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Multiple Ways of Processing in Questionnaires

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Abstract

In social sciences when questionnaires are used, there is a new tool, the bar instead of Likert scale. The bar has been suggested by Vougiouklis & Vougiouklis in 2008, who have proposed the replacement of Likert scales, usually used in questionnaires, with bar. This new tool, gives the opportunity to researchers to elaborate the questionnaires in different ways, depending on the filled questionnaires and of course on the problem. Moreover, we improve the procedure of the filling the questionnaires, using the bar instead of Likert scale, on computers where we write down automatically the results, so they are ready for research. This new kind of elaboration is being applied on data obtained by a survey, studying the new results. The hyperstructure theory is being related with questionnaires and we study the obtained hyperstructures, which are used as an organized device of the problem and we focus on special problems.
Keywords: hyperstructures; questionnaires; bar;
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1 Basic definitions

The main object of this paper is the class of hyperstructures called H_v-structures introduced in 1990 [17], which satisfy the weak axioms where the non-empty intersection replaces the equality. Some basic definitions are the following:

In a set H equipped with a hyperoperation (abbreviation *hyperoperation = hope)* $\cdot : H \times H \to P(H) - \{\varnothing\}$, we abbreviate by *WASS* the *weak associativity*: $(xy)z \cap x(yz) \neq \varnothing, \forall x, y, z \in H$ and by *COW* the *weak commutativity*: $xy \cap yx \neq \varnothing, \forall x, y \in H.$

The hyperstructure $(H, \cdot)$ is called an *H_v-semigroup* if it is *WASS*, it is called *H_v-group* if it is reproductive H_v-semigroup, i.e., $xH = Hx = H, \forall x \in H.$

Motivation. In the classical theory the quotient of a group with respect to an invariant subgroup is a group. F. Marty from 1934, states that, the quotient of a group with respect to any subgroup is a hypergroup. Finally, the quotient of a group with respect to any partition (or equivalently to any equivalence relation) is an H_v-group. This is the motivation to introduce the H_v-structures [17], [18].

$(R, +, \cdot)$ is called an *H_v-ring* if $(+)$ and $(\cdot)$ are WASS, the reproduction axiom is valid for $(+)$ and $(\cdot)$ is *weak distributive* with respect to $(+)$:

$$x(y + z) \cap (xy + xz) \neq \varnothing,\ (x + y)z \cap (xz + yz) \neq \varnothing,\ \forall x, y, z \in R.$$

Let $(R, +, \cdot)$ be an H_v-ring, $(M, +)$ be a COW H_v-group and there exists an external hope

$$\cdot : R \times M \to P(M) : (a, x) \to ax$$

such that $\forall a, b \in R$ and $\forall x, y \in M$ we have

$$a(x + y) \cap (ax + ay) \neq \varnothing,\ (a + b)x \cap (ax + bx) \neq \varnothing,\ (ab)x \cap a(bx) \neq \varnothing,$$

then M is called an *H_v-module* over F. In the case of an H_v-field F, which is defined later, instead of an H_v-ring R, then the *H_v-vector space* is defined.

For more definitions and applications on H_v-structures one can see [2], [3], [4], [5], [6], [10], [14], [16], [18].

The main tool to study hyperstructures is the fundamental relation. In 1970 M. Koscas defined in hypergroups the relation β and its transitive closure $\beta*$. This relation connects the hyperstructures with the corresponding classical structures and is defined in H_v-groups as well. T. Vougiouklis introduced the $\gamma*$ and $\epsilon*$ relations, which are defined, in H_v-rings and H_v-vector spaces, respectively [17]. He also named all these relations $\beta*$, $\gamma*$ and $\epsilon*$, fundamental relations because they play very important role to the study of hyperstructures especially in the representation theory of them. For similar relations see [18], [22], [4].

Definition 1.1. *The fundamental relations β^*, γ^* and ϵ^*, are defined, in H_v-groups, H_v-rings and H_v-vector space, respectively, as the smallest equivalences so that the quotient would be group, ring and vector space, respectively.*

Specifying the above motivation we remark the following: Let $(G, \cdot)$ be a group and R be an equivalence relation (or a partition) in G, then $(G/R, \cdot)$ is an H_v-group, therefore we have the quotient $(G/R, \cdot)/\beta^*$ which is a group, the fundamental one. Remark that the classes of the fundamental group $(G/R, \cdot)/\beta^*$ are a union of some of the R-classes. Otherwise, the $(G/R, \cdot)/\beta^*$ has elements classes of G where they form a partition which classes are larger than the classes of the original partition R.

The way to find the fundamental classes is given by the following [17], [20], [21], [22]:

Theorem 1.1. *Let $(\boldsymbol{H}, \cdot)$ be an H_v-group and denote by **U** the set of all finite products of elements of H. We define the relation β in H by setting $x\beta y$ iff $\{x, y\} \subset$ **u** where **u** $\in$ **U**. Then β^* is the transitive closure of β.*

A well known and large class of hopes is given as follows [15], [18], [12]:

Let $(G, \cdot)$ be a groupoid then for every $P \subset G$, $P \neq \varnothing$, we define the following hopes called P-*hopes*: for all $x, y \in G$

$$\underline{P} : x\underline{P}y = (xP)y \cup x(Py),$$

$$\underline{P}_r : x\underline{P}_r y = (xy)P \cup x(yP), \underline{P}_l : x\underline{P}_l y = (Px)y \cup P(xy).$$

The $(G, \underline{P})$,$(G, \underline{P}_r)$ and $(G, \underline{P}_l)$ are called P-*hyperstructures*. The most usual case is if $(G, \cdot)$ is semigroup, then $x\underline{P}y = (xP)y \cup x(Py) = xPy$ and $(G, \underline{P})$ is a semihypergroup. We do not know what hyperstructures are $(G, \underline{P}_r)$ and $(G, \underline{P}_l)$. In some cases, depending on the choice of P, the $(G, \underline{P}_r)$ and $(G, \underline{P}_l)$ can be associative or WASS. If more operations are defined in G, then for each operation several P-hopes can be defined.

2 The bar in questionnaires

Last decades hyperstructures seem to have a variety of application not only in mathematics, but also in many other sciences [1], [2], [9], [13], [19], [25], including the social ones.

An important application which can be used in social sciences is the combination of hyperstructure theory with fuzzy theory, by the replacement of the Likert Scale by the Bar. The suggestion is the following [9]:

Definition 2.1. *In every question substitute the Likert scale with 'the bar' whose poles are defined with '0' on the left end, and '1' on the right end:*

0 ________________ 1

The subjects/participants are asked instead of deciding and checking a specific grade on the scale, to cut the bar at any point s/he feels expresses her/his answer to the specific question.

The use of the bar of Vougiouklis & Vougiouklis instead of a scale of Likert has several advantages during both the filling-in and the research processing. The final suggested length of the bar, according to the Golden Ratio, is 6.2cm, see [7], [8], [23]. Several advantages on the use of the bar instead of scale one can find in [9].

There are certain advantages concerning the use of the bar comparing to the Likert-scale during all stages of developing, filling and processing. The most important maybe advantage of the bar though is the fact that it provides the potential for different types of processing. Therefore, it gives the initiative to the researcher to explore if the given answers follow a special kind of distribution, as Gauss or parabola for example. In this case the researcher has the opportunity to correct any kind of tendency appeared, for more accurate results. A possibility of choosing among a number of alternatives is offered, by using fuzzy logic in the same way as it has already been done combining mathematical models with multivalued operation.

3 Evaluation

The following survey is based on the described theory that has been established in the department of Elementary Education of Democritus University of Thrace, in the frame of course evaluation, and especially of Algebra of first semester. The sample was 152 students, who were asked to answer questions related to the course, to the teacher and to the teaching of the course. The questionnaire used the bar, which was firstly divided into six equal-segments according to the first questionnaires which used a six-grade Likert scale.

The use of histograms helped in order to explore if the answers follow any kind of distribution or they present any kind of tendency. In this case, the bar is redivided into equal-area segments, for more accurate results.

The filling questionnaire procedure has been accomplished using computers, and especially a software developed for this purpose. Using this software the results can automatically be transferred for research elaboration. There are several advantages of the bar, the only disadvantage is to the data collection for further elaboration. The implemented program has been developed to overcome the problems raised during the data collection, inputting of data from questionnaires to processing. It eliminates the time of data collection, transferring data directly for any kind of elaboration [10].

3.1 Question category: Course

The first question category is about the course and consists of 9 questions. Gathering the answers on the bar, it is obvious that there is an upward trend, a fact that becomes even more obvious on the following histograms:

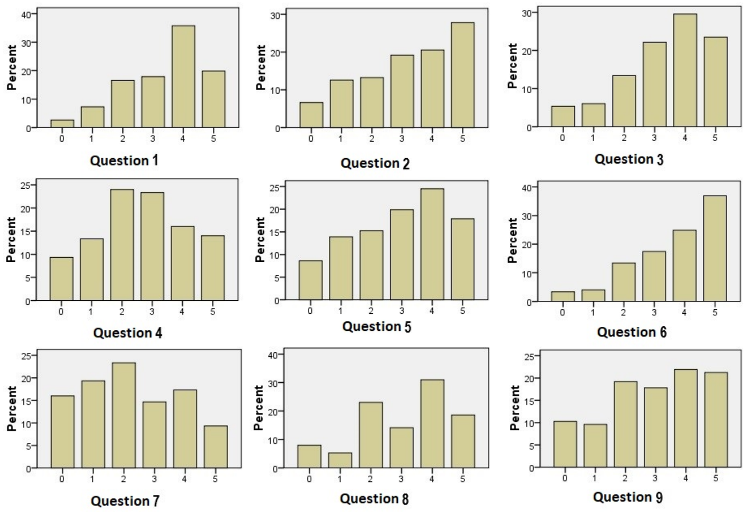

Figure 1: Question Category: Course

In the majority of the questions, on can notice a vast concentration in the last 2 or 3 grades and in some of the questions this is more obvious, as the concentration is the last grades is much higher.

More specifically, question number 1,2,3,5,6 and 9 present the biggest concentration rate in the last 2 grades, while in question 4, there is a remarkable concentration in the center of the bar.

Based mainly on this histograms and some other parameters that have been obtained by the correspondence analysis, the answers of questions 2,4 and 6 will be redistributed on the bar, which will be now divided in equal-area segments:
For question 2, the bar will be divided in 6 equal-ares segments according to the increasing-low parabola
For question 4, the be will be divided in 6 equal-ares segments according to the

Gauss distribution and,
for question 6, the bar will be divided in 6 equal-ares segments according to the increasing-upper parabola.

The new obtained histograms are the following:

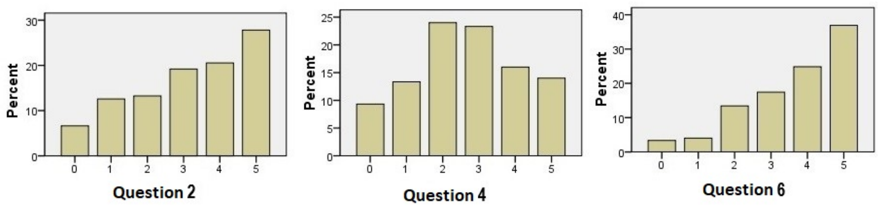

Figure 2: Equal segments

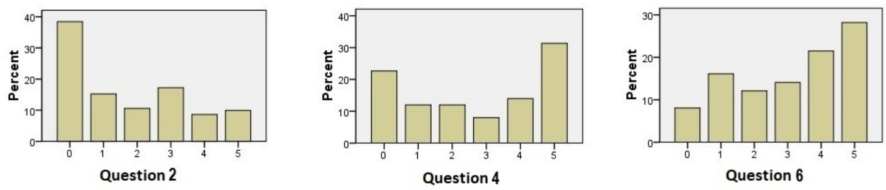

Figure 3: Equal-area segments

One can see that the use of the upper-low parabola on question 2, reveals that in question 2, more than 50% was concentrated at the last two grades of the scale, but with the new distribution there exists a tendency to the first grades. For questions number 4 and 6 the new histograms give no more information.

3.2 Question category:Teaching

The second question category consists of 6 questions relevant to the 'teaching of the lesson' and to the extend that some factors contributed to its comprehension. The related histograms are the following:

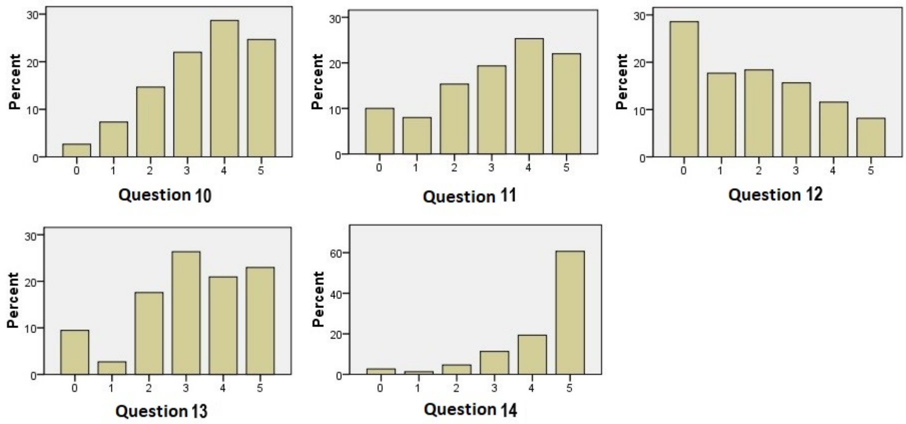

Figure 4: Question Category: Teaching

In this question category, there is also a general upward trend - with the exception of question number 12. More specifically, in questions 10, 11 and 13 the biggest concentration rate appears in the last grades, in opposition to question 12, in which the biggest rate appears in the first 2 grades. Question 14 present a vast rate in the last grade.

So, for question 12 the bar will be divided in equal-area segments according to decreasing-low parabola and for question 14, according to increasing upper parabola. The new obtained histograms are the following:

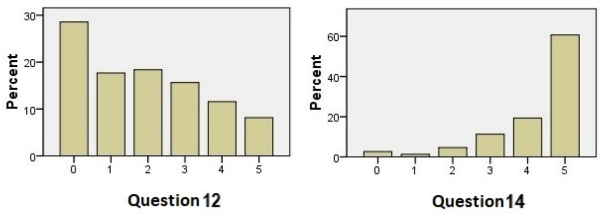

Figure 5: Equal segments

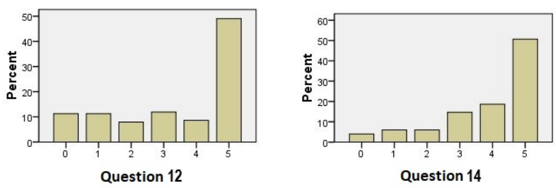

Figure 6: Equal-area segments

From the new distribution, the bar gives different results for question 12, as it reveals that the increasing-low trend not exists anymore. This fact is very important for the researcher as it gives him information he couldn't have only through the first subdivision of the bar. The second question leads to the same results.

3.3 Question category:Teacher

In the penultimate category there are 3 questions concerning the teacher.

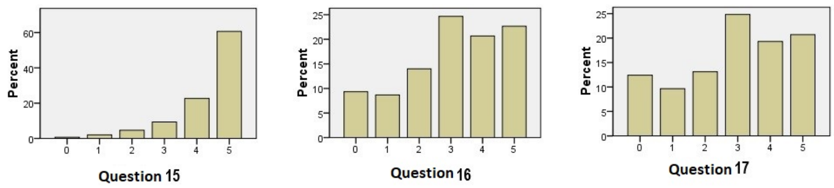

Figure 7: Question Category: Teacher

Once again, there is an obvious trend to the respondents according to the histograms, even more remarkable in the first question: there is a vast concentration rate in the last grade. Because of that, the bar will be divided into equal area segments following the increasing-upper parabola:

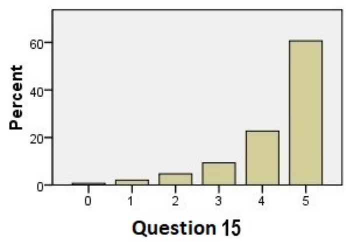

Figure 8: Equal segments

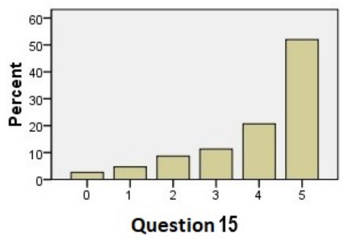

Figure 9: Equal-area segments

The new histogram is just confirming the first result.

4 Questionnares and Hyperstructures

In the research processing suppose that we want to use Likert scale through the bar dividing the continuum [0,62] into equal segments and into equal area division of Gauss distribution [9] or parabola distribution [24]. If we consider that the continuum [0,62] is divided into n segments, we can number the n segments starting with 0. We can define a hope on the segments as follows [11] :

Definition 4.1.

$$\textit{For all } i, j \in \{0, 1, ..., n-1\}, \textit{ if } e_n \textit{ the } n^{th} \textit{ segment ,then}$$

$$e_i \oplus e_j = \{e_k : x + y \in e_k, \forall x \in e_i, y \in e_j\}$$

Therefore, we can consider as an organized device the group $(Z_n, \oplus)$ where n the number of segments, as we have a modulo like hyperoperation. The multiplication tables obtained by this hyperoperation , referred in mm, are the following:

6 equal segments

0:$[0, 10.33]$, **1**:$(10.33, 20.66]$, **2**:$(20.66, 30.99]$, **3**:$(30.99, 41.32]$, **4**:$(41.32, 51.65]$, **5**:$(51.65, 62]$

$\oplus$	**0**	**1**	**2**	**3**	**4**	**5**
0	0,1	1,2	2,3	3,4	4,5	0,5
1	1,2	2,3	3,4	4,5	0,5	0,1
2	2,3	3,4	4,5	0,5	0,1	1,2
3	3,4	4,5	0,5	0,1	1,2	2,3
4	4,5	0,5	0,1	1,2	2,3	3,4
5	0,5	0,1	1,2	2,3	3,4	4,5

6 equal-area segments (Gauss distribution)

0:$[0, 22]$, **1**:$(22, 27]$, **2**:$(27, 31]$, **3**:$(31, 35]$, **4**:$(35, 40]$, **5**:$(40, 62]$,

$\oplus$	**0**	**1**	**2**	**3**	**4**	**5**
0	0,1,2,3,4,5	1,2,3,4,5	2,3,4,5	3,4,5	4,5	0,5
1	1,2,3,4,5	5	5	5	0,5	0,1
2	2,3,4,5	5	5	0,5	0	0,1,2
3	3,4,5	5	0,5	0	0	0,1,2,3
4	4,5	0,5	0	0	0	0,1,2,3,4
5	0,5	0,1	0,1,2	0,1,2,3	0,1,2,3,4	0,1,2,3,4,5

Increasing Low parabola $\mathbf{x = y^2}$

0:$[0, 34]$, **1**:$(34, 43]$, **2**:$(43, 49]$, **3**:$(49, 54]$, **4**:$(54, 58]$, **5**:$(58, 62]$

$\oplus$	**0**	**1**	**2**	**3**	**4**	**5**
0	0,1,2,3,4,5	0,1,2,3,4,5	0,2,3,4,5	0,3,4,5	0,4,5	0,5
1	0,1,2,3,4,5	0	0	0,1	0,1	0,1
2	0,2,3,4,5	0	0,1	0,1	1,2	1,2
3	0,3,4,5	0,1	0,1	1,2	1,2,3	2,3
4	0,4,5	0,1	1,2	1,2,3	2,3	3,4
5	0,5	0,1	1,2	2,3	3,4	4,5

Increasing Upper parabola $\mathbf{1 - y = (1 - x)^2}$

0:$[0, 22]$, **1**:$(22, 32]$, **2**:$(32, 40]$, **3**:$(40, 48]$, **4**:$(48, 55]$, **5**:$(55, 62]$

$\oplus$	**0**	**1**	**2**	**3**	**4**	**5**
0	0,1,2,3,	1,2,3,4	2,3,4,5	0,3,4,5	4,5,0	0,5
1	1,2,3,4	0,3,4,5	0,4,5	0	0,1	0,1
2	2,3,4,5	0,4,5	0	0,1	0,1,2	1,2
3	0,3,4,5	0	0,1	0,1,2	1,2,3	2,3
4	0,4,5	0,1	0,1,2	1,2,3	2,3	3,4
5	0,5	0,1	1,2	2,3	3,4	4,5

Decreasing low parabola $\mathbf{y = (1 - x)^2}$

0:$[0, 4]$, **1**:$(4, 8]$, **2**:$(8, 13]$, **3**:$(13, 19]$, **4**:$(19, 28]$, **5**:$(28, 62]$

$\oplus$	**0**	**1**	**2**	**3**	**4**	**5**
0	0,1	1,2	2,3	3,4	4,5	0,5
1	1,2	2,3	2,3,4	3,4	4,5	0,1,5
2	2,3	2,3,4	3,4	4,5	4,5	0,1,2,5
3	3,4	3,4	4,5	4,5	5	0,1,2,3,5
4	4,5	4,5	4,5	5	5	0,1,2,3,4,5
5	0,5	0,1,5	0,1,2,5	0,1,2,3,5	0,1,2,3,4,5	0,1,2,3,4,5

Decreasing upper parabola $\mathbf{1 - y = x^2}$

0:$[0, 7]$, **1**:$(7, 14]$, **2**:$(14, 22]$, **3**:$(22, 30]$, **4**:$(30, 40]$, **5**:$(40, 62]$

$\oplus$	**0**	**1**	**2**	**3**	**4**	**5**
0	0,1	1,2	2,3	3,4	4,5	0,5
1	1,2	2,3	2,3,4	3,4,5	4,5	0,1,5
2	2,3	2,3,4	3,4,5	4,5	5	0,1,2,5
3	3,4	3,4,5	4,5	5	0,1,5	0,1,2,3
4	4,5	4,5	5	0,1,5	0,1,2,5	1,2,3,4
5	0,5	0,1,5	0,1,2,5	0,1,2,3	1,2,3,4	2,3,4,5

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