

An Analysis of Loan Repayment Plans According to the Bank Customer Profile

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Abstract

It has been demonstrated that there exists a general preference for improvement in loan repayment plans in the way that people prefer decreasing sequences of installments as tested by Hoelzl et al. (2011). Moreover, they also demonstrated that there exists a positive correlation between financial capability and financial literacy when it is given the possibility of having a gain by investing a part of the available money. In these cases, the most financial literate consumers showed a preference for increasing loan plans instead of decreasing ones. In this vein, independently of the level of the borrowers' risk profile, we suggest that an *ad hoc* offer should be made to the customers taking into account these two characteristics by distinguishing three different levels for both personal traits: low, medium and high. Thus, we have analyzed the interest rate which makes both the decreasing and the increasing loan plans indifferent when considering that the option to invest part of the money in savings products is given. Moreover, the analysis has been carried out by considering that the loan repaid principal is variable either in arithmetic progression or in geometric progression. Thus, regarding the main repayment plans offered by banks we have analyzed which one fits better to the defined customer's profile.

Keywords: Loan Plan, Sequence of Payments, Financial Literacy, Financial Capability, Customer's Profile.

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1. Introduction

Traditionally, the offer of loans by banks has been focused on the analysis and description of the main repayment schedules and the calculation of the effective interest rate of the corresponding operation. Thus, given the rate of interest, the client may indistinctly choose a loan with constant installments, constant repaid principal or payments variable in arithmetic progression, among others. From a theoretical point of view, all these repayment methods are equivalent because the interest rate (which is the price of the operation) is the same.

However, in our opinion, neither the loan offer nor the choice of a repayment schedule should be indifferent for banks and costumers, respectively. In effect, it is necessary to previously analyze the costumer profile and the needs of the bank in order to try to satisfy both requirements. Accordingly, we are going to first examine both the main elements defining the costumer profile and the variables of interest for the management of banking institutions.

This paper starts from the conclusions obtained by Hoelzl *et al.* (2011) consisting in the proposal of some suitable advises to different consumption groups based on their preferences on loan repayment plans considered as sequences of installments. To do that, this paper is focused on the results obtained from the three studies implemented by these scholars, especially in the second one. Thus, in order to make an *ad hoc* offer to customers, the financial capability and the level of financial literacy could be the main two inputs which determine if borrowers choose falling, constant or rising installments, according to their preferences. In particular, as pointed out by Hoelzl *et al.* (2011), there is a positive correlation between the rising profile and the financial education level when a part of the monthly available cash can be saved.

In spite of the fact that financial security and consumer self-protection also require to have a good understanding of financial topics (Kozup and Hogarth, 2008), we are going to only focus on the financial literacy. Thus, with respect to the concept of financial literacy, the following definition has been considered: “Financial literacy is a measure of the degree to which one understands key financial concepts and possesses the ability and confidence to manage personal finances through appropriate, short-term decision-making and sound, long-range financial planning, while mindful of life events and changing economic conditions” (Remund, 2010). Therefore, it has been demonstrated that financial literacy is a factor which determines the financial decisions taken by customers when they have to define their credit portfolio (Disney and Gathergood, 2013). In this regard, a low level of financial literacy has direct repercussions in self-control and over-indebtedness (Gathergood, 2012). That is the reason whereby it

has been analyzed how this characteristic could be measured (Huston, 2010). Moreover, there is a relationship between financial satisfaction and financial capability for those consumers who have certain level of financial education and avoid risky decisions (Xiao *et al.*, 2014). In this vein, the effects of financial education in financial capability have been analyzed (Xiao and O’Neill, 2016). Thus, considering that financial capability is “the ability to manage and take control of their finance”, the age and the unemployment status are the two factors with most impact on the financial capability (Taylor, 2011). Different analyses show that younger adults have more financial difficulties and make worse financial plans (Atkinson *et al.*, 2006; Kempson *et al.*, 2004). Therefore, with respect to the customer profile, there are three significant variables of interest for banks:

1. The *payment capability* which can be defined as the financial potential to face the installments derived from the financial operation. Usually, it is stated that the sum of all installments corresponding to the same client must be between 30% and 40% of his/her overall incomes.
2. The *risk* which is the probability of default in the operation. The possibility of failure in payments can be covered by personal or real guarantees.
3. The *financial literacy* which implies a financial knowledge sufficient to adequately invest the extra money and an understanding of the meaning of the effective rate of the operation. On the contrary, a customer with a low financial training does not have enough financial knowledge to invest and will analyze the total amount of interests instead of the effective rate of the operation.

In our opinion, these three variables are related with the loan schedule which is most suitable for a specific borrower. Table 1 shows the relationship of variables #1 and #3 with the loan repayment plan, where the symbol \uparrow means that the principal repaid must be increasing in arithmetic progression, whilst \downarrow denotes that the plan must be decreasing. The variable #2 will be incorporated to the analysis in Section 2.

		Financial literacy		
		High	Medium	Low
Payment capability	High	\uparrow	\downarrow	\downarrow
	Medium	\uparrow	\uparrow	\downarrow
	Low	\uparrow	\uparrow	\uparrow

Table 1. Different customer profiles.

Figure 1 helps to understand the existing relationship between variables #1 and #3 and the three identified profiles (decreasing, constant and increasing).

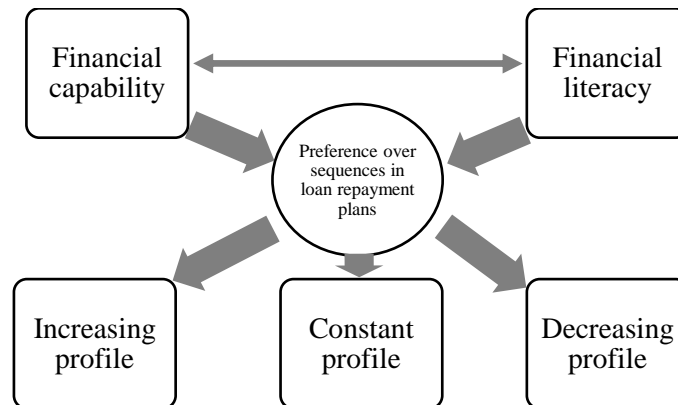


Figure 1. Relationship between variables #1 and #3 and the three considered profiles: decreasing, constant and increasing.

The organization of this paper is as follows. Section 2 shows an analysis of the interest rate according to the loan repayment plan. In Section 3 and Section 4 the main repayment plans offered by banks are analyzed by considering that the principal is repaid in arithmetic progression and geometric progression, respectively. In Section 5, the relationship between variables #1 and #3 and the three profiles is remade by considering the repayment plans obtained in Section 3 and 4. Finally, Section 6 summarizes and concludes.

2. An Analysis of the Interest Rate According to the Loan Repayment Plan

In general, in a loan with a principal C_0 to be repaid in n periods, the repayment plan can follow either a constant or a variable repaid principal (increasing or decreasing arithmetic progression). Figure 2 displays these three possible situations for a loan to be repaid in four periods where the principal is €20.

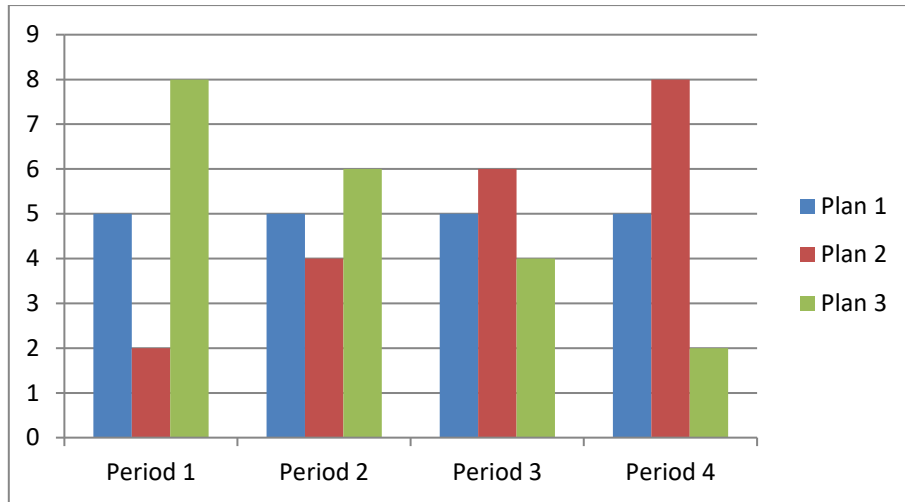


Figure 2. Evolution of the repaid principal variable in arithmetic progression: constant (in blue), increasing (in red) or decreasing (in green).

In general, the following identity holds for the three repayment plans:

$$\sum_{s=1}^n A_s = C_0.$$

Although the financial cost of these three alternatives is the same (more specifically, the interest rate i), it has been experimentally shown that people prefer the third option in which the repaid principal is a decreasing arithmetic progression, all terms amounting C_0 . Maybe this is because people prefer to leave the best for last (Kahneman *et al.*, 1993) or because the total burden in this option is lower than the corresponding one to other plans. This is consistent with a borrower characterized by a low financial training or a high financial capability. Moreover, this type of clients supposes a small risk for banking institutions (see variable #2 in Section 1) since the most part of the principal is repaid at the beginning of the operation. Therefore, a good initiative of banks could be to offer a smaller rate of interest because these clients pay more attention to the total amount of interests instead of the effective rate of the operation. The opposite can be said for clients with a high financial capability or a high financial training because they prefer a repaid principal increasing in arithmetic progression which implies a higher risk for banks and the possibility to invest the extra money. Therefore, this type of clients can support a greater rate of interest.

Therefore, the following question could be addressed: what is the new interest rate, say i' ($i' > i$), such that plan 2 is indifferent to the plan 3? If the difference involved in plan 3 is d ($d < 0$) and the common difference of the second plan is d' ($d' = 0$ or $d' > 0$), then we can require that the sum of payments corresponding to plan 3 must coincide with the sum of payments corresponding to plan 2 minus the amount A earned in the investments. As the aggregated principal repaid in both plans must coincide with C_0 , we can propose the following equality between the aggregated interest due in both plans:

$$\sum_{s=1}^n I_s = \sum_{s=1}^n I'_s - A,$$

or equivalently,

$$i \sum_{s=1}^n \left[C_0 - (s-1)A - \frac{(s-1)(s-2)}{2} d \right] = i' \sum_{s=1}^n \left[C_0 - (s-1)A' - \frac{(s-1)(s-2)}{2} d' \right] - A$$

As $A = \frac{C_0}{n} - \frac{n-1}{2}d$ and $A' = \frac{C_0}{n} - \frac{n-1}{2}d'$, then we can write:

$$\begin{aligned} & i \sum_{s=1}^n \left[C_0 - (s-1)C_0 + \frac{(s-1)(n-1)}{2}d - \frac{(s-1)(s-2)}{2}d \right] \\ &= i' \sum_{s=1}^n \left[C_0 - (s-1)C_0 + \frac{(s-1)(n-1)}{2}d' - \frac{(s-1)(s-2)}{2}d' \right] - A. \end{aligned}$$

Finally, as $n-1 > s-2$, for every $s = 1, 2, \dots, n$, we can deduce that $i' > i$.

As indicated in sections 1 and 2, the loan repayment schedule is conditioned by the customer profile and the risk supported by the banking entity. Therefore, Section 3 will be devoted to deduce the different repayment plans of a loan starting from the different variability of the repaid principal.

3. The Principal Repaid is Variable in Arithmetic Progression

In this case, the principal repaid is an arithmetic progression whose first term is A and whose common difference is d :

- $A_1 = A$,
- $A_2 = A + d$,
- $A_3 = A + 2d$,
- \vdots
- $A_n = A + (n-1)d$.

Let us analyze the structure of the periodic payments:

- $a_1 = A + C_0i$,
- $a_2 = (A + d) + (C_0 - A)i$
 $= (A + C_0i) + (d - Ai)$
 $= a_1 + d - Ai$,
- $a_3 = (A + 2d) + (C_0 - 2A - d)i$
 $= a_2 + d - (A + d)i$
 $= a_2 + d - A_2i$,
- $a_4 = (A + 3d) + (C_0 - 3A - 3d)i$
 $= a_3 + d - (A + 2d)i$
 $= a_3 + d - A_3i$,
- \vdots
- In general, $a_s = a_{s-1} + d - A_{s-1}i$.

This series is an arithmetic progression of second order. In effect,

- $d_{k-1} := a_k - a_{k-1} = d - A_{k-1}i$, and
- $d_k := a_{k+1} - a_k = d - A_ki$.

Therefore, $d_k - d_{k-1} = -di$. The general solution of an arithmetic progression of second order is:

$$a_s = \frac{r}{2}s^2 + \left(d_1 - \frac{3r}{2}\right)s + (r - d_1 + a_1), \quad (1)$$

where r is the common difference of $d_k - d_{k-1}$, where $d_k - d_{k-1} = -di$. By applying Equation (1) to our concrete case, it would remain:

$$a_s = -\frac{di}{2}s^2 + \left(d - Ai + \frac{3di}{2}\right)s + (-di - d + Ai + A + C_0i). \quad (2)$$

The outstanding principal at time s is:

$$C_s = C_0 - sA - \frac{(1+s-1)(s-1)}{2}d = C_0 - sA - \frac{s(s-1)}{2}d.$$

Therefore, the structure of the interest due is the following:

$$\begin{aligned} I_s &= C_{s-1}i = \left[C_0 - (s-1)A - \frac{(s-1)(s-2)}{2}d \right]i \\ &= C_0i - (s-1)Ai - \frac{di}{2}(s^2 - 3s + 2) \\ &= -\frac{di}{2}s^2 + \left(-Ai + \frac{3di}{2}\right)s + (C_0i + Ai - di). \end{aligned}$$

As expected, observe that $a_s = A_s + I_s$. Finally, Table 2 shows the repayment plan of this loan category.

Period	Payment a_s	Interest due I_s	Principal repaid A_s	Outstanding principal C_s
0	-	-	-	C_0
1	$a_1 = A + C_0i$	$I_1 = C_0i_1$	$A_1 = A$	$C_1 = C_0 - A$
2	$a_2 = a_1 + d - Ai$	$I_2 = C_1i_2$	$A_2 = A + d$	$C_2 = C_0 - A - d$
\vdots	\vdots	\vdots	\vdots	\vdots
n	$a_n = a_{n-1} + d - A_{n-1}i$	$I_n = C_{n-1}i_n$	$A_n = A + (n-1)d$	$C_n = 0$

Table 2. Repayment plan of a loan where the principal repaid varies in arithmetic progression.

A special case is when $d = 0$ in whose case a constant repayment plan is obtained (see Table 3).

Period	Payment a_s	Interest due I_s	Principal repaid A_s	Outstanding principal C_s
0	-	-	-	C_0
1	$a_1 = A + C_0i$	$I_1 = C_0i_1$	$A_1 = A$	$C_1 = C_0 - A$
2	$a_2 = a_1 - Ai$	$I_2 = C_1i_2$	$A_2 = A$	$C_2 = C_0 - A$
\vdots	\vdots	\vdots	\vdots	\vdots
n	$a_n = a_{n-1} - A_{n-1}i$	$I_n = C_{n-1}i_n$	$A_n = A$	$C_n = 0$

Table 3. Repayment plan of a loan where the principal repaid varies in arithmetic progression where $d = 0$.

Moreover, if $A = 0$, the American repayment plan can be obtained (see Table 4).

Period	Payment a_s	Interest due I_s	Principal repaid A_s	Outstanding principal C_s
0	-	-	-	C_0
1	$a_1 = I_1$	$I_1 = C_0i_1$	$A_1 = 0$	$C_1 = C_0$
2	$a_2 = I_2$	$I_2 = C_0i_2$	$A_2 = 0$	$C_2 = C_0$
\vdots	\vdots	\vdots	\vdots	\vdots
n	$a_n = I_n$	$I_n = C_0i_n$	$A_n = C_0$	$C_n = 0$

Table 4. Repayment plan of a loan where the principal repaid varies in arithmetic progression where $d = 0$ and $A = 0$.

4. The Principal Repaid is Variable in Geometric Progression

In this case, the principal repaid is a geometric progression whose first term is A and whose common ratio is r :

- $A_1 = A$,
- $A_2 = Ar$,
- $A_3 = Ar^2$,
- \vdots
- $A_n = Ar^{n-1}$.

Let us analyze the structure of the periodic payments:

- $a_1 = A + C_0i$,
- $a_2 = Ar + (C_0 - A)i$
 $= Ar + C_0ir - C_0ir + (C_0 - A)i$
 $= a_1r + (C_0 - A - C_0r)i$,
- $a_3 = Ar^2 + (C_0 - A - Ar)i$
 $= Ar^2 + (C_0 - A)ir - (C_0 - A)ir + (C_0 - A - Ar)i$
 $= a_2r + (C_0 - A - C_0r)i$,
- $a_4 = Ar^3 + (C_0 - A - Ar - Ar^2)i$
 $= Ar^3 + (C_0 - A - Ar)ir - (C_0 - A - Ar)ir + (C_0 - A - Ar - Ar^2)i$
 $= a_3r + (C_0 - A - C_0r)i$,
- \vdots
- In general, $a_s = a_{s-1}r + (C_0 - A - C_0r)i$.

This series is an arithmetic-geometric sequence whose general solution is:

$$a_s = a_1r^{s-1} + d \frac{1-r^{n-1}}{1-r}, \quad (3)$$

where r is the common ratio and d is the difference of the progression. By applying Equation (3) to our concrete case, it would remain:

$$\begin{aligned} a_s &= (A + C_0i)r^{n-1} + (C_0 - A - C_0r)i \frac{1-r^{n-1}}{1-r} \\ &= (A + C_0i)r^{n-1} + C_0i(1-r^{n-1}) - Ai \frac{1-r^{n-1}}{1-r} \\ &= Ar^{n-1} + C_0i - Ai \frac{1-r^{n-1}}{1-r} \end{aligned} \quad (4)$$

The outstanding principal at time s is:

$$C_s = C_0 - \sum_{k=1}^s A_k = C_0 - A \frac{1-r^s}{1-r}.$$

Therefore, the structure of the interest due is the following:

$$I_s = C_{s-1}i = \left(C_0 - A \frac{1-r^{s-1}}{1-r} \right) i$$

$$= C_0i - A \frac{1-r^{s-1}}{1-r} i.$$

As expected, observe that $a_s = A_s + I_s$. Finally, Table 5 shows the repayment plan of this loan category.

Period	Payment a_s	Interest due I_s	Principal repaid A_s	Outstanding principal C_s
0	-	-	-	C_0
1	$a_1 = A + C_0i$	$I_1 = C_0i_1$	$A_1 = A$	$C_1 = C_0 - A$
2	$a_2 = a_1r + (C_0 - A - C_0r)i$	$I_2 = C_1i_2$	$A_2 = Ar$	$C_2 = C_0 - A - Ar$
\vdots	\vdots	\vdots	\vdots	\vdots
n	$a_n = a_{n-1}r + (C_0 - A - C_0r)i$	$I_n = C_{n-1}i_n$	$A_n = Ar^{n-1}$	$C_n = 0$

Table 5. Repayment plan of a loan where the principal repaid varies according to a geometric progression.

Thus, when $r = 1 + i$, the French repayment plan can be obtained (see Table 6), where $a_1 = a_2 = \dots = a_n$ and $i_1 = i_2 = \dots = i_n$.

Period	Payment a_s	Interest due I_s	Principal repaid A_s	Outstanding principal C_s
0	-	-	-	C_0
1	$a_1 = A + C_0i = a$	$I_1 = C_0i_1$	$A_1 = A$	$C_1 = C_0 - A$
2	$a_2 = A(1+i) + (C_0 - A)i = a$	$I_2 = C_1i_2$	$A_2 = A(1+i)$	$C_2 = C_0 - A - A(1+i)$
\vdots	\vdots	\vdots	\vdots	\vdots
n	a	$I_n = C_{n-1}i_n$	$A_n = A(1+i)^{n-1}$	$C_n = 0$

Table 6. Repayment plan of a loan where the principal repaid varies according to a geometric progression and $r = 1 + i$.

5. General Discussion

In sections 3 and 4 we have obtained the main repayment plans offered by banks: the American plan, the constant principal repaid plan and the French plan. Thus, we can observe that, the same characteristics being considered, the amount of interests in the constant principal repaid plan and the French plan is less than in the American plan (Figure 3). Therefore, the American plan will be the best choice for those customers with a high level of financial literacy and a high level of financial capability since they could invest the available money and repay the principal at the end of the loan. Finally, for a low financial capability profile, the choice would be the French plan independently of the financial literacy profile.

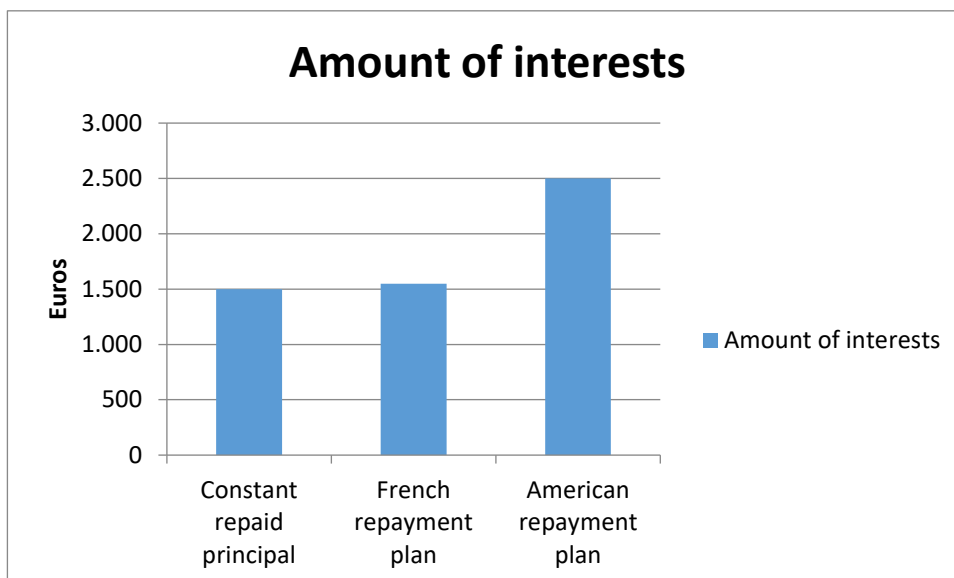


Figure 3. Amount of interests for a loan with a principal of €10,000 to be repaid in 5 years at a 5% interest rate.

Regarding now the relationship between variables #1 and #3 (shown in Table 1), we can remake this relationship by considering, in this case, that the borrowers have to choose one of the aforementioned plans (see Table 7). It is necessary to point out that the choice of the American plan and the constant principal repaid plan will depend on the possibility to invest the available money with a suitable profitability.

		Financial literacy		
		High	Medium	Low
Payment capability	High	American	Constant	Constant
	Medium	American/Constant	Constant	French
	Low	French	French	French

Table 7. Customer profiles considering the three main repayment plans (American, constant principal repaid and French).

We would like also to remark that the use of a repayment plan where the payments are distributed in different years allows combining this yearly schedule with other repayment plans inside each year. In that way, the payment schedule will be adapted according to the borrower’s financial availability inside each year.

6. Conclusions

A preference for improvement has been demonstrated for sequences of both incomes (Loewenstein and Sicherman, 1991) and outcomes (Loewenstein and Prelec, 1993). Nevertheless, focusing on loan repayment plans as sequences of installments, we have to take into account the conclusions obtained by Hoelzl *et al.* (2011) about the positive correlation found between the preferences for increasing installments and a high level of financial literacy, by considering the same financial capability. In this regard, we have suggested that banks can evaluate the level of each borrower by considering two main categories: financial literacy and financial capability. In that way, they could make a classification for each of them in three different levels: low, medium and high. Thus, according to the level of risk, we have to consider that it is higher if the loan schedule is based on increasing installments than if it is based on decreasing ones since most of principal is repaid at the beginning when considering the falling profile. Taking into account the aforementioned statement, Table 1 shows the suitability of each plan (falling or rising) according to the borrower’s level in each category. It seems that those consumers with a low level of financial literacy and high level of financial capability (low-high profile) prefer a falling plan. In those cases, banks could offer low interest rates since borrowers focus on the total amount of interests and the risk is lower. However, it is likely that customers with a high level of financial literacy and high level of financial capability (high-high profile) prefer rising plans since they are interested in investing part of their available money. Thus, in this case a higher interest rate could be offered by banks. In that way, we have obtained the

interest rate which makes equivalent both offers by considering the amount saved by the consumer with a high-high profile.

On the other hand, by considering that the principal is repaid in arithmetic progression, we have analyzed the evolution experienced by each parameter of the repayment plan during the loan lifetime. Thus, when $d = 0$, the constant principal repaid plan is obtained. Moreover, if $d = 0$ and $A = 0$, the American repayment plan is obtained. The same analysis has been done by considering that the principal is repaid in geometric progression. Here, the special case is when $r = 1 + i$, by obtaining the French repayment plan. In that way, we have considered that these three plans are the main ones offered by banks. Thus, taking into account the amount of interests of the three plans for a loan with the same characteristics, we obtained that the corresponding to the American repayment plan is quite higher than for the other plans. This leads us to conclude that this plan will be the best choice for those customers with a high-high profile since they could invest the available money and repay the principal at the end of the loan. Finally, for a low financial capability profile, the choice would be the French plan independently of the financial literacy profile.

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