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Analytical Study of Mixed Convective Flow and Heat Transfer in Vertical Channel Filled with Immiscible Viscous Fluids

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Abstract

In this paper investigation of mixed convective flow and heat transfer in vertical channel filled with immiscible viscous fluids has been carried out. The governing differential equations are solved analytically by regular perturbation method. The impact of governing parameters on velocity and temperature fields namely Grash of number, Brinkman number, perturbation parameter, viscosity ratio, width ratio, conductivity ratio, Nusselt number are investigated and represented graphically.

Keywords: mixed convective flow, heat transfer, perturbation method.

2010 AMS subject classification: 76D05, 35Q30‡

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1. Introduction

Mixed convective flow and heat transfer has importance from researchers due to their diverse application in engineering, automobile sector and various technical fields. This includes geothermal mining, nuclear reactors, heat and cold storage, wielding equipment, aircrafts Desai and Vafai [5]. The study of laminar fully developed mixed convection in a vertical channel with uniform wall temperature was one of the first attempt of Tao [10]. Recently Hamadah and Wirtz [11], Bratetta [3], Aung and Worku [1] assumed symmetric and asymmetric heating of walls of the vertical channel. Prathap kumar et al [7] studied the chemical reaction effects on mixed convection flow in vertical channel with immiscible fluids analytical as well as numerically. Umavati and Chamkha [6] analyzed mixed convection in presence of heat source or heat sink in a vertical channel. Jha and Oni [2] assumed temperature dependent viscosity to study the mixed convection flow in a vertical channel where they found that increase in viscosity parameter increases fluid velocity. With hall and ion-slip effects Srinivasacharya and Shafeeurrahaman [4] analyzed mixed convection flow in a vertical channel filled with nanofluid where they found with the increment in magnetic parameter decrease in temperature, velocity, nanoparticle concentration were occurred. Prathap kumar et al [8] analyzed and found impact of different governing parameter on mixed convective flow in a vertical channel using third kind of boundary condition where channel is filled with porous media using differential transfer method as well as perturbation method. In fluid dynamics regular perturbation method is most preferably used, Rashidi and Ganji [9].
Keeping in view of several applications of mixed convective flow in vertical channel, the aim of this paper is to investigate laminar fully developed mixed convection flow and heat transfer in vertical channel with immiscible viscous fluid analytically to extend the studies available in the literature. The governing differential equations are solved using regular perturbation method valid for small values of perturbation parameter. The thermal buoyancy force, viscous dissipation, viscosity ratio, width ratio, conductivity ratio, Nusselt number are considered to investigate their impact on flow field.

Notation:

b- ratio of thermal expansion coefficients (β2/β1)
Br - Brinkman number (μ1 U0 (1)2/k1 ΔT)
g- acceleration due to gravity
Gr - Grashof number (gβ1D13ΔT/ν12)
GR - mixed convective parameter (Gr/Re)
D - width ratio (D2/D1)
D1, D2 - width of regions
k - thermal conductivities (k1/k2)
m - ratio of viscosities (μ1/μ2)
n - ratio of densities (ρ2/ρ1)
p - pressure (assuming p1=p2=p)
P- =p+ρ0gX, difference between the pressure and hydrostatic pressure

Re - Reynolds number (D1U0(1)/ν1)
U0(i) - reference velocity
Greek Symbols
α1, α2- Thermal diffusivities
β1, β2- Coefficients of thermal expansion
ΔT- Difference in temperature (T2−T1)
Ɛ- Dimensionless parameter/perturbation parameter (GR Br)
Θ- Dimensionless temperature
θ1, θ2- Temperatures
μ1, μ2- Viscosities
ν1, ν2- Kinematics viscosities
ρ1, ρ2 - Densities
Subscripts i= 1,2 corresponding to region-I and region-II respectively.

2. Preliminaries

Consider a steady two dimensional laminar fully developed mixed convection flow in open ended vertical channel filled with immiscible viscous fluids The X axis is taken upward and parallel to the walls and Y axis is normal on it, shown in Fig 1. We consider fluid to be incompressible, and temperature between the plate and fluid is small, so that the fluid properties taken as constant except the density in the buoyancy term of equation of motion.

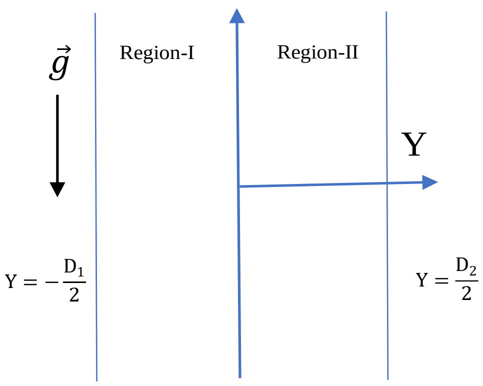

Fig.1. Physical configuration

Region −I

$$g\beta_1(T_1 - T_0) - \frac{1}{\rho_1}\frac{dP}{dX} + \frac{\mu_1}{\rho_1}\frac{d^2U_1}{dY^2} = 0 \tag{2.1}$$

$$\alpha_1\frac{d^2T_1}{dY^2} + \frac{\nu_1}{C_p}\left(\frac{dU_1}{dY}\right)^2 = 0 \tag{2.2}$$

Region −II

$$g\beta_2(T_2 - T_0) - \frac{1}{\rho_2}\frac{dP}{dX} + \frac{\mu_2}{\rho_2}\frac{d^2U_2}{dY^2} = 0 \tag{2.3}$$

$$\alpha_2\frac{d^2T_2}{dY^2} + \frac{\nu_2}{C_p}\left(\frac{dU_2}{dY}\right)^2 = 0 \tag{2.4}$$

P depends only on X

$$\frac{dP}{dY} = 0 \tag{2.5}$$

In the presence of viscous dissipation, the energy balance equation can be written as Region −I

$$\frac{d^2T_1}{dY^2} = \frac{-\nu_1}{\beta_1 g}\frac{d^4U_1}{dY^4} \tag{2.6}$$

From Eq. (2.2) and Eq. (2.6)

$$\frac{d^4U_1}{dY^4} = \frac{\beta_1 g\rho_1}{k_1}\left(\frac{dU_1}{dY}\right)^2 \tag{2.7}$$

Region −II

$$\frac{d^2T_2}{dY^2} = \frac{-\nu_2}{\beta_2 g}\frac{d^4U_2}{dY^4} \tag{2.8}$$

From Eq. (2.4) and Eq. (2.8)

$$\frac{d^4U_2}{dY^4} = \frac{\beta_2 g\rho_2}{k_2}\left(\frac{dU_2}{dY}\right)^2 \tag{2.9}$$

On account of Eq. (2.1) and Eq. (2.3) there exists a constant A such that

$$\frac{dP}{dX} = A \tag{2.10}$$

The boundary and interface conditions are

$$U_I\left(\frac{-D_1}{2}\right) = 0 = U_2\left(\frac{-D2}{2}\right),\quad U_1(0) = U_2(0),\quad T_0 = \frac{T_1 + T_2}{2},$$

$$\left.\frac{d^2U_1}{dY^2}\right|_{Y=-\frac{D_1}{2}} = \frac{A}{\mu_1} + \frac{\beta_1 g[T_2 - T_1]}{2\nu_1},\qquad \left.\frac{d^2U_2}{dY^2}\right|_{Y=\frac{D_2}{2}} = \frac{A}{\mu_2} - \frac{\beta_2 g[T_2 - T_1]}{2\nu_2},$$

$$\mu_1\frac{dU_1}{dY} = \mu_2\frac{dU_2}{dY}, \text{at } Y = 0 \quad , \quad \frac{d^3U_1}{dY^3} = \frac{1}{mnbk}\frac{d^3U_2}{dY^3}, \text{at } Y = 0$$

$$\frac{d^2U_1}{dY^2} = \frac{1}{mnb}\frac{d^2U_2}{dy^2} + \frac{A}{\mu_1}\left[1 - \frac{1}{nb}\right] \quad \text{at } Y = 0$$

$$T_1(0) = T_2(0),\qquad k_1\frac{dT_1}{dY} = k_2\frac{dT_2}{dY}, \text{at } Y = 0 \tag{2.11}$$

Equations (2.7),(2.9) and (2.11) determine the velocity distribution. They can be written in a non-dimensional form by means of following dimensionless variables

$$u_1 = \frac{U_1}{U_0^{(1)}};\ u_2 = \frac{U_2}{U_0^{(2)}};\ y_1 = \frac{Y_1}{D_1};\ y_2 = \frac{Y_2}{D_2};\quad G_r = \frac{g\beta_I\Delta TD_1^3}{\nu_1^2};\ Re = \frac{U_0^{(1)}D_1}{\nu_1}$$

$$B_r = \frac{\mu_1 {U_0^{(1)}}^2}{k_1\Delta T};\ U_0^{(1)} = \frac{-AD_1^2}{48\mu_1};\ U_0^{(2)} = -\frac{AD_2^2}{48\mu_2};$$

$$G_R = \frac{G_r}{Re} \;; \qquad \theta_1 = \frac{T_1 - T_0}{\Delta T};\; \theta_2 = \frac{T_2 - T_0}{\Delta T};\; R_T = \frac{T_2 - T_1}{\Delta T} \tag{2.12}$$

Eqs. (2.7), (2.9) becomes

Region −I

$$\frac{d^4 u_1}{dy^4} = G_R B_r \left(\frac{du_1}{dy}\right)^2 \tag{2.13}$$

Region −II

$$\frac{d^4 u_2}{dy^4} = G_R B_r\, mnbkD^4 \left(\frac{du_2}{dy}\right)^2 \tag{2.14}$$

The boundary and interface conditions are

$$u_1 U_0^{(1)} = 0\,, \text{at} \quad y = -\frac{1}{4}\;;\; u_1\left(-\frac{1}{4}\right) = 0 = u_2\left(\frac{1}{4}\right)\;;\; u_1(0) = mD^2 u_2(0)\;;$$

$$\frac{d^2u_1}{dy^2} = -48 + \frac{G_R R_T}{2}\,, \text{at } y = -\frac{1}{4}\,;\, \frac{d^2u_2}{dy^2} = -48 - \frac{G_R\, nb\, R_T}{2}\,, \text{at } y = \frac{1}{4}\;;$$

$$\frac{du_1}{dy} = D\frac{du_2}{dy}\,, \text{at } y = 0\;; \quad \frac{d^2u_1}{dy^2} = \frac{1}{nb}\left[\frac{d^2u_2}{dy^2} + 48(1 - nb)\right], \text{at } y = 0\;;$$

$$\frac{d^3u_1}{dy^3} = \frac{1}{nbkD}\frac{d^3u_2}{dy^3}\,, \text{at } y = 0 \tag{2.15}$$

$$\text{Where } D = \frac{D_2}{D_1}\,, m = \frac{\mu_1}{\mu_2}, n = \frac{\rho_2}{\rho_1}, b = \frac{\beta_2}{\beta_1}, k = \frac{k_1}{k_2}$$

Solutions

Case of Negligible of Viscous Dissipation ($\mathbf{B_r = 0}$)

The solution of Eqs. (2.13) and (2.14) can be obtained using Eq. (2.15) in the absence of viscous dissipation, that is, when the parameter, ($\mathbf{B_r = 0}$) is given by

Region −I

$$u_1 = E_1 + E_2 y + E_3 y^2 + E_4 y^3 \tag{3.1}$$

Region −II

$$u_2 = E_5 + E_6 y + E_7 y^2 + E_8 y^3 \tag{3.2}$$

Using Eq. (2.12) in Eqs. (2.1) and (2.3) the energy balance equations are

Region −I

$$\theta_1 = -\frac{1}{G_R}\left[48 + \frac{d^2u_1}{dy^2}\right] \tag{3.3}$$

Region −II

$$\theta_2 = -\frac{1}{nbG_R}\left[48 + \frac{d^2u_2}{dy^2}\right] \tag{3.4}$$

Using the expressions obtained in Eqs. (3.1) and (3.2) the energy balance Eqs. (3.3) and (3.4) becomes

Region −I

$$\theta_1 = -\frac{1}{G_R}\,[48 + 2E_3 + 6E_4 y] \tag{3.5}$$

Region −II

$$\theta_2 = -\frac{1}{nbG_R}\,[48 + 2E_7 + 6E_8 y] \tag{3.6}$$

Case of Negligible buoyancy Force $(G_R = 0)$

When the buoyancy forces are negligible $(G_R = 0)$and viscous dissipation is dominating $(B_r \neq 0)$,so that purely forced convection occurs. For this case, the solutions of Eqs. (2.13) and (2.14) can be obtained using the Eq. (2.15), the velocities are given by

Region −I

$$u_1 = F_1 + F_2 y + F_3 y^2 + F_4 y^3 \tag{3.7}$$

Region −II

$$u_2 = F_5 + F_6 y + F_7 y^2 + F_8 y^3 \tag{3.8}$$

The energy balance Eqs. (2.6) and (2.8) in non- dimensional form can also be written as

Region −I

$$\frac{d^2\theta_1}{dy^2} = -B_r\left(\frac{du_1}{dy}\right)^2 \tag{3.9}$$

Region −II

$$\frac{d^2\theta_2}{dy^2} = -B_r\, m\, k\, D^4\left(\frac{du_2}{dy}\right)^2 \tag{3.10}$$

The boundary and interface conditions are

$$\theta_1\left(-\frac{1}{4}\right) = -\frac{R_T}{2}\,; \qquad \theta_2\left(\frac{1}{4}\right) = \frac{R_T}{2}\,;$$
$$\theta_1(0) = \theta_2(0)\,; \qquad \frac{d\theta_1}{dy} = \frac{1}{kD}\frac{d\theta_2}{dy}, \text{at } y = 0\,; \tag{3.11}$$

Solving Eqs. (3.9) and (3.10) ,using Eqs (3.7) and (3.8) we obtain

Region −I

$$\theta_1 = -B_r\begin{pmatrix} G_3 y^2 + G_4 y^3 + G_5 y^4 \\ G_6 y^5 + G_7 y^6 \end{pmatrix} + G_2 y + G_1 \tag{3.12}$$

Region −II

$$\theta_2 = -m\, k\, D^4\, B_r\begin{pmatrix} G_{10} y^2 + G_{11} y^3 + G_{12} y^4 \\ + G_{13} y^5 + G_{14} y^6 \end{pmatrix} + G_9 y + G_8 \tag{3.13}$$

Combine Effects of Buoyancy Force and Viscous Dissipation

By using the perturbation method, we solve Eqs. (2.13) and (2.14) with a dimensionless parameter $|\varepsilon|$ (< 1) defined as

$$\varepsilon = G_R B_r \tag{3.14}$$

Which is independent of the reference temperature difference ΔT. The solutions are assumed in the form

$$u(y) = u_0(y) + \varepsilon u_1(y) + \varepsilon^2 u_2(y) + \cdots = \sum_{n=0}^{\infty} \varepsilon^n u_n(y) \tag{3.15}$$

Substituting Eq. (3.15) in Eqs. (2.13) and (2.14) and the coefficients of like powers of ε to obtain the zeroth and first order equations as follows
Region −I (Zeroth -order equation)

$$\frac{d^4u_{10}}{dy^4} = 0 \tag{3.16}$$

First-order equation

$$\frac{d^4u_{11}}{dy^4} = \left(\frac{du_{10}}{dy}\right)^2 \tag{3.17}$$

Region −II (Zeroth-order equation)

$$\frac{d^4u_{20}}{dy^4} = 0 \tag{3.18}$$

First-order equation

$$\frac{d^4u_{21}}{dy^4} = \mathrm{mnbk}D^4\left(\frac{du_{20}}{dy}\right)^2 \tag{3.19}$$

The corresponding boundary and interface conditions for the zeroth and first order by Eq. (2.15) reduces to

$$\begin{gathered}
u_{10}\left(-\frac{1}{4}\right) = 0 = u_{20}\left(\frac{1}{4}\right); \quad u_{11}\left(-\frac{1}{4}\right) = 0 = u_{21}\left(\frac{1}{4}\right); \\
u_{10}(0) = mD^2u_{20}(0); \qquad u_{11}(0) = mD^2u_{21}(0); \\
\frac{d^2u_{10}}{dy^2} = -48 + \frac{G_RR_T}{2}, \text{at } y = -\frac{1}{4}; \qquad \frac{d^2u_{11}}{dy^2} = 0, \text{at } y = -\frac{1}{4}; \\
\frac{d^2u_{20}}{dy^2} = -48 - \frac{nbG_RR_T}{2}, \text{at } y = \frac{1}{4}; \qquad \frac{d^2u_{21}}{dy^2} = 0, \text{at } y = \frac{1}{4}; \\
\frac{du_{10}}{dy} = D\frac{du_{20}}{dy}, \text{at } y = 0; \qquad \frac{du_{11}}{dy} = D\frac{du_{21}}{dy}, \text{at } y = 0; \\
\frac{d^2u_{10}}{dy^2} = \frac{1}{nb}\left[\frac{d^2u_{20}}{dy^2} + 48(1 - nb)\right], \text{at } y = 0; \\
\frac{d^2u_{11}}{dy^2} = \frac{1}{nb}\left[\frac{d^2u_{21}}{dy^2}\right], \text{at } y = 0; \qquad \frac{d^3u_{10}}{dy^3} = \frac{1}{nbkD}\frac{d^3u_{20}}{dy^3}, \text{at } y = 0; \\
\frac{d^3u_{11}}{dy^3} = \frac{1}{nbkD}\frac{d^3u_{21}}{dy^3}, \text{at } y = 0;
\end{gathered} \tag{3.20}$$

Solutions of zeroth-order Eqs. (3.16) and (3.18) using Eq. (3.20) are

$$u_{10} = C_1 + C_2y + C_3y^2 + C_4y^3 \tag{3.21}$$

$$u_{20} = B_1 + B_2y + B_3y^2 + B_4y^3 \tag{3.22}$$

Solutions of first-order Eqs. (3.17) and (3.19) using Eq. (3.20) are

$$u_{11} = P_5y^8 + P_6y^7 + P_7y^6 + P_8y^5 + P_9y^4 + \frac{P_1}{6}y^3 + \frac{P_2}{2}y^2 + P_3y + P_4 \tag{3.23}$$

$$u_{21} = Q_5y^8 + Q_6y^7 + Q_7y^6 + Q_8y^5 + Q_9y^4 + \frac{Q_1}{6}y^3 + \frac{Q_2}{2}y^2 + Q_3y + Q_4 \tag{3.24}$$

Using the velocities given by Eqs. (3.21) - (3.24) the energy balance Eqs. (3.3) and (3.4) becomes

Region −I

$$\theta_1 = -\frac{1}{G_R}\left[\begin{array}{c} 48+2C_3+6C_4 y \\ +\varepsilon\left(\begin{array}{c} 56P_5y^6+42P_6y^5+30P_7y^4 \\ +20P_8y^3+12P_9y^2+P_1\,y+P_2 \end{array}\right) \end{array}\right] \quad (3.25)$$

Region −II

$$\theta_2 = -\frac{1}{nbG_R}\left[\begin{array}{c} 48+2B_3+6B_4 y \\ +\varepsilon\left(\begin{array}{c} 56Q_5y^6+42Q_6y^5+30Q_7y^4 \\ +20Q_8y^3+12Q_9y^2+Q_1\,y+Q_2 \end{array}\right) \end{array}\right] \quad (3.26)$$

Heat Transfer

The wall heat transfer expression in terms of the Nusselt number is

$$Nu_- = (1+D)\frac{d\theta_1}{dy}\,, \quad \text{at } y=-\frac{1}{4}$$

$$Nu_+ = \left(1+\frac{1}{D}\right)\frac{d\theta_1}{dy}\,, \text{ at } y=\frac{1}{4}$$

$$Nu_- = -\frac{(1+D)}{G_R}\,\left[6C_4-\varepsilon\left(\frac{21}{64}P_5-\frac{105}{128}P_6+\frac{15}{8}P_7-\frac{15}{4}P_8+6P_9-P_1\right)\right] \quad (3.27)$$

$$Nu_+ = -\frac{\left(1+\frac{1}{D}\right)}{nbG_R}\,\left[6B_4+\varepsilon\left(\frac{21}{64}Q_5+\frac{105}{128}Q_6+\frac{15}{8}Q_7+\frac{15}{4}Q_8+6Q_9-Q_1\right)\right] \quad (3.28)$$

3. Results and discussions

Investigation of laminar mixed convection flow in vertical channel filled with immiscible viscous fluid has been done analytically by using a regular perturbation method taking the product of the thermal Grashof number (GR=Gr/Re) and Brinkman number Br as perturbation parameter. And solution are valid only for small values of perturbation parameter ε(<1). Viscous dissipation term is also included in the energy equations.

The flow fields are evaluated in case of asymmetric heating (RT=1) and are represented graphically in Fig 2-8. The velocity and temperature fields in the absence of Brinkman number (Br=0) are obtained for different values of thermal Grashof number (GR) and are depicted in Fig.2a, Fig.2b respectively. For negative value of GR velocity field increases in region-I and decreases in region II whereas for positive values of GR velocity decreases in region I and increases in region II. One can also observe that flow was an increasing function for value GR (>0) and decreasing function for GR (<0). But temperature field decreases in both the regions for all different values of GR.

The dimensionless temperature field θ is obtained and is shown in Fig.3 for different values of Brinkman number Br in case of negligible Buoyancy force (GR=0). As Brinkman number increases temperature field is also increases in both regions.
The velocity and temperature fields are obtained at GR=±500 for different values of ε and are shown in Fig.4a, Fig,4b respectively. The velocity field is an increasing function of ε for upward flow ε (>0) and decreasing function of ε for downward flow ε(<0). Whereas temperature field increases for both ε (>0) and ε (<0). The perturbation parameter ε is more effective on velocity field than temperature field. From Fig.4a it can also pointed out that at cold (left) and hot (right) walls reversal flow occurs for upward and downward flow respectively.
The effect of viscosity ratio m, width ratio D and conductivity ratio k on flow field evaluated for the values of GR =100, and ε=0.01 in case of asymmetric heating (RT=1). The effect of viscosity ratio m on the velocity and temperature fields are shown in Fig.5a, Fig.5b respectively. As the viscosity ratio m increases the flow field increases in both regions. Temperature increases from cold to hot walls for all values of m.
The effect of width ratio D on the velocity and temperature fields are shown in Fig.6a, Fig.6b respectively. Effect of the width ratio is similar to effect of viscosity ratio on flow field as D increases both velocity and temperature fields increases.
The effect of conductivity ratio k on the fields of velocity and temperature are depicted in Fig.7a, Fig.7b respectively. As k increases, the velocity and temperature fields reduce in both the regions, this means larger the conductivity ratio smaller the flow rate is. The Nusselt number at the cold wall (Nu-) and hot wall (Nu+) for |ε| is shown in Fig.8.The Nu- is an increasing function of |ε| and Nu+ is a decreasing function of |ε|

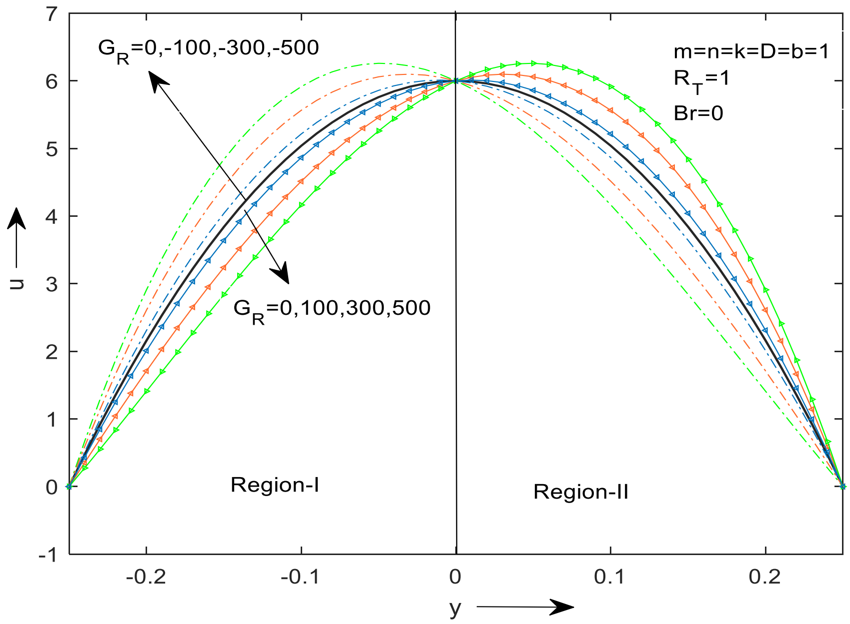

Fig. 2a. Velocity profiles for different values of G_R

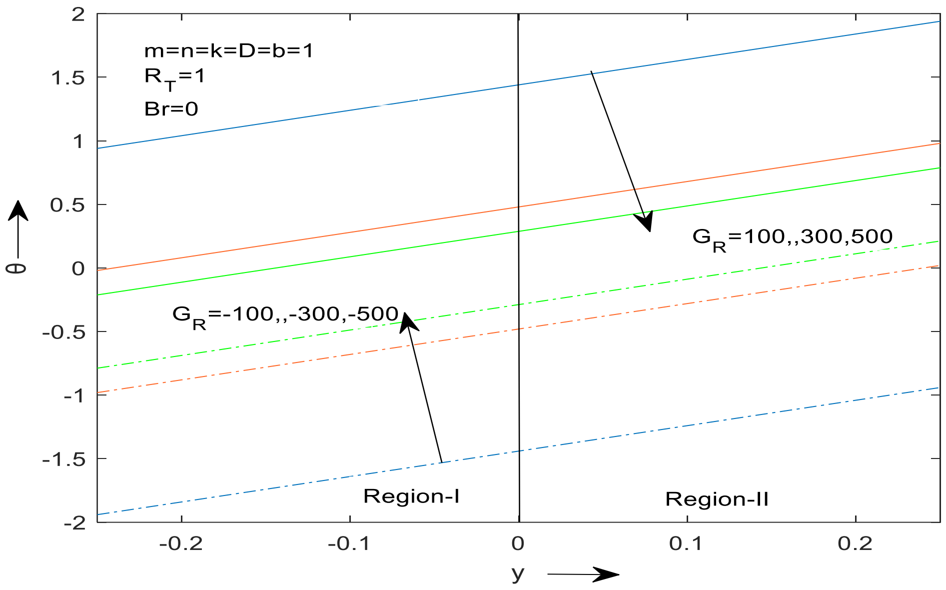

Fig. 2b. Temperature profiles for different values of G_R

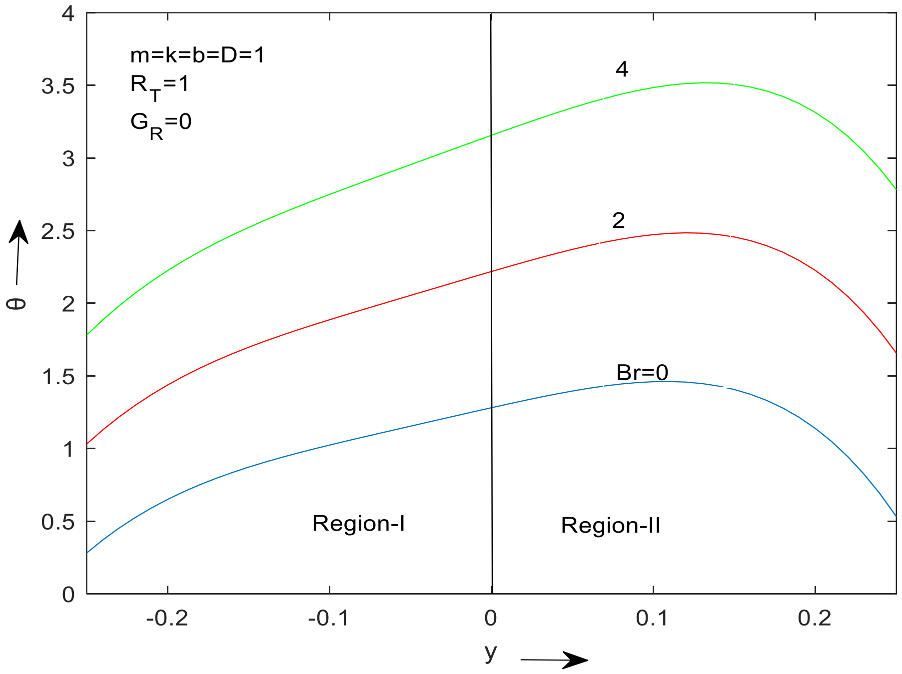

Fig. 3. Temperature profile for different values of B_r

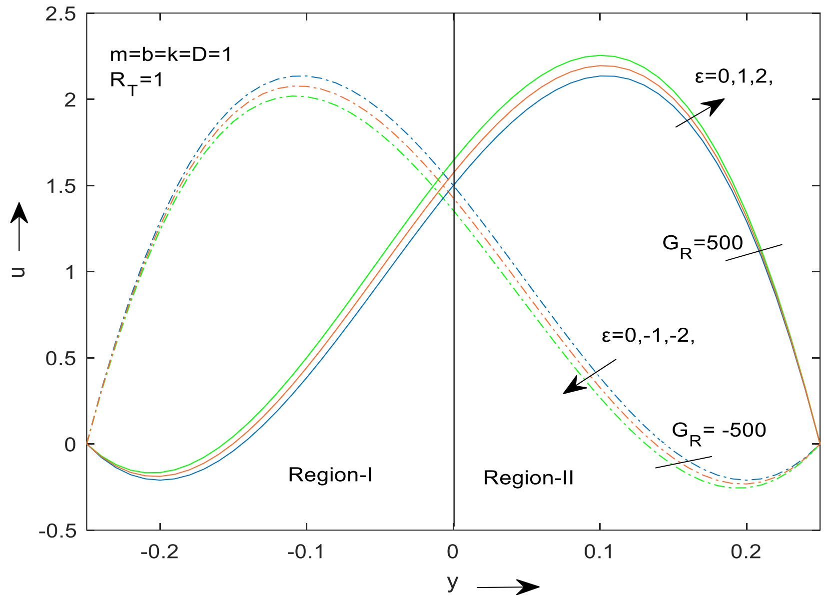

Fig. 4a. Velocity profile for different values of ε.

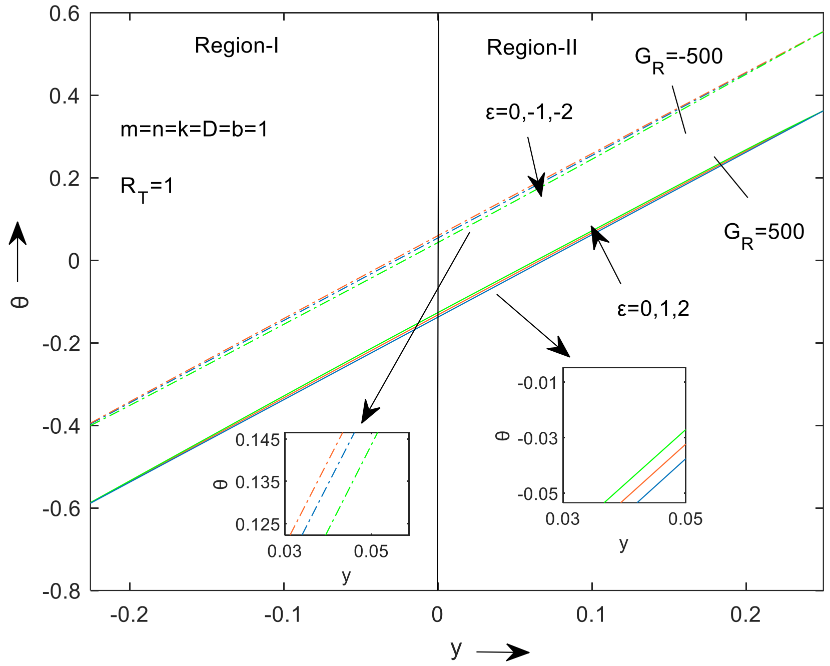

Fig. 4b. Temperature profile for different values of ε.

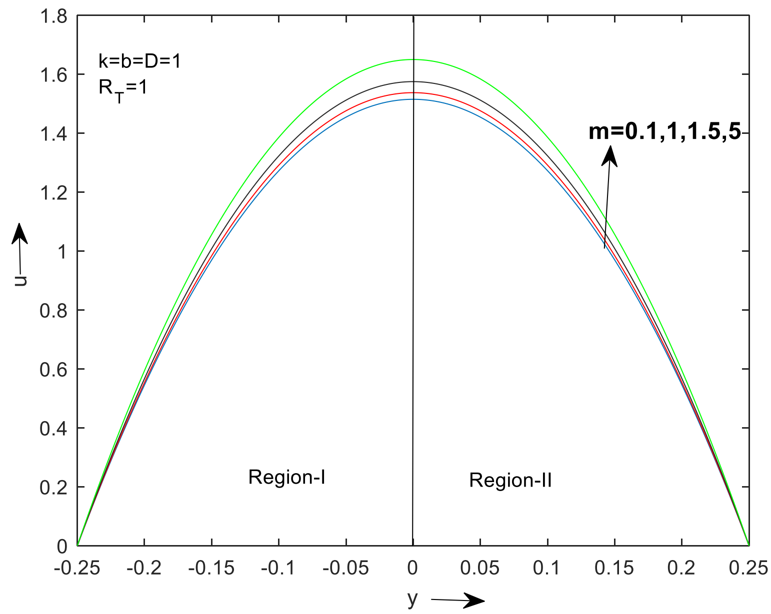

Fig. 5a. Velocity profile for different values of m.

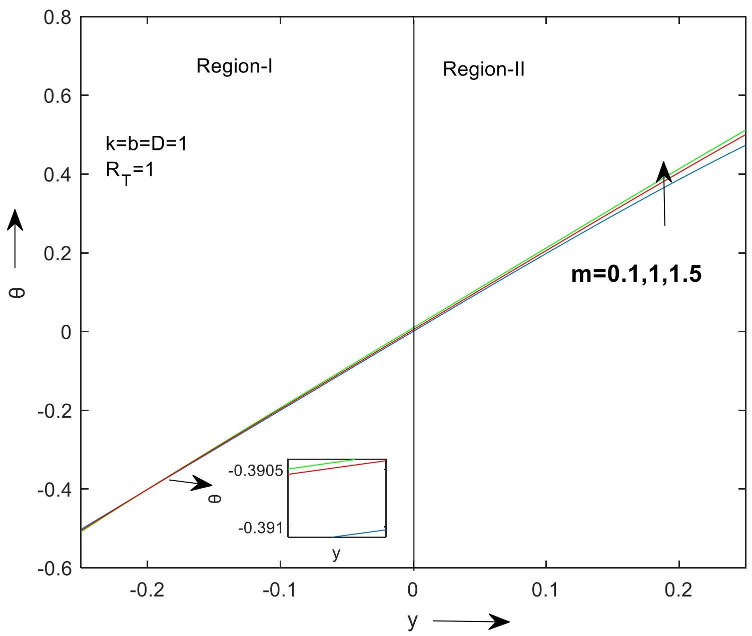

Fig. 5b. Temperature profile for different values of m.

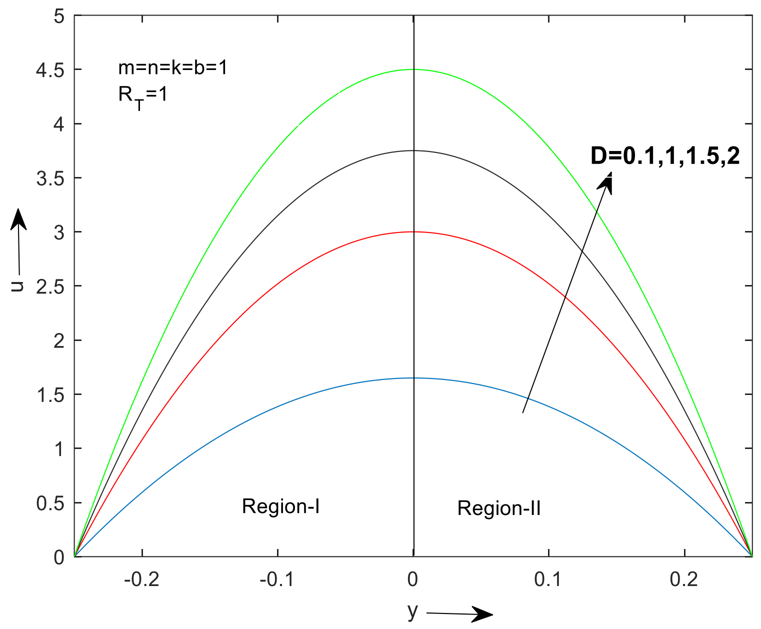

Fig. 6a. Velocity profile for different values of D.

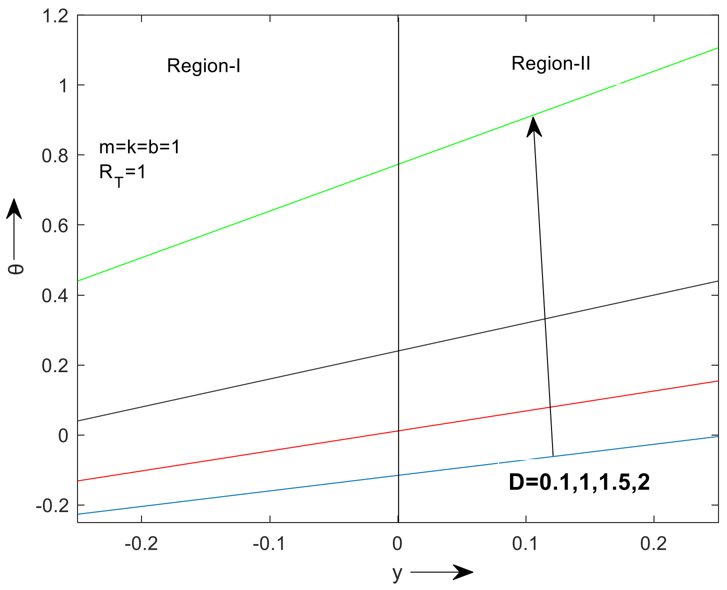

Fig. 6b. Temperature profile for different values of D.

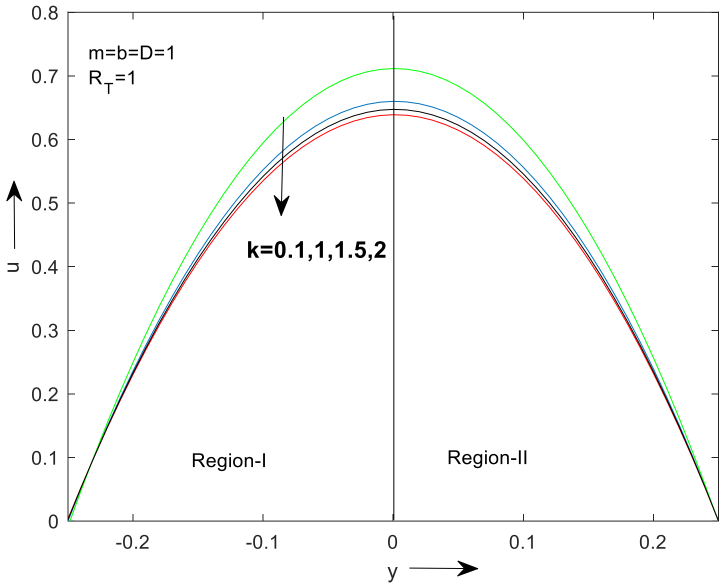

Fig. 7a. Velocity profile for different values of k

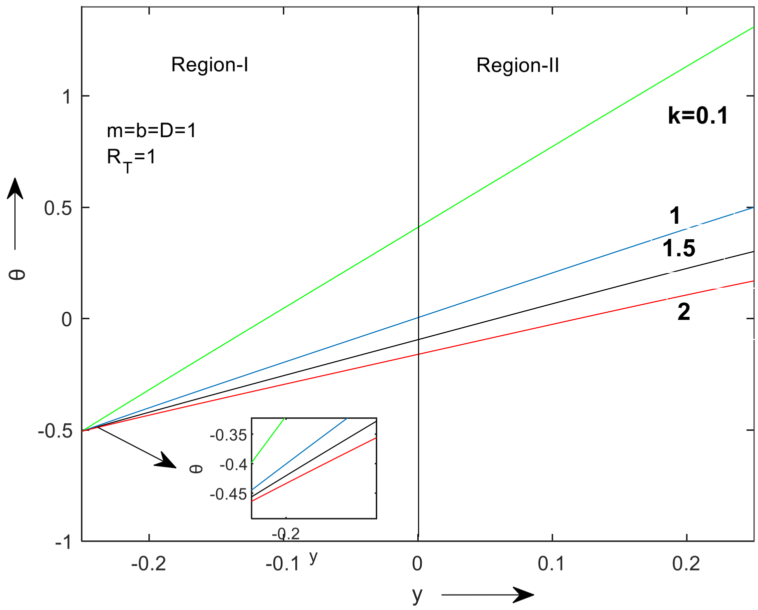

Fig. 7b. Temperature profile for different values of k

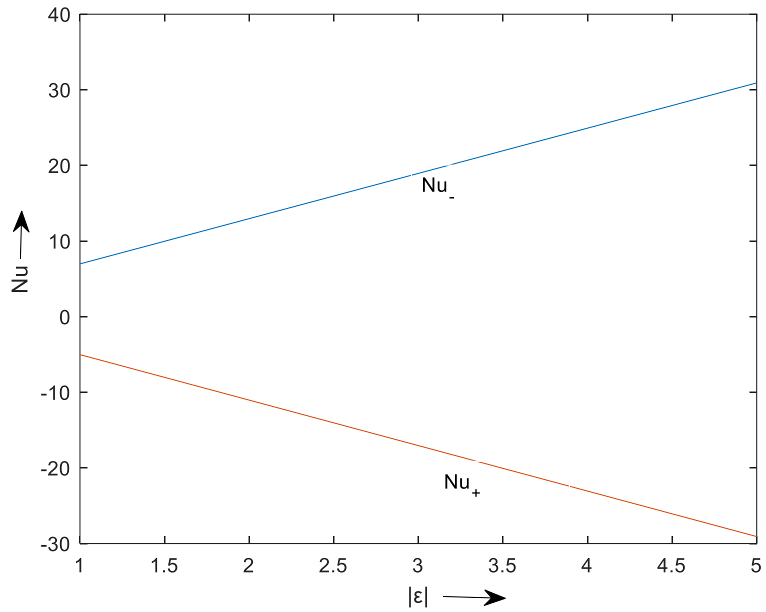

Fig. 8. Nusselt number Vs |ε|.

4. Conclusions

The problem of mixed convective flow and heat transfer in vertical channel filled with immiscible viscous fluids was analyzed analytically by regular perturbation method and represented graphically. The conclusions made are, the flow was an increasing function of perturbation parameter ε for upward flow and decreasing function of ε for downward flow, viscosity ration m and width ratio D enhance the velocity and temperature fields where as the larger the value of conductivity ratio k, smaller the fields of velocity and temperature.

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