RATIO MATHEMATICA 27 (2014) 3-25 ISSN:1592-7415

Multi-criteria media mix decision model for advertising multiple product with segment specific and mass media

Sugandha Aggarwal[a], Anshu Gupta[b], P.C. Jha[a]

[a]Department of Operational Research, University of Delhi,
Delhi-110007, India
sugandha_or@yahoo.com
anshu@aud.ac.in
[b]SBPPSE, Ambedkar Univeristy Delhi,
Delhi-110006, India
jhapc@yahoo.com

Abstract

Judicious media planning decisions are crucial for successful advertising of products. Media planners extensively use mathematical models supplemented with market research and expert opinion to devise the media plans. Media planning models discussed in the literature largely focus on single products with limited studies related to the multi-product media planning. In this paper we propose a media planning model to allocate limited advertising budget among multiple products advertised in a segmented market and determine the number of advertisements to be given in different media. The proposed model is formulated considering both segment specific and mass media vehicles to maximize the total advertising reach for each product. The model also incorporates the cross product effect of advertising of one product on the other. The proposed formulation is a multi-objective linear integer programming model and interactive linear integer goal programming is discussed to solve the model. A real life case study is presented to illustrate the application of the proposed model.

Key words: Multiple products, Mass advertising, Segment Specific advertising, Spectrum effect, Media Planning, Multi-objective decision making, Interactive Approach.

2000 AMS: 90B60, 90C10, 90C29.

1 Introduction

A firm's market share and profit are driven by consumer demand and spending. Advertising carried by the firms to promote their products play a crucial role in fuelling consumer demand. It is through media that consumers receive advertising messages. It acts as a link between the advertisers and the consumers. Media such as television, radio, newspapers, magazines, and the internet act as distributors of the advertising messages. Media planning is a challenging process and the media choices are made such that the advertising objectives are met. The goal of a media planner is to reach the target audience with the right message through the right media. Advertising reach and frequency are the critical elements in setting up a media plan [19]. This study proposes a mathematical programming media allocation model to maximize the advertising reach of a firm that markets multiple products advertised through different media in a segmented market.

There are two major aspects of media planning, viz. selection of the media and allocation of the advertising budget. A media planner focuses on reaching its target customers with a right message that can convert them into potential buyers. The target market of a product can be taken as uniform or it can be bifurcated in to various segments based on the customer profile characteristics. When the market is considered as uniform, the advertising is carried at the mass level through the media that could reach all the customers. Though, the customers in the target market possesses some common characteristics that identify them as the potential customers still there exist differences in how they respond to the products and the advertising messages. If the product is advertised only at the mass level with a uniform advertising strategy, due to the differential behaviour of the potential market customers it may not be effective in influencing the customers to buy the product. In the recent years firms have tried to reach its customers with advertising that is tailored with respect to their individual characteristics so that the advertising not only reaches them but also convert them into potential buyers. Segmentation is an important concept of marketing that helps the advertisers to develop a media plan with respect to the customer's characteristics. Given the importance of segment driven marketing, importance of mass marketing can't be undermined as it creates a wider spectrum of reach. Hence the marketers choose to adopt the advertising strategy such that the product is advertised using mass media and also with segment driven advertising media. The reach obtained in segments can thus be obtained both from segment specific advertising and mass advertising. The model proposed in this paper incorporates this idea and develops a media plan that allocates advertising budget for both mass and segment specific media.

Companies are increasingly extending their products into product lines that are related or fall into distant categories. Marketing product lines instead of single product gives a competitive edge to the firms. It helps in meeting the diversified demand of products that are related which customer tend to use together and also provides a variety to the customers. Firms have limited resources in terms of value that can be used for advertising. For the case of single product advertised in a segmented market, the segments compete for media budget allocation among themselves and with mass media allocation. If a firm markets several products the competition for advertising budget first exists between the products and then at the segment and mass level. At any instant of time if several products are marketed by a firm advertising reach of an individual product no longer remains independent of other products. Due to substitution or complimentary effect that one product may have on other the advertising reach is also affected. Very limited research has investigated media planning model for multiple products jointly [16] .

In this paper we propose a multi-objective linear integer media planning model to allocate advertising budget between several products marketed by a firm through various media in a segmented market. The model allocates media budget and also determines the number of advertisements for each product, in all chosen media both at segment and mass level. It also incorporates cross product effect of advertising among products and maximizes the total advertising reach taking all products together. Interactive goal programming technique is discussed to solve the formulated model.

The paper is organized as follows: literature review is carried in section 2. In section 3, the model formulation and solution procedure are discussed. A case study is presented in section 4 illustrating the solution methodology. Concluding remarks are made in section 5.

2 Literature Review

The researchers have worked on various aspects of media planning such as the models for media selection, models concerning the "timing" aspect, market segmentation studies, budget allocation models, media scheduling models, media effectiveness models.

Broadbent [3] presented a review of the simulation and optimization procedures for the media planning models. The author discussed a number of media planning models and classified them into two approaches: mathematical model approach and algorithmic approach. A linear programming media allocation model was proposed by Bass and Lonsdale [1] with an objective to maximize the media exposures for one product for a single time unit. Authors explored the influence of several types of constraints on the model solution. Little and Lodish [14] formulated a media planning model based on a heuristic search algorithm to select and schedule media maximizing the total market response in different segments over the several time periods. Zufryden [20] developed media planning models with an objective of maximizing sales and determine the optimal media schedule over time. They considered stochastic response behavior in the objective function and later developed heuristics for solving the model [21]. Dwyer and Evans [7] proposed an optimization model for to select the best set of mailing lists in the direct mail advertising maximizing the proportion of customers reachable with direct mail pieces. The formulated binary integer model is solved through the branch and bound algorithm.

Korhonen et al. [12] proposed an evolutionary approach to media selection model. The model constraints and objective have interchangeable role in this approach. The iterations are performed for different set of objectives and constraints, computing the decision maker's value function in each iteration. Then the value function most suited to the decision maker is chosen as the solution. The study was carried out for a software company in Finland. Doyle and Saunders [6] developed a model to determine the spending on the promotion of multiple products for a retail store. The model optimally allocates budget to the promotional campaigns where each campaign is for a specific product. They considered cross product effect of advertising campaigns that lags or leads a particular campaign for up to four periods. A logarithmic linear regression model was proposed by the authors. Danaher and Rust [5] developed a model with an objective of maximizing the return on investment considering the diminishing return on the advertising and calculated the optimal amount of expenditure on the media campaign.

A media planning model based on the analytic hierarchy process was developed by Kwak et al. [13]. The model is developed to allocate the budget in the media categories and determine the number of advertisements for different media categories for digital products. Three criterion customer, advertising and budget were considered to be fulfilled through the model. The solution methodology based on pre-emptive goal programming technique was used. Buratto et al. [4] analyzed the media selection problem to choose an advertising channel for the pre-launch campaign for a new product (as cited in [11]). Authors considered a segmented market with several advertising channels that have different diffusion spectra and efficiencies. The problem is analyzed in two steps. First, an optimal control problem is solved explicitly in order to determine the optimal advertising policy for each channel. Then a maximum profit channel is chosen. They discussed a simulation where the choice of a newspaper among six Italian newspapers is presented.

Grosset and Viscolani [8] proposed a dynamic profit maximizing advertising model comparing the model performance under two strategies viz. 1) single medium advertising for a segmented market, that reaches segments with the same message but with varying effectiveness and 2) advertising independently for each segment through a single segment specific medium. The profit is measured in terms of goodwill where the growth of goodwill depends on the advertising effort and the goodwill decays due to forgetting of the advertised brand. Viscolani [18] proposed a non-linear programming advertising model for a segmented market to select a set of advertising media with an objective of maximizing profit. Using the approach similar to the Grosset and Viscolani [8] they suggested to use multiple media.

Hsu et al. [9] gave a fuzzy model using genetic algorithm to determine the optimum advertising mix and the number of insertions in different promotional instruments based on linguistic preferences of the domain experts. Bhattacharya [2] proposed an integer programming model to determine the optimal number of insertions in different media with an objective of maximizing the reach to the target population for a single product. Jha et al. [10] extended the model for the multiple products and a segmented market. Saen [17] proposed a model for the selection of media through the approach of data envelopment analysis in presence of flexible factors and imprecise data. Royo et al. [16] proposed an advertising budget allocation model for multiple products considering cross elasticity of products. They optimised the investment on advertising in multiple media for multiple products. This model was further extended by Royo et al. [16] under stochastic environment. Jha et al. [11] proposed an integer linear programming model of media planning for a single product advertised with multiple media with mass and segment specific advertising strategies. The model is developed with reach maximization objective.

As discussed above an extensive literature has been developed on optimization of media planning decisions. Most of the researchers have focused on media planning models for single product. In the present age, firms market several products simultaneously to provide product variety to the customer. The advertisement budget is to be divided among the products judiciously. In case of multi-product offering it is also observed that the one product advertising affects the advertising of other product[16]. The effect can either be substitutive or complimentary. It is important to measure and take account of this effect in media planning decisions. This necessitates joint media planning for the range of products such that the advertising budget can be shared between the products judiciously at the same time accounting for the cross-product effect of advertising which is considered in this paper. The study carried also integrates concept of media planning for multiple products with segmentation aspect. Another distinguished feature of the study is that we consider two types of advertising strategies viz. mass and segment specific in the model development. This differentiation between advertising strategies has been recently carried in some recent studies [11]. Both strategies affect advertising message reach in the potential market in different manner. While the mass advertising spread reach over the entire market widely, segment specific advertising plays crucial role in targeting segments.

The model developed in this paper maximizes the total reach of all the products taking in to consideration budgetary restrictions and bounds on the decision variables. The reach function is formulated considering the cross product effect of advertising. The model is tested on a real life case study.

3 Model Development

3.1 Notation

i	index for segments ($i = 0, 1, ..., N$)
j	index for advertising media ($j = 1, 2, ..., M_i$)
k	index for media options ($k = 1, 2, ..., K_{ij}$)
l	index for slot in a media ($l = 1, 2, ..., L_{ij}$)
p	index for products ($p = 1, 2, ..., P$)
q	index for customer profile characteristics ($q = 1, 2, ..., Q$)
jkl	j^{th} media, k^{th} media option, l^{th} slot
a^p_{ijkl}	reach per advertisement for p^{th} product in i^{th} segment, jkl^{th} media driver
C_{ijkl}	average number of readers/viewers of jkl^{th} media driver in segment i
c_{ijkl}	cost of inserting one advertisement in jkl^{th} media driver in segment i
v^p_{ijkl}	lower bound on the number of advertisements in jkl^{th} media driver of segment i for p^{th} product
u^p_{ijkl}	upper bound on the number of advertisements in jkl^{th} media driver of segment i for p^{th} product
x^p_{ijkl}	decision variable denoting the number of advertisements to be given in jkl^{th} media driver of segment i for p^{th} product
e^p_{irjkl}	percentage of people who follow jkl^{th} media driver in segment i, and are p^{th} product's potential customers possessing r^{th} profile characteristic.
α^p_{ijk}	spectrum effect of k^{th} media option of j^{th} mass media vehicle on i^{th} segment for p^{th} product; ; $0 < \alpha^p_{ijk} < 1$
w_{rp}	relative importance of r^{th} customer profile characteristic for p^{th} product
r	minimum proportion of budget allocated for mass advertisement
G	total advertising budget
Z_p	total reach of p^{th} product
A_p	reach component solely due to advertisement of product p
θ_{pf}	constant of proportionality representing CPE of advertising of product p on reach of product f

3.2 Model Formulation

Assuming a firm markets P products in a segmented market and the segments index vary from 1 to N and index 0 represents the mass media.

The mathematical model to maximize the total reach of advertising for each product through the mass and segment specific media is formulated as follows

$$\text{Vector Maximize} Z = [Z_1, Z_2, ..., Z_P]^T \tag{1}$$

subject to

(P1)

$$\sum_{p=1}^{P}\sum_{i=0}^{N}\sum_{j=1}^{M_i}\sum_{k=1}^{K_{ij}}\sum_{l=1}^{L_{ij}} c_{ijkl}x^p_{ijkl} \le G \tag{2}$$

$$\sum_{p=1}^{P}\sum_{j=1}^{M_0}\sum_{k=1}^{K_{0j}}\sum_{l=1}^{L_{0j}} c_{0jkl}x^p_{0jkl} \ge rG \tag{3}$$

$$x^p_{ijkl} \ge v^p_{ijkl}\ \forall p = 1, 2, ...P; i = 0, 1, 2, \ldots N; j = 1, 2, \ldots M_i; k = 1, 2, \ldots K_{ij}; l = 1, 2, \ldots L_{ij} \tag{4}$$

$$x^p_{ijkl} \le u^p_{ijkl}\ \forall p = 1, 2, ...P; i = 0, 1, 2, \ldots N; j = 1, 2, \ldots M_i; k = 1, 2, \ldots K_{ij}; l = 1, 2, \ldots L_{ij} \tag{5}$$

$$x^p_{ijkl} \ge 0 \text{ and integers}$$
$$\forall p = 1, 2, ...P; i = 0, 1, 2, \ldots N; j = 1, 2, \ldots M_i; k = 1, 2, \ldots K_{ij}; l = 1, 2, \ldots L_{ij} \tag{6}$$

where

$$Z_p = A_p + \sum_{\substack{f=1\\ f\neq p}}^{P} \theta_{pf}A_f \tag{7}$$

$$A_p = \sum_{i=1}^{N}\left(\sum_{j=1}^{M_i}\sum_{k=1}^{K_{ij}}\sum_{l=1}^{L_{ij}} a^p_{ijkl}x^p_{ijkl} + \sum_{j=1}^{M_0}\sum_{k=1}^{K_{0j}}\sum_{l=1}^{L_{0j}} \alpha^p_{ijkl}\left(a^p_{0jkl}x^p_{0jkl}\right)\right) \tag{8}$$

$$a^p_{ijkl} = \left\{\sum_{r=1}^{R} w_{rp}\, e^p_{irjkl}\right\} C_{ijkl} \tag{9}$$

Equation (1) represented by Z is a vector of objective functions with the components denoting the advertising reach of each product p. Component of Z denoted by Z_p (expressed mathematically as (7)) represents the combined reach from advertising for the product p and the cross product effect from advertising of other products. Where the reach expected to obtain from advertising for the product p is expressed as A_p (given by (8)). The individual advertising reach of each product as given by equation (8) is the sum of the reach from segment specific advertising and spectrum effect of the mass advertising in the segments. The per unit advertisement reach as given in equation (9) is the product of the readership/viewership of the media driver and the relative proportion of potential customers among them.

Equation (2) represents the budgetary constraint. Knowing the importance of mass advertising it is likely that media planner specify a lower bound on the budget to be spent on mass advertising as otherwise very little budget may be allocated to the mass media. Equation (3) represents the lower bound constraint on the mass media budget allocation. Constraint (4) and (5) are the lower and upper bounds specified by the media planner on the number of advertisements in different media for different products to ensure the diversity in advertising budget allocations rather than allocating the entire budget to some specific set of media. Constraint (6) imposes the decision variable to take integral values.

In the literature authors have suggested to formulate evolutionary model [12] wherein constraints and objectives roles can interchange. This allows flexibility to the decision maker, tradeoff the model variables and ensures that an efficient solution is obtained. In this direction in order to obtain an efficient solution and ensure some minimum reach for every product first we solve the model (P1) for each reach objective one by one to obtain the advertising reach aspirations for all products. These aspirations are set as lower bound constraints on reach objective and the resulting model is formulated as follows

$$\begin{aligned}
&\text{Vector Maximize} Z = [Z_1, Z_2, ..., Z_P]^T \\
&\text{subject to constraints (2)-(6) and} \\
&Z_p \geq Z_p^* \; \forall p = 1, 2, ...P
\end{aligned} \tag{P2}$$

Weighted sum multi-objective model using scalar weights μ_p; $\sum \mu_p = 1$; $(p = 1, 2, ...P)$ according to the relative importance of the products [15] is formulated using (P2) to obtain the media planning model as given in (P3)

$$\begin{aligned}
&\text{Maximize} \sum_{p=1}^{P} \mu_p Z_p \\
&\text{subject to constraints (2)-(6) and} \\
&Z_p \geq Z_p^* \; \forall p = 1, 2, ...P
\end{aligned} \tag{P3}$$

The weights in the model (P3) can be given by the decision maker or computed through the interactive approach (discussed in detail in [11]). The linear integer optimization model (P3) is solved by coding on LINGO optimization modelling software. The solution to model (P3) may result in infeasibility due to high aspirations on reach objective for products. Further a goal linear integer model is formulated for model (P2) to obtain a compromised solution and trade off the reach aspirations and budget.

Solution Methodology: Goal Programming

In goal programming, the solution is obtained such that the deviations from the goals are minimized. Deviations may be either positive or negative. Problem (P2) is solved in two stages. In Stage 1 the model is solved to minimize the deviations of the rigid constraints and in the second stage goal deviations are minimized incorporating the solution of first stage. The formulations of the two stages of goal programming are given as follows

Stage 1

$$\text{Minimize } \rho_1 + \eta_2 + \sum_{p=1}^{P}\sum_{i=0}^{N}\sum_{j=1}^{M_i}\sum_{k=1}^{K_{ij}}\sum_{l=1}^{L_{ij}} \eta_{ijkl}^{p} + \sum_{p=1}^{P}\sum_{i=0}^{N}\sum_{j=1}^{M_i}\sum_{k_j=1}^{K_{ij}}\sum_{l_j=1}^{L_{ij}} \rho'^{p}_{ijkl} \qquad \text{(P4)}$$

subject to constraints

$$\sum_{p=1}^{P}\sum_{i=0}^{N}\sum_{j=1}^{M_i}\sum_{k=1}^{K_{ij}}\sum_{l=1}^{L_{ij}} c_{ijkl}x_{ijkl}^{p} + \eta_1 - \rho_1 = A \qquad (10)$$

$$\sum_{p=1}^{P}\sum_{j=1}^{M_0}\sum_{k=1}^{K_{0j}}\sum_{l=1}^{L_{0j}} c_{0jkl}x_{0jkl}^{p} + \eta_2 - \rho_2 = rA \qquad (11)$$

$$x_{ijkl}^{p} + \eta_{ijkl}^{p} - \rho_{ijkl}^{p} = v_{ijkl}^{p}$$

$$\forall p = 1,2,...P; i = 0,1,2,\dots N; j = 1,2,\dots M_i; k = 1,2,\dots K_{ij}; l = 1,2,\dots L_{ij} \qquad (12)$$

$$x^p_{ijkl} + \eta^p_{ijkl} - \rho^p_{ijkl} = u^p_{ijkl}$$
$$\forall p = 1, 2, ...P; i = 0, 1, 2, \ldots N; j = 1, 2, \ldots M_i; k = 1, 2, \ldots K_{ij}; l = 1, 2, \ldots L_{ij} \tag{13}$$

$$x^p_{ijkl} \geq 0 \text{ and integers}$$
$$\forall p = 1, 2, ...P; i = 0, 1, 2, \ldots N; j = 1, 2, \ldots M_i; k = 1, 2, \ldots K_{ij}; l = 1, 2, \ldots L_{ij} \tag{14}$$

$$\eta^p_{ijkl}, \rho^p_{ijkl}, \eta'^p_{ijkl}, \rho'^p_{ijkl} \geq 0$$
$$\forall p = 1, 2, ...P; i = 0, 1, 2, \ldots N; j = 1, 2, \ldots M_i; k = 1, 2, \ldots K_{ij}; l = 1, 2, \ldots L_{ij} \tag{15}$$

$$\eta_i, \rho_i \geq 0 \forall i = 1, 2 \tag{16}$$

Stage 2

$$\begin{aligned} &\text{Minimize } g(\eta, \rho, X) = \sum_{p=1}^{P} \lambda_{p+2} \eta_{p+2} \\ &\text{subject to constraints (10)-(15) and} \\ &Z_p + \eta_{p+2} - \rho_{p+2} = Z_p^* \quad \forall p = 1, 2, ...P \\ &\eta_i, \rho_i \geq 0 \quad \forall i = 1, 2, ..., (P+2) \end{aligned} \tag{P5}$$

where $g(\eta, \rho, X)$ is objective function of (P5) and η_{p+2}, ρ_{p+2}, are negative and positive deviational variables of goals for p^{th} product objective function.

4 Case Study

To illustrate the application of the proposed model a case study is presented in this section demonstrating the media planning decision of a firm marketing five products (P1-P5) in the market. The name of the firm has not been disclosed due to the commercial confidentiality. The firm has to devise an advertising plan for its products for a period of one quarter. On the basis of geographic segmentation, the market for all the products is divided into fourteen segments (say S1-S14). The company wants to promote all products at the mass level as well as at the segment level. The firm's potential market is described on the basis of demographic characteristics: gender and income level, that is the potential market to which these products are targeted to, are females belonging to middle class group.

For segment level advertising in each segment, up to four newspapers (RNP1-RNP4), and two television channels (RCH1, RCH2) are selected. For the mass advertising four newspapers (NNP1-NNP4), and two television channels (NCH1, NCH2) are selected. Each of these media is chosen based on the potential market preferences, expert opinion and the market research. Further in each media there are different slots, such as in case of newspapers we can advertise on front page (FP) and/or other pages (OP). Similarly in case of television, slots can be classified as prime time (PT) and other time (OT). The total budget given by the firm for the media planning is Rs. 800 millions. The minimum proportion of the budget allocated to mass media is set as 30 %.

The data given by the firm is confidential and used with appropriate rescaling (given in Tables 3-12 in the appendix). The potential customer profile matrix corresponding to each media is computed for all segments by conducting a survey of on a sample. The percentage profile matrix computed for product 1 is given in Table 3. Similarly profile matrices are computed for all products. The weights defining relative importance of the potential customer profile characteristics gender and income level is given in Table 4. The values of the relative importance are inferred from the primary and secondary data with expert opinion. The cross product effect coefficient matrix is shown in Table 5. Table 6 gives the spectrum effect coefficient of the mass media on the various segments of the potential market.

The cost of advertisement in newspapers is measured in per square cm and an advertising space of 4cm x 6cm is considered. In case of television advertisement rates are given per 10 second slot and 30 second advertisement duration is preferred by the media planner. The advertising costs used in the study are given in Table 7. The media planner has also provided the lower and upper bounds on number of advertisements to be given for different products in different media as given in Table 8-12. These bounds are set to ensure the minimum visibility of ads in every media and distribute the advertising resources judiciously such that all chosen media can be used for advertising.

The optimization model (P1) is coded on LINGO optimization modelling software. In order to compute the target goals on the reach objectives for each product, first model (P1) is solved for each of the five products as a single objective model taking reach objective of one product at a time. The branch and bound method is used in the software to solve the model. Using these aspirations as the lower bounds on reach objectives for all products, the media planning model (P3) is coded. As the scalar weights of relative importance of product are not known, so we use interactive technique (for details of the method reader may refer [11]) to determine these weights. For the first iteration of interactive technique, 125 ($=V$) dispersed weighing vectors are generated randomly such that the components of each vector lie in the range $[0, 1]$ and the sum of all the components of each vector is equal to one. Taking a suitable value of d (computed using mathematical expression given in the algorithm) and through forward filtering approach 10 non-dominated distinct vectors (W) are filtered with L_2 metric distances between each set of vector. The problem (P3) is solved for all these 10 filtered weighing vectors. The model shows infeasibility with these filtered weighting vectors. Thus we form an interactive weighted sum goal programming model for (P1) to obtain a compromised solution using the reach targets as goals on the reach objectives.

The goal programming model is solved in two stages. In stage 1, the deviations corresponding to the rigid constraints are minimized and in stage 2 the deviations from the reach goals are minimized. First, the model (P4) is coded and solved in LINGO. In the next stage of goal programming, model (P5) is coded incorporating the solution obtained in stage 1. The weights given to the reach deviations are determined using interactive approach. Using the ten non-dominated distinct vectors generated earlier, the problem (P5) is solved 10 times. The solution and the objective function values are tabulated for all the runs and 5 ($=P$) best criterion vectors are filtered from 10 runs which are presented to the decision maker. On discussion with the decision maker, most preferred solution is selected. Using the weighing vector corresponding to the selected solution, the reduction factor is calculated and a new interval is formed between which new generation of weighting vectors is generated and the iteration is repeated. Five iterations of the interactive approach is carried based on the termination criteria ($t \lesssim k$) of the algorithm. Note that the parameters of the interactive algorithms are defined in Jha et al. [11] and same notations are used in this paper. All the calculations are carried out on a computing device with Intel Core Duo 1.40 GHz processor and 4 GB RAM. The average time taken to solve each problem is 2-4 seconds.

It can be seen from solution in Table 1 that as we move from iteration 1 to iteration 5, the total reach obtained from all the five products together improves. But the percentage change in the total objective function value decreases in successive iterations (except one iteration). As per the termination criteria of the interactive algorithm should converge in five iterations and we can see that the solutions of iteration 4 and 5 are very close to each other (% change=.09%), so the algorithm is terminated in five iterations.

The budget is fully utilized with 24.27 % of the total budget allocated to newspaper and the rest of 75.73% to TV. With these budget allocations among media it is expected to obtain approximately 20% of the reach from newspaper advertising and rest 80% from TV advertising. The distribution of budget among mass and segment level media is 31% and 69% (approx.) respectively. The product wise percentage allocation of the total budget and expected reach is given in Table 2. The optimal number of advertisements for different media for all the products is given in Table 13-17 in the appendix.

Table 1: Iteration parameters and the solution obtained

		Iteration 1 ($h=0$)	Iteration 2 ($h=1$)	Iteration 3 ($h=2$)	Iteration 4	Iteration 5 ($h=4$)
Interval width	$[\underline{\lambda}_1^{h+1}, \overline{\lambda}_1^{h+1}]$	$[0,1]$	$[0,.732]$	$[0,.536]$	$[0.056,.449]$	$[0.0715,.359]$
	$[\underline{\lambda}_2^{h+1}, \overline{\lambda}_2^{h+1}]$	$[0,1]$	$[0,.732]$	$[0,.536]$	$[0,.392]$	$[0.101,.389]$
	$[\underline{\lambda}_3^{h+1}, \overline{\lambda}_3^{h+1}]$	$[0,1]$	$[0,.732]$	$[0,.536]$	$[0,.392]$	$[0.013,.301]$
	$[\underline{\lambda}_4^{h+1}, \overline{\lambda}_4^{h+1}]$	$[0,1]$	$[0,.732]$	$[0.015,.552]$	$[0,.392]$	$[0.0848,.373]$
	$[\underline{\lambda}_5^{h+1}, \overline{\lambda}_5^{h+1}]$	$[0,1]$	$[0,.732]$	$[0,.536]$	$[0.075,.468]$	$[0.0104,.298]$
D		0.066	0.0545	0.0536	0.0445	0.026
Reduction Factor		0.732	0.536	0.392	0.2877	0.2108
Vector Selected		Vector 1 [0.1039 0.1854 0.2556 0.1966 0.2584]	Vector 5 [0.1624 0.1752 0.1178 0.2834 0.2611]	Vector 1 [0.2524 0.1577 0.1956 0.1230 0.2713]	Vector 1 [0.2154 0.2447 0.1569 0.2287 0.1543]	Vector 39 [0.2025 0.1600 0.2225 0.2485 0.1725]
Reach		1858245000	1882864000	1899541000	1916792000	1918657000
% increase in Reach		-	1.32%	0.88%	0.91%	0.09%

Table 2: Product wise allocations from iteration 5

Products	Reach Achieved	Reach aspired	% reach achieved from aspired	% budget utilized
P1	641926400	720325700	89%	0.33%
P2	313761900	469876000	67%	0.16%
P3	572235900	607432200	94%	0.27%
P4	192188100	266835600	72%	0.13%
P5	198544300	310087000	64%	0.11%

5 Conclusion

A media planning model is proposed in this study to allocate advertising budget jointly among multiple products advertised in a segmented market. Media vehicles are chosen with respect to two types of advertising strategies namely, segment driven and mass media advertising. Segment specific media targets the segment potential while the mass media reaches the wider market with spectrum effect on the segments. The model determines the number of advertisement to be given in each of the media within the bounds suggested by media planner. When several products are advertised by a firm to serve the diverse need of a market, advertising of one product shows the cross product effect on other products. The study considers this effect in the model. Model applicability and solution methodology based on interactive linear integer goal programming is discussed with a case study and LINGO is used for computational support. The proposed model incorporates the cross product effect of advertising of a firms own products. Effect of competitive products can also be included in the future studies. The scope of the model is limited to media planning for a single period. The model can be further extended for dynamic media planning incorporating the retention and diminishing effect of advertising.

Appendix

Table 3: Customer percentage profile matrix for newspapers and television for product 1

Segment	RNP1				RNP2				RNP3				RNP4				RCH1				RCH2			
	gender		income		gender		income		gender		income		gender		income		gender		income		gender		income	
	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT	PT	OT	PT	OT
S1	0.29	0.13	0.12	0.04	0.35	0.08	0.12	0.06	0.2	0.14	0.19	0.06	0.24	0.13	0.15	0.09	0.23	0.19	0.15	0.12	0.27	0.17	0.13	0.08
S2	0.3	0.15	0.14	0.09	0.2	0.1	0.1	0.08	0.15	0.12	0.19	0.1	0.27	0.1	0.09	0.12	0.25	0.12	0.14	0.08	0.22	0.1	0.08	0.03
S3	0.29	0.14	0.16	0.07	0.19	0.07	0.12	0.04	0.17	0.15	0.18	0.09	0.22	0.1	0.1	0.05	0.32	0.14	0.21	0.13	0.27	0.13	0.14	0.09
S4	0.15	0.08	0.15	0.08	0.15	0.06	0.08	0.04	0.25	0.12	0.18	0.07	–	–	–	–	0.4	0.23	0.21	0.11	0.23	0.07	0.15	0.06
S5	0.27	0.17	0.2	0.1	0.1	0.05	0.07	0.03	0.15	0.11	0.18	0.06	0.33	0.09	0.07	0.06	0.37	0.15	0.22	0.08	0.18	0.06	0.12	0.03
S6	0.22	0.11	0.13	0.06	0.14	0.06	0.07	0.03	0.21	0.17	0.14	0.04	0.26	0.12	0.18	0.06	0.39	0.2	0.19	0.08	0.24	0.1	0.12	0.09
S7	0.3	0.18	0.18	0.08	0.24	0.14	0.2	0.12	0.26	0.17	0.13	0.05	–	–	–	–	0.3	0.2	0.13	0.09	0.16	0.07	0.11	0.08
S8	0.31	0.17	0.19	0.08	0.12	0.07	0.12	0.06	0.28	0.14	0.16	0.09	0.27	0.08	0.17	0.11	0.31	0.14	0.15	0.09	0.19	0.11	0.13	0.11
S9	0.26	0.12	0.16	0.06	0.25	0.1	0.16	0.07	0.21	0.13	0.17	0.1	–	–	–	–	0.28	0.2	0.17	0.11	0.25	0.16	0.18	0.1
S10	0.29	0.13	0.17	0.07	0.2	0.13	0.12	0.06	0.22	0.14	0.18	0.07	–	–	–	–	0.33	0.18	0.19	0.11	0.27	0.19	0.16	0.1
S11	0.28	0.13	0.13	0.08	0.2	0.13	0.15	0.07	0.22	0.17	0.15	0.09	–	–	–	–	0.31	0.13	0.13	0.1	0.25	0.2	0.14	0.09
S12	0.23	0.13	0.15	0.08	0.15	0.1	0.1	0.05	0.2	0.16	0.2	0.08	0.22	0.12	0.07	0.07	0.46	0.15	0.19	0.07	0.32	0.12	0.23	0.13
S13	0.28	0.13	0.18	0.08	0.23	0.1	0.13	0.07	–	–	–	–	–	–	–	–	0.33	0.17	0.3	0.22	0.27	0.19	0.23	0.11
S14	0.25	0.11	0.17	0.1	0.21	0.08	0.12	0.05	–	–	–	–	–	–	–	–	0.25	0.17	0.13	0.08	0.13	0.08	0.08	0.06
	RNP1				RNP2				RNP3				RNP4				RCH1				RCH2			
Mass	gender		income		gender		income		gender		income		gender		income		gender		income		gender		income	
Media	0.25	0.1	0.12	0.06	0.15	0.07	0.11	0.03	0.16	0.06	0.12	0.06	0.15	0.07	0.11	0.06	0.3	0.2	0.21	0.11	0.22	0.16	0.19	0.09

Table 4: Weights

	CR1	CR2
P1	0.65	0.35
P2	0.6	0.4
P3	0.36	0.64
P4	0.3	0.7
P5	0.55	0.45

Table 5: Cross Product Effect Matrix

Product	P1	P2	P3	P4	P5
P1	0	0.0109	0.034	0.02345	0.0034
P2	0.0234	0	0.0234	0.009	0.0054
P3	0.0195	0.0134	0	0.0156	0.00493
P4	0.0214	0.0093	0.0041	0	0.0067
P5	0.0145	0.011	0.0013	0.0078	0

Table 6: Spectrum effect coefficient of national newspapers and TV channels on regions

Segments	NNP1	NNP2	NNP3	NNP4	NCH1	NCH2
S1	0.09	0.12	0.1	0.12	0.1	0.11
S2	0.08	0.09	0.13	0.1	0.09	0.06
S3	0.12	0.09	0.09	0.15	0.12	0.1
S4	0.06	0.04	0.05	0.03	0.1	0.11
S5	0.07	0.07	0.08	0.09	0.03	0.04
S6	0.13	0.09	0.1	0.06	0.1	0.08
S7	0.03	0.06	0.04	0.07	0.04	0.05
S8	0.07	0.07	0.07	0.07	0.07	0.05
S9	0.1	0.14	0.1	0	0.04	0.04
S10	0.04	0.05	0.04	0.06	0.05	0.05
S11	0.06	0.06	0.05	0.02	0.04	0.05
S12	0.08	0.08	0.12	0.15	0.1	0.12
S13	0.04	0.02	0.01	0.05	0.05	0.05
S14	0.03	0.02	0.02	0.03	0.07	0.1

Table 7: Ad cost in different media

Segments	RNP1		RNP2		RNP3		RNP4		RCH1		RCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
S1	3750	1944	2423	1385	1400	1000	1719	1665	65480	26968	27548	12988
S2	2751	917	2221	1610	1650	900	1300	650	40400	19800	26400	12000
S3	3940	2225	2138	950	2040	1060	1285	1045	43628	15376	30464	13980
S4	1750	500	900	400	790	380	–	–	33800	12220	20908	9964
S5	3310	1572	2500	1200	1767	1010	1375	1100	19384	9408	14000	9100
S6	3800	2000	2331	1665	2200	1340	1400	900	45928	16480	41948	21472
S7	1200	600	1150	670	1100	550	–	–	8924	5948	6600	3200
S8	1200	600	1160	600	1000	550	900	500	14400	9000	8924	3964
S9	3700	1800	3960	2100	2500	1450	–	–	30980	12500	16700	9700
S10	1700	1000	1650	900	1450	850	–	–	17848	8956	14956	6980
S11	2500	1200	1640	1040	1100	870	–	–	8948	3980	5948	2980
S12	2920	1530	2100	1400	1100	890	2047	1575	41448	18984	30464	17476
S13	1800	900	1000	500	–	–	–	–	12250	6700	9945	4350
S14	700	527	595	424	–	–	–	–	34080	18900	21000	12340
Mass Media	NNP1		NNP2		NNP3		NNP4		NCH1		NCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
	9800	5640	8690	4250	6900	3540	5500	2900	104390	61019	86814	46570

Table 8: Upper and lower bounds on advertisements in different media for P1

Segments	RNP1		RNP2		RNP3		RNP4		RCH1		RCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
S1	[1, 22]	[12, 85]	[1, 15]	[12, 65]	[1, 20]	[12, 70]	[1, 12]	[11, 68]	[8, 36]	[18, 92]	[6, 39]	[15, 85]
S2	[1, 14]	[9, 50]	[1, 12]	[8, 41]	[1, 12]	[7, 42]	[1, 13]	[7, 48]	[7, 34]	[16, 88]	[4, 33]	[13, 73]
S3	[1, 24]	[11, 90]	[1, 16]	[9, 69]	[1, 20]	[10, 75]	[1, 15]	[8, 62]	[7, 39]	[19, 94]	[5, 38]	[14, 83]
S4	[1, 14]	[4, 56]	[1, 10]	[3, 44]	[1, 12]	[4, 42]	–	–	[6, 33]	[14, 78]	[6, 37]	[16, 81]
S5	[1, 20]	[7, 81]	[1, 15]	[5, 72]	[1, 18]	[6, 76]	[1, 13]	[4, 70]	[4, 31]	[8, 65]	[3, 25]	[7, 57]
S6	[1, 18]	[8, 76]	[1, 13]	[7, 61]	[1, 15]	[6, 65]	[1, 14]	[7, 60]	[7, 38]	[17, 85]	[5, 36]	[13, 79]
S7	[1, 12]	[6, 65]	[1, 11]	[5, 75]	[1, 14]	[6, 72]	–	–	[6, 31]	[10, 68]	[3, 31]	[10, 68]
S8	[1, 12]	[9, 49]	[1, 10]	[7, 45]	[1, 8]	[5, 40]	[1, 8]	[4, 38]	[7, 32]	[12, 72]	[3, 33]	[10, 72]
S9	[1, 18]	[10, 76]	[1, 14]	[11, 58]	[1, 14]	[8, 60]	–	–	[5, 33]	[12, 64]	[3, 29]	[8, 64]
S10	[1, 13]	[6, 49]	[1, 10]	[5, 40]	[1, 10]	[4, 42]	–	–	[4, 33]	[11, 62]	[3, 26]	[9, 59]
S11	[1, 19]	[9, 82]	[1, 14]	[7, 70]	[1, 14]	[6, 75]	–	–	[5, 34]	[12, 66]	[4, 27]	[10, 62]
S12	[1, 16]	[8, 71]	[1, 12]	[7, 55]	[1, 15]	[6, 71]	[1, 14]	[8, 63]	[8, 36]	[16, 79]	[7, 40]	[16, 86]
S13	[1, 12]	[4, 48]	[1, 9]	[3, 40]	–	–	–	–	[3, 31]	[11, 64]	[3, 25]	[12, 57]
S14	[1, 14]	[5, 64]	[1, 11]	[4, 40]	–	–	–	–	[5, 32]	[14, 85]	[5, 29]	[14, 65]
Mass Media	NNP1		NNP2		NNP3		NNP4		NCH1		NCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
	[1, 18]	[12, 84]	[1, 12]	[12, 64]	[1, 15]	[12, 70]	[1, 12]	[12, 64]	[8, 39]	[20, 94]	[8, 25]	[17, 86]

Table 9: Upper and lower bounds on advertisements in different media for P2

Segments	RNP1		RNP2		RNP3		RNP4		RCH1		RCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
S1	[1, 11]	[6, 42]	[1, 10]	[7, 36]	[1, 8]	[6, 30]	[1, 10]	[7, 32]	[7, 36]	[18, 77]	[5, 34]	[14, 65]
S2	[1, 8]	[4, 30]	[1, 6]	[3, 26]	[1, 7]	[4, 26]	[1, 6]	[4, 26]	[7, 38]	[16, 80]	[4, 32]	[13, 64]
S3	[1, 12]	[6, 49]	[1, 12]	[3, 31]	[1, 12]	[5, 35]	[1, 12]	[4, 32]	[8, 39]	[16, 82]	[6, 34]	[12, 66]
S4	[1, 8]	[4, 32]	[0, 7]	[2, 27]	[0, 7]	[2, 25]	–	–	[7, 29]	[13, 64]	[3, 35]	[12, 62]
S5	[1, 11]	[3, 49]	[1, 10]	[3, 32]	[1, 9]	[3, 36]	[1, 10]	[3, 31]	[5, 29]	[11, 65]	[3, 25]	[7, 59]
S6	[1, 13]	[5, 44]	[1, 12]	[4, 29]	[1, 12]	[4, 34]	[1, 12]	[3, 32]	[6, 33]	[15, 78]	[7, 33]	[10, 66]
S7	[1, 8]	[4, 29]	[0, 9]	[3, 21]	[1, 8]	[2, 25]	–	–	[7, 26]	[12, 63]	[3, 27]	[7, 52]
S8	[1, 9]	[6, 27]	[0, 8]	[4, 25]	[0, 6]	[3, 23]	[0, 6]	[3, 22]	[5, 28]	[12, 64]	[3, 31]	[8, 55]
S9	[1, 10]	[7, 45]	[1, 9]	[6, 42]	[1, 9]	[5, 35]	–	–	[3, 32]	[11, 72]	[3, 25]	[5, 61]
S10	[1, 8]	[4, 28]	[0, 6]	[3, 24]	[0, 7]	[2, 26]	–	–	[3, 27]	[10, 74]	[4, 29]	[8, 60]
S11	[1, 10]	[6, 50]	[0, 10]	[4, 48]	[0, 11]	[3, 36]	–	–	[2, 27]	[8, 71]	[3, 26]	[8, 49]
S12	[1, 9]	[4, 35]	[0, 8]	[2, 32]	[0, 6]	[1, 30]	[0, 8]	[2, 32]	[7, 36]	[14, 68]	[4, 33]	[12, 64]
S13	[1, 8]	[3, 28]	[0, 6]	[4, 26]	–	–	–	–	[5, 27]	[8, 65]	[5, 23]	[9, 59]
S14	[1, 8]	[4, 26]	[0, 4]	[3, 24]	–	–	–	–	[7, 33]	[14, 64]	[6, 33]	[16, 64]
Mass Media	NNP1		NNP2		NNP3		NNP4		NCH1		NCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
	[1, 12]	[10, 49]	[0, 10]	[9, 32]	[1, 12]	[8, 48]	[1, 10]	[9, 40]	[8, 39]	[18, 82]	[5, 33]	[16, 72]

Table 10: Upper and lower bounds on advertisements in different media for P3

Segments	RNP1		RNP2		RNP3		RNP4		RCH1		RCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
S1	[1, 9]	[8, 44]	[1, 7]	[6, 40]	[1, 8]	[7, 38]	[1, 7]	[6, 43]	[8, 39]	[16, 75]	[7, 33]	[13, 65]
S2	[1, 6]	[5, 40]	[0, 5]	[4, 30]	[1, 5]	[3, 35]	[1, 4]	[3, 28]	[7, 36]	[14, 68]	[5, 32]	[10, 57]
S3	[1, 10]	[4, 45]	[0, 9]	[2, 40]	[1, 9]	[4, 32]	[1, 8]	[3, 27]	[8, 38]	[17, 78]	[7, 34]	[12, 62]
S4	[1, 6]	[8, 40]	[1, 7]	[5, 30]	[1, 6]	[6, 32]	–	–	[8, 38]	[14, 73]	[6, 35]	[14, 64]
S5	[1, 7]	[4, 44]	[0, 6]	[3, 29]	[0, 7]	[3, 32]	[1, 6]	[3, 27]	[2, 31]	[5, 50]	[3, 22]	[7, 47]
S6	[1, 8]	[7, 43]	[1, 7]	[5, 32]	[1, 6]	[6, 30]	[1, 5]	[5, 23]	[7, 36]	[15, 70]	[6, 28]	[11, 59]
S7	[1, 6]	[5, 40]	[0, 4]	[4, 25]	[1, 5]	[4, 27]	–	–	[3, 33]	[10, 55]	[6, 25]	[8, 49]
S8	[1, 6]	[4, 40]	[1, 5]	[3, 27]	[1, 6]	[3, 32]	[1, 5]	[3, 27]	[6, 35]	[12, 63]	[5, 26]	[10, 52]
S9	[0, 9]	[7, 42]	[1, 8]	[7, 35]	[0, 7]	[5, 32]	–	–	[3, 31]	[7, 52]	[3, 23]	[7, 50]
S10	[1, 7]	[4, 40]	[1, 9]	[5, 25]	[1, 7]	[3, 23]	–	–	[4, 33]	[10, 61]	[6, 25]	[8, 51]
S11	[1, 8]	[7, 43]	[0, 7]	[5, 32]	[1, 7]	[5, 27]	–	–	[5, 34]	[8, 57]	[4, 26]	[11, 55]
S12	[1, 7]	[6, 43]	[1, 5]	[5, 34]	[1, 6]	[4, 29]	[1, 5]	[5, 26]	[8, 36]	[16, 73]	[7, 32]	[16, 66]
S13	[1, 5]	[5, 40]	[0, 4]	[4, 28]	–	–	–	–	[5, 34]	[9, 59]	[3, 21]	[10, 48]
S14	[1, 6]	[4, 40]	[0, 4]	[3, 31]	–	–	–	–	[7, 37]	[13, 65]	[6, 29]	[12, 61]
Mass Media	NNP1		NNP2		NNP3		NNP4		NCH1		NCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
	[1, 10]	[8, 47]	[1, 8]	[6, 38]	[1, 10]	[6, 45]	[1, 10]	[6, 40]	[8, 39]	[17, 78]	[7, 35]	[13, 66]

Table 11: Upper and lower bounds on advertisements in different media for P4

Segments	RNP1		RNP2		RNP3		RNP4		RCH1		RCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
S1	[1, 5]	[3, 20]	[1, 3]	[3, 20]	[1, 4]	[2, 16]	[0, 3]	[3, 19]	[4, 36]	[14, 65]	[4, 27]	[12, 57]
S2	[0, 4]	[2, 18]	[0, 2]	[1, 16]	[0, 3]	[1, 14]	[0, 2]	[1, 13]	[5, 35]	[14, 61]	[3, 27]	[8, 49]
S3	[0, 4]	[3, 17]	[0, 2]	[2, 13]	[0, 3]	[2, 14]	[0, 2]	[2, 14]	[5, 37]	[10, 70]	[4, 27]	[10, 55]
S4	[0, 3]	[2, 15]	[0, 1]	[1, 13]	[1, 2]	[2, 13]	–	–	[5, 37]	[13, 64]	[4, 26]	[11, 57]
S5	[1, 4]	[2, 16]	[1, 3]	[1, 12]	[0, 3]	[2, 14]	[0, 2]	[1, 12]	[2, 27]	[9, 51]	[3, 25]	[7, 41]
S6	[0, 5]	[3, 20]	[1, 3]	[2, 18]	[1, 2]	[3, 14]	[0, 3]	[2, 18]	[4, 32]	[12, 63]	[3, 25]	[9, 51]
S7	[0, 4]	[2, 18]	[1, 3]	[2, 16]	[1, 3]	[2, 14]	–	–	[3, 31]	[11, 55]	[3, 26]	[7, 43]
S8	[1, 3]	[2, 16]	[0, 2]	[3, 16]	[0, 2]	[2, 15]	[1, 1]	[3, 13]	[4, 31]	[13, 60]	[3, 27]	[7, 46]
S9	[1, 5]	[3, 20]	[0, 3]	[2, 18]	[0, 3]	[3, 16]	–	–	[2, 29]	[10, 52]	[3, 24]	[7, 42]
S10	[1, 4]	[1, 15]	[1, 3]	[2, 14]	[1, 4]	[1, 12]	–	–	[3, 33]	[13, 59]	[3, 27]	[7, 45]
S11	[0, 5]	[3, 20]	[1, 3]	[3, 18]	[0, 4]	[3, 19]	–	–	[3, 34]	[11, 56]	[3, 25]	[8, 49]
S12	[1, 5]	[3, 19]	[1, 4]	[3, 18]	[1, 3]	[2, 19]	[0, 3]	[3, 18]	[5, 33]	[14, 68]	[4, 24]	[12, 59]
S13	[0, 4]	[2, 17]	[0, 2]	[2, 15]	–	–	–	–	[3, 38]	[12, 57]	[3, 23]	[7, 47]
S14	[0, 4]	[2, 15]	[0, 2]	[1, 13]	–	–	–	–	[4, 33]	[12, 59]	[3, 26]	[10, 54]
Mass Media	NNP1		NNP2		NNP3		NNP4		NCH1		NCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
	[1, 5]	[3, 20]	[1, 3]	[3, 20]	[1, 4]	[3, 18]	[1, 4]	[3, 20]	[5, 37]	[14, 70]	[4, 27]	[12, 59]

Table 12: Upper and lower bounds on advertisements in different media for P5

Segments	RNP1		RNP2		RNP3		RNP4		RCH1		RCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
S1	[0, 5]	[3, 18]	[0, 4]	[2, 13]	[0, 5]	[2, 17]	[0, 4]	[2, 14]	[8, 38]	[12, 70]	[7, 29]	[11, 57]
S2	[0, 4]	[2, 15]	[0, 3]	[2, 13]	[0, 4]	[2, 9]	[0, 3]	[2, 6]	[7, 38]	[10, 68]	[6, 28]	[10, 51]
S3	[0, 5]	[2, 17]	[0, 3]	[2, 12]	[0, 4]	[2, 13]	[0, 2]	[2, 12]	[8, 37]	[13, 70]	[7, 29]	[10, 55]
S4	[0, 4]	[1, 13]	[0, 2]	[1, 10]	[0, 2]	[1, 10]	–	–	[7, 37]	[10, 64]	[6, 29]	[10, 55]
S5	[0, 3]	[3, 14]	[0, 3]	[2, 11]	[0, 3]	[2, 12]	[0, 3]	[2, 11]	[2, 31]	[5, 57]	[3, 25]	[5, 45]
S6	[0, 4]	[2, 16]	[0, 3]	[2, 12]	[0, 3]	[2, 13]	[0, 3]	[1, 14]	[7, 35]	[12, 62]	[7, 27]	[9, 51]
S7	[0, 3]	[2, 14]	[0, 2]	[1, 7]	[0, 2]	[2, 12]	–	–	[6, 33]	[7, 59]	[5, 25]	[7, 49]
S8	[0, 4]	[2, 15]	[0, 2]	[2, 8]	[0, 3]	[2, 10]	[0, 2]	[1, 8]	[6, 34]	[9, 62]	[6, 27]	[8, 51]
S9	[0, 5]	[3, 18]	[0, 4]	[3, 13]	[0, 3]	[2, 12]	–	–	[3, 31]	[6, 58]	[3, 24]	[7, 47]
S10	[0, 4]	[2, 15]	[0, 3]	[2, 12]	[0, 2]	[2, 11]	–	–	[4, 33]	[9, 60]	[5, 25]	[8, 51]
S11	[0, 5]	[3, 18]	[0, 4]	[2, 12]	[0, 3]	[2, 11]	–	–	[5, 34]	[7, 61]	[4, 24]	[9, 47]
S12	[0, 5]	[3, 17]	[0, 4]	[2, 16]	[0, 3]	[2, 15]	[0, 3]	[2, 11]	[7, 37]	[13, 66]	[6, 27]	[11, 55]
S13	[0, 3]	[1, 13]	[0, 2]	[1, 7]	–	–	–	–	[5, 34]	[8, 62]	[2, 24]	[8, 45]
S14	[0, 3]	[2, 13]	[0, 2]	[2, 8]	–	–	–	–	[7, 36]	[10, 62]	[7, 29]	[11, 52]
Mass Media	NNP1		NNP2		NNP3		NNP4		NCH1		NCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
	[0, 5]	[3, 18]	[0, 4]	[3, 13]	[0, 5]	[3, 15]	[0, 5]	[3, 18]	[8, 37]	[13, 70]	[7, 23]	[9, 57]

Table 13: Optimal number of advertisements in different media for P1

Segments	RNP1		RNP2		RNP3		RNP4		RCH1		RCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
S1	1	12	15	65	20	70	1	11	36	92	39	85
S2	14	50	1	8	1	42	13	48	34	88	33	13
S3	24	90	16	9	20	75	15	8	39	94	38	83
S4	14	56	1	3	12	42	–	–	33	78	37	81
S5	1	7	1	5	18	76	1	4	31	8	25	7
S6	18	76	1	7	1	6	14	7	38	85	17	13
S7	12	65	11	75	14	72	–	–	31	68	31	68
S8	12	49	10	45	8	40	8	38	32	72	33	72
S9	18	76	14	11	1	8	–	–	5	64	29	8
S10	1	6	1	5	1	4	–	–	33	62	26	59
S11	1	9	1	7	1	6	–	–	34	66	27	62
S12	1	71	1	7	1	6	14	8	36	79	40	86
S13	1	4	1	3	–	–	–	–	31	64	25	57
S14	1	5	1	4	–	–	–	–	32	85	29	14
Mass Media	NNP1		NNP2		NNP3		NNP4		NCH1		NCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
	18	84	12	12	15	70	12	64	39	94	25	86

Table 14: Optimal number of advertisements in different media for P2

Segments	RNP1		RNP2		RNP3		RNP4		RCH1		RCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
S1	1	6	10	7	8	30	1	7	7	18	5	14
S2	1	30	1	3	1	4	6	26	38	16	4	13
S3	1	6	1	3	12	35	12	4	39	82	6	12
S4	1	32	0	2	0	25	–	–	29	64	35	12
S5	1	3	1	3	1	36	1	3	5	11	3	7
S6	1	5	1	4	1	4	1	3	33	15	7	10
S7	1	4	0	3	8	25	–	–	26	63	27	52
S8	9	27	0	4	6	23	6	3	28	64	31	55
S9	1	7	1	6	1	5	–	–	3	11	3	5
S10	1	4	0	3	0	2	–	–	27	10	4	8
S11	1	6	0	4	0	3	–	–	27	71	26	49
S12	1	4	0	2	0	1	8	2	36	14	33	12
S13	1	3	0	4	–	–	–	–	27	65	23	9
S14	1	4	0	3	–	–	–	–	33	14	33	16
Mass Media	NNP1		NNP2		NNP3		NNP4		NCH1		NCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
	12	49	0	9	12	48	1	9	39	18	33	16

Table 15: Optimal number of advertisements in different media for P3

Segments	RNP1		RNP2		RNP3		RNP4		RCH1		RCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
S1	1	8	7	40	8	38	1	6	39	75	33	65
S2	6	40	5	4	1	35	4	28	36	68	32	57
S3	10	4	0	40	9	32	6	3	38	78	34	62
S4	6	40	7	30	6	32	–	–	38	73	35	64
S5	1	4	0	3	7	32	1	3	31	50	3	7
S6	8	43	7	5	6	6	5	5	36	70	6	11
S7	6	5	4	25	5	27	–	–	33	55	25	49
S8	6	40	5	27	6	32	5	27	35	63	26	52
S9	0	7	1	7	0	5	–	–	31	52	3	7
S10	1	4	1	5	1	3	–	–	33	61	25	51
S11	1	7	0	5	1	5	–	–	34	57	26	55
S12	1	43	1	5	1	4	5	5	36	73	32	66
S13	1	5	0	4	–	–	–	–	34	59	21	48
S14	1	4	0	3	–	–	–	–	37	65	29	61
Mass Media	NNP1		NNP2		NNP3		NNP4		NCH1		NCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
	10	47	8	38	10	45	10	40	39	78	35	66

Table 16: Optimal number of advertisements in different media for P4

Segments	RNP1		RNP2		RNP3		RNP4		RCH1		RCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
S1	1	3	3	3	4	16	0	3	4	14	4	12
S2	0	18	0	1	3	1	2	13	35	14	3	8
S3	0	3	0	2	3	14	2	2	37	70	4	10
S4	3	15	0	1	2	13	–	–	37	64	26	11
S5	1	2	1	1	3	2	0	1	2	9	3	7
S6	5	3	1	2	1	3	3	2	32	63	3	9
S7	0	2	3	16	3	14	–	–	31	55	26	43
S8	3	16	2	3	2	15	1	13	31	60	27	46
S9	1	3	0	2	3	3	–	–	2	10	3	7
S10	1	1	1	2	1	1	–	–	33	13	3	7
S11	0	3	1	3	0	3	–	–	34	56	25	49
S12	1	3	1	3	3	2	3	18	33	14	24	12
S13	0	2	0	2	–	–	–	–	38	57	23	47
S14	0	2	0	1	–	–	–	–	33	12	26	10
Mass Media	NNP1		NNP2		NNP3		NNP4		NCH1		NCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
	5	20	3	20	4	18	4	3	37	14	27	12

Table 17: Optimal number of advertisements in different media for P5

Segments	RNP1		RNP2		RNP3		RNP4		RCH1		RCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
S1	0	3	0	2	5	17	0	2	8	12	7	11
S2	0	2	0	2	0	2	3	6	38	10	6	10
S3	0	2	0	2	4	13	2	2	37	70	7	10
S4	0	1	0	1	2	10	–	–	37	64	29	10
S5	0	3	0	2	3	2	0	2	2	5	3	5
S6	0	2	0	2	0	2	0	1	35	12	7	9
S7	0	2	0	1	2	12	–	–	33	57	25	49
S8	4	15	0	2	3	10	2	8	34	9	27	51
S9	0	3	0	3	0	2	–	–	3	6	3	7
S10	0	2	0	2	0	2	–	–	33	9	5	8
S11	0	3	0	2	0	2	–	–	34	61	24	47
S12	0	3	0	2	0	2	3	2	37	13	27	11
S13	0	1	0	1	–	–	–	–	34	62	24	8
S14	0	2	0	2	–	–	–	–	7	10	7	11
Mass Media	NNP1		NNP2		NNP3		NNP4		NCH1		NCH2	
	FP	OP	FP	OP	FP	OP	FP	OP	PT	OT	PT	OT
	0	3	0	3	5	3	0	3	37	13	23	9

Acknowledgment

The authors are very much thankful to the Editors and anonymous referees for their valuable comments and suggestions which have helped in revising the contents and bringing the paper to the current form.

References

[1] F M. Bass and R. T. Lonsdale, *An exploration of linear programming in media selection*, Journal of Marketing Research (1966), 179-188.

[2] U. K. Bhattacharya, *A chance constraints goal programming model for the advertising planning problem*, European Journal of Operational Research 192 (2009), 382-395.

[3] S. Broadbent, *Media planning and computers by 1970: A review of the use of mathematical models in media planning*, Applied Statistics (1966), 234-256.

[4] A. Buratto, L. Grosset and B. Viscolani, *Advertising a new product in a segmented market*, European Journal of Operational Research 175 (2006), 1262-1267.

[5] P. J. Danaher and R. T. Rust, *Determining the optimal return on investment for an advertising campaign*, European Journal of Operational Research 95 (1996), 511-521.

[6] P. Doyle and J. Saunders, *Multiproduct advertising budgeting*, Marketing Science 9 (1990), 97-113.

[7] F. R. Dwyer and J. R. Evans, *A branch and bound algorithm for the list selection problem in direct mail advertising*, Management Science 27 (1981), 658-667.

[8] L. Grosset and B. Viscolani, *Advertising in a segmented market: comparison of media choices*, IMA Journal of Management Mathematics 19 (2008), 219-226.

[9] T. H. Hsu, T. N. Tsai and P. L. Chiang, *Selection of the optimum promotion mix by integrating a fuzzy linguistic decision model with genetic algorithms*, Information Sciences 179 (2009), 41-52.

[10] P. C. Jha, R. Aggarwal and A. Gupta, *Optimal media planning for multi-products in segmented market*, Applied Mathematics and Computation 217 (2011), 6802-6818.

[11] P.C. Jha, S. Aggarwal, A. Gupta and R. Sarker, *Multi-criteria media mix decision model for advertising a single product with segment specific and mass media*, to appear in Journal of Industrial and Management Optimisation (2015).

[12] P. Korhonen, S. C. Narula and J. Wallenius, *An evolutionary approach to decision-making, with an application to media selection*, Mathematical and Computer Modelling 12 (1989), 1239-1244.

[13] N.K. Kwak, C.W. Lee and J.H. Kim, *An MCDM model for media selection in the dual consumer/industrial market*, European Journal of Operational Research 166 (2005), 255-265.

[14] J. D. Little and L. M. Lodish, *A media planning calculus*, Operations Research 17 (1969), 1-35.

[15] R. T. Marler and J.S. Arora, *The weighted sum method for multi-objective optimization: new insights*, Structural and multidisciplinary optimization 41 (2010), 853-862.

[16] C.B. Royo, H. Zhang, L.A. Blanco and J. Almagro, *Multistage multiproduct advertising budgeting*, European Journal of Operational Research 225 (2013), 179-188.

[17] R.F. Saen, *Media selection in the presence of flexible factors and imprecise data*, Journal of the Operational Research Society 62 (2011), 1695-1703.

[18] B. Viscolani, *Advertising decisions for a segmented market*, Optimization 58 (2009), 469-477.

[19] W. Wells, J. Burnett and S.E. Moriarty, *Advertising: Principles and Practice*, Seventh Edition Pearson Prentice Hall, 2005.

[20] F.S. Zufryden, *Media scheduling: a stochastic dynamic model approach*, Management Science 19 (1973), 1395-1406.

[21] F.S. Zufryden, *Media scheduling and solution approaches*, Operational Research Quarterly (1975), 283-295.