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Elongation of Sets in Soft Lattice Topological Spaces

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Abstract

The aim of this paper, we investigate some Lattice sets such as soft lattice exterior, soft lattice interior, soft lattice boundary and soft lattice border sets in soft lattice topological spaces which are defined over a soft lattice L with a fixed set of parameter A and it is also a generalization of soft topological spaces. Further, we develop and continue the initial views of some soft lattice sets, which are deep-seated for further research on soft lattice topology and will consolidate the origin of the theory of soft topological spaces.
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1 Introduction

The concept of soft theory was first originated by Molodstov in 1999, which is deal with unpredictable problems meanwhile modeling results in engineering cases such as medical sciences, economics, etc., In 2003, Maji. et. al.[8] studied and discussed the fundamental ideas of soft theory. Following stage of soft set linked with netrosophic sets are introduced by Parimala Mani et. al.[9] in 2018 and Also, we introduced the new notion of neutrosophic complex $\alpha\Psi$-connectedness in neutrosophic complex topological spaces and investigate some of its properties in 2022[5]. In 2019[13], several new generalizations of nano open sets be introduced and investigated by Nethaji, Ochanan.
The study of soft topological spaces (on short $\mathfrak{S}.\mathfrak{T}.\mathfrak{S}$) is instated by Shabir and Naz[14] in 2011. They discussed $\mathfrak{S}.\mathfrak{T}$ on the collection θ on soft set (on short $\mathfrak{S}.\mathfrak{S}$) over U. Accordingly, they discussed fundamental notions of $\mathfrak{S}.\mathfrak{T}.\mathfrak{S}$ such as soft open (on short $\mathfrak{S}.\mathfrak{O}$), soft closed (on short $\mathfrak{S}.\mathfrak{C}$), $\mathfrak{S}$ closure, $\mathfrak{S}$ neighborhood of a point, $\mathfrak{S}$ T_i spaces, for (i $=1, 2, 3, 4$), $\mathfrak{S}$ regular spaces, $\mathfrak{S}$ normal spaces, and their specific features are also established. Therefore, in 2011[1], Naim Cagman, Serkan Karatas, and Serdar Enginoglu investigated a topology with $\mathfrak{S}.\mathfrak{S}$ called $\mathfrak{S}.\mathfrak{T}$ and its corresponding features. Then they present the foundation of the theory $\mathfrak{S}.\mathfrak{T}.\mathfrak{S}$. The $\mathfrak{S}.\mathfrak{T}.\mathfrak{S}$ may be the initial stage for the concepts of the soft mathematical opinion of structures which are the foundation of $\mathfrak{S}.\mathfrak{S}$. theoretic operation.
From the concept of $\mathfrak{S}.\mathfrak{S}$, the idea of soft lattices (on short $\mathfrak{S}.\mathfrak{L}$) has arisen. In 2010[7], F. Li studied and defined this conviction of $\mathfrak{S}.\mathfrak{L}$ and primary operations of results on $\mathfrak{S}.\mathfrak{L}$. Additional, an application of $\mathfrak{S}.\mathfrak{S}$ to lattices has executed by E. Kuppusamy in 2011. A different approach towards $\mathfrak{S}.\mathfrak{L}$ can be seen in E. Kuppusamy apart from what F. Li has done. Further, the operation and the properties of $\mathfrak{S}.\mathfrak{L}$ were studied by V. D. Jobish. et. al.[4] in 2013. Many theorems related to various types of unions, intersections, and complements including De Margon's Laws are obtained. In 2020[12], M. Parimala et. al explained the $nI\alpha g$ closed sets in nano ideal toplogical spaces with various prevailing closed sets.
Currently, topology depends toughly on the thoughts of the soft theory. Recently, $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$ was first investigated by Sandhya. et. al.[11] in 2021 that are discussed throughout an $\mathfrak{S}.\mathfrak{L}$ 'L' with a fixed set of parameters 'A' and it is also a generalization of $\mathfrak{S}.\mathfrak{T}.\mathfrak{S}$. They detailed discussed the concept of Soft L - open (on short $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{O}$), soft L - closed (on short $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{C}$), $\mathfrak{S}.\mathfrak{L}$ - closure, $\mathfrak{S}.\mathfrak{L}$ – interior point, and $\mathfrak{S}.\mathfrak{L}$ - neighborhood. In this paper, we continue investigating a soft L – interior (on short $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{I}$), soft L – exterior (on short $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{E}$), soft L - boundary (on short $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{B}$), and soft L - border (on short $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{Bor}$) which are basics for stimulating research on $\mathfrak{S}.\mathfrak{T}.\mathfrak{S}$ and will build up the fountain of the theory of $\mathfrak{S}.\mathfrak{T}.\mathfrak{S}$.

2 Preliminaries

Definition 2.1 (5,7). *Let's take U be a whole set and A be a set parameters. A pair (F, A), where F is a map from A to $\wp(U)$ is called a $\mathfrak{S}.\mathfrak{S}$ over U. Here, the $\mathfrak{S}.\mathfrak{S}$ is simply represented by f_A.*

Example 2.1. *Let say that there are 6 cars in the whole world $U = \{\mathfrak{w}_1, \mathfrak{w}_2, \mathfrak{w}_3, \mathfrak{w}_4, \mathfrak{w}_5, \mathfrak{w}_6\}$ is the set of cars under regard and that $A = \{\rho_1, \rho_2, \rho_3, \rho_4, \rho_5\}$ is a set of parameters denoted as colors.*
The $r_a, (a = 1, 2, 3, 4, 5)$ it means the parameters 'Red', 'Blue', 'Black', 'White', and 'Ash,' respectively.

Consider the mapping f_A given by 'cars' (.), where (.) is to be complete in by one of the parameters $r_a \in E$. For instance, $f_A(\rho_1)$ means ' Cars (Colors)'.

Suppose that $B = \{\rho_1, \rho_2, \rho_5\} \subseteq A$ and $f_A(\rho_1) = \{\mathfrak{w}_1, \mathfrak{w}_4\}$, $f_A(\rho_2) = U$, and $f_A(\rho_5) = \{\mathfrak{w}_2, \mathfrak{w}_4, \mathfrak{w}_5\}$ Then, we can view the $\mathfrak{S}.\mathfrak{S}$ F_A as consisting of the following collection of approximations:

$F_A = \{(\rho_1, \{\mathfrak{w}_1, \mathfrak{w}_4\}), (\rho_2, U), (\rho_5, \{\mathfrak{w}_2, \mathfrak{w}_4, \mathfrak{w}_5\})\}.$

Definition 2.2 (2,7). *In two $\mathfrak{S}.\mathfrak{S}$ f_A, g_A over U, we say that*

(i) f_A is a soft subset of g_A if

(a) $A \subseteq B$, and (b) $\forall \rho \in A$, λ (ρ)= μ (ρ) are equal to estimations.

(ii) f_A is soft equal set to g_A denoted by $f_A = g_A$ if $f_A \subseteq g_A$ and $g_A \subseteq f_A$

Definition 2.3 (7). *Let $A = \{\rho_1, ... \rho_n\}$ be a parameters. The 'Not set of A', denoted by ΓA $is\ defined\ as$ $\Gamma A = \{\Gamma\rho_1, ..., \Gamma\rho_n\}$, $\Gamma\rho_i$ means not $\rho_i\ \forall\ i = 1, 2, 3 \ldots n$.*

Definition 2.4 (7,9). *Complement of a $\mathfrak{S}.\mathfrak{S}$ f_A over U, represented by f'_A is defined as $f'_A = (F', \Gamma A)$, $F' : \Gamma A \longrightarrow \wp(U)$ such that(on short $\mathfrak{s}.\mathfrak{t}$) $F'(\Gamma\rho) = U - F(\rho)$, $\forall\, \Gamma\rho \in \Gamma A$.*

Definition 2.5 (9). *The relative complement of a $\mathfrak{S}.\mathfrak{S}$ f_A over U, stand for f_A^C is defined as $(f_A)^C = (F^C, A)$, $F^C : A \longrightarrow \wp(U)$ $\mathfrak{s}.\mathfrak{t}$ $F^C(\rho) = U - F(\rho)$, $\forall\, \rho \in A$.*

Definition 2.6 (7,9). *Let f_A be a $\mathfrak{S}.\mathfrak{S}$ over U , then f_A is Null $\mathfrak{S}.\mathfrak{S}$ if $\forall\, \rho \in A, F(\rho) = \phi$ and is denoted by ϕ_A.*
Let f_A be a $\mathfrak{S}.\mathfrak{S}$ over U ,then f_A is absolute $\mathfrak{S}.\mathfrak{S}$ represented by U_A, if $\forall\, \rho \in A, F(\rho) = U$. Also, $U_A^C = \phi_A$ and $\phi_A^C = U_A$.

Definition 2.7 (2,7)**.** *Union of two $\mathfrak{S}.\mathfrak{S}$ f_A, g_B over U is the $\mathfrak{S}.\mathfrak{S}$ h_C,*

$$C = A \bigcup B \text{ and } \forall\, \rho \in C, \kappa(\rho) = \begin{cases} \lambda(\rho), & \text{if} \quad \rho \in A - B \\ \mu(\rho), & \text{if} \quad \rho \in B - A \\ \lambda(\rho) \bigcup \mu(\rho), & \text{if} \quad \rho \in A \bigcap B \end{cases}$$

We write $f_A \bigcup g_B = h_C$.

Definition 2.8 (2,7)**.** *The intersection of two $\mathfrak{S}.\mathfrak{S}$ f_A, g_B over a whole set U is the $\mathfrak{S}.\mathfrak{S}$ h_C, here $C = A \bigcap B$ and $\forall\, e \in C, \kappa(\rho) = \lambda(\rho)$ or $\mu(\rho)$. We mark done $f_A \bigcap g_B = h_c$.*

Definition 2.9 (1)**.** *Consider $\mathfrak{F}_{\mathfrak{A}}, \mathfrak{G}_{\mathfrak{A}} \in \mathfrak{S}.\mathfrak{S}$ (U, A). The soft symmetric difference of these sets is the $\mathfrak{S}.\mathfrak{S}$. $\mathfrak{H}_{\mathfrak{A}} \in$ to $\mathfrak{S}.\mathfrak{S}$.(U, A), here the map $\mathfrak{H} : A \to \wp(U)$ defined as follows:*
$\mathfrak{h}(\rho) = ((\mathfrak{f}(\rho) \setminus \mathfrak{g}(\rho)) \bigcup ((\mathfrak{g}(\rho) \setminus (\mathfrak{f}(\rho))$ for each $\rho \in A$. We mark down $\mathfrak{H}_{\mathfrak{A}} = \mathfrak{F}_{\mathfrak{A}} \,\Delta\, \mathfrak{G}_{\mathfrak{A}}$.

Definition 2.10 (3,6,10)**.** *A sublatice of a lattice L is a non-void subset of L that is a lattice with the same meet and join operation as L, ie., $\alpha, \beta \in L$ implies $\alpha \bigwedge \beta$, $\alpha \bigvee \beta \,\forall\, \alpha, \beta \in L$.*

Definition 2.11 (3,6,10)**.** *A Complete lattice L and A is the parameters of the $\mathfrak{S}.\mathfrak{L}$ over L. The triplet $M = (f, A, L)$, $f : A \to \wp(L)$ is $\mathfrak{S}.\mathfrak{L}$ if f(ρ) is the sublattice of L for each $\rho \in A$. Then the $\mathfrak{S}.\mathfrak{L}$ is represented by f_A^L.*

Definition 2.12 (10)**.** *Two $\mathfrak{S}.\mathfrak{L}$. f_A^L and g_A^L over L its difference is denoted by $h_A^L = f_A^L \setminus g_A^L$, is stated as $\mathfrak{h}(\rho) = ((\mathfrak{f}(\rho) \setminus \mathfrak{g}(\rho)) \,\forall \rho \in A$.*

Definition 2.13 (10)**.** *Let us consider L be any complete lattice and A be the non void set of parameters. Let θ contains complete members, uniquely complemented $\mathfrak{S}.\mathfrak{L}$. over L, then θ is $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}$, then the condition hold:*

(i) $\phi_A, L_A \in \theta$.

(ii) $\bigcup_{a \in n} \eta_a \in \theta, \forall \{\eta_a : a \in n\} \subseteq \theta$

(iii) $\eta_1 \bigcap \eta_2 \in \theta, \forall\, \eta_1, \eta_2 \in \theta$.

Then the triplet (L, θ, A) is called a $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$. (soft L – space or soft L – topological space) over L. The members of θ are called soft lattice open sets in L. Also, a soft lattice (f_A^L) is called soft lattice closed if the relative complement $(f_A^L)^C$ belongs to θ.

3 Extension of $\mathfrak{S}.\mathfrak{L}$ - sets

Definition 3.1. *In $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$, the $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{I}$ of (f_A^L) is the union of all $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{O}$ sets contained in f_A^L denoted by $(f_A^L)^\circ$.*
i.e., $(f_A^L)^\circ = \bigcup \{g_A^L \,:\, g_A^L \in \theta \text{ and } g_A^L \subseteq f_A^L\}$.

Theorem 3.1. *Let (L, θ, A) be a $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$ over L and f_A^L , g_A^L are $\mathfrak{S}.\mathfrak{L}.$ over L. Then,*

(i) $\phi_A^\circ = \phi_A \text{ and } L_A = L_A^\circ$

(ii) $(f_A^L)^\circ \subseteq (f_A^L)$

(iii) $f_A^L \text{ is a } \mathfrak{S}.\mathfrak{L} - \mathfrak{O} \text{ set} \iff (f_A^L)^\circ = f_A^L$

(iv) $((f_A^L)^\circ)^\circ = (f_A^L)^\circ$

(v) $f_A^L \subseteq g_A^L \Rightarrow (f_A^L)^\circ \subseteq (g_A^L)^\circ$

(vi) $(f_A^L)^\circ \bigcap (g_A^L)^\circ = (f_A^L \bigcap g_A^L)^\circ$

(vii) $(f_A^L)^\circ \bigcup (g_A^L)^\circ \subseteq (f_A^L \bigcup g_A^L)^\circ$

Proof
Results (i), (ii) are trival.

(iii) If (f_A^L) is $\mathfrak{S}.\mathfrak{L}-\mathfrak{O}$ set, (f_A^L) is itself a $\mathfrak{S}.\mathfrak{L}-\mathfrak{O}$ set contained in (f_A^L). Since, $(f_A^L)^\circ$ is the largest $\mathfrak{S}.\mathfrak{L} - \mathfrak{O}$ set contained in (f_A^L) , $(f_A^L) = (f_A^L)^\circ$.
Conversely, Suppose that $(f_A^L) = (f_A^L)^\circ$. Since $(f_A^L)^\circ$ is a $\mathfrak{S}.\mathfrak{L} - \mathfrak{O}$ set, so (f_A^L) is $\mathfrak{S}.\mathfrak{L} - \mathfrak{O}$ set over L.

(iv) since $(f_A^L)^\circ$ is $\mathfrak{S}.\mathfrak{L} - \mathfrak{O}$ set, by (iii) $((f_A^L)^\circ)^\circ = (f_A^L)^\circ$.

(v) suppose that $(f_A^L) \subseteq (g_A^L)$. Since,$(f_A^L)^\circ \subseteq (f_A^L) \subseteq (g_A^L)$. $(f_A^L)^\circ$ is a $\mathfrak{S}.\mathfrak{L} - \mathfrak{O}$ subset of (g_A^L), so by the definition of $(g_A^L)^\circ$, $(f_A^L)^\circ \subseteq (g_A^L)^\circ$.

(vi) we have $(f_A^L \bigcap g_A^L) \subseteq f_A^L$ and $(f_A^L \bigcap g_A^L) \subseteq g_A^L$. This implies (by v) $(f_A^L \bigcap g_A^L)^\circ \subseteq (f_A^L)^\circ$and $(f_A^L \bigcap g_A^L)^\circ \subseteq (g_A^L)^\circ$ so that, $(f_A^L \bigcap g_A^L)^\circ \subseteq (f_A^L)^\circ \bigcap (g_A^L)^\circ$.
Also, since $(f_A^L)^\circ \subseteq f_A^L \; and \; (g_A^L)^\circ \subseteq g_A^L \; implies$
$(f_A^L)^\circ \bigcap (g_A^L)^\circ \subseteq (f_A^L \bigcap g_A^L)$ so that, $(f_A^L \bigcap g_A^L)^\circ$ is the largest $\mathfrak{S}.\mathfrak{L} - \mathfrak{O}$ subsets of $(f_A^L \bigcap g_A^L)$. Hence, $(f_A^L)^\circ \bigcap (g_A^L)^\circ \subseteq (f_A^L \bigcap g_A^L)^\circ$.
Thus, $(f_A^L)^\circ \bigcap (g_A^L)^\circ = (f_A^L \bigcap g_A^L)^\circ$.

(vii) Since, $f_A^L \subseteq (f_A^L \bigcup g_A^L)$ and, $g_A^L \subseteq (f_A^L \bigcup g_A^L)$.
So, by (v) $(f_A^L)^\circ \subseteq (f_A^L \bigcup g_A^L)^\circ$ and $(g_A^L)^\circ \subseteq (f_A^L \bigcup g_A^L)^\circ$. So that $(f_A^L)^\circ \bigcup (g_A^L)^\circ \subseteq (f_A^L \bigcup g_A^L)^\circ$.

Example 3.1. *Now the given example to show that the statement of theorem 1(v) may be strict or equal, Let* $L = \{S_{l_1}, S_{l_2}, S_{l_3}, S_{l_4}, S_{l_5}, S_{l_6}, S_{l_7}, S_{l_8}\}$;$A = \{\rho_1, \rho_2\}$;
$\theta = \{f_{1A}^L, f_{2A}^L, f_{3A}^L, f_{4A}^L, f_{5A}^L, L_A, \phi_A\}$

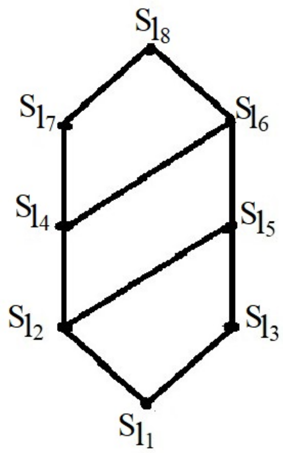

Figure 1: Complete lattice

$f_{1A}^L = \{(\rho_1, \{S_{l_4}, S_{l_7}, S_{l_8}\}), (\rho_2, \{S_{l_3}, S_{l_6}\})\}$,
$f_{2A}^L = \{(\rho_1, \{S_{l_6}, S_{l_8}\}), (\rho_2, \{S_{l_1}, S_{l_4}\})\}$
$f_{3A}^L = \{(\rho_1, \{S_{l_4}, S_{l_6}, S_{l_7}, S_{l_8}\}), (\rho_2, \{S_{l_1}, S_{l_3}, S_{l_4}, S_{l_6}\})\}$,
$f_{4A}^L = \{(\rho_1, \{S_{l_8}\}), (\rho_2, \phi)\}$
and $f_{5A}^L = \{(\rho_1, \{S_{l_4}, S_{l_7}\}), (\rho_2, \{S_{l_3}, S_{l_6}\})\}$

For Equal Condition,

We choose any two $\mathfrak{S}.\mathfrak{L}$ *from figure:1,*

$f_C^L = \{(\rho_1, \{S_{l_6}, S_{l_8}\}), (\rho_2, \{S_{l_1}, S_{l_4}\})\}$ *and*
$g_C^L = \{(\rho_1, \{S_{l_1}, S_{l_6}, S_{l_7}, S_{l_8}\}), (\rho_2, \{S_{l_1}, S_{l_3}, S_{l_4}, S_{l_6}\})\}$

$(f_C^L)^\circ = f_{2A}^L$ *and* $(g_C^L)^\circ = f_{2A}^L$.

Hence, $f_A^L \subset g_A^L\ implies (f_A^L)^\circ = (g_A^L)^\circ$.

For inclusion condition,

We choose any two $\mathfrak{S}.\mathfrak{L}$ *from figure:1,*

$f_D^L = \{(\rho_1, \{S_{l_8}\}), (\rho_2, \{S_{l_3}, S_{l_6}\})\}$ *and*
$g_D^L = \{(\rho_1, \{S_{l_4}, S_{l_7}, S_{l_8}\}), (\rho_2, \{S_{l_3}, S_{l_6}\})\}$

$(f_D^L)^\circ = f_{4A}^L$ *and* $(g_D^L)^\circ = f_{1A}^L$.

Hence, $f_A^L \subset g_A^L\ implies (f_A^L)^\circ \subset (g_A^L)^\circ$.

Example 3.2. *Now the given example to show that the statement of theorem 1(vii) may be strict or equal, Let us consider the lattice and* $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}$ *given in Example: 3.1*

For inclusion Condition,

We choose any two $\mathfrak{S}.\mathfrak{L}$ *from figure:1,*

$f_C^L = \{(\rho_1, \{S_{l_6}, S_{l_8}\}), (\rho_2, \{S_{l_1}, S_{l_3}, S_{l_6}\})\}$ *and*
$g_C^L = \{(\rho_1, \{S_{l_4}, S_{l_7}, S_{l_8}\}), (\rho_2, \{S_{l_1}, S_{l_3}, S_{l_4}, S_{l_6}\})\}$

$(f_C^L)^\circ = f_{4A}^L$ *and* $(g_C^L)^\circ = f_{1A}^L$*, which implies* $(f_C^L)^\circ \bigcup (g_C^L)^\circ = f_{1A}^L$.

$(f_C^L \bigcup g_C^L)^\circ\ is\ f_{3A}^L$.

Hence, $(f_A^L) \circ \bigcup (g_A^L)^\circ \subset (f_A^L \bigcup g_A^L)^\circ$.

For equal condition,

We choose any two $\mathfrak{S}.\mathfrak{L}$ *from figure:1,*

$f_C^L = \{(\rho_1, \{S_{l_6}, S_{l_8}\}), (\rho_2, \{S_{l_1}, S_{l_4}\})\}$ *and*
$g_C^L = \{(\rho_1, \{S_{l_1}, S_{l_6}, S_{l_7}, S_{l_8}\}), (\rho_2, \{S_{l_1}, S_{l_3}, S_{l_4}, S_{l_6}\})\}$

$(f_C^L)^\circ = f_{2A}^L$ *and* $(g_C^L)^\circ = f_{2A}^L$*, which implies* $(f_C^L)^\circ \bigcup (g_C^L)^\circ = f_{2A}^L$.

Hence, $(f_A^L) \circ \bigcup (g_A^L)^\circ = (f_A^L \bigcup g_A^L)^\circ$.

Definition 3.2. *Let (L, θ, A) be a* $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$ *over L, then the* $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{E}$. *of* $\mathfrak{S}.\mathfrak{L}$ f_A^L *is denoted by* $(f_A^L)_\circ$ *and is defined as* $(f_A^L)_\circ = ((f_A^L)^C)^\circ$.

Theorem 3.2. *Let f_A^L and g_A^L be $\mathfrak{S}.\mathfrak{L}$ of a $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$ (L, θ, A). Then,*

(i) $(f_A^L \bigcup g_A^L)_\circ = (f_A^L)_\circ \bigcap (g_A^L)_\circ$.

(ii) $(f_A^L)_\circ \bigcup (g_A^L)_\circ \subseteq (f_A^L \bigcap g_A^L)_\circ$.

(iii) $f_A^L \subseteq g_A^L$ *implies* $(f_A^L)_\circ \supseteq (g_A^L)_\circ$.

Proof

(i) $(f_A^L \bigcup g_A^L)_\circ = ((f_A^L \bigcup g_A^L)^C)^\circ = ((f_A^L)^C \bigcap (g_A^L)^C)^\circ = ((f_A^L)^C)^\circ \bigcap ((g_A^L)^C)^\circ = (f_A^L)_\circ \bigcap (g_A^L)_\circ$

(ii) $(f_A^L)_\circ \bigcup (g_A^L)_\circ = ((f_A^L)^C)^\circ \bigcup ((g_A^L)^C)^\circ \subseteq ((f_A^L)^C \bigcup (g_A^L)^C)^\circ = ((f_A^L \bigcap g_A^L)^C)^\circ = (f_A^L \bigcap g_A^L)_\circ$

(iii) $(g_A^L)_\circ = ((g_A^L)^C)^\circ \subseteq ((f_A^L)^C)^\circ = (f_A^L)_\circ$

Example 3.3. *Now the given example to show that the statement of theorem 2(ii) may be strict or equal, Let* $L_A = \{S_{l_1}, S_{l_2}, S_{l_3}, S_{l_4}, S_{l_5}, S_{l_6}, S_{l_7}\}$; $A = \{\rho_1, \rho_2\}$; $\theta = \{f_{1A}^L, f_{2A}^L, f_{3A}^L, f_{4A}^L, L_A, \phi_A\}$

$f_{1A}^L = \{(\rho_1, \{S_{l_3}, S_{l_6}\}), (\rho_2, \{S_{l_4}, S_{l_5}\})\}$, $f_{2A}^L = \{(\rho_1, \{S_{l_6}\}), (\rho_2, \phi)\}$
$f_{3A}^L = \{(\rho_1, \{S_{l_2}, S_{l_3}, S_{l_5}, S_{l_6}\}), (\rho_2, \{S_{l_4}, S_{l_5}, S_{l_6}\})\}$ *and*
$f_{4A}^L = \{(\rho_1, \{S_{l_2}, S_{l_5}, S_{l_6}\}), (\rho_2, \{S_{l_6}\})\}$

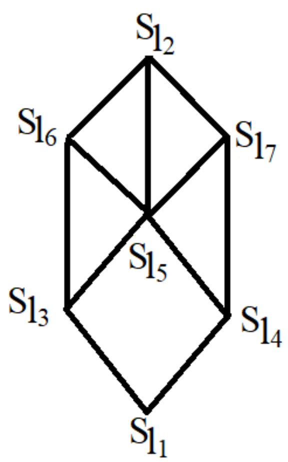

Figure 2: Complete lattice

For inclusion condition,

Now we take any two $\mathfrak{S}.\mathfrak{L}$ *from the figure:2,*

$f_C^L = \{(\rho_1, \{S_{l_2}, S_{l_6}\}), (\rho_2, \{S_{l_6}\})\}$ *and*
$g_C^L = \{(\rho_1, \{S_{l_2}, S_{l_3}, S_{l_5}\}), (\rho_2, \{S_{l_4}, S_{l_5}\})\}$

Then, $(f_C^L \bigcap g_C^L) = \{(\rho_1, \{S_{l_2}\}), (\rho_2, \phi)\}$. $(f_C^L)_\circ = \phi_A$ *and* $(g_C^L)_\circ = f_{2A}^L$, *which implies* $(f_C^L)_\circ \bigcup (g_C^L)_\circ = f_{2A}^L$. $(f_C^L \bigcap g_C^L)_\circ\, is\, f_{1A}^L$.

Hence, $(f_A^L)_\circ \bigcup (g_A^L)_\circ \subset (f_A^L \bigcap g_A^L)_\circ$.

For Equal condition,

Now we take any two $\mathfrak{S}.\mathfrak{L}$ *from the figure:2,*

$f_C^L = \{(\rho_1, \{S_{l_5}, S_{l_7}\}), (\rho_2, \{S_{l_4}, S_{l_7}\})\}$ *and*
$g_C^L = \{(\rho_1, \{S_{l_3}, S_{l_5}, S_{l_7}\}), (\rho_2, \{S_{l_4}, S_{l_5}, S_{l_7}\})\}$
Then, $(f_C^L \bigcap g_C^L) = \{(\rho_1, \{S_{l_5}, S_{l_7}\}), ((\rho_2, \{S_{l_4}, S_{l_7}\})\}$.

$(f_C^L)_\circ = f_{2A}^L$ *and* $(g_C^L)_\circ = f_{2A}^L$, *which implies* $(f_C^L)_\circ \bigcup (g_C^L)_\circ = f_{2A}^L$. $(f_C^L \bigcap g_C^L)_\circ\, is\, f_{2A}^L$.

Hence, $(f_A^L)_\circ \bigcup (g_A^L)_\circ = (f_A^L \bigcap g_A^L)_\circ$.

Example 3.4. *Now the given example to show that the statement of theorem 2(iii) may be strict or equal, Let us consider the lattice and* $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}$ *given in Example: 3.3*

For Equal condition,

Now we take any two $\mathfrak{S}.\mathfrak{L}$ *from the figure:2,*

$f_D^L = \{(\rho_1, \{S_{l_3}, S_{l_6}\}), (\rho_2, \{S_{l_4}, S_{l_5}, S_{l_6}\})\}$ *and*
$g_D^L = \{(\rho_1, \{S_{l_1}, S_{l_3}, S_{l_5}, S_{l_6}\}), (\rho_2, \{S_{l_4}, S_{l_5}, S_{l_6}\})\}$

$(f_D^L)_\circ = \phi_A$ *and* $(g_D^L)_\circ = \phi_A$.

Hence, $f_A^L \subseteq g_A^L\, implies\, (f_A^L)_\circ = (g_A^L)_\circ$.

For inclusion condition,

Now we take any two $\mathfrak{S}.\mathfrak{L}$ *from the figure:2,*

$f_B^L = \{(\rho_1, \{S_{l_4}, S_{l_5}\}), (\rho_2, \{S_{l_2}, S_{l_3}, S_{l_6}, S_{l_7}\})\}$ *and*
$g_B^L = \{(\rho_1, \{S_{l_2}, S_{l_4}, S_{l_5}, S_{l_7}\}), (\rho_2, \{S_{l_1}, S_{l_2}, S_{l_3}, S_{l_5}, S_{l_6}, S_{l_7}\})\}$

$(f_B^L)_\circ = f_{1A}^L$ *and* $(g_B^L)_\circ = f_{2A}^L$.

Hence, $f_A^L \subseteq g_A^L \; implies \; (f_A^L)_\circ \supset (g_A^L)_\circ$.

Definition 3.3. *In* $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$*, then the* $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{B}$ *of* $\mathfrak{S}.\mathfrak{L}$ f_A^L *is denoted by* $(f_A^L)^B$ *and is defined as* $(f_A^L)^B = \overline{f_A^L} \bigcap \overline{(f_A^L)^C}$.

Theorem 3.3. *Let (L, θ, A) be a* $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$*:*

(i) $(f_A^L)^B \bigcap (f_A^L)^\circ = f_\phi^L$

(ii) $(f_A^L)^B \bigcap (f_A^L)_\circ = f_\phi^L$

Proof

(i) $(f_A^L)^B \bigcap (f_A^L)^\circ = (\overline{f_A^L} \bigcap \overline{(f_A^L)^C}) \bigcap (f_A^L)^\circ$
$= \overline{f_A^L} \bigcap \overline{(f_A^L)^C} \bigcap (f_A^L)^\circ = f_\phi^L$

(ii) $(f_A^L)^B \bigcap (f_A^L)_\circ = \overline{f_A^L} \bigcap \overline{(f_A^L)^C} \bigcap (f_A^L)^C)^\circ = \overline{f_A^L} \bigcap \overline{(f_A^L)^C} \bigcap (\overline{(f_A^L)})^C$
$= f_\phi^L$

Example 3.5. *Now the given example for find the boundary, Let us consider the lattice and* $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}$ *given in Example: 3.1*

Now we take any $\mathfrak{S}.\mathfrak{L}$ *from the figure:1,*

$f_C^L = \{(\rho_1, \{S_{l_2}, S_{l_8}\}), (\rho_2, \{S_{l_1}, S_{l_3}\})\}$ *Then,* $(f_C^L)^B = (f_{4A}^L)^C$

Definition 3.4. *Let (L, θ, A) be a* $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$ *over L, then the* $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{Bor}$ *of* $\mathfrak{S}.\mathfrak{L}$ f_A^L *is denoted by* $(f_A^L)^\bullet$ *and is defined as* $(f_A^L)^\bullet = f_A^L - (f_A^L)^\circ$.

Theorem 3.4. *Let (L, θ, A) be a* $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$*. Then the following hold:*

(i) $(f_A^L)^{\bullet} = A \bigcap \overline{(L_A - f_A^L)}$

(ii) $(\phi_A^L)^{\bullet} = \phi_A^L$

(iii) $(f_A^L)^{\bullet} \subseteq ((f_A^L)^{\circ})^C$

(iv) $(f_A^L)^{\bullet} \subseteq f_A^L \subseteq \overline{f_A^L}$

Proof

(i) $f_A^L \bigcap ((f_A^L)^{\circ})^C = f_A^L \bigcap \overline{(f_A^L)^{\ C}} = f_A^L \bigcap \overline{(L_A - f_A^L)}$

(ii) $\phi_A^L \bigcap ((\phi_A^L)^{\circ})^C = (\phi_A^L) \bigcap \overline{(\phi_A^L)^C} = \phi_A^L$

(iii) $f_A^L - (f_A^L)^{\circ} = f_A^L \bigcap ((f_A^L)^{\circ})^C \subseteq ((f_A^L)^{\circ})^C$

(iv) By definition of $(f_A^L)^{\bullet}$, $(f_A^L)^{\bullet} \subseteq f_A^L$.
we know that,$f_A^L \subset \overline{f_A^L}$ Therefore,$(f_A^L)^{\bullet} \subseteq f_A^L \subseteq \overline{f_A^L}$

Example 3.6. *Now the given example to show that the statement of theorem 4(iv) may be strict or equal, Let us consider the lattice and* $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}$ *given in Example: 3.3*

We choose any two $\mathfrak{S}.\mathfrak{L}$ *from the figure:2,*

$g_B^L = \{(\rho_1, \{S_{l_2}, S_{l_3}, S_{l_5}, S_{l_6}\}), (\rho_2, \{S_{l_4}, S_{l_5}, S_{l_6}\})\}$

Now, the Closure of g_B^L *is* L_A *, Then border of* g_B^L*, is* ϕ_A

Hence,$(g_B^L)^{\bullet} \subset g_B^L \subset \overline{g_B^L}$.

4 Conclusions

In the present work, we defined and discussed some $\mathfrak{S}.\mathfrak{L}$ – sets of $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$. We extended some basic results relating to $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{I}$, $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{E}$, $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{B}$, and $\mathfrak{S}.\mathfrak{L}$ - $\mathfrak{Bor}$ of $\mathfrak{S}.\mathfrak{L}.\mathfrak{T}.\mathfrak{S}$. In the interior section, idempotent and monotonicity results are held. Formerly the intersection of the boundary and interior soft lattice gives the null set and the intersection of the boundary and exterior soft lattice should not give the non-empty soft sets. In end, this paper is the inception of a novel structure. Further, we learned a few viewpoints, it will be needed to carry out a new seeking work to build future applications.

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