RATIO MATHEMATICA 23 (2012), 3–20 ISSN: 1592-7415

General ω-hyperstructures and certain applications of those

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Abstract

The aim of this paper is to investigate general hyperstructures construction of which is based on ideas of A. D. Nezhad and R. S. Hashemi. Concept of general hyperstructures considered by the above mentioned authors is generalized on the case of hyperstructures with hyperoperations of countable arity. Specifications of treated concepts to examples from various fields of the mathematical sturctures theory are also included.

Key words: Action of a hyperstructure on a set, general n-hyperstructure, transformation hypergroup, Fredholm integral operator, ordinary and partial differential operator

MSC2010: Primary: 20N20; Secondary: 37L99, 68Q70

1 Preliminaries

A resent book [7] contains a wealth of applications of hyperstructure theory developed since 1934, see [20]. There are applications to the following subjects: geometry, hypergraphs, binary relations, lattices, fuzzy sets and rough sets, automata, cryptography, combinatoric, codes, artificial intelligence, and probability. In section 2 and 3 we give some basic definition and then, we consider three types of actions. In section 4 we present some applications. In section 5 there are described some applications of formerly investigated hyperstructures and corrected certain mistake from [14]. There is a connection between automata and dynamical systems realized by infinite automata without outputs and discrete dynamical systems.They are also shortly called as action of semigroups or groups on a given phase (or state) set. In connection with non-deterministic automata or with multifunctions (relations) on algebraic structures and topological spaces seems to be natural to investigate actions of multistructures on sets of various objects. Various generalizations of the above mentioned classical concepts are possible. Some motivating factors are coming from the general system theory [11, 23]; one illustrating example below is just based on the concept of a general time system. In [8, 9] there are investigated various binary relation and hyperstructures.

Notice, the binary relation on a binary hyperstructure, e.g. on a semihypergroup, are quite natural created by inner translations.

We use [1, 3, 7, 10, 21, 22, 26] for terminology and notations which are not define here. We suppose that the reader is familiar with some well-known notation such as presentation in hyperstructure theory. The following facts are some definitions and propositions in the theory of hyperstructure which we need for formulation of our results and in the proofs of the main results. For x from an ordered set H we denote by $[x)_{\leq} = \{y \in H \mid x \leq y\}$ the *upper end* generated by x.

The following lemma is called Ends Lemma.

Lemma 1.1. [3, 6, 24, 25] *Let $(H, \circ, \leq)$ be an ordered semigroup. Let $a \star b = [a \circ b)_{\leq}$ for any $a, b \in H$. The following conditions are equivalent:*

1) *For any pair $a, b \in H$ there exists a pair $c, d \in H$ such that $b \circ c \leq a$, $c \circ d \leq a$.*

2) *A hypergroupoid $(H, \star)$ associated with $(H, \circ, \leq)$ satisfies the associativity law and the reproduction axioms, i.e., $(H, \star)$ is a hypergroup.*

Dually we can define the Beginnings Lemma:

Lemma 1.2. [6] *Let $(H, \circ, \leq)$ be an ordered semigroup. Let $a \star b = (a \circ b]_{\leq}$ for any $a, b \in H$. The following conditions are equivalent:*

1) *For any pair $a, b \in H$ there exists a pair $c, d \in H$ such that $b \circ c \geq a$, $c \circ d \geq a$.*

2) *A hypergroupoid $(H, \star)$ associated with $(H, \circ, \leq)$ satisfies the associativity law and the reproduction axioms, i.e., $(H, \star)$ is a hypergroup.*

Quasi-order hypergroups have been introduced and studied by J. Chvalina. The following definition can be found e.g. in [7, 12, 24].

Definition 1.1. *A hypergroup $(H, \star)$ such that the following condition are satisfied:*

1) $a \in a^2 = a^3$ *for any* $a \in H$,

2) $a \star b = a^2 \cup b^2$ *for any pair* $a, b \in H$ *is called* a quasi-order hypergroup. *If moreover the unique square root condition:*

3) $a, b \in H$, $a^2 = b^2$ *implies* $a = b$

is satisfied then $(H, \star)$ *is called* an order hypergroup.

Definition 1.2. [19] *A hypergroup* $(G, \star)$ *is called* a transposition hypergroup *if it satisfies the transposition axiom: For all* $a, b, c, d \in G$ *the relation* $b \setminus a \cap c/d \neq \emptyset$ *implies* $a \star d \cap b \star c \neq \emptyset$.
The sets $b \setminus a = \{x \in G | a \in b \star x\}, c/d = \{x \in G | c \in x \star d\}$ *are called* left and right extensions, *respectively.*

Definition 1.3. [15, 16, 17] *Let X be a set,* $(G, \bullet)$ *be a (semi)hypergroup and* $\pi \colon X \times G \to X$ *a mapping such that*

$$\pi(\pi(x,t), s) \in \pi(x, t \bullet s), \ \ \textit{where } \pi(x, t \bullet s) = \{\pi(x,u); u \in t \bullet s)\} \qquad (1)$$

for each $x \in X$, $s, t \in G$. *Then* (X, G, π) *is called a* discrete transformation (semi)hypergroup *or an* action of the (semi)hypergroup G *on the phase set* X. *The mapping* π *is usually said to be simply* an action.

Remark 1.1. [16] *The condition* (1) *used above is called* Generalized Mixed Associativity Condition, *shortly GMAC.*

2 General hyperstuctures and ω-hyperstructures

Throughout this paper, the symbol X, Y will denote two non-empty sets, where $P^*(X \cup Y)$ denotes the set of all non-empty subsets of $X \cup Y$.

A general hyperstructure is formed by two non-empty sets X, Y together with a hyperoperation

$$* : X \times Y \longrightarrow P^*(X \cup Y), \quad (x, y) \mapsto x * y \subseteq (X \cup Y) \smallsetminus \emptyset.$$

Remark 2.1. *A general hyperoperation* $* : X \times Y \longrightarrow P^*(X \cup Y)$ *yields a map of powersets determined by this hyperoperation. Thus the map* $\otimes : P^*(X) \times P^*(Y) \longrightarrow P^*(X \cup Y)$ *is defined by* $A \otimes B = \bigcup\limits_{a \in A, b \in B} a * b$.

Conversely an general hyperoperation on $P^*(X) \times P^*(Y)$ yields a general hyperoperation on $X \times Y$, defined by $x * y = \{x\} \otimes \{y\}$.

In the above definition if $A \subseteq X,\ B \subseteq Y,\ x \in X,\ y \in Y$, then we define,

$$A * y = A * \{y\} = \bigcup_{a \in A} a * y, \quad x * B = \{x\} * B = \bigcup_{b \in B} x * b,$$

$$A \otimes B = \bigcup_{a \in A, b \in B} a * b.$$

Remark 2.2. *If* $X = Y = H$*, then we obtain the classical hyperstructure theory. The concept of general hyperstructure with a hyperoperation which is a mapping*

$$* : X \times Y \longrightarrow P^*(X \cup Y)$$

mentioned above (used by A. D. Nezhad and R. S. Hashemi) allows straightforward generalization onto case of "hyperoperation of an arbitrary finite arity" *or* direct generalization to hyperoperation of countable arity.

Let us define the general ω-hyperstructure:

Definition 2.1. *Let* ω *be the smallest infinite countable ordinal. As usually, we consider the smallest infinite ordinal* ω *as the set of all smaller ordinals, i.e. as the domain of all finite ordinals (non-negative integers). Let* $\{X_k; k \in \omega\}$ *be a system of non-empty sets. By an* general ω-hyperstructure *we mean the pair* $(\{X_k; k \in \omega\}, *_\omega)$*, where* $*_\omega \colon \prod\limits_{k \in \omega} X_k \to \mathcal{P}^*\Big(\bigcup\limits_{k \in \omega} X_k\Big)$ *is a mapping assigning to any sequence* $\{x_k\}_{k \in \omega} \in \prod\limits_{k \in \omega} X_k$ *a non-empty subset* $*_\omega(\{x_k\}_{k \in \omega}) \subset \bigcup\limits_{k \in \omega} X_k$.

Similarly as above, with this hyperoperation there is associated a mapping of power sets

$$\otimes_\omega \colon \prod_{k \in \omega} \mathcal{P}^*(X_k) \to \mathcal{P}^*\Big(\bigcup_{k \in \omega} X_k\Big)$$

defined by

$$\otimes_\omega\big(\{A_k\}_{k \in \omega}\big) = \bigcup \Big\{ *_\omega\big(\{x_k\}_{k \in \omega}\big); \big(\{x_k\}_{k \in \omega}\big) \in \prod_{k \in \omega} A_k \Big\}.$$

Let us formulate the special case:

Definition 2.2. [13] *Let $n \in \omega$ be an arbitrary positive integer, $n \geq 1$. Let $\{X_k; k = 1, \dots, n\}$ be a system of non-empty sets. By a* general n-hyperstructure *we mean the pair*

$$(\{X_k; k = 1, \dots, n\}, *_n),$$

where $*_n \colon \prod_{k=1}^{n} X_k \to \mathcal{P}^*\Big(\bigcup_{k=1}^{n} X_k\Big)$ is a mapping assigning to any n-tuple $(x_1, \dots, x_n) \in \prod_{k=1}^{n} X_k$ a non-empty subset $*_n(x_1, \dots, x_n) \subset \bigcup_{k=1}^{n} X_k$.

Similarly as above, with this hyperoperation there is associated a mapping of power sets

$$\otimes_n \colon \prod_{k=1}^{n} \mathcal{P}^*(X_k) \to \mathcal{P}^*\Big(\bigcup_{k=1}^{n} X_k\Big)$$

defined by

$$\otimes_n(A_1, \dots, A_n) = \bigcup \Big\{ *_n(x_1, \dots, x_n); (x_1, \dots, x_n) \in \prod_{k=1}^{n} A_k \Big\}.$$

This construction is based on an idea of Nezhad and Hashemi for $n = 2$. Hyperstructures with n-ary hyperoperations are investigated among others in [2, 27].

Definition 2.3. *Let $\mathbb{G}_1(\omega) = \big(\{X_k; k \in \omega\}, *_\omega\big)$, $\mathbb{G}_2(\omega) = \big(\{Y_k; k \in \omega\}, \bullet_\omega\big)$, be a pair of general ω-hyperstructures. By a* good homomorphism *$H \colon \mathbb{G}_1(\omega) \to \mathbb{G}_2(\omega)$ we mean any system of mappings $H = \{h_k \colon X_k \to Y_k\}$ such that the following diagram is commutative:*

$$\begin{array}{ccc} \prod X_k & \xrightarrow{*_\omega} & \mathcal{P}^*\big(\bigcup_{k \in \omega} X_k\big) \\ {\scriptstyle \prod_{k \in \omega} h_k} \downarrow & & \downarrow {\scriptstyle \varphi^\sharp} \\ \prod Y_k & \xrightarrow{\bullet_\omega} & \mathcal{P}^*\big(\bigcup_{k \in \omega} Y_k\big) \end{array} \tag{D1}$$

Here $\prod_{k \in \omega} h_k(\{x_k\}_{k \in \omega}) = \{h_k(x_k)\}_{k \in \omega}$ for any sequence $\{x_k\}_{k \in \omega}$ and $\varphi^\sharp \colon \mathcal{P}^*\big(\bigcup_{k \in \omega} X_k\big) \to \mathcal{P}^*\big(\bigcup_{k \in \omega} Y_k\big)$ is the lifting of a mapping $\varphi \colon \bigcup_{k \in \omega} X_k \to \bigcup_{k \in \omega} Y_k$ defined by the mathematical induction. For $x \in X_0$ we put $\varphi(x) = h_0(x)$. Suppose $\varphi \colon \bigcup_{j=0}^{k} X_j \to \bigcup_{j=0}^{k} Y_j$ is well-defined. Then for any $x \in X_{k+1} \setminus \bigcup_{j=0}^{k} X_j$

we put $\varphi(x) = h_{k+1}(x)$. Then using mathematical induction the mapping $\varphi\colon \bigcup_{k\in\omega} X_k \to \bigcup_{k\in\omega} Y_k$ is well-defined.

If all mappings $h_k \in H$ are bijections (or isomorphism if all H_k, Y_k are endowed with some structures) we call the H the isomorphism of ω- hyperstructures $G_1(\omega)$, $G_2(\omega)$.

3 General ω-hyperstructures created by ordered sets and by differential operators

As a certain generalization of the general n-hyperstructure from [13], Example 3.2, we will construct the following structure:

Example 3.1. *Consider a countable system of pairwise disjoint ordered sets $(X_k, \leq_k)$, $k \in \omega$ and for $x \in X_k$ let us denote $[x)_k = \{y \in X_k; x \leq_k y\}$, i.e. $[x)_k$ is the principal end generated by the element x within the ordered set $(X_k, \leq_k)$. Further, put*

$$*_\omega\big(\{x_k\}_{k\in\omega}\big) = \bigcup_{k\in\omega} [x_k)_k$$

for any sequence $*_\omega\big(\{x_k\}_{k\in\omega}\big) \in \prod_{k\omega} X_k$. Then $*_\omega\big(\{x_k\}_{k\in\omega}\big) \subseteq \bigcup_{k\in\omega} X_k$, thus

$$\mathbb{G}(\omega) = \big(\{X_k; k \in \omega\}, *_\omega\big)$$

is a general ω-hyperstructure in the sense of the above definition.
If $\mathbb{H}(\omega) = \big(\{Y_k; k \in \omega\}, \bullet_\omega\big)$ is a general ω-hyperstructure such that $(Y_k, \preceq_k)$, $k \in \omega$ are pairwise disjoint ordered sets and

$$\bullet_\omega(\{y_k\}_{k\in\omega}) = \bigcup_{k\in\omega} [y_k)_k \subseteq \bigcup_{k\in\omega} Y_k$$

for any sequence $\{y_k\}_{k\in\omega} \in \prod_{k\in\omega} Y_k$ we consider a system $h_k\colon (X_k, \leq_k) \to (Y_k, \preceq_k)$, $k \in \omega$, of strongly isotone mappings, i.e. for any $x \in X_k$ there holds $h_k\big([x_k)_k\big) = \big[h_k(x_k)\big)_k$. Then denoting $H = \{h_k\colon X_k \to Y_k; k \in \omega\}$ we obtain that H is a good homomorphism of the general ω-hyperstructure; $\mathbb{G}(\omega)$ into the general ω-hyperstructure $\mathbb{H}(\omega)$.

Indeed, consider an arbitrary sequence $\{x_k\}_{k\in\omega} \in \prod_{k\in\omega} X_k$. As above denote by $\varphi\colon \mathcal{P}^*\big(\bigcup_{k\in\omega} X_k\big) \to \mathcal{P}^*(\bigcup_{k\in\omega} Y_k)$ the lifting of the mapping $\varphi\colon \bigcup_{k\in\omega} X_k \to \bigcup_{k\in\omega} Y_k$

induced by the system $\{h_k\colon X_k \to Y_k; k \in \omega\}$*—here in such a way that* $\varphi|X_k = h_k$*. Then for any sequence* $\{x_k\}_{k\in\omega} \in \prod\limits_{k\in\omega} X_k$ *we have*

$$\begin{aligned}\varphi\big(*_\omega(\{x_k\}_{k\in\omega})\big) &= \varphi\Big(\bigcup_{k\in\omega}[x_k)_k\Big) = \bigcup_{k\in\omega}\varphi\big([x_k)_k\big) = \bigcup_{k\in\omega} h_k\big([x_k)_k\big)\\ &= \bigcup_{k\in\omega}\big[h_k(x_k)_k\big) = \bullet_\omega\big(\{h_k(x_k)\}_{k\in\omega}\big)\\ &= \bullet_\omega\Big(\prod_{k\in\omega} h_k\big(\{x_k\}_{k\in\omega}\big)\Big),\end{aligned}$$

i.e. $\varphi \circ *_\omega = \bullet_\omega \circ \prod\limits_{k\in\omega} h_k$*, thus the diagram*

$$\begin{array}{ccc} \prod X_k & \xrightarrow{\;*_\omega\;} & \mathcal{P}^*\Big(\bigcup\limits_{k\in\omega} X_k\Big)\\ {\scriptstyle \prod\limits_{k\in\omega} h_k}\Big\downarrow & & \Big\downarrow{\scriptstyle \varphi^\sharp}\\ \prod Y_k & \xrightarrow{\;\bullet_\omega\;} & \mathcal{P}^*\Big(\bigcup\limits_{k\in\omega} Y_k\Big)\end{array} \tag{D2}$$

is commutative.

From the above example there follows immediately the following assertion.

Proposition 3.1. *Let* $(X_k, \le_k)$*,* $(Y_k, \preceq_k)$*,* $k \in \omega$*, be two countable collections of pairwise disjoint ordered sets and* $\mathbb{G}(\omega)$*,* $\mathbb{H}(\omega)$ *be the corresponding* ω*-general hyperstructures. Suppose* $(X_k, \le_k) \cong (Y_k, \preceq_k)$ *for each* $k \in \omega$ *and* $h_k\colon (X_k, \le_k) \to (Y_k, \preceq_k)$ *are corresponding order-isomorphisms. Then we have* $\mathbb{G}(\omega) \cong \mathbb{H}(\omega)$*.*

Example 3.2. *Let* $J \subset \mathbb{R}$ *be an open interval,* $\mathbb{C}^n(J)$ *be the ring (with respect to usual addition and multiplication of functions) of all real functions* $f\colon J \to \mathbb{R}$ *with continuous derivatives up to the order* $n \ge 0$ *including. Denote*

$$L(p_0, p_1, \ldots, p_{n-1})\colon \mathbb{C}^n(J) \to \mathbb{C}^n(J)$$

the linear differential operator defined by

$$L(p_0, p_1, \ldots, p_{n-1})(y) = \frac{\mathrm{d}^n y(x)}{\mathrm{d}x^n} + \sum_{s=0}^{n-1} p_s(x)\frac{\mathrm{d}^s y(x)}{\mathrm{d}x^s}$$

where $y \in \mathbb{C}^n(J)$ *and* $p_s \in \mathbb{C}^n(J)$*,* $s = 0, 1, \ldots, n-1$*. In accordance with* [4, 5] *we put*

$$\mathbb{LA}_n(J) = \{L(p_0, \ldots, p_{n-1}); p_k \in \mathbb{C}^n(J)\}.$$

Instead of $L(p_{1,0}, p_{1,1}, \ldots, p_{1,n-1})$ *we write* $L(\vec{p}_1)$*. We put* $L(\vec{p}_1) \leq L(\vec{p}_2)$ *whenever*
$L(\vec{p}_j) = L(p_{j,0}, \ldots, p_{j,n-1})$*,* $j = 1, 2$*,* $p_{1,s}(x) \leq p_{2,s}(x)$*,* $s = 0, 1, \ldots, n-1$*,* $x \in J$ *and* $p_{1,0}(x) \equiv p_{2,0}(x)$*. Defining*

$$*_n\big(L(\vec{p}_1), L(\vec{p}_2), \ldots, L(\vec{p}_n)\big) = \bigcup_{k=1}^{n} \{L(\vec{p}) \in \mathbb{LA}_k(J); L(\vec{p}_k) \leq L(\vec{p})\}$$

for any n-tuple $\big(L(\vec{p}_1), L(\vec{p}_2), \ldots, L(\vec{p}_n)\big) \in \prod_{k=1}^{n} \mathbb{LA}_k(J)$ *we obtain that* $\mathbb{L}(n) = \big(\{\mathbb{LA}_k(J); k = 1, 2, \ldots, n\}, *_n\big)$ *is a general n-hyperstructure.*

Of course, $\mathbb{LA}_1(J)$ *is the set of all first-order linear differential operators of the form* $L(p_0)(y) = y'(x) + p_0(x)y$*, where* $p_0 \in \mathbb{C}(J)$ *and* $y \in \mathbb{C}^1(J)$*. Evidently* $\mathbb{LA}_j(J) \cap \mathbb{LA}_k(J) = \emptyset$ *whenever* $j \neq k$*.*

It is to be noted that if $k, m \in \{1, 2, \ldots, n\}$ are fixed different integers then setting $X = \mathbb{LA}_k(J)$, $Y = \mathbb{LA}_m(J)$ we obtain from the above construction an example of a general hyperstructure in sense of Nezhad and Hashemi. If, moreover $X = Y = \mathbb{LA}_n(J)$ then the resulting general hyperstructure is an order hypergroup of linear differential n-order operators in the sense of [3], chap. IV, or [4, 5].

Theorem 3.1. *Let* ω *be the smallest infinite countable ordinal,* $J \subseteq \mathbb{R}$ *be an open interval. If*

$$\mathbb{L}(J; \omega) = \Big(\prod_{k\in\omega} \mathbb{LA}_k(J), *_\omega, \mathcal{P}^*\big(\bigcup_{k\in\omega} \mathbb{LA}_k(J)\big)\Big)$$

is the general ω*-hyperstructure of ordinary linear differential operators and*

$$\mathbb{S}(J; \omega) = \Big(\prod_{k\in\omega} \mathbb{VA}_k(J), \bullet_\omega, \mathcal{P}^*\big(\bigcup_{k\in\omega} \mathbb{VA}_k(J)\big)\Big)$$

is the general ω*-hyperstructure of solution spaces of linear ordinary homogeneous differential equations associated with* $\mathbb{L}(J; \omega)$*. Then we have*

$$\mathbb{L}(J; \omega) \cong \mathbb{S}(J; \omega),$$

i.e. in the commutative diagram

$$\begin{array}{ccc} \prod_{k\in\omega} \mathbb{LA}_k(J) & \xrightarrow{*_\omega} & \mathcal{P}^*\Big(\bigcup_{k\in\omega} \mathbb{LA}_k(J)\Big) \\ {\scriptstyle \prod_{k\in\omega}\Phi_k}\Big\downarrow & & \Big\downarrow{\scriptstyle \varphi^\sharp} \\ \prod_{k\in\omega} \mathbb{VA}_k(J) & \xrightarrow{\bullet_\omega} & \mathcal{P}^*\Big(\bigcup_{k\in\omega} \mathbb{VA}_k(J)\Big) \end{array} \tag{D3}$$

arrows $\prod_{k\in\omega} \Phi_k$*,* $\varphi^\sharp$ *are bijections.*

Proof. By [4, 5] we have $\Phi_k\colon \mathbb{LA}_k(J) \to \mathbb{VA}_k(J)$ is a group-isomorphism for any $k \in \omega$ thus $\prod\limits_{k\in\omega} \Phi_k\colon \prod\limits_{k\in\omega} \mathbb{LA}_k(J) \to \prod\limits_{k\in\omega} \mathbb{VA}_k(J)$ is a bijection. Since $\{\mathbb{LA}_k(J); k \in \omega\}, \{\mathbb{VA}_k(J); k \in \omega\}$ are pairwise disjoint families we have that the mapping $\varphi\colon \bigcup\limits_{k\in\omega} \mathbb{LA}_k(J) \to \bigcup\limits_{k\in\omega} \mathbb{VA}_k(J)$ such that $\varphi|\mathbb{LA}_k(J) = \Phi_k$, $k \in \omega$ is a well-defined bijection hence the bijection $\mathcal{P}^*\Big(\bigcup\limits_{k\in\omega} \mathbb{LA}_k(J)\Big) \to \mathcal{P}^*\Big(\bigcup\limits_{k\in\omega} \mathbb{VA}_k(J)\Big)$ is also well-defined.

Now, for an arbitrary sequence $\{L_n\}_{n\in\omega} \in \prod\limits_{k\in\omega} \mathbb{LA}_k(J)$ we obtain that

$$\bullet_\omega\Big(\big(\prod_{k\in\omega}\Phi_k\big)\{L_n\}_{n\in\omega}\Big) = \bullet_\omega\big\{\Phi_n(L_n)\big\}_{n\in\omega} = \bullet_\omega\{V_n\}_{n\in\omega} =$$
$$= \varphi\Big(*_\omega\big\{\Phi_n^{-1}(L_n)\big\}_{n\in\omega}\Big) = \varphi^\sharp\big(*_\omega\{L_n\}_{n\in\omega}\big),$$

since the hyperoperation "$\bullet_\omega$" is associated with the hyperoperation "$*_\omega$" . Therefore the diagram D3 in the Theorem 3.1 is commutative. □

Let $\big\{(S_k, \cdot, \leq_k); k \in \omega\big\}$ be a system of quasi-ordered semigroups. Define a mapping $\odot_\omega\colon \prod\limits_{k\in\omega} S_k \to \mathcal{P}^*\Big(\bigcup\limits_{k\in\omega} S_k\Big)$ by the rule

$$\odot_\omega(x_1,\dots,x_n) = \bigcup_{k\in\omega}[x_k^2)_{\leq_k}$$

for any sequence $\{x_k\}_{k\in omega} \in \prod\limits_{k\in\omega} S_k$. Then the general ω-hyperstructure is called the *general ω-hyperstructure determined by the Ends Lemma* or shortly *EL-determined general ω-hyperstructure.*

Corollary of Theorem 3.1

Let ω be the smallest infinite countable ordinal, $\omega \neq 0$, $J \subseteq \mathbb{R}$ be an open interval. Let $\mathbb{L}^{EL}(J;n) = \Big(\prod\limits_{k\in\omega} \mathbb{LA}_k(J), *_n, \mathcal{P}^*\big(\bigcup\limits_{k\in\omega} \mathbb{LA}_k(J)\big)\Big)$ be the EL-determined general ω-hyperstructure of all linear ordinary differential operators of all orders $k \in \omega$.

Let $\mathbb{S}^{EL}(J;n) = \Big(\prod\limits_{k\in\omega} \mathbb{VA}_k(J), *_n, \mathcal{P}^*\big(\bigcup\limits_{k\in\omega} \mathbb{VA}_k(J)\big)\Big)$ be the EL-determined general ω-hyperstructure of solutions of homogeneous linear ordinary differential equations $Ly = 0$, $L \in \bigcup\limits_{k\in\omega} \mathbb{LA}_k(J)$. Then $\mathbb{L}^{EL}(J;n) \cong \mathbb{S}^{EL}(J;n)$.

In the above construction we can use a finite sequence of positive integers $\{m_1, m_2, \dots, m_n\}$ and then define the ω-hyperoperation $\odot_\omega\big(\{x_k\}_{k\in\omega}\big) = \bigcup\limits_{k\in\omega} [x_k^{m_k})_{\leq_k}$ for any ω sequence $\{x_k\}_{k\in\omega} \in \prod\limits_{k\in\omega} S_k$.

4 General R-hyperstuctures (or L-hyperstuctures)

In the paper [13], due to ideas of Nezhad and Hashemi, there are considered General R-hyperstuctures (or L-hyperstuctures) in the following sense. A general Right hyperstructure (or Left-hyperstructure) consist of two non-empty sets X, Y together with a hyperoperation:

$$\begin{aligned} *_R : X \times Y &\longrightarrow P^*(X) \qquad \text{or} \qquad *_L \quad : X \times Y \longrightarrow P^*(Y) \\ (x, y) &\mapsto x *_R y \subseteq X, \qquad\qquad\qquad (x, y) \mapsto x *_L y \subseteq Y. \end{aligned}$$

To be more precise a *general Right hyperstructure (or Left hyperstructure)* is the quadruple $(X, Y, \mathcal{P}^*(X), *_R)$ or $(X, Y, \mathcal{P}^*(X), *_L)$, shortly *general R-hyperstructure* or *general L-hyperstructure.*

The set of points $Y_R x = \{x * y : y \in Y\}$ that can be reached from a given point $x \in X$ by the R-hyperoperation of two non-empty sets X, Y, is called the R-hyperorbit of x.

If $Y_R x = X$ for all $x \in X$. Then the set Y is said to be R-hypertransitive on Y. If $Y_R x = \{x\}$. Then x is called a R-hyperfixed point to the R-hyperoperation. The set $\{y \in Y : x *_R y = \{x\}\}$, is called R-hyperisotopy set at x.

Example 4.1. *Let $X \neq \emptyset$ be an arbitrary set, $f\colon X \to X$ be a mapping, i.e. the pair (X, f) is a monounary algebra. Put $Y = \mathbb{N}$ (the set of all positive integers) and define $*_R^f\colon X \times Y \to \mathcal{P}^*(X)$ by the rule $x *_R^f n = \{f^k(x); k \in \mathbb{N}, n \leq k\}$. Then the quadruple $(X, Y, \mathcal{P}^*(X), *_R^f)$ is a general Right hyperstructure, i.e. R-hyperstructure. (Here, f^k is the k-th iteration of f).*

Example 4.2. *Let T be a linearly ordered set (i.e. a chain) with the least element. Then T is called a* time scale *or* time axis. *Suppose $A \neq \emptyset \neq B$ are arbitrary sets and S is a binary relation between sets of mappings (impulses) A^T, B^T, i.e. $S \subset A^T \times B^T$. Then the triad (A^T, B^T, S) is called a* general time system *with input space A^T, the output space B^T and with input-output relation (or the transition relation) S—cf.[23]. Now, denote $X = A^T$, $Y = B^T$ and define $*_L^S\colon X \times Y \to \mathcal{P}^*(Y)$ by $x *_L^S y = S(x) = \{u \in Y; x\, S\, u\}$ for any pair of time-impulses $x\colon T \to A, y\colon T \to B$. Then we obtain the quadruple $(X, Y, \mathcal{P}^*(X), *_L^S)$ which is a general Left hyperstructure, i.e. a general L-hyperstructure.*

In the above mentioned classical monography [23] as a general system is considered a relation $S \subseteq \prod_{i \in I} V_i$ on non-void abstract sets. However—in detail—the index set I is decomposed into two subsets I_x, I_y and by *input-output* system is ment a relation $S \subseteq X \times Y$, where $X = \prod_{i \in I_x} V_i$ is called the input object and $Y = \prod_{i \in I_y} V_i$ is termed as the output object. (cf. [23], Definition 1.2).

In the connection with general ω-hyperstructures we introduce input-output systems with inner evaluated input and output objects. Denote $K = \{1, 2, \dots, n\}$. By an IEO-*general system*, i.e. an *Input-Output general system with Inner Evaluated Objects* we mean a qudruple $(\prod_{k \in K} X_k, \prod_{k \in K} Y_k, \mathrm{R}, F_{\mathrm{R}})$, where X_k, Y_k are non-empty sets, $\mathrm{R} \subseteq \prod_{k \in K} X_k \times \prod_{k \in K} Y_k$ is a domain full binary relation, i.e. $Dom\, \mathrm{R} = \prod_{k \in K} X_k = \{x \in \prod_{k \in K} X_k; \exists y \in \prod_{k \in K} : x \,\mathrm{R}\, y\}$, $*_x \colon \prod_{k \in K} X_k \to \mathcal{P}(\bigcup_{k \in K} X_k)$, $*_y \colon \prod_{k \in K} Y_k \to \mathcal{P}(\bigcup_{k \in K} Y_k)$ are mappings (i.e. n-ary multioperations)—thus $(\prod_{k \in K} X_k, *_x)$,$(\prod_{k \in K} Y_k, *_y)$ are general n-ary hyperstructures and $F_{\mathrm{R}} \colon \mathcal{P}(\bigcup_{k \in K} X_k) \to \mathcal{P}(\bigcup_{k \in K} Y_k)$ is a mapping defined in this way:

For a non-empty set $M \subset \bigcup_{k \in K} X_k$, i.e. $M \in \mathcal{P}(\bigcup_{k \in K} X_k)$ we denote $M_X^- = \big\{[x_1, \dots, (x_1, \dots, x_n)_n] \in \prod_{k \in K} X_k; *_X(x_1, \dots, (x_1, \dots, x_n)_n) \cap M \neq \emptyset\big\}$. Then we put $F_{\mathrm{R}}(M) = *_y\big(\mathrm{R}(M_X^-\big) \subset \bigcup_{k \in K} Y_k$. If $M_X^- = \emptyset$ we define $F_{\mathrm{R}}(M) = \emptyset$. (This definition includes the case $F_{\mathrm{R}}(\emptyset) = \emptyset$). Hence the diagram

$$\begin{array}{ccc}
\prod_{k \in K} X_k & \xrightarrow{*_X} & \mathcal{P}\big(\bigcup_{k \in K} X_k\big) \\
{\scriptstyle \mathrm{R}}\big\downarrow & & \big\downarrow{\scriptstyle F_{\mathrm{R}}} \\
\prod_{k \in K} Y_k & \xrightarrow{*_Y} & \mathcal{P}\big(\bigcup_{k \in k} Y_k\big)
\end{array} \tag{D4}$$

is commutative.

Now, in the case when $X_k = A$, $Y_k = B$ for all $k \in K$, then denoting the index set K by T (the time set-continuous, which means a linearly ordered continuum with the minimal element, or discrete $\mathbb{N}_0$) we obtain $\prod_{k \in K} X_k = A^T$, $\prod_{k \in K} Y_k = B^T$ and $\bigcup_{k \in K} X_k = A$ $\bigcup_{k \in K} Y_k = B$. Thus the above IEO-general system turns to the above mentioned general time system (here with an inner evaluation) $(A^T, B^T, R, *_A, *_B)$, where $F_R \circ *_A = *_B \circ R$.

By a homomorphism of an IEO-general system $(\prod_{k\in K} X_k, \prod_{k\in K} Y_k, \mathrm{R}, *_X, *_Y)$ into another IEO-general system $(\prod_{k\in K} U_k, \prod_{k\in K} V_k, \mathrm{S}, *_U, *_V)$ we mean a quadruple of mappings $H = [\Phi_{XU}, \Phi_{YV}, \Phi_{PXU}, \Phi_{PYV}]$, $\Phi_{XU} \colon \prod_K X_k \to \prod_K U_k$; $\Phi_{YV} \colon \prod_K Y_k \to \prod_K V_k$; $\Phi_{PXU} \colon \mathcal{P}(\bigcup_K X_k \to \mathcal{P}(\bigcup_K U_k)$; $\Phi_{PYV} \colon \mathcal{P}(\bigcup_K Y_k) \to \mathcal{P}(\bigcup_K V_k)$ such that the diagram

$$
\begin{tikzcd}
\prod_{k\in K} U_k \arrow[rrr, "*_U"] \arrow[ddd, "S"'] & & & \mathcal{P}\Big(\bigcup_{k\in K} U_k\Big) \arrow[ddd, "F_S"] \\
& \prod_{k\in K} X_k \arrow[ul, "\Phi_{XU}"] \arrow[r, "*_X"] \arrow[d, "R"'] & \mathcal{P}\Big(\bigcup_{k\in K} X_k\Big) \arrow[ur, "\Phi_{PXU}"'] \arrow[d, "F_R"] & \\
& \prod_{k\in K} Y_k \arrow[dl, "\Phi_{YU}"'] \arrow[r, "*_Y"] & \mathcal{P}\Big(\bigcup_{k\in K} Y_k\Big) \arrow[dr, "\Phi_{PYV}"] & \\
\prod_{k\in K} V_k \arrow[rrr, "*_V"] & & & \mathcal{P}\Big(\bigcup_{k\in K} V_k\Big)
\end{tikzcd} \tag{D5}
$$

is commutative. In concrete cases of modelling special systems, the used objects and their mappings take a concrete interpretation.

4.1 L-hyperoperation (or R-hyperoperation) of a hyperstucture on a non-empty set

In this paragraph, we recall two definitions of a L-hyperoperation (or R-hyperoperation) of a hyperstucture on a non-empty set. Let us make our point clear with some examples.

Definition 4.1. [13] *Let $(G, \star)$ be a hyperstructure and X be a non-empty set.*

A generalized L-hyperaction *of G on X is a L-hyperoperation $\psi : G \times X \longrightarrow \mathcal{P}^*(X)$ such that the following axioms are satisfied:*

1) For all $g, h \in G$ and $x \in X$, $\psi(g \star h, x) \subseteq \psi(g, \psi(h, x))$,

2) For all $g \in G$, $\psi(g, X) = X$.

For any $g \in G$ and $A \subseteq X$, $\psi(g, A) = \bigcup_{x \in A} \psi(g, x)$, also for any $x \in X$ and $B \subseteq G$, $\psi(B, x) = \bigcup_{b \in B} \psi(b, x)$. If in the axiom 1) of definition the equality holds, the generalized R-hyperaction is called strong.

As application of the above concepts we mention the classical *interval binary hyperoperation on a linearly ordered group.* See [18]. In detail if $(G, \cdot, \leq)$ is a linearly ordered group then we define a binary hyperoperation $*\colon G \times G \to \mathcal{P}^*(G)$ by

$$\begin{aligned} a * b &= \big[\min\{a, b\}\big)_{\leq} \cap \big(\max\{a, b\}\big]_{\leq} = \big[\min\{a, b\}, \max\{a, b\}\big]_{\leq} \\ &= \big\{x \in G; \min\{a, b\} \leq x \leq \max\{a, b\}\big\} \end{aligned}$$

(which is a closed interval) where $\min\{a, b\}$, $\max\{a, b\}$ is the least element, the greatest element of the set $\{a, b\}$, respectively. It is easy to verify that the obtained hypergroupoid $(G, *)$ is an extensive commutative hypergroup. This hypergroup we obtain even in the case if we restrict ourselves onto the set G_+ of all positive elements of the linearly ordered group $(G, \cdot, \leq)$, (cf. the proof of Proposition 4.1).

Proposition 4.1. *Let $(G, \cdot, \leq)$ be a linearly ordered group, G_+ be its subset of all positive elements (i.e. the positive cone) endowed with the interval binary hyperoperation "$*_L$". Define a mapping*

$$\psi_G\colon G_+ \times G \to \mathcal{P}^*(G)$$

by

$$\psi_G(a, b) = (a + b]_{\leq} = \{x \in G; x \leq a + b\}$$

for all pairs $(a, b) \in G_+ \times G$. Then the quadruple $\big(G_+, G, \mathcal{P}^*(G), \psi_G\big)$ is the generalized L-hyperoperation of the commutative extensive hypergroup $(G, *_L)$ on the group $(G, +, \leq)$.

Proof. For the proof see [13]. □

5 Homomorphism of transformation semihypergroups

Definition 5.1. *Let (X, G, ψ), (Y, H, η) be two generalized transformation semihypergroups (GTS). A pair of mappings $\Phi = [\mu, \varphi]$ such that $\mu\colon G \to H$ is a homomorphism of semihypergroups and $\varphi\colon X \to Y$ is a mapping, is said to be a homomorphism of GTS (X, G, ψ) into GTS (Y, H, η) if for any pair $[g, x] \in G \times X$ the equality*

$$\eta\bigl(\mu(g), \varphi(x)\bigr) = \varphi\bigl(\psi(g, x)\bigr)$$

is satisfied, i.e. the diagram, where $\varphi^*\colon \mathcal{P}^*(X) \to \mathcal{P}^*(Y)$ is the corresponding liftation of the mapping $\varphi\colon X \to Y$,

$$\begin{array}{ccc} G \times X & \xrightarrow{\psi} & \mathcal{P}^*(X) \\ {\scriptstyle \mu\times\varphi}\big\downarrow & & \big\downarrow{\scriptstyle \varphi^*} \\ H \times Y & \xrightarrow{\omega} & \mathcal{P}^*(Y) \end{array} \tag{D6}$$

commutes.

Example 5.1. *Let X, Y be equivalent non-empty sets and $f\colon X \to X$, $h\colon Y \to Y$ be mappings such that mono-unary algebras $(X, f) \cong (Y, h)$. Denote $G = \{f^n; n \in \mathbb{N}_0\}$, $H = \{h^n, n \in \mathbb{N}_0\}$ and define binary hyperoperations*

$$\star\colon G \times G \to \mathcal{P}^*(G), \qquad \bullet\colon H \times H \to \mathcal{P}^*(H),$$

by

$$f^n \star f^m = \{f^k; k \in \mathbb{N}_0, m+n \le k\} \quad \text{and} \quad h^n \bullet h^m = \{h^k; k \in \mathbb{N}_0, m+n \le k\}.$$

Define mappings $\psi\colon G \times X \to \mathcal{P}^*(X)$, $\eta\colon H \times Y \to \mathcal{P}^*(Y)$, by the same rule $\psi(f^n, x) = \bigl\{f^k(x); k \in \{0, n, n+1, n+2, \dots\}\bigr\}$, $\omega(h^n, y) = \bigl\{h^k(y); k \in \{0, n, n+1, n+2, \dots\}\bigr\}$. Suppose $\xi\colon (X, f) \to (Y, h)$ is an isomorphism and $\varphi\colon (X, f) \to (Y, h)$ a homomorphism of the mono-unary algebra (X, f) onto the mono-unary algebra (Y, h). Denote $\Phi = [\mu, \varphi]$ the pair of mappings such that $\mu(f^n) = \xi \circ f^n \circ \xi^{-1}$. Then Φ is a II-homomorphism of the generalized transformation semihypergroup (X, G, ψ) into the GTS (Y, H, ω).

Indeed, for an arbitrary pair $[f^n, x] \in G \times X$ we have

$$\begin{aligned} \varphi\bigl(\psi(f^n, x)\bigr) &= \varphi\{x, f^n(x), f^{n+1}(x), \dots\} = \bigl\{\varphi(x), \varphi\bigl(f^n(x)\bigr), \varphi\bigl(f^{n+1}(x)\bigr), \dots\bigr\} \\ &= \bigl\{\varphi(x), \varphi\bigl(h^n(x)\bigr), \varphi\bigl(h^{n+1}(x)\bigr), \dots\bigr\} = \omega\bigl(h^n, \varphi(x)\bigr) \\ &= \omega\bigl(\xi \circ f^n \circ \xi^{-1}, \varphi(x)\bigr) = \omega\bigl(\mu(f^n), \varphi(x)\bigr) = \omega\bigl(\mu \times \varphi)[f^n, x]\bigr). \end{aligned}$$

The following example of generalized transformation hypergroup is based on consideration published in [4, 5].

Example 5.2. *Let $J \subset \mathbb{R}$ be an open interval and denote $C^\infty(J)$ the ring of all infinitely differentiable functions on J. Let us consider the set $\mathbb{LA}_n(J)$, $n \in \mathbb{N}$, of linear differential operators of the n-th order in the form*

$$L(p_0, \ldots, p_{n-1}) = \frac{d^n}{dx^n} + \sum_{k=0}^{n-1} p_k(x) \frac{d^k}{dx^k}.$$

Where $p_k \in C^\infty(J)$, $k = 0, 1, \ldots, n-1$; $L(p_0, \ldots, p_{n-1})\colon C^\infty(J) \longrightarrow C^\infty(J)$, thus

$$L(p_0, \ldots, p_{n-1})(f) = f^{(n)}(x) + p_{n-1}(x) f^{(n-1)}(x) + \cdots + p_0(x) f(x),\ f \in C^\infty(J).$$

Let δ_{ij} stand for the Kronecker symbol δ. For any but fixed $m \in \{0, 1, \ldots, n-1\}$ we denote by

$$\mathbb{LA}_n(J)_m = \big\{L(p_0, \ldots, p_{n-1}) | p_k \in C^\infty(J), p_m > 0\big\}.$$

Shortly we put $\mathbf{p} = (p_0(x), \ldots, p_{n-1}(x)), x \in J$ and on the set $\mathbb{LA}_n(J)_m$ we define a binary operation "$\circ_m$" and a binary relation $\leq_m$ in this way:

$$L(\mathbf{p}) \circ_m L(\mathbf{q}) = L(\mathbf{u})$$

where $u_k(x) = p_m(x) q_k(x) + (1 - \delta_{km}) p_k(x), x \in J, 0 \leq k \leq n-1$, and

$$L(\mathbf{p}) \leq_m L(\mathbf{q})$$

whenever $p_k(x) \leq q_k(x), k \neq m, k \in \{0, 1, \ldots, n-1\}, p_m(x) = q_m(x), x \in J$.

It is easy to verify that $(\mathbb{LA}_n(J)_m, \circ_m, \leq_m)$ is an ordered noncommutative group with the neutral element $D(\mathbf{w})$, where $D(\mathbf{w}) = (w_0, \ldots, w_{n-1})$, $w_k(x) = \delta_{km}$. An inverse to any $D(\mathbf{q})$ is $D^{-1}(\mathbf{q}) = \left(\frac{-q_0}{q_m}, \ldots, \frac{1}{q_m}, \ldots, \frac{-q_{n-1}}{q_m}\right)$.

Let $(\mathbb{Z}, +, \leq)$ be an additive group of all integers with an usual ordering "$\leq$". Then by Lemma 1.1 the structure $(\mathbb{Z}, \star)$, where

$$\star\colon \mathbb{Z} \times \mathbb{Z} \longrightarrow \mathcal{P}^*(\mathbb{Z})$$

was defined by $k \star l = [k + l)_\leq$ is a hypergroup.

For fixed $D(\mathbf{q}) \in \mathbb{LA}_n(J)_m$ we define an action $\psi_q : \mathbb{Z} \times \mathbb{LA}_n(J)_m \longrightarrow \mathcal{P}^*(\mathbb{LA}_n(J)_m)$ as follows,

$$\psi_q(k, L(\mathbf{p})) = \{L^t(\mathbf{q}) \circ_m L(\mathbf{p}) | t \leq k\}.$$

So $(\mathbb{LA}_n(J)_m, \mathbb{Z}, \varphi_q)$ is a generalized transformation hypergroup.

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