

Extensions of Markov's Constructive Continuum and Uniform Continuity of Constructive Functions

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We consider everywhere defined constructive functions (c.f.) on the closed unit constructive interval. As is well-known by the famous Zaslavsky-Tseitin Theorem such a c.f. can be effectively nonuniformly continuous. In this case it can not be extended to a classical continuous function. In reality, in every known counter-example a singularity could be discovered already on the level of pseudonumbers. Let us recall that a pseudonumber is a recursive sequence of rationals that is a Cauchy sequence classically. Pseudonumbers can be considered as ϕ' (Δ_2)-computable numbers as well. Let D be the set of all constructive real numbers (Markov's Continuum in the title), D_1 the set of all pseudonumbers. A c.f. f is said to be 1-complete if it can be extended to a computable (and so continuous) function over D_1 .

Theorem 1 *There is a 1-complete c.f. that is effectively nonuniformly continuous.*

This result is rather precise as a c.f. continuously extendible to ϕ'' -computable numbers is uniformly continuous classically.

As is well known every c.f. can be computed on D by a Kleene operator (partial-recursive operator). The following result together with Theorem 1 shows that there is an essential difference between Markov's and Kleene's Computability over D_1 .

Theorem 2 *A c.f. f is constructively uniformly continuous iff there is a 1-complete Kleene operator that computes f .*

References

- [1] B.A. Kushner, Some Extensions of Markov's Constructive Continuum and their Applications to the Theory of Constructive Functions, The L.E.J.Brouwer Centenary Symposium, North-Holland Publ. Co., Amsterdam, 1982, Pp. 261–273
- [2] B.A. Kushner, Lectures on Constructive Mathematical Analysis. (Translation from the Russian), AMS, Providence, Rhode Island, 1984