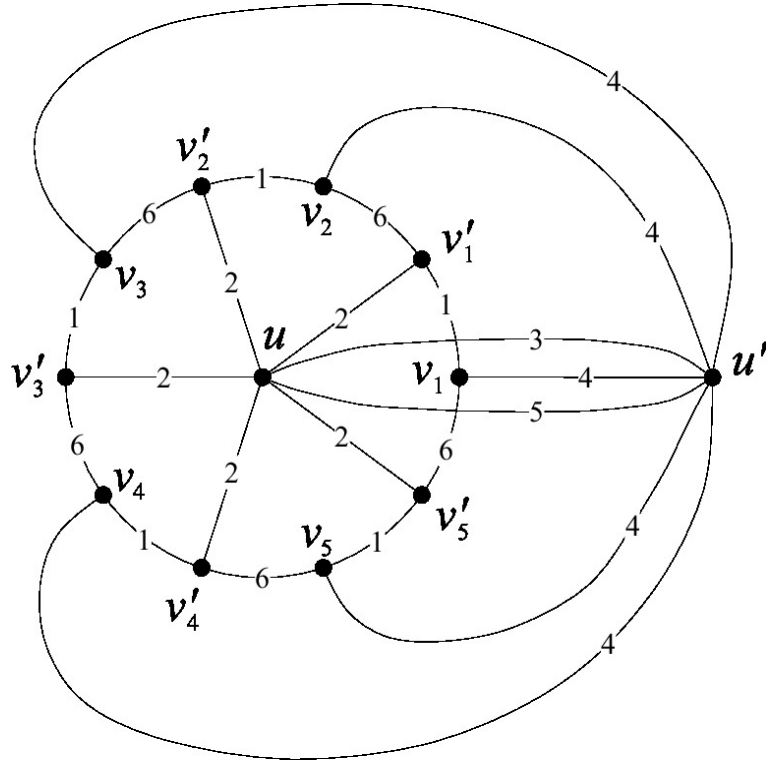


Fig. 3. Wheel graph for odd n (here $n=12$).

with

$$F^{(0)} = \{(v_i, v'_i) : t_G((v_i, v'_i)) = 1, \quad i = 1, 2, \dots, k\}, \quad (24)$$

$$F^{(1a)} = \{(v'_i, u) : t_G((v'_i, u)) = 2, \quad i = 1, 2, \dots, k\}, \quad (25)$$

$$e_a = (u, u'), \quad t_G(e_a) = 3, \quad (26)$$

$$F^{(1b)} = \{(v_i, u') : t_G((v_i, u')) = 4, \quad i = 1, 2, \dots, k\}, \quad (27)$$

$$e_b = (u, u'), \quad t_G(e_b) = 5, \quad (28)$$

$$F^{(1)} = F^{(1a)} \cup F^{(1b)} \cup \{e_a, e_b\}, \quad (29)$$

$$F^{(2)} = \{(v'_i, v_{i+1}) : t_G((v'_i, v_{i+1})) = 6, \quad i = 1, 2, \dots, k\}. \quad (30)$$

Here the vertices u and u' are connected by two edges e_a and e_b . The graph G for $n = 12$ vertices is shown in Fig. 3. Constructing the edge-disjoint folded paths, one obtains

$$|F^{(0)}| = (n - 2)/2, \quad |F^{(1)}| = n, \quad |F^{(2)}| = (n - 2)/2, \quad (31)$$

$$p = 3, \quad r_0 = r_1 = r_2 = 1, \quad q = 3, \quad (32)$$

which results $w \geq k + 3$ and

$$\sum_{i=0}^{k+3} |F^{(i \bmod 3)}| = \begin{cases} \frac{1}{3}(2n - 1)k + \frac{5}{2}n - 4, & \text{if } (k \bmod 3) = 0 \\ \frac{1}{3}(2n - 1)(k - 1) + \frac{7}{2}n - 5, & \text{if } (k \bmod 3) = 1 \\ \frac{1}{3}(2n - 1)(k - 2) + 4n - 5, & \text{if } (k \bmod 3) = 2 \end{cases} \quad (33)$$

From Eq. (22) for odd n and Eq. (33) for even n the following theorem holds.

Theorem 2: *In k fault-tolerant gossip schemes the upper bound of $\tau(n, k)$ minimum needed calls satisfies the following condition:*

$$\tau(n, k) \leq \frac{2}{3}nk + O(n). \quad (34)$$

4. Special Cases: $k = 1, 2$

Finally, we consider the special cases when $k = 1$ and $k = 2$. In these cases the construction of the k fault-tolerant graph becomes simpler. We present it here, without considering the details:

$$\tau(n, 1) \leq 2n - 3 + \left\lfloor \frac{n}{2} \right\rfloor, \quad \tau(n, 2) \leq 2n - 3 + n. \quad (35)$$

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Wheel գրաֆների վրա հիմնված k -հուսալիությանը gossip գրաֆներ

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Անփոփում

Gossip խնդիրը (հեռախոսների խնդիրը) ինֆորմացիայի տարածման խնդիր է, որտեղ հաղորդակցության ցանցի n հանգույցներից յուրաքանչյուրը տիրապետում է որևէ եզակի ինֆորմացիայի, որը պետք է փոխանցվի բոլոր մնացած հանգույցներին՝ օգտագործելով հանգույցների գույգերի միջև երկկողմանի հաղորդակցությունը: Տրված երկու հանգույցների միջև զանգի ժամանակ դրանք փոխանակում են այդ պահին իրենց հայտնի ողջ

ինֆորմացիան: Այս հոդվածում մենք դիտարկում ենք k հուսալիությամբ gossip խնդիրը, որը ընդհանուր gossip խնդրի ընդհանրացումն է, երբ թույլատրված են զանգերի ամենաշատը k պատահական խափանումներ: Խնդիրը կայանում է զանգերի այն $\tau(n, k)$ նվազագույն քանակը գտնելու մեջ, որի դեպքում կապահովվի ինֆորմացիայի ամբողջական տարածումը: Մենք նախագծել ենք k հուսալիությամբ վթարակայուն gossip սխեմա (զանգերի հաջորդականություն), որպեսզի գտնենք $\tau(n, k)$ -ի համար վերին սահմաններ, որոնք կրարելավեն նախորդ արդյունքները n -ի և k -ի որոշակի փոքր արժեքների դեպքում:

k -толерантные gossip графы основанные на wheel графов

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Аннотация

Проблемой gossip (проблемой телефонов) является проблема распространения информации, где каждый из n узлов сети связи имеет уникальный фрагмент информации, который должен быть передан всем остальным узлам с помощью двусторонней связи (телефонные звонки) между парами узлов. После вызова между данными двумя узлами, они обмениваются всей информацией, известной им в данный момент. В этой статье, мы исследуем k -отказоустойчивую gossip проблему, которая является обобщением задачи gossip, где наиболее k произвольных сбойных вызовов разрешены. Проблема в том, чтобы найти минимальное количество звонков $\tau(n, k)$, необходимых для обеспечения полного распространения информации. Мы построили k -отказоустойчивую gossip схему (последовательности вызовов) с целью найти верхние границы $\tau(n, k)$, которая улучшает ранее известные результаты для некоторых малых значений n и k .