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IFG$^{\#}\alpha$-CS in intuitionistic fuzzy topological spaces

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Abstract

The primary aim of this prospectus is to introduce and study the basic properties of Intuitionistic fuzzy generalized $\#\alpha$-closed sets, Intuitionistic fuzzy generalized $^{\#}\alpha$-open sets. Here we, compare the g $^{\#}\alpha$- closed sets with the existing closed sets with proper examples given.

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1 Introduction

Earlier g alpha closed sets has been introduced in general topology, fuzzy topology, supra topology, nano topology, we wish to introduce g $^{\#}\alpha$ in intuitionistic fuzzy topological spaces. Following authors motivated me to further continue my research in intuitionistic fuzzy topology. A. Zadeh A.Zadeh [1965] initiated the concept of fuzzy sets in which the values are taken between 0 and 1. Further Atanassov [1986] established the idea of IFS by generalizing fuzzy sets, here similar to fuzzy topology values are taken between 0 and 1 but it is defined as membership value and non-membership value. Later on IFTS was initiated using the notion of IFSs which was proposed by Coker Coker [1997], using the membership and non-membership values it was applied in general topological axioms. In continuation of above we initiate IFG $^{\#}\alpha$ -closed sets and IFG alpha-open sets and establish its characterization and find the weaker and stronger forms of topology by comparing it to other existing sets and also find whether it satisfies the topological axioms as union and finite intersection properties

2 Preliminaries

In this segment, few basic definitions and results are reviewed.

Definition 2.1. *Thakur and Chaturvedi [2008]*Let A be an IFS in (X,τ), is proposed to be IFGCS if cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is an IFOS in X.

Similarly, IFαGCS and IFGαCS Kalamani et al. [2012], , IFGSPCS Santhi and Jayanthi [2010] , IFGSCS Santhi and Sakthivel [2009], IFGSRCS Anitha and Mohana [2018], were introduced. With the help of above closed sets we, initiate new set IFG$^{\#}\alpha$- closed set.

3 IFG$^{\#}\alpha$- closed sets

Definition 3.1. *An IFS C in (X, τ) is proposed to be an IFG$^{\#}\alpha$-closed set if $\alpha cl(C) \subseteq C$, whenever $C \subseteq U$ and U is an IFGOS in (E, τ). The family of all IFG$^{\#}\alpha$CS of an IFTS (E, τ) is defined by IFG$^{\#}\alpha$C(X).*

Example 3.1. *Consider E = {p, q}, τ = {$0_\sim$, J, $1_\sim$} is IFT on E, in that $J =< e, (0.3, 0.2), (0.5, 0.6) >$. In this the only α-open sets are $0_\sim$, $1_\sim$, J. At that point IFS, $C =< e, (0.1, 0), (0.6, 0.8) >$ an IFG$^{\#}\alpha$CS in (E, τ).*

Theorem 3.1. *Every IFCS is IFG$^{\#}\alpha$CS, but reverse implication is not possible.*

Proof. ConsiderC is IFCS in (E, τ). Suppose an IFS, $C \subseteq U$ where U is IFGOS. Considering $\alpha cl(C) \subseteq cl(C)$ and C is an IFCS in E, $\alpha cl(C) \subseteq cl(C) = C \subseteq U$ and U is IFGOS. That is $\alpha cl(C) \subseteq U$. Consequently C is IFG$^{\#}\alpha$CS in E. □

Example 3.2. *Let E = {p, q} and let τ = {$0_{\sim}$, J, $1_{\sim}$} is an IFT on E, where $J =< e, (0.4, 0.1), (0.5, 0.6) >$. Let $C =< e, (0.3, 0.1), (0.7, 0.9) >$ be an IFS in E. Here C is an IFG$^{\#}\alpha$CS but not IFCS in (E, τ).*

Theorem 3.2. *Every IFαCS is IFG$^{\#}\alpha$CS but, reverse implication is not possible.*

Proof. Consider C is a $IF\alpha CS$ in E. Suppose an IFS, $C \subseteq U$, where U is IFGOS. Considering C is IFαCS, α cl(C) = C. Hence $\alpha cl(C) \subseteq U$ once $C \subseteq U$, U is IFGOS. Consequently IFG$^{\#}\alpha$CS in E. □

Example 3.3. *Let E = {p, q} and let τ = {$0_{\sim}$, J, $1_{\sim}$} be IFT, we have $J =< e, (0.2, 0.4), (0.6, 0.5) >$. Consider $C =< e, (0.2, 0.4), (0.6, 0.5) >$ be an IFS on E. Then C is IFG$^{\#}\alpha$CS but not IFα CS in (E, τ).*

Theorem 3.3. *Every IFRCS is IFG$^{\#}\alpha$CS but, reverse implication is not possible.*

Proof. Consider C is an IFRCS. We know that C=cl(int(C)) using definition. This signifies cl(C)=cl(int(C)). Consequently cl(C)=C. By which C is IFCS in E. C is an IFG$^{\#}\alpha$CS in E. □

Example 3.4. *Consider E = {p, q}, τ = {$0_{\sim}$, J, $1_{\sim}$} be IFT, we have $J =< e, (0.4, 0.4), (0.5, 0.5) >$. Here an IFS,$C =< e, (0.2, 0.2), (0.7, 0.8) >$ is an IFG$^{\#}\alpha$CS but not IFRCS in (E, τ).*

Theorem 3.4. *Every IFG$^{\#}\alpha$CS is IFSGCS but, reverse implication is not possible.*

Proof. Consider C an IFG$^{\#}\alpha$CS. Suppose an IFS, $C \subseteq U$ where U is IFSO set. Since, every IFSO set is IFGO set and C be an IFG$^{\#}\alpha$CS. We have $scl(C) \subseteq \alpha cl(C) \subseteq U$. Therefore C is IFSGCS. □

Example 3.5. *Let E = {p, q} τ = {$0_{\sim}$, J, $1_{\sim}$} be IFT we have $J =< e, (0.5, 0.7), (0.6, 0.7) >$. Let $C =< e, (0.5, 0.6), (0.8, 0.9) >$. Then C is an IFSGCS but not IFG$^{\#}\alpha$CS in (E, τ).*

Theorem 3.5. *Every IFG$^{\#}\alpha$CS is IFGSCS but, reverse implication is not possible.*

Proof. Consider C an IFG$^{\#}\alpha$CS in (E, τ). Suppose an IFS, $C \subseteq U$ where U is IFOS. Since, every IFOS set is an IFGOS and A be an IFG$^{\#}\alpha$CS. We have $scl(C) \subseteq \alpha cl(C) \subseteq U$. Therefore, C is IFGSCS set.

□

Example 3.6. *Let E = $\{p, q\}$, τ = $\{0_\sim$, J, $1_\sim\}$ be IFT, we have $J =< e, (0.1, 0.2), (0.6, 0.6) >$. Let, $C =< e, (0, 0.2), (0.9, 0.6) >$. Here C is an IFGSCS but not an IFG$^{\#}\alpha$CS.*

Theorem 3.6. *Every IFG$^{\#}\alpha$CS is IFGSRCS in (E, τ) but, reverse implication is not possible.*

Proof. Consider C an IFG$^{\#}\alpha$CS in (E, τ). Suppose an IFS, $C \subseteq U$ where U is an IFROS. Since, every IFROS set is an IFGSROS and C be an IFG$^{\#}\alpha$CS. We have $scl(C) \subseteq \alpha cl(C) \subseteq U$. Therefore, C is IFGSRCS set. □

Example 3.7. *Let E = $\{p, q\}$, τ = $\{0_\sim$, J, $1_\sim\}$, here $J =< e, (0.3, 0.6), (0.7, 0.4) >$. Consider, $C =< e, (0.3, 0.4), (0.7, 0.6) >$. Here C is an IFGSRCS but not an IFG$^{\#}\alpha$CS.*

Remark 3.1. *For any two IFG$^{\#}\alpha$CS intersection is also IFG$^{\#}\alpha$CS.*

Proof. Consider C and D any two IFG$^{\#}\alpha$CS. That is I$\alpha cl(C) \subseteq G$. Once $C \subseteq G$ and G is IFGOS and is I$\alpha cl(D) \subseteq G$ whenever $D \subseteq G$ and G is IFGOS.
Now, I$\alpha cl(C \subseteq D) = I\alpha cl(C) \cap I\alpha cl(D) \subseteq G$, where $(C \cap D) \subseteq G$ and G is IFGOS. Thus, intersection of any two IFG$^{\#}\alpha$- closed set is IFG$^{\#}\alpha$CS. □

Theorem 3.7. *Let (E, τ) be IFTS. Then for every C $\subseteq$ IFG$^{\#}\alpha$CS(E) and for every D $\in$ IFS(E), C $\subseteq$ D $\subseteq$ αcl(C) implies D $\in$ IFG$^{\#}\alpha$CS(E).*

Proof. Consider IFS $D \subseteq U$ and U be $\mathcal{IFGOS}$, considering $C \subseteq D$, $C \subseteq U$ and C is IFG$^{\#}\alpha$CS, $\alpha cl(C) \subseteq U$. By assumption, $D \subseteq \alpha cl(C)$, α cl(D) $\subseteq$ α cl(C) $\subseteq$ U. Consequently $\alpha cl(D) \subseteq U$. Thus C is IFG$^{\#}\alpha$CS of E. □

Theorem 3.8. *Consider E an IFTS. Then IFGO(E) = IFGC(E) if and only if every IFS in E an IFG$^{\#}\alpha$CS in E.*

Proof. ***Necessity*:** Assume IFGO(E) = IFGC(E). Consider $C \subseteq G$, G an IFGOS. This signifies $\alpha cl(C) \subseteq \alpha cl(G)$. Considering G an IFGOS in E, by assumption G is IFGCS in E, $\alpha cl(C) \subseteq G$. This signifies $\alpha cl(C) \subseteq G$. Consequently C is IFG$^{\#}\alpha$CS in E.
***Sufficiency*:** Assume, every IFS is IFG$^{\#}\alpha$CS. Consider G$\in$ IFO(E), we have G$\in$ IFGO(E) and so $C \subseteq G$ also G is IFOS in E, by assumption $\alpha cl(C) \subseteq G$. That is G$\in$ IFGC(E). Accordingly IFGO(E)$\subseteq$ IFGCS(E). Consider C$\in$ IFGC(E), we have C^c is IFGOS in E. But IFGO(E)$\subseteq$ IFGC(E). Consequently $C^c \in$ IFGC(E). Here C$\in$ IFGO(E). Consequently IFGC(E)$\subseteq$ IFGO(E). We know that IFGO(E) $\subseteq$ IFGC(E). □

Theorem 3.9. *Let C be IFG$^{\#}\alpha$CS of E, then αcl(C)-C contains no non-empty IFGCS.*

Proof. Suppose C is IFG$^{\#}\alpha$CS of E and consider F to be non-empty IFGCS of E, and so F $\subseteq$ αcl(C)-C. We have A $\subseteq$ E - F. Considering C is IFG$^{\#}\alpha$CS and E-F is IFGOS, and so $\alpha cl(C) \subseteq E - F$. This signifies $F \subseteq E - \alpha cl(C)$. Also $F \subseteq (E - \alpha cl(C)) \cap (\alpha cl(C) - C) \subseteq (E - \alpha cl(C)) \cap \alpha cl(C) = \phi$. Consequently F is empty. □

Theorem 3.10. *Let $C \subseteq D \subseteq E$ and assume that C is IFG$^{\#}\alpha$CS in E then C is an IFG$^{\#}\alpha$CS relative to D.*

Proof. Here we have, $C \subseteq D \subseteq E$ also C an IFG$^{\#}\alpha$CS. Considering $C \subseteq D \cap F$ where F is IFGOS in E. Since C is an IFG$^{\#}\alpha$CS in E, $C \subseteq F$ implies, $\alpha cl(C) \subseteq F$. It follows that $D \cap \alpha cl(C) \subseteq D \cap F = F$. Thus C is an IFG$^{\#}\alpha$CS relative to D. □

4 IFG$^{\#}\alpha$-open sets

In this segment, we define and establish the idea of IFG$^{\#}\alpha$ –open sets (briefly IFG$^{\#}\alpha$OS) in IFTS and establish its characterizations.

Definition 4.1. A subset B of IFTS E is proposed to be an IFG$^{\#}\alpha$-open if A^c is IFG$^{\#}\alpha$-open set.

Theorem 4.1. *Consider E an IFTS we have,*

(i) . Every IF-open set is IFG$^{\#}\alpha$OS.

(ii) . Every IF_{α}-open set is IFG$^{\#}\alpha$OS.

(iii) . Every IFR-open set is IFG$^{\#}\alpha$OS.

Proof. Proof is obvious □

Theorem 4.2. *Let (E, τ)be the IFTS then,*

(i) *Every IFG$^{\#}\alpha$-open set is IFSGOS.*

(ii) *Every IFG$^{\#}\alpha$-open set is IFGSPOS.*

(iii) *Every IFG$^{\#}\alpha$-open set is IFGSOS.*

(iv) *Every IFG$^{\#}\alpha$-open set is IFGSROS.*

Proof. Proof is obvious □

Theorem 4.3. *An IFS C of IFTS E is IFG$^{\#}\alpha$OS on the condition that $D \subseteq \alpha int(C)$ at any moment D is IFGCS in E also $D \subseteq F$.*

Proof. **Necessity:** Let C is IFG$^{\#}\alpha$OS in E. Consider D be IFGCS in E also $D \subseteq C$. We have D^c is IFGOS in E in this extent $C^c \subseteq D^c$. Considering C^c is IFG$^{\#}\alpha$CS, then $\alpha cl(C^c) \subseteq D^c$. So $(\alpha cl(C))^c \subseteq D^c$. Consequently $D \subseteq \alpha cl(C)$.
Sufficiency: Consider $D \subseteq \alpha int(C)$ at any moment D is IFGCS also $D \subseteq C$. We have $C^c \subseteq D^c$ also D^c an IFGOS. By assumption, $(\alpha cl(C))^c \subseteq D^c$.Therefore C^c is IFG$^{\#}\alpha$CS of E. Consequently C is IFG$^{\#}\alpha$OS. □

Theorem 4.4. *Consider E an IFTS. We have for all C $\subseteq$ IFG$^{\#}\alpha$OS also probably D $\in$ IFS(E), $\alpha int(C) \subseteq D \subseteq C$ signifies D$\in$ IFG$^{\#}\alpha$OS.*

Proof. Here $\alpha int(C) \subseteq D \subseteq C$ implies $C^c \subseteq D^c \subseteq (\alpha int(C))^c$. Consider $D^c \subseteq F$ also F is IFGOS in E. Considering $C^c \subseteq D^c$, $C^c \subseteq F$. Since C^c is IFG$^{\#}\alpha$CS, $\alpha cl(C^c) \subseteq F$ and so $D^c \subseteq (\alpha int(C))^c = \alpha cl(C^c)$. Consequently $\alpha cl(D^c) \subseteq \alpha cl(C^c) \subseteq F$. Accordingly D^c is IFG$^{\#}\alpha$CS in $\mathcal{E}$. This signifies D is IFG$^{\#}\alpha$OS in E. Therefore D $\in$ IFG$^{\#}\alpha$CS. □

5 Conclusions

Here we have derived a new concept of closed set called IFG $^{\#}\alpha$ CS. We have proved that IFG$^{\#}\alpha$CS is stronger than IFCS, IFCS, IFRCS. Also, IFGSCS, IFSGCS and IFGSRCS is stronger than IFG$^{\#}\alpha$CS. Further, it satisfies the intersection axiom but it does not satisfy union property. Thus we can conclude that IFG$^{\#}\alpha$CS does not form a topology. This concept can further be extended to continuous functions, irresolute functions, various forms of continuous functions such as completely continuous, perfectly continuous, contra continuous, almost continuous, slightly continuous and spaces can be introduced which are normal space and regular space, also connectedness can be described. Application can be done based on membership and non-membership values and find the MCDM problems.

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