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BIBO stability and decomposition analysis of signals and system with convolution techniques

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Abstract

In this paper control system's stability is arrived based on Bounded Input Bounded Output (BIBO) when bounded input is given in the form of discrete values. The control system allows the state estimation constraints to reach the convergence even when fluctuations in the parameters of the input system occur. To overcome this DTFT (Discrete Time Fourier Transform) is used when the signal is completely absolutely summable. Stability of the LTI (Linear time invariant) system is showed and is depending on the absolute summable of their impulse response. Simultaneously for continuous signal the stability occurs if it is absolutely integrable. In addition to that the linearity and time-invariance properties are discussed. This provide a new way to decompose the periodic signals into Fourier series by convolving the fundamental signals. Continuous and discrete time signals are focused in this paper to get linear time invariant system (LTI) through complex exponentials. Finally filtering techniques were used to eliminate the noisy frequency component in a signal.
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1 Introduction

Difference equations are described the evolution of applied mathematics phenomenon of latest technology from artificial intelligence to IoT. Higher order difference equations and time invariant systems are focusing more real world problems in the diverse fields like communication technology Srinivasan et al. [2022]. When we classify these difference equations, each category of classification are solving specific engineering and technological problems R.P.Agarwal [2000], Thamvichai and T.Bose [2002]. Various methodologies are used to resolve error reduction and quality enhancement techniques. In such cases analytical solutions are possible to address particular specified problems Kayar and KaymakçalanBistritz [2022].

To analyze these difference equations finite difference methods can be utilized. In particular when the errors are magnified, then the difference equations will be unstable and for smaller components these equation guarantees the stability Camouzis and Ladas [2008], Liu1 et al. [2022]. When the larger components are involved, tri-diagonal system (based on Crank-Nicolson Method) will be compared with other explicit and implicit methods with higher derivatives Camouziss and G.Ladas [2010], Liu [2008].

Time Delay is one of the reasons for instability which appears in dynamical systems such as biological systems, chemical systems, communication systems, nuclear systems, electrical systems, etc., and it is one of the key performances in these systems Bose [1995], Kaczorek [2011], Oppenheim et al. [2009]. As the signal is impulsive, it goes to infinity at any time and hence, the system is unstable even when an input is bounded but an output is infinite.

Bounded Signal is a signal which is having a finite value at all instants of time. In general a signal is bounded if it has finite value $M > 0$, and the signal does not exceed M, i.e. $|y(n)| \leq M, \forall n \in Z$ for discrete-time signals.

This paper is structured as follows: Section II provides the fundamental concepts of LTI systems in time as well as frequency domain. The systems represents through linear time invariant difference equations are discussed in section III. Section IV dealt with the development of Fourier series in difference equations with time dependent variable. Section V depicts the Filtering Techniques Related to LTI systems that change the shape of the input signal. Finally section VI concludes the paper.

2 LTI systems

2.1 Time domain conditions

Theorem 2.1.1 (Sufficient condition) : The BIBO stability of discrete time LTI system and its impulse response is absolutely summable i.e. $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$.
Proof :Consider the following By convolution,

$$y[n] = \sum_{n=-\infty}^{\infty} h[k]x[n-k] \tag{1}$$

Then

$$|y[n]| = |\sum_{n=-\infty}^{\infty} h[n-k]x[k]| \tag{2}$$

Applying triangular inequality

$$|y[n]| = ||x||_{\infty} \sum_{n=-\infty}^{\infty} |h[k]| \tag{3}$$

Therefore $h[n]$ is absolutely summable.

Theorem 2.1.2 (sufficient condition of continuous time) : The BIBO stability of continuous time LTI is $\int_{-\infty}^{\infty} |h(t)|dt < \infty$.

Proof : Consider the output function

$$|y(t)| = |\int_{-\infty}^{\infty} x(t-T)h(T)dT|, \tag{4}$$

$$\leq \int_{-\infty}^{\infty} M|h(T)|dT$$

$$= M\int_{-\infty}^{\infty} |h(T)|dT \tag{5}$$

Hence the proof.

2.2 LTI systems - frequency domain conditions

Discrete Time signal

In general, for BIBO stability a unit circle in the Z-plane must contain all the poles of a system. The condition for stability can be obtained by the above time domain condition

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=-\infty}^{\infty} |h[n]||e^{-jwn}|$$

Continuous Time signal In the continuous case, Laplace transform must include the imaginary axis. For BIBO stability s-plane must contain all the poles of a system. The condition for stability can be obtained by the above condition

$$\int_{-\infty}^{\infty} |h(t)|dt = \int_{-\infty}^{\infty} |h(t)||e^{-jwt}|dt \quad (6)$$

$$= \int_{-\infty}^{\infty} |h(t)|(e^{-st}|dt \quad (7)$$

where $s = \sigma + jw$ and $Re(s) = \sigma = 0$

In addition to above condition, the continuous time signals is convergence if it encounter the following Dirichlet conditions

- if the interval is finite then x is of bounded variation ,
- for all finite number of points, x is continuous, if the interval is finite.

These conditions are satisfied by the periodic interval. Assume that

$$\sum_{m=0}^{\infty} a^m = \frac{1}{(1-a)} \quad (8)$$

where $|a| < 1$. On multiplying $1 - a$ both side we get

$$\sum_{m=0}^{\infty} a^m - a \sum m = 0^{\infty} a^m = 1$$

$a^0 = 1$, since $|a| < 1$, the sums converge. As an example, if $h(t) = a^t u(t)$ for every $t \in R$, where $a > 0$. Since the integral is infinite if $a \geq 1$, it is unstable and it is finite if $0 < a < 1$, and thus

$$\int_{0}^{\infty} a^t dt = \frac{-1}{ln\ a}$$

Therefore, if $0 < a < 1$, the system becomes stable.

3 Representation of systems via LTI difference equations

The output form of difference equation of discrete time system is defined like Haung and P. M.Knopf [2012]

$$y[n] = y[n-1] + x[n]$$

If the system $h[n]$ is LTI system then

$$y[n] = x[n] * h[n] \tag{9}$$

In causal system, the impulse response is zero for all $t < 0, n < 0$ in both discrete and continuous system respectively Alzabut et al. [2021], Oppenheim et al. [2009], Thamvichai and T.Bose [2002]. For example consider the following system,

$$y[n] - \frac{1}{2}y[n-1] = x[n]$$

If $x[n] = \delta\ [n]$, then we have

$$y[0] = 1$$
$$y[1] = \frac{1}{2}$$
$$y[2] = \frac{1}{4}$$
$$\cdots$$
$$y[n] = (\frac{1}{2})^n$$

Then

$$h[n] = (\frac{1}{2})^n\ u[n]$$

4 Development of Fourier series in different equations

In this section we calculate the transform signals through Fourier series Hu [2011], Bistritz [2004]. We consider some simple transformations with time variable. The ouput function $y(t)$ is

$$y(t) = e^{st}\int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau$$

The input signal $x(t) = cosw_0t$, where $w_0 > 0$, and then

$$x(t) = \frac{1}{2}e^{jw_0t} - \frac{1}{2}e^{-jw_0t}$$

Filtering is used to modify or eliminate some frequency components in the discrete signals. Consider

$$H(e^{jw}) = \frac{1}{(1 - ae^{(-jw)})}$$

Now define $y(t)$ as

$$y(t) = e^{(-at)} \int_0^1 e^{a\ \tau} d\tau$$

$$y(t) = \frac{1}{a}[1 - e^{(-at)}]$$

Here $x(\tau)$ and $h(t - \tau)$ does not overlap, and hence $y(t) = 0$.

5 Conclusions

This paper analyzed the stability of the BIBO system in time as well as frequency domain. Based on the state of time domain, a linear time invariant system is stable only if its impulse response is absolutely stable. In addition, it is focused the decomposition of signals into LTI system through suitable examples. The development of these techniques has been used in implementation of time-varying convolution filters. The signals are convolved to produce the linear time invariant system and non overlapping systems.

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