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A NEW MATHEMATICAL APPROACH ORIENTED TO FRIENDLY COMPUTER EVALUATION OF AIR-GAP FUNCTIONS OF INDUCTION MACHINES

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SUNTO - In questo lavoro si illustra un metodo di analisi delle macchine elettriche che affronta e risolve nel dominio del tempo, il problema della determinazione delle grandezze elettriche e magnetiche di macchina e dell'analisi del loro funzionamento a regime stazionario e in transitorio. Questo metodo trae spunto dalla teoria matematica delle Distribuzioni per 9introdurre una serie di funzioni, ricorrenti nell'analisi dei fenomeni elettromagnetici di macchina, e particolarizzare per esse dele operazioni algebriche ed integro differenziali. Si perviene ad un nuovo approccio di studio delle macchine elettriche che fornisce una serie di algoritmi di immediata implementazione numerica e di semplice interpretazione fisica.

ABSTRACT - In the paper a method, for the evaluation of the main electric and magnetic quantities in electrical machines, is proposed. This technique, approaches and solves any problem directly in the time domain.

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INTRODUCTION

The field of research of electrical machines, especially with respect to the induction motors, is particularly interested, in the last years, in the searching of suitable mathematical models oriented to different expectations and requests of the specialists. In fact the great development of the power electronics and the signal electronics, together with the availability of advanced technological microprocessors has given a strong impulse towards the fulfilment of very sophisticated and complex control algorithms in order to exploit the great dynamic possibilities of such a machine, suitable in different field of applications (industrial tools, electrical traction, actuators and so on). An efficient control system cannot avoid, especially when status observers are adopted, a full analysis of the electromagnetic interactions in the machine in order to achieve the expected accuracy of the regulation and control. On the contrary, after this extended analysis of the fundamental relationships, a simplification phase must be developed in order to limit the complexity of the calculus of the microprocessors. For these reasons, usually, the starting point of the modelling phase of induction machine, is represented by a Fourier's analysis of air-gap flux density, by the consideration of the only fundamental space harmonic and by the manipulation of the resultant relationships by means of different techniques concerning the vectorial analysis (mono-axis development on Gauss plan or two-axis approach according to Park theory) [1,2,3,4]. These approaches, often, far the modellisation of the induction machine by the basic relationships of classical electrodynamics concepts producing an analysis structure for specialist only. This one should not be a problem if the only specialist are interested in this approach. Unfortunately, many schools of electrical machines use the final Fourier's approach as starting one in the standard courses developing, many often, a limited vision of the problem towards the students. Starting from this approach many of the characteristics of the windings are loosen, unbalanced configurations cannot be analysed and the behaviour of the machine outside of the symmetrical and standard conditions becomes an un-proposable question for a lot of student. For these reasons in this paper a new analysis approach is suggested. This approach develops the basic electromagnetic relationships by the analysis of the windings and the geometric parameters by means of the known rules of the classic electrodynamics and by introducing simple functions that are combinations of canonical distributions (impulses, steps, and superior order functions) [5]. This approach is not new in the initial definition of the quantities. In fact the analysis of electrical machines by means of Riemann's transform, in the initial phase, is the same. But the Riemann approach brings the formulation in an analysis space very far from the starting electromagnetic structure. The anti-transforming operations are usually

laborious and complex and therefore this method does not find a full adoption in the standard courses of electrical machine and this approach too becomes a specialist only approach.
These ones are educational questions about the teaching of electrical machines.
Another question is the opportunity and the possibility to set up a general purpose computing code in order to solve, the induction machines outside the standard balance and symmetrical situations. This problem has been already solved by considering a full Fourier's approach with all the significant space harmonics. This method often becomes complex and laborious because the significant order harmonics depends on the considered un-symmetry and at the first time only a specialist might find them.
The method in this note presented allows a friendly computer evaluation of the air-gap distributions when the rotor speed is constant; the code can be implemented in a very original technique and used by non-specialist too [6].
In this way the Authors hope to near the study of the induction motors to the electromagnetic machine structures allowing a general mathematical formalization suitable, especially, for a computer analysis of the non-symmetrical and unbalanced configurations [7,8,9].

GENERALITIES ABOUT THE MATHEMATICAL TOOLS

The canonical functions will be defined in the following forms:

impulsive function:

$$A\delta(x - x_0) \qquad \text{(A is a constant)} \tag{1}$$

step function:

$$A\,\mathrm{u}(x - x_0) = \int_{-\infty}^{x} A\delta(\chi - x_0)d\chi \tag{2}$$

linear function:

$$A\,\mathrm{l}(x - x_0) = \int_{-\infty}^{x} A\,\mathrm{u}(\chi - x_0)d\chi = A(x - x_0)\,\mathrm{u}(x - x_0) \tag{3}$$

and the linear combination functions:

$$A\,\mathrm{D}(\chi, \mathrm{X}, x) = A\delta\left(x - \chi + \frac{\mathrm{X}}{2}\right) - A\delta\left(x - \chi - \frac{\mathrm{X}}{2}\right) \tag{4}$$

$$A\,\mathrm{W}(\chi, \mathrm{X}, x) = A\,\mathrm{u}\left(x - \chi + \frac{\mathrm{X}}{2}\right) - A\,\mathrm{u}\left(x - \chi - \frac{\mathrm{X}}{2}\right) \tag{5}$$

$$A\,\mathrm{L}(\chi, \mathrm{X}, x) = A\,\mathrm{l}\left(x - \chi + \frac{\mathrm{X}}{2}\right)\mathrm{W}(\chi, \mathrm{X}, x) \tag{6}$$

The consideration of these last three functions are suitable for the characteristic structure of standard induction machines.
The function W will be called unitary window function and a lot of electrical and magnetical functions in the considered machine will be expressed by means of this one. For this reason in the following some particular mathematical properties of this window function will be presented and discussed.

Translation rule:

$$\mathrm{W}(\chi - z, \mathrm{X}, x) = \mathrm{W}(\chi, \mathrm{X}, x + z) \tag{7}$$

Product rule:

$$\begin{aligned} &\mathrm{f}(x) = \mathrm{W}(\chi_f, \mathrm{X}_f, x) \\ &\mathrm{g}(x) = \mathrm{W}(\chi_g, \mathrm{X}_g, x) \\ &\mathrm{p}(x) = \mathrm{f}(x)\,\mathrm{g}(x) = \mathrm{W}\left(\frac{h+k}{2}, (k-h)\,\mathrm{u}(k-h), x\right) \\ &k = \min\left(\chi_f + \frac{\mathrm{X}_f}{2}, \chi_g + \frac{\mathrm{X}_g}{2}\right) \\ &h = \max\left(\chi_f - \frac{\mathrm{X}_f}{2}, \chi_g - \frac{\mathrm{X}_g}{2}\right) \end{aligned} \tag{8}$$

Integration rule:

$$\begin{aligned} &\int_a^b W(\chi, \mathrm{X}, x)\,dx = (k-h)\,\mathrm{u}(k-h) \\ &k = \min(b, \chi + \mathrm{X}/2) \\ &h = \max(a, \chi - \mathrm{X}/2) \end{aligned} \tag{9}$$

A particular property is suitable according to the derivative of a particular integral expression:
(where ω is a constant)

$$\Phi(t) = \int_{c_0 + \omega t}^{d_0 + \omega t} \beta(t)\, \mathrm{W}(\chi, \mathrm{X}, x)\, dx \tag{10}$$

and the derivative is evaluated with respect to the variable t so that:

$$\varepsilon = -\frac{d\Phi(t)}{dt} \tag{11}$$

and by considering previous relationships

$$\varepsilon = -\frac{d}{dt}\left[\beta(t)\varphi(t)\, \mathrm{u}(\varphi(t))\right] \tag{12}$$

where :

$$\varphi(t) = \left[\min\left(d_0 + \omega t; \chi + \frac{\mathrm{X}}{2}\right) - \max\left(c_0 + \omega t; \chi - \frac{\mathrm{X}}{2}\right)\right] \tag{13}$$

The real field of the variable t is divided into two different domains: the first is characterised by the negative value of φ; the second is characterised by no negative values of φ. For the first values the function u is zero and the product too; for the second set of values u is unitary and therefore it will be:

$$\varepsilon = -\frac{d}{dt}\left[\beta(t)\varphi(t)\right] \tag{14}$$

$$\varepsilon = -\beta(t)\frac{d\varphi(t)}{dt} - \varphi(t)\frac{d\beta(t)}{dt} \tag{15}$$

The function $\varphi(t)$, for the second set of values of t, will be represented by:

$$\varphi(t) = \omega \mathrm{L}\left(\frac{t_1 + t_2}{2}, t_2 - t_1, t\right) + h\, \mathrm{W}\left(\frac{t_2 + t_3}{2}, t_3 - t_2, t\right) - \omega \mathrm{L}\left(\frac{t_3 + t_4}{2}, t_4 - t_3, t\right) \tag{16}$$

where the parameters depend on the values of X and d_0-c_0; in fact when X>d_0-c_0 it will be:

$$h = d_0 - c_0 \tag{17}$$

$$\begin{aligned} t_1 &\to d_0 + \omega t_1 = \chi - \frac{X}{2} \\ t_2 &\to c_0 + \omega t_2 = \chi - \frac{X}{2} \\ t_3 &\to d_0 + \omega t_3 = \chi + \frac{X}{2} \\ t_4 &\to c_0 + \omega t_4 = \chi + \frac{X}{2} \end{aligned} \tag{18}$$

and when $X<d_0-c_0$ it will be:

$h = X$

$$\begin{aligned} t_1 &\to d_0 + \omega t_1 = \chi - \frac{X}{2} \\ t_2 &\to d_0 + \omega t_2 = \chi + \frac{X}{2} \\ t_3 &\to c_0 + \omega t_3 = \chi - \frac{X}{2} \\ t_4 &\to c_0 + \omega t_4 = \chi + \frac{X}{2} \end{aligned} \tag{19}$$

when $X= d_0-c_0$ then $t_2=t_3$.

The value of the derivative will be:

$$\begin{aligned} \frac{d\varphi}{dt} &= \omega \mathrm{W}\left(\frac{t_1+t_2}{2}, t_2-t_1, t\right) + \omega(t-t_1)[\delta(t-t_1)-\delta(t-t_2)] + \\ &+ h\mathrm{D}\left(\frac{t_2+t_3}{2}, t_3-t_2, t\right) - \omega \mathrm{W}\left(\frac{t_3+t_4}{2}, t_4-t_3, t\right) - \omega(t-t_4)[\delta(t-t_3)-\delta(t-t_4)] \end{aligned} \tag{20}$$

and for the previous relationships:

$$\frac{d\varphi}{dt} = \omega\left[\mathrm{W}\left(\frac{t_1+t_2}{2}, t_2-t_1, t\right) - \mathrm{W}\left(\frac{t_3+t_4}{2}, t_4-t_3, t\right)\right] \tag{21}$$

where the values of the parameters must be evaluated according to defined constraints.

MATHEMATICAL APPROACH

Let's consider a typical configuration of standard electrical machines as represented in fig. 1 with the following assumptions :

a) the permeability of the iron is infinite with respect to the air;
b) the linear currents densities are represented by impulsive functions;
c) the significant values of B in the air-gap will be the values evaluated over the internal surface of stator;
d) for sake of simplicity only a machine with one slot for pole and for phase will be considered and with only one pole pair.

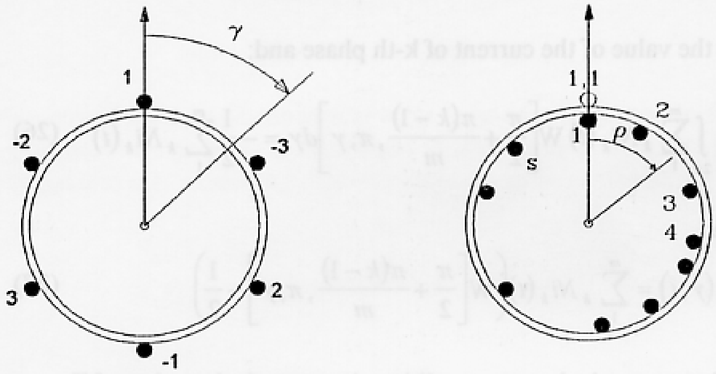

Fig.1 The considered structure of stator and rotor conductors.

These last restrictions don't deprive the method of its wide generality.
By considering the function that expresses the value of B on the internal surface of stator with respect to the angular abscissa γ only, it is convenient the adoption and the computation of the air-gap function Λ as:

$$\Lambda(\gamma,t)=\frac{\mu_0 B(\gamma,t)}{\int_H^K \underline{B}(x,y,z,t)\bullet \underline{t}dl} \tag{22}$$

H and K represent, respectively, the point of stator on the abscissa and the point on the rotor where a field line, starting from H, falls.
With the same assumption about t the significance of γ, it is suitable the consideration of the function Γ:

$$\Gamma(\gamma,t)=\frac{1}{\mu_0}\int_H^K \underline{B}(x,y,z,t)\bullet \underline{t}dl \tag{23}$$

so that:

$$B(\gamma,t)=\Lambda(\gamma,t)\Gamma(\gamma,t) \tag{24}$$

For a squirrel cage induction motor the value of Λ is constant and Γ is the superimposition of the stator Γ_s and rotor Γ_r functions.

For an m-phases symmetrical stator winding, with N conductor for slot, by introducing the function W, it is possible to evaluate Γ_s as:

$$\Gamma_s(\gamma,t)=\Gamma_0(t)\ +\sum_1^m{}_k N i_k(t)\,\mathrm{W}\left[\frac{\pi}{2}+(k-1)\frac{\pi}{m},\pi,\gamma\right] \tag{25}$$

where $i_k(t)$ is the value of the current of k-th phase and:

$$\Gamma_0(t)=-\frac{1}{2\pi}\int_{2\pi}\sum_1^m{}_k N i_k(t)\,\mathrm{W}\left[\frac{\pi}{2}+\frac{\pi(k-1)}{m},\pi,\gamma\right]d\gamma=-\frac{1}{2}\sum_1^m{}_k N i_k(t) \tag{26}$$

and, therefore:

$$\Gamma_s(\gamma,t)=\sum_1^m{}_k N i_k(t)\left(\mathrm{W}\left[\frac{\pi}{2}+\frac{\pi(k-1)}{m},\pi,\gamma\right]-\frac{1}{2}\right) \tag{27}$$

In the case of asymmetrical stator condition, in general, the value of Γ_S can be evaluated as:

$$\Gamma_s(\gamma,t)=\Gamma_0(t)+\sum_1^m{}_k N_k i_k(t)\,\mathrm{W}[\chi_k,\mathrm{X}_k,\gamma] \tag{28}$$

where:

$$\Gamma_0(t)=-\frac{1}{2\pi}\int_{2\pi}\sum_1^m{}_k N_k i_k(t)\,\mathrm{W}[\chi_k,\mathrm{X}_k,\gamma]d\gamma=-\frac{1}{2\pi}\sum_1^m{}_k N_k i_k(t)\mathrm{X}_k \tag{29}$$

and therefore:

$$\Gamma_s(\gamma,t)=\sum_1^m{}_k N_k i_k(t)\left\{\mathrm{W}[\chi_k,\mathrm{X}_k,\gamma]-\frac{\mathrm{X}_k}{2\pi}\right\} \tag{30}$$

Both for symmetrical condition and in non-symmetrical condition the stator flux density in the air-gap can be evaluated as the product of Λ_0 ($\Lambda_0=\mu_0/\delta_c$) for a characteristic sum of a W function and a constant value:

$$B_s(\gamma,t)=\Lambda_0\sum_{1}^{m}{}_k N_k i_k(t)\left\{W[\chi_k,X_k,\gamma]-\frac{X_k}{2\pi}\right\} \tag{31}$$

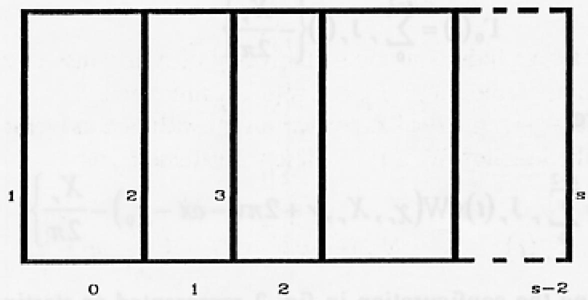

Fig.2 Rotor cage

By considering an asymmetrical rotor cage as represented in fig. 2 and the external rings as equipotential sides, it is possible to associate to each mash (from 0 to s-2) a current J. The bar current can be evaluated as:

$$\begin{aligned}
&I_1(t)=J_0(t)\\
&I_2(t)=J_1(t)-J_0(t)\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
&I_{s-1}(t)=J_{s-2}(t)-J_{s-3}(t)\\
&I_s(t)=-J_{s-2}(t)
\end{aligned} \tag{32}$$

and, by considering a rotor reference the function Γ_r as:

$$\Gamma_r(\rho,t)=\sum_{0}^{s-2}{}_r J_r(t)\,W(\chi_r,X_r,\rho)+\Gamma_0 \tag{33}$$

where χ_r and X_r represent the centre and the amplitude of the r-th mesh (for r= 0 to s-2).
For an angular speed ω the same function can be evaluated with respect to a stator reference as:

$$\Gamma_\nu(\gamma,t)=\sum_{0}^{s-2}{}_r J_r(t)\,W(\chi_r,X_r,\gamma+2\pi\nu-\omega t-\gamma_0)+\Gamma_0 \tag{34}$$

where the index ν represents the number of full turns of the rotor after t=0:

ν = integer (t/T) with T= 2π/ω

By imposing the null value of the flux of B all around the rotor surface the value of Γ_0 will be:

$$\Gamma_0(t)=\sum_{0}^{s-2}{}_r\,J_r(t)\left\{-\frac{X_r}{2\pi}\right\} \tag{35}$$

and, therefore

$$\Gamma_{\nu r}(\gamma,t)=\sum_{0}^{s-2}{}_r\,J_r(t)\left\{W(\chi_r,X_r,\gamma+2\pi\nu-\omega t-\gamma_0)-\frac{X_r}{2\pi}\right\} \tag{36}$$

By considering the configuration in fig. 2 represented as starting condition will be $\gamma_0=0$.
In conclusion it will be:

$$B_{\nu r}(\gamma,t)=\Lambda_0\sum_{0}^{s-2}{}_r\,J_r(t)\left\{W(\chi_r,X_r,\gamma+2\pi\nu-\omega t-\gamma_0)-\frac{X_r}{2\pi}\right\} \tag{37}$$

For a most suitable utilisation of previous relationships it is convenient to express the function $B(\rho,t)$ as the superimposition of window function by evaluating the amplitude of the flux density $\mathbf{B}_\varsigma$ between two consecutive bars:

$$B_r(\rho,t)=\sum_{0}^{s-1}{}_\varsigma\,\mathbf{B}_\varsigma(t)\,W(\chi_\varsigma,X_\varsigma,\rho) \tag{38}$$

with

$$\mathbf{B}_\varsigma(t)=\Lambda_0\left(J_\varsigma(t)\left(1-\frac{X_r}{2\pi}\right)-\sum_{0}^{s-2}{}_{r\neq\varsigma}\,J_r(t)\frac{X_r}{2\pi}\right) \tag{39}$$

and $J_{s-1}=0$

The computation of the e.m.f. can be operated by considering previous expressions. The static e.m.f. produced by the no-constant field between two consecutive bars can be evaluated as:

$$e_\varsigma^{(R,R)}=-X_\nu l\frac{D}{2}\frac{d}{dt}\left(\mathbf{B}_\varsigma(t)\right) \tag{40}$$

where l *is* the machine length and D is the diameter and $0\le\varsigma\le s-2$.

The e.m.f. produced by the ς-th "slice" of rotor field in the k-th stator phase, when the rotor speed is ω, can be computed by adopting the shown property of the derivative of integral expression as a succession of time-varying functions as:

$$e_{\varsigma,k}^{(R,S)} = -N\, l \frac{D}{2} \left\{ \begin{array}{l} \omega \mathrm{B}_\varsigma(t) \left[\mathrm{W}\left(\frac{t_{1(\varsigma,k)} + t_{2(\varsigma,k)}}{2}, t_{2(\varsigma,k)} - t_{1(\varsigma,k)}, t \right) - \mathrm{W}\left(\frac{t_{3(\varsigma,k)} + t_{4(\varsigma,k)}}{2}, t_{4(\varsigma,k)} - t_{3(\varsigma,k)}, t \right) \right] + \\ \frac{d\mathrm{B}_\varsigma(t)}{dt} \left[\begin{array}{l} \omega \mathrm{L}\left(\frac{t_{1(\varsigma,k)} + t_{2(\varsigma,k)}}{2}, t_{2(\varsigma,k)} - t_{1(\varsigma,k)}, t \right) + \mathrm{X}_{r\varsigma}\, \mathrm{W}\left(\frac{t_{2(\varsigma,k)} + t_{3(\varsigma,k)}}{2}, t_{3(\varsigma,k)} - t_{2(\varsigma,k)}, t \right) + \\ -\omega \mathrm{L}\left(\frac{t_{3(\varsigma,k)} + t_{4(\varsigma,k)}}{2}, t_{4(\varsigma,k)} - t_{3(\varsigma,k)}, t \right) \end{array} \right] \end{array} \right\} \quad 41)$$

The induced e.m.f. in the h-th stator phase by the k-th current can be evaluated by means the following notation:

$$e_{k,h}^{(S,S)} = -N^2 \frac{D}{2} l \Lambda_0 \mu\left(\lambda_{k,h}\right) \frac{d}{dt} i_k(t) \quad (42)$$

where $\mu(\lambda_{k,h})$ is the measure of the following interval $\lambda_{k,h}$:

$$\mu(\lambda_{k,h}) = \left[\chi_k - \frac{\mathrm{X}_k}{2}, \chi_k + \frac{\mathrm{X}_k}{2} \right] \cap \left[\chi_h - \frac{\mathrm{X}_h}{2}, \chi_h + \frac{\mathrm{X}_h}{2} \right] \quad (43)$$

(in the case of symmetrical three-phase machine this value is a generic element of the following matrix:

$$\mu(\lambda_{\kappa,\eta}) = \pi \begin{pmatrix} 1 & 1/3 & -1/3 \\ 1/3 & 1 & 1/3 \\ -1/3 & 1/3 & 1 \end{pmatrix} \quad (44)$$

The evaluation of the e.m.f. in the ς-th rotor mesh by the k-th stator phase can be developed by considering that the stator flux density has been expressed as sum of two different terms. The contribute of the first one {W} in the e.m.f. can be evaluated by considering the previous property of the derivative of integral expression so that:

$$e_{k,\varsigma}^{(S,R)\{W\}} = -\Lambda_0 lN \frac{D}{2}\left\{\begin{array}{l} \omega i_k(t)\left[W\left(\frac{t_{1(k,\varsigma)}+t_{2(k,\varsigma)}}{2}, t_{2(k,\varsigma)}-t_{1(k,\varsigma)}, t\right) - W\left(\frac{t_{3(k,\varsigma)}+t_{4(k,\varsigma)}}{2}, t_{4(k,\varsigma)}-t_{3(k,\varsigma)}, t\right)\right] + \\ \frac{di_k(t)}{dt}\left[\begin{array}{l} \omega L\left(\frac{t_{1(k,\varsigma)}+t_{2(k,\varsigma)}}{2}, t_{2(k,\varsigma)}-t_{1(k,\varsigma)}, t\right) + X_{\pi\varsigma} W\left(\frac{t_{2(k,\varsigma)}+t_{3(k,\varsigma)}}{2}, t_{3(k,\varsigma)}-t_{2(k,\varsigma)}, t\right) + \\ -\omega L\left(\frac{t_{3(k,\varsigma)}+t_{4(k,\varsigma)}}{2}, t_{4(k,\varsigma)}-t_{3(k,\varsigma)}, t\right) \end{array}\right] \end{array}\right\} \tag{45}$$

the second one {C} is:

$$e_{k,\varsigma}^{(S,R)\{C\}} = \Lambda_0 lN \, X_\varsigma \frac{D}{2}\frac{X_k}{2\pi}\frac{di_k(t)}{dt} \tag{46}$$

Obviously it will be:

$$e_\varsigma = \sum_1^m {}_k\left(e_{k,\varsigma}^{(S,R)\{W\}} + e_{k,\varsigma}^{(S,R)\{C\}}\right) + e_\varsigma^{(R,R)} \tag{47}$$

Therefore the balance voltage equations in the k-th stator phase and in the ς-th rotor mesh will be:

$$v_k(t) = Ri_k(t) + L\frac{d}{dt}i_k(t) - \sum_1^m {}_h e_{k,h}^{(S,S)} - \sum_0^{s-1} {}_\varsigma e_{\varsigma,k}^{(R,S)} \tag{48}$$

$$e_\varsigma = -R_{\varsigma+1}J_{\varsigma-1} + \left(R_{\varsigma+1}+R_{\varsigma+2}\right)J_\varsigma - R_{\varsigma+2}J_{\varsigma+1} - L_{\varsigma+1}\frac{d}{dt}J_{\varsigma-1} + \left(L_{\varsigma+1}+L_{\varsigma+2}\right)\frac{d}{dt}J_\varsigma - L_{\varsigma+2}\frac{d}{dt}J_{\varsigma+1} \tag{49}$$

where R and L are, respectively, the resistance and the leakage inductance of stator phase while R_ς and L_ς are the resistance and the leakage inductance of the ς-th rotor bar.

The resulting differential equations systems has as unknown the instantaneous values of phases current (stator) and of mesh currents (rotor).

COMPUTER PROGRAMMING IMPLEMENTATION

The mathematical approach presented in previous section suggests an original structure for the computer implementation. Standard programming routines base oneself on a differential equations resolve that gives the amplitude of unknown currents by solving an ordinary differential equations system. The unknown currents are the instantaneous values in the air-gap m.m.f. treated in terms of space Fourier's analysis. In case of symmetrical conditions, by considering the fundamental space harmonic and by adopting particular techniques of equivalence the system is linear with constant coefficient, if the speed is constant. In non-symmetrically conditions or in the eventuality of non isotropic air-gap configuration (cases for which the consideration of only the fundamental harmonic produces a no suitable solution because of the inaccuracy of results) it is necessary to take into account an higher number of space harmonics and notwithstanding particular assumptions there is a major complexity of the parameters computation. The difficulty and complexity of this computation depends on the degree of non-symmetrical conditions (electrical and magnetic) and on the number of space harmonics that have been considered in any case. This computation, in general, is an heavy and with an high probability of mistake and usually represents a phase that hardly can be generalised and implemented by an automatic programming routine. In fact often this procedure can be developed only by expert operators.

For the reconstruction of the air-gap functions, after the system resolution, a Fourier series sum must be computed for each time instant and space abscissa. This operation, that represents the fundamental relationship at the base of the method, is called only at this time so that the procedure needs different approach with respect to the analysis phase of the problem and to the synthesis phase.

The mathematical approach of previous section allows a development of the analysis of the problem and the synthesis of results by using the same logical and computational techniques. Furthermore the implementation of automatic programming routine, starting from the geometrical configuration of machine and the material properties, is possible in order to render all the procedures very friendly and adapt to inexpert operators also in the case of unbalanced and asymmetrical configuration and with a very high level of accuracy that is comparable only with the results get by means of the most complex procedure.

The preliminary steps of the present procedure are:

a) identification of stator areas and electrical connections;

b) identification of rotor areas and materials;

c) automatic generation of the Dynamic Connection Matrix (DCM);

d) set up of initial conditions.

At this point the identification of unknowns, the generation of the different terms of e.m.fs and the evaluation of the parameters of the differential equations system are possible for all the instant of integration. Therefore a standard resolve can be adopted. At each step of integration the reconstruction of air-gap function can be available by using the relevant element of the DCM. The elements of this matrix, in fact, perform a double target: the identification of system parameters and the organisation of the different terms of air-gap distribution. The amplitude of these terms are represented by the solution of the differential equations.

Let's analysing better the characteristic of the DCM.
This matrix has the informations about all the possible configuration of the rotor surface in front of stator surface with respect to the different angular positions get by rotation. The stator areas are defined as the cylindrical surfaces included between two consecutive winding side of the same phase while the rotor areas are defined as the cylindrical surface between two consecutive bars. DCM has n_r rows and n_c columns where n_r represents the number of all the possible situations in a full round and n_c represents the number of stator areas. All the situations are performed in the DCM in consecutive order. The element of the i-j site is the set of the rotor areas that are in front of the j-th stator area during the i-th situation of reciprocal position. Therefore the elements of the i-th row are equal to the elements of the (i-1)-th row except one: the different element is the one has produced the change of configuration. Obviously, for particular symmetrical conditions it would be possible that the change of configuration involves two or more elements of rotor area.3
A simple example can give an easy idea of the organisation of such a matrix DCM.
Let's consider a non-symmetrical electrical structure as shown in fig.3 where 1s and 2s have been called the stator areas (two phases and one slot for pole and phase). By means of the number 1,2 and 3 have been named the three different rotor surfaces (rotor with non-symmetrical cage of three bars).

$$DCM = \begin{Vmatrix} 3,1,2 & 1,2,3 \\ 3,1 & 1,2,3 \\ 3,1 & 1,2 \\ 3,1 & 3,1,2 \\ 3,1 & 3,1 \\ 3 & 3,1 \\ 2,3 & 3,1 \\ 2,3 & 2,3,1 \\ 2,3 & 2,3 \\ 1,2,3 & 2,3 \\ 1,2,3 & 1,2,3 \\ 1,2 & 1,2,3 \end{Vmatrix}$$

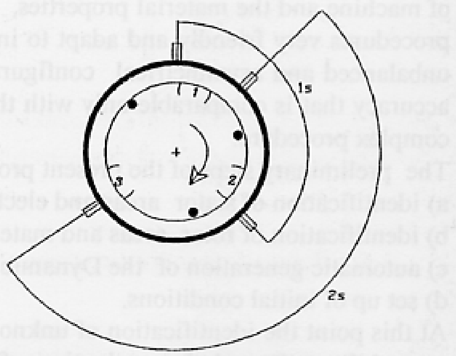

Fig. 3 Non symmetrical electrical structure of induction motor and relevant Dynamic Connection Matrix

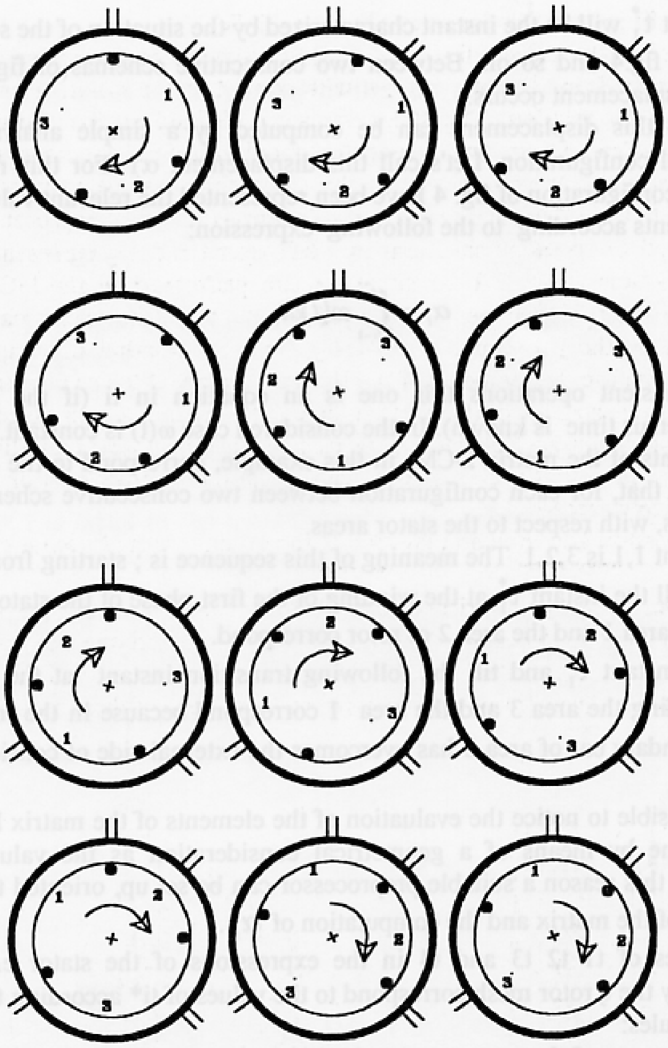

Fig. 4 Sequence of rotor positions in order to evaluate the transition instants

Let's consider as initial instant $t_0=0$ the instant characterised by the configuration rotor/stator of the first schema of fig.4. In this schema there is the superimposition of the rotor bar between the first and the third rotor area with the first slot of the first phase. By considering a full rotation of the rotor it is possible to determine all the transition instants t_i^* for which a change of reciprocal situation rotor/stator occurs. In the considered case twelve different transition instants can be evaluated.

The instant t_1^* will be the instant characterized by the situation of the second schema of fig.4 and so on. Between two consecutive schemas of fig.4 an angular displacement occurs.
In general this displacement can be computed by a simple analysis of geometrical configuration. Let's call this displacement $\alpha 1$. For this reason near each configuration of fig. 4 have been represented the relevant values of displacements according to the following expression:

$$\alpha_i = \int_{t_{i-1}^*}^{t_i^*} \omega(t)dt$$

During transient operations this one is an equation in ti (if the speed function versus time is known). In the considered case $\omega(t)$ is constant.
The elements of the matrix DCM, in this example, correspond to the set of rotor areas that, for each configuration between two consecutive schema of fig.4 occurs, with respect to the stator areas.
The element 1,1 is 3,2,1. The meaning of this sequence is ; starting from the instant 0 till the instant t_1^* at the winding of the first phase of the stator the area 3, the area 1 and the area 2 of rotor correspond.
After the instant t_1^* and till the following transition instant at the same stator winding the area 3 and the area 1 correspond because in the instant t_1^* the boundary bar of area 2 has overcomes the external side of considered winding.
As it is possible to notice the evaluation of the elements of the matrix DCM can be done by means of a geometrical consideration as the values of $\alpha_i$too. For this reason a suitable preprocessor can be set up, oriented to the definition of the matrix and the computation of α_i .
The values of t1 t2 t3 and t4 in the expressions of the stator e.m.f.s produced by the ς rotor mesh correspond to the values of ti* according to the following rules:
in the sequence of the rotor sectors, in front of a stator reference, with a positive value of speed according to the following schema

$$\Rightarrow \_\_\_ | \overset{\varsigma - 1}{\_\_\_\_\_\_} | \overset{\varsigma}{\_\_\_\_\_\_} | \overset{\varsigma + 1}{\_\_\_\_\_\_} | \_\_\_ \Rightarrow$$

in the k-th DCM column

a) case $X < d_0 - c_0$

$t_1 = t_i^*$ i is equal to the number of the row where for the first time the number ς appears

$t_2 = t_i^*$ i is equal to the number of the row where for the first time the number ς -1 appears
$t_3 = t_i^*$ i is equal to the number of the row where for the first time the number ς + 1 disappears
$t_4 = t_i^*$ i is equal to the number of the row where for the first time the number ς disappears

b) case $X > d_0 - c_0$

$t_1 = t_i^*$ i is equal to the number of the row where for the first time the number ς appears
$t_2 = t_i^*$ i is equal to the number of the row where for the first time the number ς +1 disappears
$t_3 = t_i^*$ i is equal to the number of the row where for the first time the number ς - 1 appears
$t_4 = t_i^*$ i is equal to the number of the row where for the first time the number ς disappears

The values of t1 t2 t3 and t4 in the expressions of the rotor e.m.f.s (ς mesh) produced by the k-th stator winding are equal to the values of ti* that appear in the relationship of the stator e.m.f.s (winding k) produced by the ς rotor mesh as it is easy to verify.
According to the previous rules, for the considered example, it will be;

$$
\begin{array}{lll}
e_{1,1}^{(R,S)} \quad \textbf{(a)} & e_{2,1}^{(R,S)} \quad \textbf{(a)} & e_{3,1}^{(R,S)} \quad \textbf{(b)} \\
t_1 = \dot{t}_9 & t_1 = \dot{t}_6 & t_1 = \dot{t}_0 \\
t_2 = \dot{t}_0 & t_2 = \dot{t}_9 & t_2 = \dot{t}_5 \\
t_3 = \dot{t}_1 & t_3 = \dot{t}_{11} & t_3 = \dot{t}_6 \\
t_4 = \dot{t}_5 & t_4 = \dot{t}_1 & t_4 = \dot{t}_{11}
\end{array}
$$

$$
\begin{array}{lll}
e_{1,2}^{(R,S)} \quad \textbf{(a)} & e_{2,2}^{(R,S)} \quad \textbf{(a)} & e_{3,2}^{(R,S)} \quad \textbf{(a)} \\
t_1 = \dot{t}_{10} & t_1 = \dot{t}_7 & t_1 = \dot{t}_3 \\
t_2 = \dot{t}_3 & t_2 = \dot{t}_{10} & t_2 = \dot{t}_7 \\
t_3 = \dot{t}_4 & t_3 = \dot{t}_2 & t_3 = \dot{t}_8 \\
t_4 = \dot{t}_8 & t_4 = \dot{t}_4 & t_4 = \dot{t}_2
\end{array}
$$

And for the expressions of the c.m.f.s in the rotor meshes;

$$e_{1,1}^{(S,R)} \quad \begin{array}{l} t_1 = t_9^* \\ t_2 = t_0^* \\ t_3 = t_1^* \\ t_4 = t_5^* \end{array} \qquad e_{1,2}^{(S,R)} \quad \begin{array}{l} t_1 = t_6^* \\ t_2 = t_9^* \\ t_3 = t_{11}^* \\ t_4 = t_1^* \end{array} \qquad e_{1,3}^{(S,R)} \quad \begin{array}{l} t_1 = t_0^* \\ t_2 = t_5^* \\ t_3 = t_6^* \\ t_4 = t_{11}^* \end{array}$$

$$e_{2,1}^{(S,R)} \quad \begin{array}{l} t_1 = t_{10}^* \\ t_2 = t_3^* \\ t_3 = t_4^* \\ t_4 = t_8^* \end{array} \qquad e_{2,2}^{(S,R)} \quad \begin{array}{l} t_1 = t_7^* \\ t_2 = t_{10}^* \\ t_3 = t_2^* \\ t_4 = t_4^* \end{array} \qquad e_{2,3}^{(S,R)} \quad \begin{array}{l} t_1 = t_3^* \\ t_2 = t_7^* \\ t_3 = t_8^* \\ t_4 = t_2^* \end{array}$$

The knowledge of the transition instants in each function of the e.m.f.s expressions allows the right evaluation of the coefficient of the differential equation for the computation of the electrical unknowns.
All the operations have been developed in order to produce a friendly approach on a computing unit without any lose of accuracy, in non-symmetrical condition too. These operations, in fact, don't need a pre-computational phase (complex winding factors, Fourier's expansions and so on) but start from a geometrical knowledge of the system and from the values of the electrical parameters (phases and bars resistance and leakage inductances).

CONCLUSION

In the paper a new approach has been presented, in the field of the computation of electrical unknown of an induction machine, very suitable to solve non-symmetrical condition too by means of a very friendly computer evaluation. This one represents the first step that allows the theoretical formalisation of the problem.
In order to simplify the presentation of the mathematical method in the paper only the case will constant value of the speed has been considered . In the future the same methodology will be approach by a most general point of view by considering the mechanical transient and variable function $\omega(t)$.
Next phases will approach the extension of these concepts to a most complex induction machine structure and the study of feasibility of an oriented software, particularly suitable for the mentioned problem.

LIST OF SYMBOLS

μ_0	air gap permeability
χ	central abscissa of a generic window function
c_0	generalised first integration limit
d_0	generalised second integration limit
δ_c	air gap ;
X	generalised range of window function
B	air-gap flux density

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