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Perfect edge domination in vague graphs

M. Kaliraja[*]
P. Kanibose[†]
A. Ibrahim[‡]

Abstract

In this paper, we introduce the notions of perfect edge domination set and perfect edge domination number in vague graphs. Also, we introduce the definitions of connected perfect edge domination set and connected perfect edge dominating number. Moreover, we investigate some related properties of these concepts with comprehensive results and illustrations.

Keywords: Vague graphs; Edge dominating set; Perfect edge domination set; Perfect edge domination number; Connected perfect edge domination set; Connected perfect edge domination number.

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[*] Assistant Professor, P.G. and Research Department of Mathematics, H. H. The Rajah's College, Pudukkotai, Affiliated to Bharathidasan University, Trichirappalli, Tamilnadu, India; mkr.maths009@gmail.com.

[†]Research Scholar, P. G. and Research Department of Mathematics, H. H. The Rajah's College, Pudukkotai, Affiliated to Bharathidasan University, Trichirappalli, Tamilnadu, India; kanibose77@gmail.com.

[‡] Assistant Professor, P.G. and Research Department of Mathematics, H. H. The Rajah's College, Pudukkotai, Affiliated to Bharathidasan University, Trichirappalli, Tamilnadu, India; ibrahimaadhil@yahoo.com; dribra@hhrc.ac.in

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1. Introduction

Edge domination sets in graphs are a phenomenon in research that has gained prominence due to the extreme scope of offering research solutions with varied dimensions. With the aid of perfect edge dominance and successful edge dominance in graphs, Chin Lung Lua, Ming-Tat Koa and Chuan Yi Tangb [3] investigated perfect edge dominance in graphs. R. S. Chitra and N. Prabhavathi [2] performed an analysis of perfect edges, perfect edge covering, and perfect edge vertex domination sets in graphs. The concept of ideal domination sets was proposed by S. Revathi, P.J. Jayalakshmi, and C.V. R. Harinarayanan [10].The notion of connected edge domination in fuzzy graphs was suggested by C.Y. Ponnappan, S. Basheer Ahamed, and P. Surulinathan [7]. P. Karpagam and V. Revathi [6] introduced the idea of connected edge perfect domination in fuzzy graphs with the idea of the principle of connected edge dominance in fuzzy graphs. S. Revathi, C.V.R. Harinarayanan, and R. Muthuraj [9] introduced an advanced idea of perfect domination in intuitionistic fuzzy graphs. W. L. Gau and D. J. Buehrer [4] proposed the concept of a vague set. R.A. Borzooeiy and H. Rashmanlou [1] introduced the notion of domination in vague graphs, and obtained strong domination numbers with applications. The definition of dominating sets in vague graphs was efficiently used in vague graphs by Yahya Talebi and Hossein Rashmanlou [11]. Recently, the authors [5] explored the concepts of edge dominance, independent edge domination in vague graphs, and obtained its related properties.

In this paper, we present the notion of a perfect edge domination set and the perfect edge domination number of the vague graphs. Further, we introduce the connected perfect edge domination set and connected perfect edge dominating number. Also, we obtained some relevant properties.

2. Preliminaries

In this section, we will show some basic definitions and properties that are helpful in developing our main results.

Definition 2.1[4] A vague set P in the universe of discourse X is characterized by two membership functions given by

i. A truth membership function $t_P: X \to [0,\ 1]$,

ii. A false membership function $f_P: X \to [0,\ 1]$.

Where $t_P(x)$ is lower bound of the grade of membership of x derived from the 'evidence for x', and $f_P(x)$ is a lower bound of the negation of x derived from the 'evidence against x' and $t_P(x) + f_P(x) \leq 1$. Thus the grade of membership of x in the vague set P is bounded by a subinterval $[\ t_P(x),\ 1 - f_P(x)]$ of [0, 1]. The vague set P is written as $P = \{(x, [\ t_P(x), f_S(x)])/x \in X\}$, where the interval $[\ t_P(x),\ 1 - f_S(x)]$ is called the value of x in the vague set P.

Definition 2.2[1] A vague graph is of the form $G = (P, Q)$, where

i. A sequence of distinct vertices $P = \{v_1, v_2, \dots v_n\}$, such that $t_P : P \to [0,1]$ and $f_P: P \to [0,\ 1]$ are truth and false membership functions, respectively such that $0 \leq t_P(x) + f_P(x) \leq 1$. for all $x \in P$.

ii. A vague relation of the vague subsets $X \times Y$ is an expression R, defined by $R = \{\{(x, y), t_R(x, y), f_R(x, y)\}/x \in X, y \in Y\}$, where $t_R: X \times Y \to [0,1]$ are$f_R: X \times Y \to [0,1]$, which satisfies the condition $0 \leq t_R(x, y) + f_R(x, y) \leq 1$ for all $(x, y) \in X \times Y$.

Definition 2.3[1] Let $G = (P, Q)$ be a vague graph, where $P = (t_{P,} f_P)$ is a vague set of vertex and $Q = (t_{Q,} f_Q)$ is a set edge in G, then for every $uv \in Q$, such that $t_Q(uv) \leq \min\{t_Q(u), t_Q(v)\}$ and $f_Q(uv) \leq \max\{f_Q(u), f_Q(v)\}$.

Definition 2.4 [1] Let $G = (P, Q)$ be a vague graph. If cardinality of the arcs $t_Q(v_i v_j) = \min\{t_P(v_i), f_P(v_j)\}$ and $f_Q(v_i v_j) = \max\{t_P(v_i), f_P(v_j)\}$ for all $v_i v_j \in$, then G is called a complete vague graph.

Definition 2.5[10] An arc of a vague graph $G = (P, Q)$ is said to be a strong edge in G. If $t_Q(uv) \geq (t_Q)^\infty(uv)$ and $f_Q(uv) \leq (f_Q)^\infty(uv)$.

Where i. $(t_Q)^\infty(uv) = sup\{(t_Q)^r(uv): r = 1,2, \dots, n\}$

ii . $(f_Q)^\infty(uv) = inf\{(f_Q)^r(uv): r = 1,2, \dots, n\}$.

Definition 2.6[10] Let e_i be an edge in a vague graph $G = (P, Q)$. Then the neighborhood of e_j is representing by $N(e_i) = \{\ e_j \in Q/(u, v) \text{ is a strong arc}\}$.

Definition 2.7[7] Let G = (P, Q) be a vague graph and e_i , $e_j \in Q$. If a strong arc e_i is adjacent to e_j. Then, we say that $e_{i,}$ dominates e_j . It is denoted by D.

Definition 2.8[1] An edge e_j in a vague graph G = (P, Q) is called a neighbor of $e_i \in D$ with respect to D, if $N(e_j) \cap D = \{e_i\}$.

Definition 2.9[1] Let G = (P, Q) be a vague graph. If neighborhood degree is defined by

i. The minimum neighborhood degree of G is $\delta(G) = min\{d_N(e), e \in Q\}$

ii. The maximum neighborhood degree of G is $\Delta(G) = max\{d_N(e), e \in Q\}$.

Definition 2.10[4] Two vertices v_i and v_j in a vague graph $G = (P, Q)$ are called a strong neighborhood G. If either one of the conditions are hold,

i. $t_Q(v_i v_j) > 0, f_Q(v_i v_j) > 0$

ii. $t_Q(v_i v_j) = 0,\ f_Q(v_i v_j) > 0$

iii. $t_Q(v_i v_j) > 0, f_Q(v_i v_j) = 0.$

Definition 2.11[10] Let $G = (P, Q)$ be a vague graph. Then number of edge (the cardinality of Q) is called the order size of a vague graph and is denoted by

$$O(S) = \sum_{v_i v_j \in Q} \left(\frac{1 + t_Q(v_i v_j) - f_Q(v_i v_j)}{2} \right) \text{ for all } v_i v_j \in Q.$$

Definition 2.12[4] Two edges in a vague graph $G = (P, Q)$ is celled an independent if there is no any strong arcs between them.

Definition 2.13[4] Let $G = (P, Q)$ be a vague graph. If the sub graph is induced by D has an isolated edge.

Definition 2.14[4] A edge in vague graph $G = (P, Q)$is an isolated edge, if it is not adjacent to any strong arc in G.

3. Main Results

In this section, we introduce perfect edge domination set, perfect edge domination number, connected perfect edge domination set and connected perfect edge dominating number of vague graphs, and obtain some properties with illustrations.

Definition 3.1 Let $G = (P, Q)$ be a vague graph and D be an edge dominating set in G. If for every edge of $Q(G) - D$ is adjacent to exactly one edge in D, then D is called a perfect edge domination set in G.

Example 3.2 Let $G = (P, Q)$ be a vague graph as shown in the figure 3.1. Consider the edge set $Q = \{e_{1,} e_{2,} e_{3,} e_{4,} e_5\}$. We have $e_{1,} e_4$ and e_5 are strong arcs in G.

Here, $\{e_5\}$ and $\{e_{1,}e_4\}$ are edge dominating sets in G. Now, $D = \{e_5\}$ is a perfect edge domination in G. Since, $\{e_5\}$ is dominates the all other neighbor edges in G.

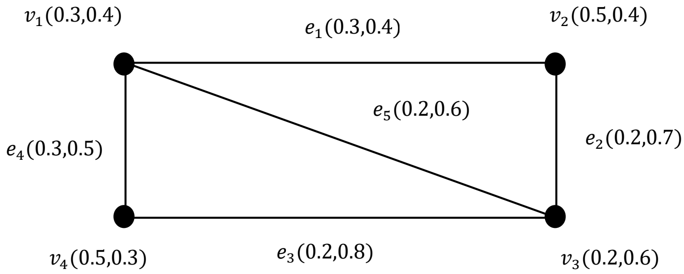

Figure 3.1: Perfect edge domination set

Definition 3.3 Let $G = (P, Q)$ be a vague graph. If it has a minimum perfect edge dominating set of vague graph G. Then it is called a perfect edge dominating number of G, and it is denoted by $\gamma_p(D)$ in G.

Example 3.4 Let $G = (P, Q)$ be a vague graph as shown in the figure 3.2. From the edge set $Q = \{e_{1,}e_{2,}e_{3,}e_{4,}\}$, we see that $\{e_1\}$ and $\{e_4\}$ are strong arc G. Then $\{e_1\}$ and $\{e_4\}$ are a perfect edge dominating sets of G. Then minimum perfect edge dominating number is $\gamma_p(D)$ = 0.70.

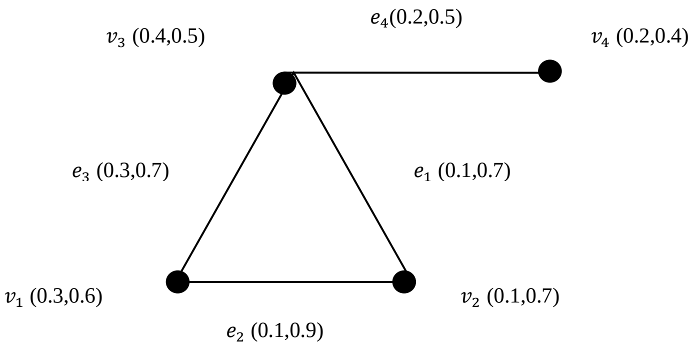

Figure 3.2: Minimum perfect edge dominating number.

Proposition 3.5 Let $G = (P, Q)$ be a vague graph with at least one isolated edge, then perfect edge dominating set does not exist.

Proof: Let D be a minimal perfect edge dominating set and e_i be the path of D, since G has at least one isolated edge. Incase $Q(G) - D$ is a perfect edge dominating set of vague graph, it has e_j as neighborhood of perfect edge domination set D. There exist a complement path of vague graph is denoted by $N(e_j)$.

From the definition 2.6, we have $|N(e_j) \cap D|$=1, but in this vague graph $|N(e_j) \cap D| \neq$1.

This is a contradiction. Therefore, $Q(G) - D$ not a perfect dominating set. ■

Proposition 3.6 Let $G = (P, Q)$ be a complete vague graph. Then, every edge in D is a perfect edge dominating set.

Proof: Let $G = (P, Q)$be a complete vague graph and let $e_i \in D$ be a edge dominating set in G. Then every edges in the graph is $t_Q(v_i v_j) = \min\{t_P(v_i), f_P(v_j)\}$ and $f_Q(v_i v_j) = \max\{t_P(v_i), f_P(v_j)\}$ for all $v_i v_j \in Q$.

Therefore, every path in G is a strong arc and complement edges are $e_j \in Q(G) - D$ adjacent to exactly one edge in D is said to be a perfect edge domination set in vague graph. Here, D is a perfect edge dominating set of vague graph G, and then every complete vague graph G is a perfect edge dominating set. ■

Example 3.7 Let $G = (P, Q)$ be a complete vague graph as shown in the figure 3.3. From the edge set $Q = \{e_{1,} e_{2,} e_{3,} e_{4,} e_5\}$ is an edge dominating set. Then, we have $e_{1,} e_4$ and e_5 which are strong arcs of the vague graph.

Here $\{e_5\}$ and $\{e_{1,} e_4\}$ are edge dominating set of G, for $D = \{e_5\}$ is a perfect edge domination in vague graph.

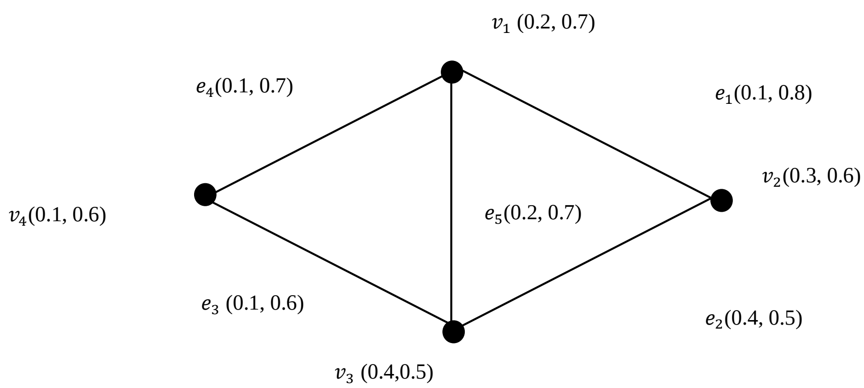

Figure 3.3: Perfect edge domination set

Proposition 3.8 Let $G = (P, Q)$ be a vague graph, then D is a minimal perfect edge dominating set. If for each $Q(G) - D$ is not a perfect edge dominating set.

Proof: Let G be a vague graph and has a minimal perfect edge domination set D. From the definition 3.1, if D is minimum, the arcs must be strong. Suppose e_i and e_j are any two edges adjacent in G, but $e_i \in D$ is a minimum perfect edge domination set of a vague graph. Then e_j may or may not have any strong in this graph. Thus, each edge in e_j has no strong neighbor of edge in $Q(G) - D$. Hence, $Q(G) - D$ is not perfect edge domination set of vague graph G. ■

Definition 3.9 Let $G = (P, Q)$ be a vague graph and S is an edge dominating set of G is connected perfect edge domination set with$\langle S \rangle$ is connected.

Example 3.10 Let $G = (P, Q)$ be a vague graph as shown in the figure3.4. From the edge set to $Q = \{e_{1,}e_{2,}e_{3,}e_{4,}e_5\}$, which are strong arcs of G. Here $\{e_1, e_2\}$ and $\{e_{3,}e_4\}$ are edge dominating sets of G. Then $D = \{e_1, e_4\}$ is connected perfect edge domination in G.Since, $\{e_1, e_4\}$ is a dominating set in G.

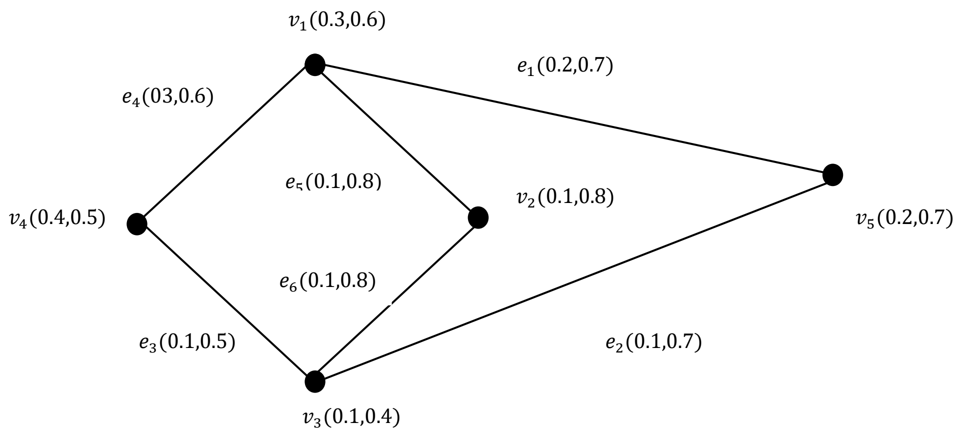

Figure 3.4: Connected perfect edge domination

Connected strong perfect edge dominating set $Q_{cs} = \{e_1, e_2\}$ is connected, then $Q - Q_{cs} = \{e_3, e_4, e_5, e_6\}$ is connected

$$N(e_3) = \{e_2, e_4, e_6\} \cap \{e_1, e_2\} = e_{2.}$$

$$N(e_5) = \{e_1, e_4, e_5\} \cap \{e_1, e_2\} = e_{1.}$$

$$N(e_6) = \{e_1, e_5, e_3\} \cap \{e_1, e_2\} = e_2.$$

$$N(e_3) = \{e_1, e_5, e_3\} \cap \{e_1, e_2\} = e_1.$$

Definition 3.11 Let $G = (P, Q)$ be a vague graph. Then the smallest cardinality number of the arc in any edge connected perfect edge dominating set of G is called connected perfect edge domination number, which is denoted as γ_{cp}.

Proposition3.12 Let $G = (P, Q)$be a vague graph and $\langle S \rangle$ be a minimal connected edge perfect dominating set of G,then $Q(G) - \langle S \rangle$ is also a connected edge perfect dominating set of G.

Proof: Let $\{e_1, e_2, e_3, \dots, e_n\}$ be an edge set of vague graph $G$and M be a minimal connected edge dominating set of $Q(G)$. If $Q(G) - M$ is not a connected perfect edge dominating set. Then from, the definition 3.1, we have D has minimum set of arcs that should be strong. if e_i and e_j are any two edges adjacent in G, but $e_i \in D$ is a minimum perfect edge domination set of a vague graph, then e_j is may (or) may not have any strong arc in this graph, Thus, every edge in e_j has no strong neighbor of edge in $Q(G) - D$. Hence $Q(G) - D$ is not a perfect edge domination set of vague graph G. ■

Example 3.13 Let $G = (P, Q)$ be a vague graph as shown in the figure 3.5. From the edge set $Q = \{e_1, e_2, e_3, e_4, e_5, e_6,\}$, we see that e_1, e_2 and e_5 are strong arcs in the graph G.Then $\{e_1, e_5\}$ are a connected perfect edge dominating set of G. Thus, $\{e_1, e_5\}$ is minimum connected perfect edge dominating number is $\gamma_{cp}(D)$ = 0.50.

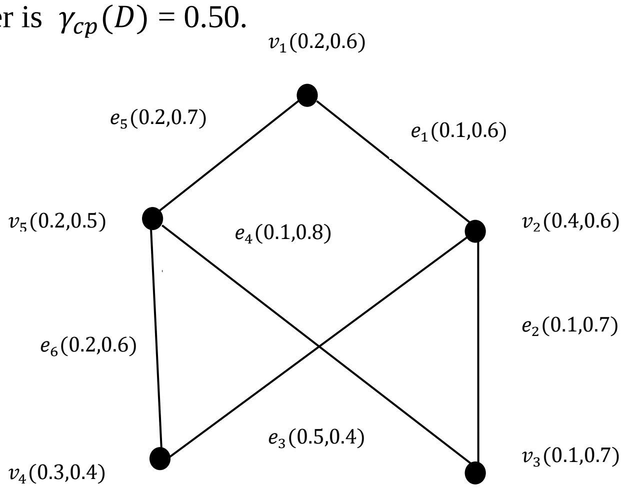

Figure 3.5: Minimum perfect edge dominating number

Remark 3.14 Let $G = (P, Q)$be a vague graph and $\langle S \rangle$ be a minimal connected edge perfect dominating set of G, then $Q(G) - \langle S \rangle$ is a minimal connected edge perfect dominating set of G.

Proposition 3.15 Let $G = (P, Q)$ be a vague graph and D is a connected perfect edge domination set and it is without isolated edges, then $\frac{r}{\Delta_c(G)+1} \geq \gamma_{CP}(G)$.

Proof: Let G be a vague graph and D be a connected perfect edge domination set. From the definition 3.9 of maximum neighbourhood and minimum neighbourhood of connected perfect edge domination set, then

$$|D|\Delta_c(G) \leq \sum_{e \in D} d_Q(e) = \sum_{e \in D} |N(e)|$$

$$\leq \left| \bigcup_{e \in D} N(e) \right|$$

$$\leq |Q - D|$$

$$\leq r - |D|$$

$$|D|\Delta_c(G) + |D| \leq r$$

Therefore, $\frac{r}{\Delta_c(G)+1} \geq \gamma_{CP}(G)$. ■

Proposition 3.16 Every connected perfect edge dominating set of a vague graph $G = (P, Q)$ is not independent.

Proof: Let G be a vague graph and D be a minimal connected perfect edge dominating set, then path of graphs have strong arcs. Clearly, every edges in the graph are satisfied by the condition of $t_Q(uv) \geq (t_Q)^\infty(uv)$ and $f_Q(uv) \leq (f_Q)^\infty(uv)$.

Suppose, we assume that a vague graph D contains an isolated edge e_i in G. Since G is a connected perfect edge dominating set, e_i is a strong neighborhood of at least one edge $D - \{e_j\}$. Thus, $D - \{e_j\}$ is an edge dominating set which contradicts in the minimum edge domination set D. Hence a vague graph is not independent. ■

Proposition 3.17 Let $G = (P, Q)$ be a vague graph and S_c be a minimal connected perfect edge dominating set of G.

Proof: Let S_c be a minimal connected perfect edge dominating set of vague graph $G = (P, Q)$. If e_j is not dominated by edge e_i in S_c and the induced vague graph $< S_c >$ is disconnected. We know that from the definition of connected perfect dominating set, if every edge $e_j \in Q - S_c$ is perfect dominated by exactly one edge e_i in S_c and the induced vague sub graph $< S_c >$ is connected. So, we have every edge $e_j \in Q - S_c$ is dominated by some edge e_i in $S_c$which is a contradiction. Therefore, S_c is connected dominating set. ■

4. Conclusions

In this study, we have introduced the notions of perfect edge domination set and perfect edge domination number of vague graphs. Furthermore, we have investigated some related properties with suitable examples. Moreover, we have introduced the notion of connected perfect domination set and connected perfect edge domination number of the vague graph. Finally, we have obtained some properties.

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