# Changing and Unchanging strong efficient edge domination number of some standard graphs when a vertex is removed or an edge is added

M. Annapoopathi[1]
N. Meena[2]

**Abstract**

Let $G = (V, E)$ be a simple graph. A subset S of E(G) is a strong (weak) efficient edge dominating set of G if $|N_s[e] \cap S| = 1$ for all $e \in E(G)$($|N_w[e] \cap S| = 1$ for all $e \in E(G)$) where $N_s(e) = \{f \,/\, f \in E(G)$, f is adjacent to e & deg f $\geq$ deg e$\}$($N_w(e) = \{f \,/\, f \in E(G)$, f is adjacent to e & deg f $\leq$ deg e$\}$) and $N_s[e]=N_s(e) \cup \{e\}$($N_w[e] = N_w(e) \cup \{e\}$). The minimum cardinality of a strong efficient edge dominating set of G (weak efficient edge dominating set of G) is called a strong efficient edge domination number of G and is denoted by $\gamma'_{se}(G)$ ($\gamma'_{we}(G)$).When a vertex is removed or an edge is added to the graph, the strong efficient edge domination number may or may not be changed. In this paper the change or unchanged of the strong efficient edge domination number of some standard graphs are determined, when a vertex is removed or an edge is added.

**Keywords:** Domination, edge domination, strong edge domination, efficient edge domination, strong efficient edge domination.

**AMS subject classification:** 05C69[3]

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[1] Reg.No:17231072092002, Research Scholar, Department of Mathematics, P.G. & Research Department of Mathematics, The M.D.T. Hindu College, Tirunelveli. (Affiliated to Manonmaniam Sundaranar University, Tirunelveli – 627 012, Tamil Nadu, India) and Assistant Professor, National Engineering College, Kovilpatti - 628503, Tamil Nadu, India. Email: annapoopathi.nec@gmail.com

[2] Assistant Professor, P.G. & Research Department of Mathematics, the M.D.T. Hindu College, Tirunelveli. (Affiliated to Manonmaniam Sundaranar University, Tirunelveli – 627 012, Tamil Nadu, India). Email: meena@mdthinducollege.org

[3] Received on July 10, 2022. Accepted on October 15, 2022. Published on January 30, 2023. doi: 10.23755/rm.v45i0.1024. ISSN: 1592-7415. eISSN: 2282-8214. 

# 1. Introduction

It is meant by the graph that it is a finite, undirected graph without loops and multiple edges. The concept of domination in graphs was introduced by Ore. Two volumes on domination have been published by T. W. Haynes, S. T. Hedetniemi and P. J. Slater [4, 5]. Let G be a graph with vertex set V and edge set E. A subset D of V (G) is called a strong dominating set of G if every vertex in V- D is strongly dominated by at least one vertex in D. Similarly, a set D is a subset of V (G) is called a weak dominating set of G if every vertex in V- D is weakly dominated by at least one vertex in D. The strong (weak) domination number $\gamma_s(G)$ $(\gamma_w(G))$ respectively of G is the minimum cardinality of a strong (weak) dominating set of G. A subset D of V (G) is called an efficient dominating set of G if for every vertex u ∈ V (G), $|N[u] \cap D| = 1$ [1, 2]. Edge dominating sets were also studied by Mitchell and Hedetniemi [6, 7]. A subset F of edges in a graph G = (V, E) is called an edge dominating set of G if every edge in E-F is adjacent to at least one edge in F. The edge domination number $\gamma'(G)$ of a graph G is the smallest cardinality among all minimum edge dominating sets of G. The degree of an edge uv is defined to be deg u + deg v - 2. An edge uv is called an isolated edge if deg uv = 0. A subset F of E is called an efficient edge dominating set if every edge in E is either in F or dominated by exactly one edge in F. The cardinality of minimum efficient edge dominating set is called the edge domination number of G. Motivated by these definitions; strong efficient edge domination in graphs is defined as follows. A subset S of E(G) is a strong (weak) efficient edge dominating set of G if $|N_s[e] \cap S| = 1$ for all $e \in E(G)$ [ $|N_w[e] \cap S| = 1$ for all $e \in E(G)$] where $N_s(e) = \{ f/f \in E(G) \;\&\; deg\, f \geq \deg\, e\}$ ( $N_w(e) = \{f/f \in E(G) \;\&\; deg\, f \leq \deg\, e\}$ ) and $N_s[e] = N_s(e) \cup \{e\}$ $(N_w[e] = N_w(e) \cup \{e\})$.The minimum cardinality of a strong efficient edge dominating set of G (weak efficient edge dominating set of G) is called as a strong efficient edge domination number of G (weak efficient edge domination number of G ) and also denoted by $\gamma'_{se}(G)$ $(\gamma'_{we}(G))$.

**Definition 1.1.** Let $G = (V, E)$ be a simple graph. Let $E(G) = \{e_1, e_2, e_3, e_4, \dots \dots \dots \dots . e_n \}$. An edge $e_i$ is said to be the full degree edge if and only if deg $e_i$ = n-1.

**Observation 1.2.** $\gamma'_{se}(G) = 1$ if and only if G has a full degree edge.

**Observation 1.3.** $\gamma'_{se}\left(K_{1,n}\right) = 1,\ n \geq 1$ and $\gamma'_{se}\left(D_{r,s}\right) = 1,\ r, s \geq 1$

**Theorem 1.4.** For any path $P_m$ , $\gamma'_{se}( P_m ) = \begin{cases} n, & if\ m = 3n+1, n \geq 1 \\ n+1, & if\ m = 3n, n \geq 2 \\ n+1, & if\ m = 3n+2, n \geq 1 \end{cases}$

**Theorem 1.5.** $\gamma'_{se}(C_{3n}) = n,\ \forall n \in N$

**Theorem 1.6.** Let $W_m$ be a wheel graph. Then $W_m$ has a strong efficient edge dominating set if and only if m = 3n, n ≥ 2 and $\gamma'_{se}(W_{3n}) = n$, n ≥ 2.

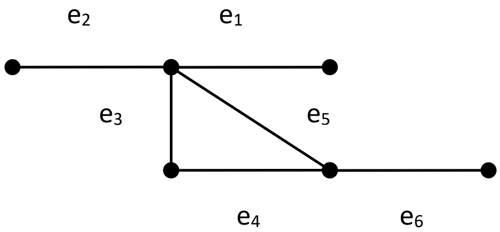
# 2. Main Results

**Definition 2.1.** $(E'_{se})^0(G) = \{e \in E(G) / (\gamma'_{se})(G+e) = (\gamma'_{se})(G)\}$

$(E'_{se})^+(G) = \{e \in E(G) / (\gamma'_{se})(G+e) > (\gamma'_{se})(G)\}$ ,

$(E'_{se})^-(G) = \{e \in E(G) / (\gamma'_{se})(G+e) < (\gamma'_{se})(G)\}$

**Example 2.2.** Consider the following graph



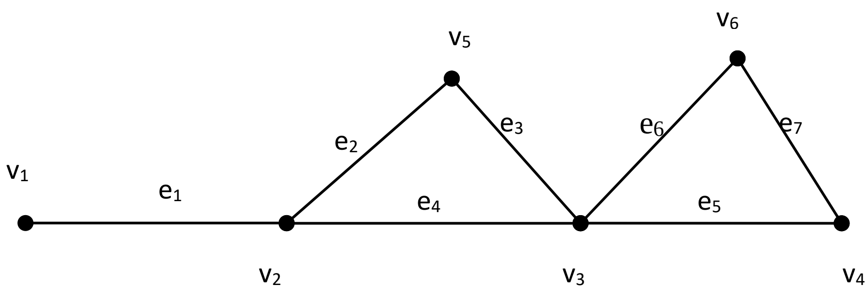

Since $e_5$ is the full degree edge of G, $\gamma'_{se}(G)$ =1. $\{e_3, e_6\}$ is the unique strong efficient edge dominating set of G – $e_5$. Therefore $(\gamma'_{se})(G-e_5) = 2 > (\gamma'_{se})(G)$ and $e_5$ is the full degree edge of G – $e_3$ and $\gamma'_{se}(G-e_3) = 1 = \gamma'_{se}(G)$. Hence $\gamma'_{se}(G) = (\gamma'_{se})(G-e_3)$.

**Definition 2.3.** $(V'_{se})^0(G) = \{v \in V(G) / \gamma'_{se}(G-v) = \gamma'_{se}(G)\}$

$(V'_{se})^+(G) = \{v \in V(G) / \gamma'_{se}(G-v) > \gamma'_{se}(G)\}$ ,

$(V'_{se})^+(G) = \{v \in V(G) / \gamma'_{se}(G-v) < \gamma'_{se}(G)\}$

**Example 2.4.** Consider the following graph




S= $\{e_4, e_7\}$ is the strong efficient edge dominating set of G and $\gamma'_{se}(G)$ =2. $\gamma'_{se}(G-v_i) = \gamma'_{se}(G), i=1,3$ and $\gamma'_{se}(G-v_i) < \gamma'_{se}(G), i \neq 1,3$

**Theorem 2.5.** Let G = $P_{3n}$, $n \geq 1$. Then $(V_{se}^{'})^{+}(G) = \phi$

**Proof: Case (1):** Let G = $P_{3n}$, $n \geq 1$. Let v be the end vertex of G. Then G – v = $P_{3n-1}$. $\gamma_{se}^{'}(P_{3n-1}) = \gamma_{se}^{'}(P_{3(n-1)+2}) = n-1+1 = n$ and $\gamma_{se}^{'}(G) = n+1$. Therefore $\gamma_{se}^{'}(G-v) < \gamma_{se}^{'}(G)$. Hence $v_i \notin (V_{se}^{'})^{+}(G)$.

**Case (2):** Let $v = v_{3k}, 1 \leq k \leq n-1$. Thus G – v = $P_{3k-1} \cup P_{3n-3k}$ and $\gamma_{se}^{'}(P_{3k-1}) = k$, $\gamma_{se}^{'}(P_{3n-3k}) = \gamma_{se}^{'}(P_{3(n-k)}) = n-k+1$. Therefore $\gamma_{se}^{'}(G-v) = \gamma_{se}^{'}(P_{3k-1}) + \gamma_{se}^{'}(P_{3n-3k})$ = $k+n-k+1 = n+1 = \gamma_{se}^{'}(G)$. Hence $v \in (V_{se}^{'})^{0}(G)$.

**Case (3):** Let $v = v_{3k+1}, 1 \leq k \leq n-1$. Thus G – v = $P_{3k} \cup P_{3n-3k-1}$ and $\gamma_{se}^{'}(P_{3k}) = k+1$, $\gamma_{se}^{'}(P_{3n-3k-1}) = \gamma_{se}^{'}(P_{3(n-k-1)+2}) = n-k$. Therefore $\gamma_{se}^{'}(G-v) = \gamma_{se}^{'}(P_{3k}) + \gamma_{se}^{'}(P_{3n-3k-1})$ = $k+1+n-k = n+1 = \gamma_{se}^{'}(G)$. Hence $v \in (V_{se}^{'})^{0}(G)$.

**Case (4):** Let $v = v_{3k+2}, 1 \leq k \leq n-2$. Thus G – v = $P_{3k+1} \cup P_{3n-3k-2}$ and $\gamma_{se}^{'}(P_{3k+1}) = k$, $\gamma_{se}^{'}(P_{3n-3k-2}) = \gamma_{se}^{'}(P_{3(n-k-1)+1}) = n-k-1$. Therefore $\gamma_{se}^{'}(G-v) = \gamma_{se}^{'}(P_{3k+1}) + \gamma_{se}^{'}(P_{3n-3k-2})$ $= k+n-k-1 = n-1 < \gamma_{se}^{'}(G)$. Hence $v \in (V_{se}^{'})^{-}(G)$.

**Case (5):** when v = $v_2$ or $v_{3n-1}$. Thus G-v = $P_{3n-2} \cup P_1$ having no strong efficient dominating set. From the above given the cases it is identified that, $(V_{se}^{'})^{+}(G) = \phi$

**Theorem 2.6.** Let G = $P_{3n+1}$, $n \geq 1$. Then $(V_{se}^{'})^{-}(G) = \phi$

**Proof: Case (1):** Let G = $P_{3n+1}$, $n \geq 1$. Let v be the end vertex of G. Then G – v = $P_{3n}$. $\gamma_{se}^{'}(G-v) = n+1$ but $\gamma_{se}^{'}(G) = n$. Therefore $\gamma_{se}^{'}(G-v) > \gamma_{se}^{'}(G)$. Hence $v \in (V_{se}^{'})^{+}(G)$

**Case (2):** Let $v = v_{3k}, 1 \leq k \leq n-1$. Thus G – v = $P_{3k-1} \cup P_{3n+1-3k}$. Therefore $\gamma_{se}^{'}(G-v) = \gamma_{se}^{'}(P_{3k-1}) + \gamma_{se}^{'}(P_{3n+1-3k})$ = $\gamma_{se}^{'}(P_{3(k-1)+2}) + \gamma_{se}^{'}(P_{3(n-k)+1})$ = $k+n-k = n = \gamma_{se}^{'}(G)$. Hence $v \in (V_{se}^{'})^{0}(G)$.

**Case (3):** Let $v = v_{3k+1}, 1 \leq k \leq n-1$. Thus G – v = $P_{3k} \cup P_{3n-3k}$. Therefore $\gamma_{se}^{'}(G-v) = \gamma_{se}^{'}(P_{3k}) + \gamma_{se}^{'}(P_{3(n-k)})$ = $k+1+n-k+1 = n+2 > n = \gamma_{se}^{'}(G)$. Hence $v \in (V_{se}^{'})^{+}(G)$.

**Case (4):** Let $v = v_{3k+2}, 1 \leq k \leq n-2$. Thus G – v = $P_{3k+1} \cup P_{3n-3k-1}$ = $P_{3k+1} \cup P_{3(n-k-1)+2}$ and, Therefore $\gamma_{se}^{'}(G-v) = \gamma_{se}^{'}(P_{3k+1}) + \gamma_{se}^{'}(P_{3(n-k-1)+2})$ $= k+n-k-1+1 = n$ $= \gamma_{se}^{'}(G)$. Hence $v \in (V_{se}^{'})^{0}(G)$.

**Case (5):** when v = $v_2$ or $v_{3n}$. G-v = $P_{3n-1} \cup P_1$ which has no strong efficient dominating set. From the above all the cases, $(V_{se}^{'})^{-}(G) = \phi$

**Theorem 2.7.** Let G = $P_{3n+2}$, $n \geq 1$. Then $(V_{se}^{'})^{-}(G) = \phi$

**Proof: Case (1):** Let G = $P_{3n+2}$, $n \geq 1$. Let v be the end vertex of G. Then G – v = $P_{3n+1}$. $\gamma'_{se}(G-v) = \gamma'_{se}(P_{3n+1}) = n$ but $\gamma'_{se}(G) = n+1$. Therefore $\gamma'_{se}(G-v) < \gamma'_{se}(G)$. Hence $v \notin (V'_{se})^{-}(G)$

**Case (2):** Let $v = v_{3k}, 1 \leq k \leq n-1$. Thus G – v = $P_{3k-1} \cup P_{3n+2-3k}$. Therefore $\gamma'_{se}(G-v) = \gamma'_{se}(P_{3k-1}) + \gamma'_{se}(P_{3n+2-3k}) = \gamma'_{se}(P_{3(k-1)+2}) + \gamma'_{se}(P_{3(n-k)+2}) = k+n-k+1 = n+1 = \gamma'_{se}(G)$. Hence $v \in (V'_{se})^{0}(G)$.

**Case (3):** Let $v = v_{3k+1}, 1 \leq k \leq n-1$. Thus G – v = $P_{3k} \cup P_{3n-3k+1}$. Therefore $\gamma'_{se}(G-v) = \gamma'_{se}(P_{3k}) + \gamma'_{se}(P_{3(n-k)+1}) = k+1+n-k = n+1 = \gamma'_{se}(G)$. Hence $v \in (V'_{se})^{0}(G)$.

**Case (4):** Let $v = v_{3k+2}, 1 \leq k \leq n-2$. Thus G – v = $P_{3k+2} \cup P_{3n-3k-1} = P_{3k+2} \cup P_{3(n-k-1)+2}$ and, Therefore $\gamma'_{se}(G-v) = \gamma'_{se}(P_{3k+2}) + \gamma'_{se}(P_{3(n-k-1)+2}) = k+2+n-k-1 = n+1 = \gamma'_{se}(G)$. Hence $v \in (V'_{se})^{0}(G)$.

**Case (5):** when v = $v_2$ or $v_{3n+1}$. G-v = $P_{3n-2} \cup P_1$ which has no strong efficient dominating set. From the above all the cases, $(V'_{se})^{-}(G) = \phi$

**Theorem 2.8.** Let $G = C_{3n}, n \geq 1$. Then $(V'_{se})^{0}(G) = V(G)$.

**Proof:**

Let $G = C_{3n}, n \geq 1$. Let $v \in V(G)$. Then $\gamma'_{se}(G)$ =n, G – v = $P_{3n-1}$ and $\gamma'_{se}(P_{3n-1}) = \gamma'_{se}(P_{3(n-1)+2}) = n$ Therefore $\gamma'_{se}(G-v) = \gamma'_{se}(G)$. Hence $(V'_{se})^{0}(G) = V(G)$.

**Theorem 2.9.** Let $G = K_{1,n}, n \geq 2$. Then $(V'_{se})^{0}(G) = V(G)$.

**Proof:**

Let $G = K_{1,n}, n \geq 2$. Let $v \in V(G)$. Then $\gamma'_{se}(G)$=1, G – v = $K_{1,n-1}$ and $\gamma'_{se}(K_{1,n-1}) = 1$. Therefore $\gamma'_{se}(G-v) = \gamma'_{se}(G)$. Hence $(V'_{se})^{0}(G) = V(G)$.

**Theorem 2.10.** Let $G = D_{r,s}$, $r,s \geq 1$. Then $|(V'_{se})^{0}(G)| = r+s-2$.

**Proof:**

Let $G = D_{r,s}$, $r,s \geq 1$. Let V (G) = $\{u, v, u_i, v_j / 1 \leq i \leq r, 1 \leq j \leq s\}$, $E(G) = \{uv, uu_i, vv_j / 1 \leq i \leq r, 1 \leq j \leq s\}$. Let v = $u_i$ or $v_j$. Then $\gamma'_{se}(G)$=1, G – v = $D_{r-1,s}$ = $D_{r,s-1}$ and $\gamma'_{se}(D_{r-1,s}) = \gamma'_{se}(D_{r,s-1}) = 1$. Therefore $\gamma'_{se}(G-v) = \gamma'_{se}(G)$. Hence $(V'_{se})^{0}(G) = V(G) - \{u, v\}$. Therefore $|(V'_{se})^{0}(G)| = r+s-2$

**Theorem 2.11.** Let $G = W_{3n}$, $n \geq 2$. Then $v \in (V'_{se})^{0}(G)$ if $v \in (K_1)$ and $v \in (V'_{se})^{-}(G)$ if $v \in V(C_{3n-1})$

**Proof:**

Let $G = W_{3n}, n \geq 2$. Let V(G) = $\{v, v_i / 1 \leq i \leq 3n\}$, $E(G) = \{vv_j, v_i v_{i+1}, v_{3n} v_1 / 1 \leq j \leq 3n, 1 \leq i \leq 3n-1\}$

**Case (1):** G – v = $C_{3n}$. Therefore $\gamma'_{se}(G - v) = \gamma'_{se}(C_{3n}) = n$. Hence $v \in (V'_{se})^0(G)$.

**Case (2):** Let v = $v_i$, $1 \leq i \leq 3n$. G – v = $F_{3n-1}$. Therefore $\gamma'_{se}(G - v) = \gamma'_{se}(F_{3n-1}) = \gamma'_{se}(F_{3(n-1)+2}) = n$. but $\gamma'_{se}(G) = 2n$. Hence $\gamma'_{se}(G - v) < \gamma'_{se}(G)$. Therefore $v \in (V'_{se})^-(G)$

**Theorem 2.12.**

Let G = $P_{3n}$, $n \geq 2$. Let e = uv be any edge incident with any vertex of G and G' = G+e. Then $\gamma'_{se}(G + e) = \gamma'_{se}(G) - 1$ if e is incident with $u_1$ or $u_{3i}$, $1 \leq i \leq 3n-2$ and $\gamma'_{se}(G + e)$ has no strong efficient edge dominating set if e is incident with $u_2$, $u_{3n-1}$, $u_{3i+2}$, $1 \leq i \leq 3n-2$

**Proof:**

Let G = $P_{3n}$, $n \geq 2$. V(G) $= \{u_i / 1 \leq i \leq 3n\}$, $E(G) = \{e_i = u_i u_{i+1} / 1 \leq i \leq 3n-1\}$. Let e = uv be the new edge incident with any vertex of G and G' = G+e.

**Case 1:** Let e be an end edge of G'. Then G' = $P_{3n+1}$. Therefore $\gamma'_{se}(G') = \gamma'_{se}(P_{3n+1}) = n$ but $\gamma'_{se}(G) = n+1$. Therefore $\gamma'_{se}(G + e) < \gamma'_{se}(G)$. Hence $e \in (E'_{se})^-(G)$.

**Case 2:** Let the edge e be incident with the vertex $u_2$. Let S be a strong efficient edge dominating set of G'. Suppose $n \geq 2$. Among all the edges, the edge $e_2$ have maximum degree. It must belong to S. It strongly efficiently dominates e, $e_3$, $e_1$. Also the edges $e_5$, $e_8$, $e_{11}$, …, $e_{3n-4}$ belong to S. If the edge $e_{3n-2}$ belongs to S, then $|N_S[e_{3n-3}] \cap S| = |\{e_{3n-4}, e_{3n-2}\}| = 2 > 1$, a contradiction. Hence G' has no strong efficient edge dominating set. The proof is similar if the edge e is added at the vertex $u_{3n-1}$.

**Case 3:** Let the edge e be incident with the vertex $u_3$. $e_2$ and $e_3$ are the only maximum degree edges. Hence any strong efficient edge dominating set contains either $e_2$ or $e_3$. Then $S_1 = \{e_1, e_3, e_6, …., e_{3n-3}, e_{3n-1}\}$, $S_2 = \{e_2, e_4, e_7, …. e_{3n-2}\}$ are the strong efficient edge dominating sets of G'. Therefore $|S_1|$=n+1, $|S_2|$=n. Hence $\gamma'_{se}(G') = n < \gamma'_{se}(G)$. Therefore $e \in (E'_{se})^-(G)$. The proof is similar if the edge e is incident with the vertex $u_{3i}$, $2 \leq i \leq n-1$

**Case 4:** Let the edge e be incident with the vertex $u_4$. Then S = $\{e_2, e_4, e_7, …. e_{3n-2}\}$ is the unique strong efficient edge dominating set of G' and $\gamma'_{se}(G') = n$. Therefore $\gamma'_{se}(G + e) < \gamma'_{se}(G)$. Hence $e \in (E'_{se})^-(G)$. The proof is similar if the edge e is incident with the vertex $u_{3i+1}$, $2 \leq i \leq n-1$

**Case 5:** Let the edge e be incident with the vertex $u_5$. Let S be a strong efficient edge dominating set of G'. The edge $e_4$ & $e_5$ are the only maximum degree edges. If the edge $e_4$ belongs to S then no edge in S to strongly efficiently dominate $e_2$. If the edge $e_5$ belongs to S then $e_2$, $e_8$, $e_{11}$, …., $e_{3n-4}$ belongs to S and there is no edge in S to strongly efficiently dominate $e_{3n-2}$. Hence strong efficient edge dominating set does not exists. Proof is similar if the edge e is incident with the vertex $u_{3i+2}, 2 \le i \le n-2$. From all the above cases, $\left(E_{se}^{'}\right)^0(G) = \phi$

Remark: Let n = 1, G'=$K_{1,3}$. $\gamma_{se}^{'}(G') = 1 = \gamma_{se}^{'}(G)$. Therefore $e \in \left(E_{se}^{'}\right)^0(G)$

**Theorem 2.13.**
Let G = $P_{3n+1}$, $n \ge 1$. Let e = uv be any edge incident with any vertex of G and G' = G+e. Then $\gamma_{se}^{'}(G+e) = \gamma_{se}^{'}(G)$ if e is incident with $u_2$ or $u_{3i}, 1 \le i \le n-1$, $u_{3j+2}, 1 \le j \le n-2$ and $\gamma_{se}^{'}(G+e) = \gamma_{se}^{'}(G)+1$ if e is incident with $u_1$, $u_{3n}$, $u_{3i+1}, 1 \le i \le n-1$.

**Proof:**
Let G = $P_{3n+1}$, $n \ge 1$. V(G) $=\{u_i / 1 \le i \le 3n+1\}$, $E(G) = \{e_i = u_i u_{i+1} / 1 \le i \le 3n\}$. Let e = uv be the any edge incident with any vertex of G and G' = G+e.

**Case 1:** Let e be an end edge of G'. Then G' = $P_{3n+2}$. Therefore $\gamma_{se}^{'}(G') = \gamma_{se}^{'}(P_{3n+2}) = n+1$ but $\gamma_{se}^{'}(G) = n.$ Therefore $\gamma_{se}^{'}(G+e) > \gamma_{se}^{'}(G)$. Hence $e \in (E_{se}^{'})^{-}(G)$. Therefore $\gamma_{se}^{'}(G+e) = \gamma_{se}^{'}(G)+1$

**Case 2:** Let the edge e be incident with the vertex $u_2$. Suppose $n \ge 1$. S = {$e_2$, $e_5$, $e_8$, …. $e_{3n-1}$} is the unique strong efficient edge dominating sets of G' and |S| = n. Therefore $\gamma_{se}^{'}(G') = \gamma_{se}^{'}(G) = n$ Hence $e \in (E_{se}^{'})^0(G)$. The proof is similar if the edge e is incident with the vertex $u_{3n}$.

**Case 3:** Let the edge e be incident with the vertex $u_3$. $e_2$ and $e_3$ are the only maximum degree edges. Hence any strong efficient edge dominating set contains either $e_2$ or $e_3$ .If $e_2$ belongs to S then S = {$e_2$, $e_5$, $e_8$, …., $e_{3n-3}$, $e_{3n-1}$} is the unique strong efficient edge dominating sets of G'and |S| =n. Hence $\gamma_{se}^{'}(G') = n = \gamma_{se}^{'}(G)$. Therefore $(E_{se}^{'})^{+}(G) = \phi$. If the edge $e_3$ belongs to S then there is no edge in S to strongly efficiently dominate $e_3$. The proof is similar if the edge e is incident with the vertex $u_{3i}, 2 \le i \le n-1$.

**Case 4:** Let the edge e be incident with the vertex $u_4$. Then $S_1$ = {$e_1$, $e_3$, $e_5$, …. $e_{3n-1}$} , $S_2$ = {$e_2$, $e_4$, $e_7$, …. $e_{3n-2}$, $e_{3n}$} are the strong efficient edge dominating sets of G' and $|S_1|=|S_2|$=n+1. Therefore $\gamma_{se}^{'}(G') = n+1$ but $\gamma_{se}^{'}(G) = n.$ Therefore $\gamma_{se}^{'}(G+e) > \gamma_{se}^{'}(G)$. Hence $e \in (E_{se}^{'})^{+}(G)$. The proof is similar if the edge e is incident with the vertex $u_{3i+1}, 2 \le i \le n-1$

**Case 5:** Let the edge e be incident with the vertex $u_5$. Let S be a strong efficient edge dominating set of G'. The edge $e_4$ & $e_5$ are the only maximum degree edges. If the edge $e_4$ belongs to S then no edge in S to strongly efficiently dominate $e_2$. If the edge $e_5$ belongs to S then $e_2$, $e_8$, $e_{11}$, …., $e_{3n-4}$ belongs to S and there is no edge in S to strongly efficiently dominate $e_{3n-2}$. Hence strong efficient edge dominating set does not exists.

Proof is similar if the edge e is incident with the vertex $u_{3i+2}$, $2 \le i \le n-2$. From all the above cases, $\left(E'_{se}\right)^0(G) = \phi$

**Theorem 2.14.**
Let G = $P_{3n+2}$ $n \ge 1$. Let e = uv be any edge incident with any vertex of G and G' = G+e. Then $\gamma'_{se}(G+e) = \gamma'_{se}(G)$ if e is incident with all $u_i / 1 \le i \le 3n+2$ except $u_1$, $u_{3n}$ and $\gamma'_{se}(G+e) = \gamma'_{se}(G)+1$ if e is incident with $u_1$, $u_{3n}$.

**Proof:**
Let G = $P_{3n+2}$, $n \ge 1$. V(G) $= \{u_i / 1 \le i \le 3n+2\}$, $E(G) = \{e_i = u_i u_{i+1} / 1 \le i \le 3n+1\}$. Let e = uv be the any edge incident with any vertex of G and G' = G+e.

**Case 1:** Let e be an end edge of G'. Then G' = $P_{3n+3}$=$P_{3(n+1)}$. Therefore $\gamma'_{se}(G') = \gamma'_{se}(P_{3(n+1)}) = n+2$ but $\gamma'_{se}(G) = n+1$. Therefore $\gamma'_{se}(G+e) > \gamma'_{se}(G)$. Hence $e \in (E'_{se})^+(G)$. Therefore $\gamma'_{se}(G+e) = \gamma'_{se}(G)+1$

**Case 2:** Let the edge e be incident with the vertex $u_2$. Suppose $n \ge 1$. S = {$e_2$, $e_5$, $e_8$, …. $e_{3n-1}$, $e_{3n+1}$} is the unique strong efficient edge dominating sets of G' and |S| = n+1. Therefore $\gamma'_{se}(G') = \gamma'_{se}(G) = n+1$. Hence $e \in (E'_{se})^0(G)$. The proof is similar if the edge e is incident with the vertex $u_{3n}$.

**Case 3:** Let the edge e be incident with the vertex $u_3$. $e_2$ and $e_3$ are the only maximum degree edges. Hence any strong efficient edge dominating set contains either $e_2$ or $e_3$ .If $e_2$ belongs to S then S = {$e_2$, $e_5$, $e_8$, …., $e_{3n-3}$, $e_{3n-1}$, $e_{3n+1}$} is the unique strong efficient edge dominating set of G' and |S| = n+1. Hence $\gamma'_{se}(G') = n+1 = \gamma'_{se}(G)$ . Therefore $e \in (E'_{se})^0(G)$. If $e_3$ belongs to S then S = {$e_1$, $e_3$, $e_6$, ….,$e_{3n}$ } is the unique strong efficient edge dominating set of G' and |S| = n+1. Hence $\gamma'_{se}(G') = n+1 = \gamma'_{se}(G)$ . Therefore $e \in (E'_{se})^0(G)$. The proof is similar if the edge e is incident with the vertex $u_{3i}$, $2 \le i \le n-1$.

**Case 4:** Let the edge e be incident with the vertex $u_4$. The edge $e_3$ & $e_4$ are the only maximum degree edges. Hence any strong efficient edge dominating set S contains either $e_3$ or $e_4$. If the edge $e_3$ belongs to S then S = {$e_1$, $e_3$, $e_6$, ….,$e_{3n}$ } is the unique strong efficient edge dominating set of G' and |S| = n+1. Hence $\gamma'_{se}(G') = n+1 = \gamma'_{se}(G)$ . Therefore $e \in (E'_{se})^0(G)$. If the edge $e_4$ belongs to S then there is no edge in S to strongly efficiently dominate $e_{3n}$. Hence strong efficient edge dominating set does not exist. The proof is similar if the edge e is incident with the vertex $u_{3i+1}$, $2 \le i \le n-1$

**Case5:** Let the edge e be incident with the vertex $u_5$. The edge $e_4$ & $e_5$ are the only maximum degree edges. Hence any strong efficient edge dominating set S contains either $e_4$ or $e_5$. If the edge $e_4$ belongs to S then there is no edge in S to strongly efficiently dominate $e_2$. If the edge $e_5$ belongs to S then {$e_2$, $e_5$, $e_8$, …., $e_{3n-3}$, $e_{3n-1}$, $e_{3n+1}$}} is the unique strong efficient edge dominating set of G' and |S| = n+1. Hence

$\gamma'_{se}(G') = n+1 = \gamma'_{se}(G)$ . Therefore $e \in (E'_{se})^0(G)$. Proof is similar if the edge e is incident with the vertex $u_{3i+2}$, $2 \le i \le n-2$.

**Theorem 2.15.**
Let G = $C_{3n}$, $n \ge 1$. Let e = uv be any edge incident with any vertex of G and G' = G+e. Then $\gamma'_{se}(G) = \gamma'_{se}(G+e)$.

**Proof:** Let G = $C_{3n}$, $n \ge 1$. Let e = uv be the new edge. V(G') $= \{u_i, u / 1 \le i \le 3n\}$, $E(G') = \{e_i = u_i u_{i+1} / 1 \le i \le 3n-1, e_{3n} = u_{3n}u_1, e = u_i v\}$ and G' = G + e. Let the edge e be incident with the vertex $u_1$. $e_1$ and $e_{3n}$ are the maximum degree edges and they are adjacent. Any strong efficient dominating set contains $e_1$ or $e_{3n}$. Then $S_1 = \{e_1, e_4, e_7, \ldots. e_{3n-2}\}$, $S_2 = \{e_3, e_6, e_9, \ldots. e_{3n-3}, e_{3n}\}$ are the strong efficient edge dominating sets of G' and $|S_1|=|S_2|=$ n, $n \ge 1$. $\gamma'_{se}(G') = n, n \ge 1$. No other strong efficient edge dominating set exists without $e_1$ and $e_{3n}$. Therefore $\gamma'_{se}(G) = \gamma'_{se}(G+e) = n, n \ge 1$. The proof is similar if the edge e is with any $u_i$, $2 \le i \le 3n$

**Theorem 2.16.**
Let G = $K_{1,n}$, $n \ge 1$. Let e be any edge incident with any vertex of G and G' = G+e. Then $\gamma'_{se}(G) = \gamma'_{se}(G+e)$.

**Proof:** Let G = $K_{1,n}$, $n \ge 1$. Let V(G) = $\{u_i, u / 1 \le i \le n\}$, E(G) = $\{uu_i / 1 \le i \le n\}$. V(G') $= \{u_i, u, v / 1 \le i \le n\}$.

**Case 1:** Let e be the new edge incident with u. Then G' = $K_{1,n+1}$. Therefore $\gamma'_{se}(G') = \gamma'_{se}(G) = 1$ **Case2:** Let $u_iv$ be the new edge incident with $u_i$. Then $\{uu_i\}$ is the unique strong efficient edge dominating set of G' and $\gamma'_{se}(G') = 1$. Therefore $\gamma'_{se}(G) = \gamma'_{se}(G+e) = 1$.

**Theorem 2.17.**
Let G = $D_{r,s}$, $r, s \ge 1$. Let e = xy be any edge incident with any vertex of G and G' = G+e. Then $\gamma'_{se}(G') = \begin{cases} \gamma'_{se}(G), & if \quad e = wu \ or \ wv \\ \gamma'_{se}(G)+1, & if \quad e = u_i w \ or \ v_j w \end{cases}$.

**Proof:** G = $D_{r,s}$, $r, s \ge 1$. Let e = xy be the new edge. V(G) = $\{u_i, v_j, u, v / 1 \le i \le r, 1 \le j \le s\}$, V(G') = $\{u_i, v_j, u, v, x, y / 1 \le i \le r, 1 \le j \le s\}$, $E(G') = \{e_i = uu_i, f = uv, f_j = vv_j, e = xy / 1 \le i \le r, 1 \le j \le s\}$. Then G' = G+e.

**Case 1:** If the edge e is incident with either the vertex u or the vertex v. Then G'= $D_{r+1,s}$ or G'= $D_{r,s+1}$ and $\gamma'_{se}(G') = \gamma'_{se}(G) = 1$

**Case 2:** Let the edge e be incident with the vertex $u_i$, $1 \le i \le r$ or $v_j$, $1 \le j \le s$. Then S = {uv, $u_i$w} or S = {uv, $v_j$w} is the strong efficient edge dominating set of G' and |S|=2. $\gamma'_{se}(G') = 2$. $\gamma'_{se}(G) = \gamma'_{se}(G+e) = 2$.

# 3. Conclusions

In this paper, the change or unchanged of the strong efficient edge domination number of some standard graphs are determined, when a vertex is removed or an edge is added.

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