Near mean labeling in dicyclic snakes

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Abstract

K. Palani et al. defined the concept of near mean labeling in digraphs. Let $D = (V, A)$ be a digraph where $V$the vertex is set and A is the arc set. Let $f: V \rightarrow \{0, 1, 2, \ldots, q\}$ be a 1-1 map. Define $f^*: A \rightarrow \{1, 2, \ldots, q\}$ by$f^*(e = \overrightarrow{uv}) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. Let$f^*(v) = \left|\sum_{w \in V} f^*(\overrightarrow{vw}) - \sum_{w \in V} f^*(\overrightarrow{wv})\right|$. If$f^*(v) \leq 2\ \forall\ v \in A(D)$, then f is said to be a near mean labeling of D and D is said to be a near mean digraph. In this paper, different dicyclic snakes are defined and the existence of near mean labeling in them is checked.

Keywords: Near mean labeling, digraphs, di-cyclic, di-quadrilateral, di-pentagonal, snake.

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1. Introduction

Graph theory has applications in many areas of the computing, social and natural science. The theory is also intimately related to many branches of mathematics, including matrix theory, numerical analysis, probability, topology and combinatory. Over the last 50 year graph theory has evolved into an important mathematical tool in the solution of a wide variety of problems in many areas of society. A graph labeling is an assignment of integers to the vertices or edges or both, subject to certain conditions have been motivated by practical problems, labelled graphs serve useful mathematical models for a broad range of applications such as: coding theory, including the design of good types codes, synch-set codes, missile guidance codes and convolutional codes with optimal auto correlation properties. The concept of graph labeling was introduced by Rosa in 1967 [6]. A useful survey on graph labeling by J.A. Gallian (2014) can be found in [2]. Somasundaram and Ponraj [4] have introduced the notion of mean labeling of graphs. A directed graph or digraph D consists of a finite set V of vertices and a collection of ordered pairs of distinct vertices. Any such pair (u, v) is called an arc or directed line and will usually be denoted by$\overrightarrow{uv}$. A digraph D with p vertices and q arcs is denoted by$D$ (p, q). The indegree $d^{-}(v)$ of a vertex v in a digraph D is the number of arcs having v as its terminal vertex. The outdegree $d^{+}(v)$ of v is the number of arcs having v as its initial vertex [1].

K. Palani et al. introduced the concepts of mean and near mean digraphs in [3]. In this paper, different di-cyclic snakes are introduced and the existence of near mean labeling is investigated.

2. Preliminaries

The following definition and theorem are basics which are needed for the subsequent section.

Definition 2.1: [3] Let $f: V \to \{0, 1, 2, \dots, q\}$ be a 1-1 map. Define $f^{*}: A \to \{1, 2, \dots, q\}$ by$f^{*}(e = \overrightarrow{uv}) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$. Let$f^{*}(v) = \left| \sum_{w \in V} f^{*}(\overrightarrow{vw}) - \sum_{w \in V} f^{*}(\overrightarrow{wv}) \right|$. If$f^{*}(v) \leq 2\ \forall\ v \in A(D)$, then f is said to be a near mean labeling of D and D is said to be a near mean digraph.

Definition 2.2: [5] A cyclic snake mC_n is obtained by replacing every edge of P_m byC_n.

Theorem 2.3: [3] The directed cycle $\overrightarrow{C_n}$ is a near mean digraph.

3. Main Results

In this section, the different dicyclic snakes are defined and the near mean labeling existence is verified.

Definition 3.1: In cyclic snakemC_n, orient the edges of each cycle clockwise. The resulting graph is called **Di-Cyclic Snake** and it is denoted as$\overrightarrow{mC_n}$. For$n = 3, 4, 5$, Di-Cyclic Snakes are called Di-Triangular Snake $\overrightarrow{TS_n}$, Di-Quadrilateral Snake $\overrightarrow{QS_n}$ and Di-Pentagonal Snake $\overrightarrow{PS_n}$ respectively.

Definition 3.2: In mC_n when n is even, orient the edges of the cycle alternately and call resulting graph as **Alternating Di-Cyclic Snake.** Denote it as$A\overrightarrow{mC_n}$.

Theorem 3.3: Di-Quadrilateral Snake $\overrightarrow{QS_n}$ admits near mean labeling.
Proof: Let $V\left(\overrightarrow{QS_n}\right) = \{u_i | 1 \le i \le n\} \cup \{v_i | 1 \le i \le n-1\} \cup \{w_i | 1 \le i \le n-1\}$ be the vertex set and let $A\left(\overrightarrow{QS_n}\right) = \left\{\{\overrightarrow{u_i v_i}\} \cup \{\overrightarrow{v_i u_{i+1}}\} \cup \{\overrightarrow{u_{i+1} w_i}\} \cup \{\overrightarrow{w_i u_i}\} | 1 \le i \le n-1\right\}$ be the arc set.
Define $f: V(\overrightarrow{QS_n}) \to \{0, 1, 2, \dots, (4n-4)\}$ by
$f(u_i) = 4(i-1)$ for $1 \le i \le n$
$f(v_i) = 4i - 2$ for $1 \le i \le n-1$

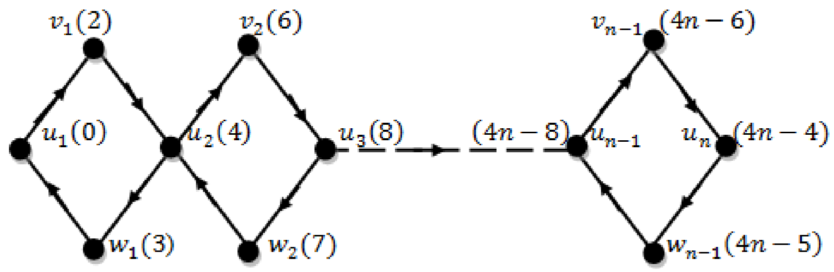

Figure 3.1: The labeling of a Di-Quadrilateral snake

For $i = 1$ to $n-1$

$$f^*(\overrightarrow{u_i v_i}) = \left\lceil \frac{f(u_i)+f(v_i)}{2} \right\rceil = \left\lceil \frac{[4(i-1)]+[4i-2]}{2} \right\rceil = \left\lceil \frac{8i-6}{2} \right\rceil = 4i - 3. \quad (3.3.1)$$

$$f^*(\overrightarrow{v_i u_{i+1}}) = \left\lceil \frac{f(v_i)+f(u_{i+1})}{2} \right\rceil = \left\lceil \frac{[4i-2]+[4(i+1-1)]}{2} \right\rceil = \left\lceil \frac{8i-2}{2} \right\rceil = 4i - 1. \quad (3.3.2)$$

$$f^*(\overrightarrow{u_{i+1} w_i}) = \left\lceil \frac{f(u_{i+1})+f(w_i)}{2} \right\rceil = \left\lceil \frac{[4(i+1-1)]+[4i-1]}{2} \right\rceil = \left\lceil \frac{8i-1}{2} \right\rceil = \frac{8i}{2} = 4i. \quad (3.3.3)$$

$$f^*(\overrightarrow{w_i u_i}) = \left\lceil \frac{f(w_i)+f(u_i)}{2} \right\rceil = \left\lceil \frac{[4i-1]+[4(i-1)]}{2} \right\rceil = \left\lceil \frac{8i-5}{2} \right\rceil = \frac{8i-4}{2} = 4i - 2. \quad (3.3.4)$$

Now $f^*(u_1) = |f^*(\overrightarrow{u_1 v_1}) - f^*(\overrightarrow{w_1 u_1})|$
$= |[4(1) - 3] - [4(1) - 2]|$ [by (3.3.1) & (3.3.4)]
$= |1 - 2| = |-1| < 2$
Therefore, $f^*(u_1) < 2$ (3.3.5)
For $i = 2$ to $n-1$
$f^*(u_i) = |[f^*(\overrightarrow{u_i v_i}) + f^*(\overrightarrow{u_i w_{i-1}})] - [f^*(\overrightarrow{w_i u_i}) + f^*(\overrightarrow{v_{i-1} u_i})]|$
$= |[4i - 3 + 4(i-1)] - [4i - 2 + 4(i-1) - 1]|$ [by (1), (3), (4) & (2)]
$= |8i - 7 - 8i + 7| = |0| < 2$
Therefore, $f^*(u_i) < 2$ for $i = 2$ to $n-1$ (3.3.6)
$f^*(u_n) = |f^*(\overrightarrow{u_n w_{n-1}}) - f^*(\overrightarrow{v_{n-1} u_n})|$
$= |4(n-1) - [4(n-1) - 1|$ [by (3.3.3) & (3.3.2)]

$= |1| < 2$

Therefore, $f^*(u_n) < 2$ (3.3.7)

For $i = 1$ to $n-1$

$f^*(v_i) = |f^*(\overrightarrow{v_i u_{i+1}}) - f^*(\overrightarrow{u_i v_i})|$

$= |(4i-1) - (4i-3)|$ [by (3.3.2) & (3.3.1)]

$= |4i - 1 - 4i + 3| = |2| = 2$

Therefore, $f^*(v_i) = 2$ for $i = 1$ to $n-1$ (3.3.8)

For $i = 1$ to $n-1$

$f^*(w_i) = |f^*(\overrightarrow{w_i u_i}) - f^*(\overrightarrow{u_{i+1} w_i})|$

$= |(4i-2) - (4i)|$ [by (3.3.4) & (3.3.3)]

$= |-2| \leq 2$

Therefore, $f^*(w_i) \leq 2$ for $i = 1$ to $n-1$ (3.3.9)

From equations (5) to (9), $f^*(u) \leq 2 \; \forall \; u \in V(\overrightarrow{QS_n})$

Hence Di-Quadrilateral Snake $\overrightarrow{QS_n}$ is a near mean digraph.

Theorem 3.4. Di-Pentagonal Snake $\overrightarrow{PS_n}$ is a near mean digraph.

Proof:

Let $V(\overrightarrow{PS_n}) = \{u_i | 1 \leq i \leq n\} \cup \{v_i | 1 \leq i \leq n-1\} \cup \{w_i | 1 \leq i \leq n-1\} \cup \{x_i | 1 \leq i \leq n-1\}$ be the vertex set and let $A(\overrightarrow{PS_n}) = \left\{\{\overrightarrow{u_i v_i}\} \cup \{\overrightarrow{v_i w_i}\} \cup \{\overrightarrow{w_i u_{i+1}}\} \cup \{\overrightarrow{u_{i+1} x_i}\} \cup \{\overrightarrow{x_i u_i}\} | 1 \leq i \leq n-1\right\}$ be the arc set.

Define $f: V(\overrightarrow{PS_n}) \to \{0, 1, 2, \ldots, (5n-5)\}$ by

$f(u_i) = 5(i-1)$ for $1 \leq i \leq n$

$f(v_i) = 5i - 3$ for $1 \leq i \leq n-1$

$f(w_i) = 5i - 1$ for $1 \leq i \leq n-1$

$f(x_i) = 5i - 2$ for $1 \leq i \leq n-1$

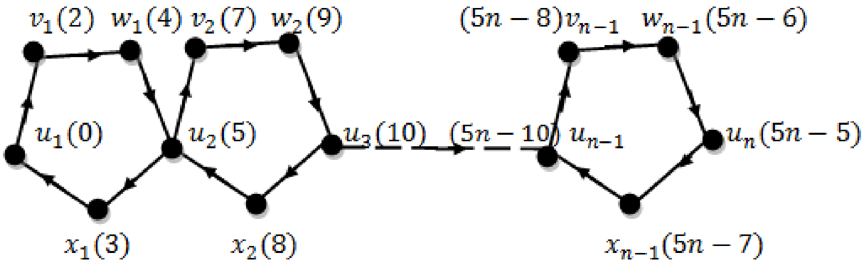

Figure 3.2. The labeling of a Di-Pentagonal snake

For $i = 1$ to $n-1$

$f^*(\overrightarrow{u_i v_i}) = \left\lceil \frac{f(u_i)+f(v_i)}{2} \right\rceil = \left\lceil \frac{[5(i-1)]+[5i-3]}{2} \right\rceil = \left\lceil \frac{10i-8}{2} \right\rceil = 5i - 4$ (3.4.1)

$f^*(\overrightarrow{v_i w_i}) = \left\lceil \frac{f(v_i)+f(w_i)}{2} \right\rceil = \left\lceil \frac{[5i-3]+[5i-1]}{2} \right\rceil = \left\lceil \frac{10i-4}{2} \right\rceil = 5i - 2$ (3.4.2)

$f^*(\overrightarrow{w_i u_{i+1}}) = \left\lceil \frac{f(w_i)+f(u_{i+1})}{2} \right\rceil = \left\lceil \frac{[5i-1]+[5(i+1-1)]}{2} \right\rceil = \left\lceil \frac{10i-1}{2} \right\rceil = \frac{10i}{2} = 5i$ (3.4.3)

$f^*(\overrightarrow{u_{i+1} x_i}) = \left\lceil \frac{f(u_{i+1})+f(x_i)}{2} \right\rceil = \left\lceil \frac{[5(i+1-1)]+[5i-2]}{2} \right\rceil = \left\lceil \frac{10i-2}{2} \right\rceil = 5i - 1$ (3.4.4)

$f^*(\overrightarrow{x_i u_i}) = \left\lceil \frac{f(x_i)+f(u_i)}{2} \right\rceil = \left\lceil \frac{[5i-2]+[5(i-1)]}{2} \right\rceil = \left\lceil \frac{10i-7}{2} \right\rceil = \frac{10i-6}{2} = 5i-3$ (3.4.5)

Next to find $f^*(u_i)$

Now $f^*(u_1) = |f^*(\overrightarrow{u_1 v_1}) - f^*(\overrightarrow{x_1 u_1})|$

$= |[5(1)-4]-[5(1)-3]|$ [by (3.4.1) & (3.4.5)]

$= |1-2| = |-1| < 2$

Therefore, $f^*(u_1) < 2$ (3.4.6)

For $i = 2$ to $n-1$

$f^*(u_i) = |[f^*(\overrightarrow{u_i v_i}) + f^*(\overrightarrow{u_i x_{i-1}})] - [f^*(\overrightarrow{x_i u_i}) + f^*(\overrightarrow{w_{i-1} u_i})]|$

$= |[5i-4+5(i-1)-1]-[5i-3+5(i-1)]|$ [by (3.4.1), (3.4.4), (3.4.5) & (3.4.3)]

$= |10i-10-10i+8| = |-2| \leq 2$

Therefore, $f^*(u_i) \leq 2$ for $i = 2$ to $n-1$ (3.4.7)

$f^*(u_n) = |f^*(\overrightarrow{u_n x_{n-1}}) - f^*(\overrightarrow{w_{n-1} u_n})|$

$= |5(n-1)-1-5(n-1)|$ [by (3.4.4) & (3.4.3)]

$= |-1| < 2$

Therefore, $f^*(u_n) < 2$ (3.4.8)

For $i = 1$ to $n-1$

$f^*(v_i) = |f^*(\overrightarrow{v_i w_i}) - f^*(\overrightarrow{u_i v_i})|$

$= |(5i-2)-(5i-4)|$ [by (3.4.2) & (3.4.1)]

$= |2| = 2$

Therefore, $f^*(v_i) = 2$ for $i = 1$ to $n-1$ (3.4.9)

For $i = 1$ to $n-1$

$f^*(w_i) = |f^*(\overrightarrow{w_i u_{i+1}}) - f^*(\overrightarrow{v_i w_i})|$

$= |(5i)-(5i-2)|$ [by (3.4.3) & (3.4.2)]

$= |2| = 2$

Therefore, $f^*(w_i) = 2$ for $i = 1$ to $n-1$ (3.4.10)

For $i = 1$ to $n-1$

$f^*(x_i) = |f^*(\overrightarrow{x_i u_i}) - f^*(\overrightarrow{u_{i+1} x_i})|$

$= |(5i-3)-(5i-1)|$ [by (3.4.3) & (3.4.2)]

$= |-2| \leq 2$

Therefore, $f^*(x_i) \leq 2$ for $i = 1$ to $n-1$ (3.4.11)

From equations (6) to (11), $f^*(u) \leq 2 \ \forall\, u \in V(\overrightarrow{PS_n})$

Hence, Di-Pentagonal Snake $\overrightarrow{PS_n}$ is a near mean digraph.

Theorem 3.5. Di-Cyclic snake $m\overrightarrow{C_n}$ is a near mean digraph for $m \geq 1$ and $n \geq 3$.

Proof: Let u_{ij} denote the jth vertex in the ith copy of $\overrightarrow{C_n}$

Here, $V(m\overrightarrow{C_n}) = \{u_{ij} | 1 \leq i \leq m, 1 \leq j \leq n\}$ and

$A(m\overrightarrow{C_n}) = \{\overrightarrow{u_{ij} u_{i(j+1)}} | 1 \leq i \leq m, 1 \leq j \leq n-1\} \cup \{\overrightarrow{u_{in} u_{i1}} | 1 \leq i \leq m\}$

Following the procedure of near mean labeling of $\overrightarrow{C_n}$ in [3], define

$f: V(m\overrightarrow{C_n}) \rightarrow \{0,1,2,\ldots,mn\}$ as below

$$f(u_{ij}) = \begin{cases} (i-1)n + 2(j-1) & \text{for} \quad 1 \le j \le \left\lfloor \frac{n}{2} \right\rfloor + 1, 1 \le i \le m \\ (i-1)n + 2\big(n-(j-1)\big) + 1 & \text{for} \quad \left\lfloor \frac{n}{2} \right\rfloor + 2 \le j \le n, 1 \le i \le m \end{cases}$$

where $\left|V(m\overrightarrow{C_n})\right| = mn - m + 1$ and $\left|A(m\overrightarrow{C_n})\right| = mn$

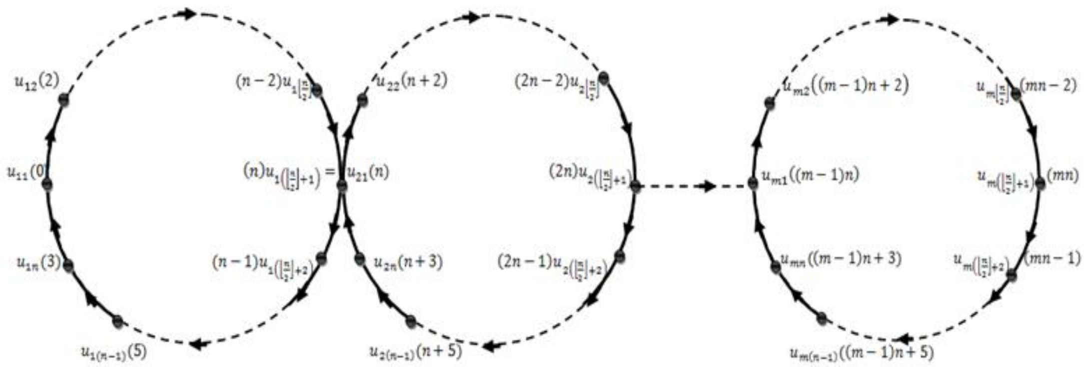

Figure 3.3: The labeling of the Di-cyclic Snake $m\overrightarrow{C_n}$ when n is even

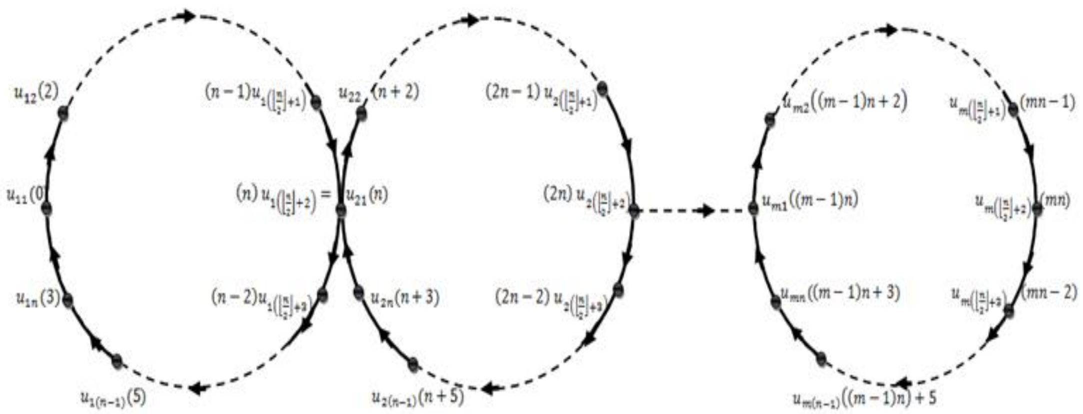

Figure 3.4: The labeling of the Di-cyclic Snake $m\overrightarrow{C_n}$ when n is odd

Now to find $f^*(\overrightarrow{u_{ij}u_{i(j+1)}})$

For $i = 1$ to $m,\ j = 1$ to $\left\lfloor \frac{n}{2} \right\rfloor$

$$f^*(\overrightarrow{u_{ij}u_{i(j+1)}}) = \left\lceil \frac{f(u_{ij}) + f(u_{i(j+1)})}{2} \right\rceil$$

$$= \left\lceil \frac{[(i-1)n+2(j-1)]+[(i-1)n+2((j+1)-1)]}{2} \right\rceil$$

$$= \left\lceil \frac{2(i-1)n+4j-2}{2} \right\rceil = (i-1)n + 2j - 1 = ni + 2j - n - 1.$$

Therefore, $f^*(\overrightarrow{u_{ij}u_{i(j+1)}}) = ni + 2j - n - 1$ for $i = 1$ to $m,\ j = 1$ to $\left\lfloor \frac{n}{2} \right\rfloor$ (3.5.1)

$$f^*\left(\overrightarrow{u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}}\right) = \left\lceil\frac{f\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}\right)+f\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}\right)}{2}\right\rceil$$

$$= \left\lceil\frac{\left[(i-1)n+2\left(\left\lfloor\frac{n}{2}\right\rfloor+1-1\right)\right]+\left[(i-1)n+2\left(n-\left(\left\lfloor\frac{n}{2}\right\rfloor+2-1\right)\right)+1\right]}{2}\right\rceil$$

$$= \left\lceil\frac{2(i-1)n+2n-1}{2}\right\rceil = \left\lceil\frac{2(i-1+1)n-1}{2}\right\rceil = \left\lceil\frac{2in-1}{2}\right\rceil = \frac{2in}{2} = ni$$

Therefore, $f^*\left(\overrightarrow{u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}}\right) = ni$ for $i = 1$ to m and $n \geq 3$ (3.5.2)

For $i = 1$ to m, $j = \left\lfloor\frac{n}{2}\right\rfloor + 2$ to $n-1$

$$f^*\left(\overrightarrow{u_{ij}u_{i(j+1)}}\right) = \left\lceil\frac{f(u_{ij})+f(u_{i(j+1)})}{2}\right\rceil$$

$$= \left\lceil\frac{[(i-1)n+2(n-(j-1))+1]+[(i-1)n+2(n-((j+1)-1))+1]}{2}\right\rceil$$

$$= \left\lceil\frac{2(i-1)n+4n-4j+4}{2}\right\rceil$$

$$= (i+1)n-2j+2 = ni-2j+n+2$$

$f^*\left(\overrightarrow{u_{ij}u_{i(j+1)}}\right) = ni-2j+n+2$ for $i = 1$ to m, $j = \left\lfloor\frac{n}{2}\right\rfloor + 2$ to $n-1$ (3.5.3)

$$f^*\left(\overrightarrow{u_{in}u_{i1}}\right) = \left\lceil\frac{f(u_{in})+f(u_{i1})}{2}\right\rceil$$

$$= \left\lceil\frac{[(i-1)n+2(n-(n-1))+1]+[(i-1)n+2(1-1)]}{2}\right\rceil$$

$$= \left\lceil\frac{2(i-1)n+2(n-n+1)+1}{2}\right\rceil = \left\lceil\frac{2(i-1)n+3}{2}\right\rceil = \frac{2(i-1)n+4}{2}$$

$$= (i-1)n+2 = ni-n+2.$$

Therefore, $f^*\left(\overrightarrow{u_{in}u_{i1}}\right) = ni-n+2$ for $i = 1$ to m and $n \geq 3$ (3.5.4)

Next to find $f^*(u_{ij})$

$$f^*(u_{11}) = \left|f^*\left(\overrightarrow{u_{11}u_{12}}\right) - f^*\left(\overrightarrow{u_{1n}u_{11}}\right)\right|$$

$= |[n(1)+2(1)-n-1]-[n(1)-n+2]|$ [by (3.5.1) & (3.5.4)]

$$= |1-2| = |-1| < 2$$

Therefore, $f^*(u_{11}) < 2$ (3.5.5)

For $i = 1$ to m, $j = 2$ to $\left\lfloor\frac{n}{2}\right\rfloor$

$$f^*(u_{ij}) = \left|f^*\left(\overrightarrow{u_{ij}u_{i(j+1)}}\right) - f^*\left(\overrightarrow{u_{i(j-1)}u_{ij}}\right)\right|$$

$= |[ni+2j-n-1]-[ni+2(j-1)-n-1]|$ [by (3.5.1)]

$$= |-1+3| = |2| = 2$$

Therefore, $f^*(u_{ij}) = 2$ for $i = 1$ to m, $j = 2$ to $\left\lfloor\frac{n}{2}\right\rfloor$ (3.5.6)

For $i = 1$ to $m-1$, two cases for $f^*(u_{ij})$ when $j = \left\lfloor\frac{n}{2}\right\rfloor + 1$ & $\left\lfloor\frac{n}{2}\right\rfloor + 2$

Case (i): n is odd.

$$f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}\right) = \left|f^*\left(\overrightarrow{u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}}\right) - f^*\left(\overrightarrow{u_{i\left\lfloor\frac{n}{2}\right\rfloor}u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}}\right)\right|$$

$= \left|ni-\left(ni+2\left\lfloor\frac{n}{2}\right\rfloor-n-1\right)\right|$ [by (3.5.2) & (3.5.1)]

$$= \left| n - 2\left\lfloor \frac{n}{2} \right\rfloor + 1 \right|$$
$$= \left| n - 2\left(\frac{n-1}{2}\right) + 1 \right| = |2| = 2$$

$f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}\right) = 2$ when n is odd (3.5.7)

$$f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}\right) = \left|\left(f^*\left(\overrightarrow{u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+3\right)}}\right) + f^*\left(\overrightarrow{u_{(i+1)1}u_{(i+1)2}}\right)\right)\right.$$
$$\left. - \left(f^*\left(\overrightarrow{u_{i\left\lfloor\frac{n}{2}\right\rfloor+1}u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}}\right) + f^*\left(\overrightarrow{u_{(i+1)n}u_{(i+1)1}}\right)\right)\right|$$
$$= \left|\left[\left(ni - 2\left(\left\lfloor\frac{n}{2}\right\rfloor + 2\right) + n + 2\right) + n(i+1) + 2 - n - 1\right] - [ni + n(i+1) - n + 2]\right|$$

[by (3.5.3), (3.5.1), (3.5.2) & (3.5.4)]

$$= \left|\left[\left(ni - 2\left(\left\lfloor\frac{n}{2}\right\rfloor\right) - 4 + n + 2\right) + (ni + n + 2 - n - 1)\right] - [ni + ni + 2]\right|$$
$$= \left|\left[2ni - 2\left(\frac{n-1}{2}\right) + n - 1\right] - [2ni + 2]\right|$$
$$= |-2| \leq 2$$

$f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}\right) \leq 2$ when n is odd. (3.5.8)

Case (ii): n is even.

$$f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}\right) = \left|\left[f^*\left(\overrightarrow{u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}}\right) + f^*\left(\overrightarrow{u_{(i+1)1}u_{(i+1)2}}\right)\right]\right.$$
$$\left. - \left[f^*\left(\overrightarrow{u_{i\left\lfloor\frac{n}{2}\right\rfloor}u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}}\right) + f^*\left(\overrightarrow{u_{(i+1)n}u_{(i+1)1}}\right)\right]\right|$$
$$= \left|[ni + (n(i+1) + 2(1) - n - 1)] - \left[\left(ni + 2\left\lfloor\frac{n}{2}\right\rfloor - n - 1\right) + (n(i+1) - n + 2)\right]\right|$$

[by (3.5.2), (3.5.1) & (3.5.4)]

$$= \left|[ni + ni + n + 2 - n - 1] - \left[ni + 2\left(\frac{n}{2}\right) - n - 1 + ni + 2\right]\right|$$
$$= |0| < 2$$

$f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}\right) < 2$ when n is even (3.5.9)

$$f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}\right) = \left|f^*\left(\overrightarrow{u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+3\right)}}\right) - f^*\left(\overrightarrow{u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}}\right)\right|$$
$$= \left|\left(ni - 2\left(\left\lfloor\frac{n}{2}\right\rfloor + 2\right) + n + 2\right) - ni\right| \quad \text{[by (3) \& (4)]}$$
$$= \left|n - 2\left\lfloor\frac{n}{2}\right\rfloor - 4 + 2\right|$$
$$= \left|n - 2\left(\frac{n}{2}\right) - 2\right|$$
$$= |-2| \leq 2$$

$f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}\right) \leq 2$ when n is even (3.5.10)

Cases (i) and (ii) imply $f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}\right) \leq 2$ for $i = 1$ to $m - 1$ (3.5.11)

and $f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}\right) \leq 2$ for $i = 1$ to $m - 1$ (3.5.12)

Now to find $f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}\right)$ and $f^*\left(u_{i\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}\right)$ for $i = m$

$$f^*\left(u_{m\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}\right) = \left|f^*\left(\overrightarrow{u_{m\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}u_{m\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}}\right) - f^*\left(\overrightarrow{u_{m\left\lfloor\frac{n}{2}\right\rfloor}u_{m\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}}\right)\right|$$
$$= \left|nm - \left(nm + 2\left\lfloor\frac{n}{2}\right\rfloor - n - 1\right)\right| \quad \text{[by (2) \& (1)]}$$
$$= \left|n - 2\left\lfloor\frac{n}{2}\right\rfloor + 1\right|$$
$$= \begin{cases} \left|n - 2\left(\frac{n-1}{2}\right) + 1\right| & \text{if } n \text{ is odd} \\ \left|n - 2\left(\frac{n}{2}\right) + 1\right| & \text{if } n \text{ is even} \end{cases}$$
$$= \begin{cases} |2| & \text{if } n \text{ is odd} \\ |1| & \text{if } n \text{ is even} \end{cases}$$

Therefore, $f^*\left(u_{m\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}\right) \leq 2$ (3.5.13)

$$f^*\left(u_{m\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}\right) = \left|f^*\left(\overrightarrow{u_{m\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}u_{m\left(\left\lfloor\frac{n}{2}\right\rfloor+3\right)}}\right) - f^*\left(\overrightarrow{u_{m\left(\left\lfloor\frac{n}{2}\right\rfloor+1\right)}u_{m\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}}\right)\right|$$
$$= \left|\left(nm - 2\left(\left\lfloor\frac{n}{2}\right\rfloor + 2\right) + n + 2\right) - nm\right| \quad \text{[by (3.5.3) \& (3.5.2)]}$$
$$= \left|n - 2\left\lfloor\frac{n}{2}\right\rfloor - 2\right|$$
$$= \begin{cases} \left|n - 2\left(\frac{n-1}{2}\right) - 2\right| & \text{if } n \text{ is odd} \\ \left|n - 2\left(\frac{n}{2}\right) - 2\right| & \text{if } n \text{ is even} \end{cases}$$
$$= \begin{cases} |-1| & \text{if } n \text{ is odd} \\ |-2| & \text{if } n \text{ is even} \end{cases}$$

Therefore, $f^*\left(u_{m\left(\left\lfloor\frac{n}{2}\right\rfloor+2\right)}\right) \leq 2$ (3.5.14)

For $i = 1$ to $m,\ j = \left\lfloor\frac{n}{2}\right\rfloor + 3$ to $n - 1$

$$f^*(u_{ij}) = \left|f^*(\overrightarrow{u_{ij}u_{i(j+1)}}) - f^*(\overrightarrow{u_{i(j-1)}u_{ij}})\right|$$
$$= \left|[ni - 2j + n + 2] - [ni - 2(j-1) + n + 2]\right| \quad \text{[by (3)]}$$
$$= |-2|$$

Therefore, $f^*(u_{ij}) \leq 2$ for $i = 1$ to $m,\ j = \left\lfloor\frac{n}{2}\right\rfloor + 3$ to $n - 1$ (3.5.15)

For $i = 1$ to m and $j = n$

$$f^*(u_{in}) = \left|f^*(\overrightarrow{u_{in}u_{i1}}) - f^*(\overrightarrow{u_{i(n-1)}u_{in}})\right|$$
$$= \left|[ni - n + 2] - [ni - 2(n-1) + n + 2]\right| \quad \text{[by (4)]}$$
$$= |ni - n + 2 - ni + 2n - 2 - n - 2| = |-2|$$

Therefore, $f^*(u_{in}) \leq 2$ (3.5.16)

Equations (3.5.5), (3.5.6) and (3.5.11) to (3.5.16) imply $f^*(u_{ij}) \leq 2$ for$1 \leq i \leq m, 1 \leq j \leq n$.

Thus, Di-Cyclic snake $m\overrightarrow{C_n}$ is a near mean digraph for $m \geq 1$ and$n \geq 3$.

Theorem 3.6. Alternating Di-Cyclic snakes are non near mean digraphs.

Proof: In an alternating Di-Cyclic snake, either $d^+(u_{21}) = 0$ and $d^-(u_{21}) = 4$ (or) $d^+(u_{21}) = 4$ and $d^-(u_{21}) = 0$.
Therefore, Corresponding to every $f: V \to \{0, 1, 2, \dots, q\}$, $\sum_{w\in V} f^*(\overrightarrow{u_{21}w}) = 0$ and $\sum_{w\in V} f^*(\overrightarrow{wu_{21}})$ is a sum of at least three positive integers (or) $\sum_{w\in V} f^*(\overrightarrow{wu_{21}}) = 0$ and $\sum_{w\in V} f^*(\overrightarrow{u_{21}w})$ is a sum of at least three positive integers.
Therefore, $f^*(u_{21}) > 2$.
Therefore, No function $f: V \to \{0, 1, 2, \dots, q\}$ is a near mean labeling.
Thus, an alternating di-cyclic snake is a non near mean digraph.

4. Conclusions

In this article, different dicyclic snakes are introduced. Also, existence of near mean labeling is verified to dicyclic snakes and its generalisation. Most of the labeling are proved only for graphs. In this way, we develop the concept of labeling into digraphs

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