Families of mappings satisfying a mixed implicit relation

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Abstract

A fixed point for a suitable map or operator is identical to the presence of a solution to a theoretical or real-world problem. As a result, fixed points are crucial in many fields of mathematics, science and engineering. The purpose of this paper is to prove unique common fixed point theorems for families of weakly compatible mappings. Given mappings satisfy common limit range property and a mixed implicit relation. Our results generalize, extend and improve the results of Imdad (2013) and Popa (2018). We provide an application for integral type contraction condition. An example is also mentioned to check the authenticity of our results.

Keywords: common fixed point, weakly compatible mappings, mixed implicit relation, almost altering distance, common limit range property.

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1 Introduction and Preliminaries

Fixed point theory is an important tool of modern mathematics as it helps to find a unique fixed point of multi-valued and single-valued mappings by restricting the condition of the domain of the function. It also helps to find the results of many differential as well as integral equations which can further be used to solve many industrial based problems. The most popular tool in fixed point theory is Banach contraction [6] principle which states that every contraction mapping on a complete metric space has a unique fixed point. Various authors have extended and generalized this principle in various directions. In 1976, Jungck [9] used the concept of commuting maps to prove a common fixed point theorem. Several authors have investigated various concepts of minimal commuting maps. A pair of self-mappings $(\mathcal{P}, \mathcal{Q})$ on a metric space $(\mathcal{X}, d)$ is said to be compatible [10] if $\lim_{n\to\infty} d(\mathcal{P}\mathcal{Q}u_n, \mathcal{Q}\mathcal{P}u_n) = 0$, whenever $\{u_n\}$ is a sequence in $\mathcal{X}$ such that $\lim_{n\to\infty} \mathcal{P}u_n = \lim_{n\to\infty} \mathcal{Q}u_n = u$ for some $u \in \mathcal{X}$. A pair of self-mappings $(\mathcal{P}, \mathcal{Q})$ on a metric space $(\mathcal{X}, d)$ is called weakly compatible [11] if $\mathcal{P}$ and $\mathcal{Q}$ commute at their points of coincidence. Pant ([13], [14], [15]) initiated the study of common fixed points for non-compatible mappings. Further, Aamri and El-Moutawakil [1] introduced $(E.A)$ property as a generalization of non-compatible mappings. A pair of self-mappings $(\mathcal{P}, \mathcal{Q})$ on a metric space $(\mathcal{X}, d)$ is said to satisfy $(E.A)$ property [1] if there exists a sequence $\{u_n\}$ in $\mathcal{X}$ such that $\lim_{n\to\infty} \mathcal{P}u_n = \lim_{n\to\infty} \mathcal{Q}u_n = u$, for some $u \in \mathcal{X}$.

In 2011, Sintunavarat and Kumam [22] introduced the concept of common limit range property.

Definition 1.1. *[22] A pair of self-mappings $(\mathcal{P}, \mathcal{Q})$ on a metric space $(\mathcal{X}, d)$ is said to satisfy the common limit range property with respect to $\mathcal{Q}$, denoted by $CLR_{\mathcal{Q}}$, if there exists a sequence $\{u_n\}$ in $\mathcal{X}$ such that*

$$\lim_{n\to\infty} \mathcal{P}u_n = \lim_{n\to\infty} \mathcal{Q}u_n = u, \text{ for some } u \in \mathcal{Q}(\mathcal{X}).$$

Thus one can note that the mappings $\mathcal{P}$ and $\mathcal{Q}$ satisfying property $(E.A)$ along with the closedness of the subspace $\mathcal{Q}(\mathcal{X})$ always have $CLR_{\mathcal{Q}}$ property with respect to $\mathcal{Q}$.

In 2013, Imdad *et al.* [8] extended the notion of common limit range property for pairs of self mappings.

Definition 1.2. *[8] Two pairs of self-mappings $(\mathcal{P}, \mathcal{Q})$ and $(\mathcal{S}, \mathcal{T})$ on a metric space $(\mathcal{X}, d)$ are said to satisfy common limit range property with respect to $\mathcal{Q}$ and $\mathcal{T}$, denoted by $(CLR)_{(\mathcal{Q},\mathcal{T})}$, if there exists two sequences $\{u_n\}$ and $\{v_n\}$ in $\mathcal{X}$ such that*

$$\lim_{n\to\infty} \mathcal{P}u_n = \lim_{n\to\infty} \mathcal{Q}u_n = \lim_{n\to\infty} \mathcal{S}v_n = \lim_{n\to\infty} \mathcal{T}v_n = t,$$

where $t \in \mathcal{Q}(\mathcal{X}) \cap \mathcal{T}(\mathcal{X})$.

In 2018, Popa *et al.* [17] introduced a new type of limit range property as follows.

Definition 1.3. *[17] Let* $\mathcal{P}, \mathcal{Q}, \mathcal{T}$ *be self mappings of a metric space* $(\mathcal{X}, d)$*. The pair* $(\mathcal{P}, \mathcal{Q})$ *is said to satisfy common limit range property with respect to* $\mathcal{T}$ *if there exists a sequence* $\{u_n\}$ *in* $\mathcal{X}$ *such that* $\lim_{n\to\infty} Pu_n = \lim_{n\to\infty} Qu_n = u$*, for some* $u \in \mathcal{Q}(\mathcal{X}) \cap \mathcal{T}(\mathcal{X})$.

Now we extend the definition 1.3 for families of mappings.

Definition 1.4. *Let* $\mathcal{Q}_1, \mathcal{Q}_2, ..., \mathcal{Q}_{2n}$ *and* $\mathcal{P}$ *be self mappings of a metric space* $(\mathcal{X}, d)$*. The pair* $(\mathcal{P}, \mathcal{Q}_1\mathcal{Q}_3...\mathcal{Q}_{2n-1})$ *is said to satisfy common limit range property with respect to* $\mathcal{Q}_2\mathcal{Q}_4...\mathcal{Q}_{2n}$ *if there exists a sequence* $\{u_n\}$ *in* $\mathcal{X}$ *such that* $\lim_{n\to\infty} Pu_n = \lim_{n\to\infty} \mathcal{Q}_1\mathcal{Q}_3...\mathcal{Q}_{2n-1}u_n = u$*, for some* $u \in \mathcal{Q}_1\mathcal{Q}_3...\mathcal{Q}_{2n-1}(\mathcal{X}) \cap \mathcal{Q}_2\mathcal{Q}_4...\mathcal{Q}_{2n}(\mathcal{X})$.

Remark 1.1. *[17] Let* $\mathcal{P}, \mathcal{Q}, \mathcal{S}$ *and* $\mathcal{T}$ *be self mappings of a metric space* $(\mathcal{X}, d)$*. If the pairs* $(\mathcal{P}, \mathcal{Q})$ *and* $(\mathcal{S}, \mathcal{T})$ *satisfy the common limit range property with respect to* $\mathcal{Q}$ *and* $\mathcal{T}$*, then* $(\mathcal{P}, \mathcal{Q})$ *satisfy the limit range property with respect to* $\mathcal{T}$*, but the converse does not hold.*

Boyd and Wong [5] introduced ϕ contraction condition and generalized the Banach contraction principle using this contraction. A self mapping $\mathcal{P}$ on a complete metric space $(\mathcal{X}, d)$ is said to satisfy ϕ contraction if $d(\mathcal{P}\alpha, \mathcal{P}\beta) \leq \phi(d(\alpha, \beta))$, for all $\alpha, \beta \in \mathcal{X}$, where $\phi : [0, \infty) \to [0, \infty)$ is an upper semi continuous function from right such that $0 \leq \phi(t) < t$ for all $t > 0$. The theorems of existence of fixed points for self mappings in Hilbert spaces satisfying ϕ-weak contraction were studied by Alber and Guerre-Delabriere [3]. Further Rhoades [21] extended this concept in complete metric space. Some fixed point results are proved in [7], [8] and in other papers for mappings with common limit range property satisfying (ϕ, ψ)-weak contractive conditions.
The following theorem is proved in [8].

Theorem 1.1. *[8] Let* P*,* Q*,* S *and* T *be self mappings of a metric space* $(\mathcal{X}, d)$ *satisfying*

$$\psi(d(Px, Qy)) \leq \psi(m(x, y)) - \phi(m(x, y)),$$

for all $x, y \in \mathcal{X}$ *and for some* ϕ*,* ψ*,*
where $m(x, y) = \max\{d(Sx, Ty), d(Sx, Px), d(Ty, Qy), d(Sx, Qy), d(Ty, Px)\}$ *and* ϕ*,* $\psi : [0, \infty) \to [0, \infty)$ *such that* ϕ *is a lower semi-continuous function and* $\phi^{-1}(0) = 0$ *and* ψ *is a non-decreasing continuous function with* $\psi^{-1}(0) = 0$.
If the pairs (P, Q) *and* (S, T) *satisfy the* $(CLR)_{(S,T)}$ *property and are weakly compatible, then* P*,* Q*,* S *and* T *have a unique common fixed point.*

Definition 1.5. *[12] An altering distance is a function $\psi : [0,\infty) \to [0,\infty)$ satisfying:*
(ψ_1): ψ is increasing and continuous,
(ψ_2): $\psi(t) = 0$ if and only if $t = 0$.

Definition 1.6. *[18] A function $\psi : [0,\infty) \to [0,\infty)$ is an almost altering distance if it satisfies:*
(ψ_1'): ψ is continuous,
(ψ_2'): $\psi(t) = 0$ if and only if $t = 0$.

Example 1.1. *Define a function $\psi : [0,\infty) \to [0,\infty)$ by*

$$\psi(t) = \begin{cases} 2t, t \in [0,1] \\ \frac{1}{1+t}, t \in (1,\infty). \end{cases}$$

Here we note that every altering distance is an almost altering distance, but converse is not true.
Various authors have unified several common fixed point theorems by using implicit functions. In 2008, Ali and Imdad [2] introduced a new class of implicit functions.

Definition 1.7. *[2] Let $\mathcal{F}$ be the family of lower semi-continuous functions $F : \mathbb{R}_+^6 \to \mathbb{R}$ which are satisfying:*
(F_1) for all $u > 0$, $F(u,0,u,0,0,u) > 0$;
(F_2) for all $u > 0$, $F(u,0,0,u,u,0) > 0$;
(F_3) for all $u > 0$, $F(u,u,0,0,u,u) > 0$;

Definition 1.8. *[17] Let $\mathcal{F}_D$ be the set of all lower semi-continuous functions $F : \mathbb{R}_+^6 \to \mathbb{R}$ which are satisfying:*
(F_{1D}) for all $u > 0$, $F(u,0,u,0,0,u) \geq 0$;
(F_{2D}) for all $u > 0$, $F(u,0,0,u,u,0) \geq 0$;
(F_{3D}) for all $u > 0$, $F(u,u,0,0,u,u) \geq 0$;

Now we provide some examples in support of definition 1.8.

1. Let $F(u_1,...,u_6) = u_1 - t\max\{u_2,u_3,u_4,u_5,u_6\}$, where $t \in [0,1]$.
2. Let $F(u_1,...,u_6) = u_1 - t\max\{u_2,u_3,u_4,\frac{u_5+u_6}{3}\}$, where $t \in [0,1]$.
3. Let $F(u_1,...,u_6) = u_1 - \alpha\max\{u_2,u_3,u_4\} - \beta(u_5+u_6)$, where $\alpha, \beta \geq 0$ and $\alpha + 2\beta < 1$.
4. Let $F(u_1,...,u_6) = u_1 - \alpha\max\{u_2,u_3,u_4,\frac{1}{2}(u_5+u_6),\frac{u_3u_4}{1+u_2},\frac{u_5u_6}{1+u_1}\}$, where $\alpha \in [0,1)$.

5. Let $F(u_1, ..., u_6) = u_1 - \max\{cu_2, cu_3, cu_4, au_5 + bu_6)\}$, where $c > 0$, $a, b \geq 0$ and $a + b + c \leq 1$.

Definition 1.9. *[17] Let $\mathcal{G}_D$ be the set of all lower semi-continuous functions $G : \mathbb{R}_+^5 \to \mathbb{R}$ such that $G(t_1, ..., t_5) > 0$ if one of $t_1, ..., t_5 > 0$.*

The following functions belong to the set $\mathcal{G}_D$.

1. $G(t_1, ..., t_5) = \max\{t_1, ..., t_5\}$.
2. $G(t_1, ..., t_5) = \max\{t_1, \frac{t_2+t_3}{2}, \frac{t_4+t_5}{2}\}$.
3. $G(t_1, ..., t_5) = t_1^2 + t_2^2 + t_3^2 + t_4^2 + t_5^2$.
4. $G(t_1, ..., t_5) = \frac{1}{t_1+t_2+t_3+t_4+t_5}$.

Definition 1.10. *[17] A function $\phi(u_1, ..., u_6, t_1, ..., t_5) = F(u_1, ..., u_6) + G(t_1, ..., t_5)$ is called a mixed implicit relation.*

The aim of this paper is to prove general fixed point theorems for families of weakly compatible mappings with common limit range property satisfying a mixed implicit relation. Our results generalize, extend and improve the results of Popa [17] and Imdad [8].

2 Main Results

In 2018, Popa *et al.* [17] proved the following theorem.

Theorem 2.1. *[17] Let $(\mathcal{X}, d)$ be a metric space and P, Q, S and T be four self mappings on $\mathcal{X}$ satisfying*

$$F(\psi(d(Px, Qy)), \psi(d(Sx, Ty)), \psi(d(Sx, Px)), \psi(d(Ty, Qy)), \psi(d(Sx, Qy)), \psi(d(Ty, Px))) + G(\psi(d(Sx, Ty)), \psi(d(Sx, Px)), \psi(d(Ty, Qy)), \psi(d(Sx, Qy)), \psi(d(Ty, Px))) \leq 0,$$

for all $x, y \in \mathcal{X}$, for some $F \in \mathcal{F}_D$, $G \in \mathcal{G}_D$ and ψ is an almost altering distance. If the pairs (P, S) and (Q, T) are weakly compatible and (P, S) and T satisfy $(CLR)_{(P,S)T}$ property, then P, Q, S and T have a unique common fixed point.

Now we extend the Theorem 2.1 for any even number of weakly compatible mappings.

Theorem 2.2. *Let $Q_1, Q_2, ..., Q_{2n}, P_0$ and P_1 be self mappings on a metric space $(\mathcal{X}, d)$, satisfying the following conditions:*
(C1) $Q_2(Q_4...Q_{2n}) = (Q_4...Q_{2n})Q_2$,
$Q_2Q_4(Q_6...Q_{2n}) = (Q_6...Q_{2n})Q_2Q_4$,
$\vdots$
$Q_2...Q_{2n-2}(Q_{2n}) = (Q_{2n})Q_2...Q_{2n-2}$,
$P_1(Q_4...Q_{2n}) = (Q_4...Q_{2n})P_1$,
$P_1(Q_6...Q_{2n}) = (Q_6...Q_{2n})P_1$,
$\vdots$
$P_1Q_{2n} = Q_{2n}P_1$,
$Q_1(Q_3...Q_{2n-1}) = (Q_3...Q_{2n-1})Q_1$,
$Q_1Q_3(Q_5...Q_{2n-1}) = (Q_5...Q_{2n-1})Q_1Q_3$,
$\vdots$
$Q_1...Q_{2n-3}(Q_{2n-1}) = (Q_{2n-1})Q_1...Q_{2n-3}$,
$P_0(Q_3...Q_{2n-1}) = (Q_3...Q_{2n-1})P_0$,
$P_0(Q_5...Q_{2n-1}) = (Q_5...Q_{2n-1})P_0$,
$\vdots$
$P_0Q_{2n-1} = Q_{2n-1}P_0$,
(C2) the pairs $(P_0, Q_1...Q_{2n-1})$ and $(P_1, Q_2...Q_{2n})$ are weakly compatible and $(P_0, Q_1....Q_{2n-1})$ and $Q_2...Q_{2n}$ satisfy $(CLR)_{(P_0,Q_1...Q_{2n-1})Q_2...Q_{2n}}$ property,
(C3)

$$\begin{aligned}
&F(\psi(d(P_0x, P_1y)), \psi(d(Q_1Q_3...Q_{2n-1}x, Q_2Q_4...Q_{2n}y)), \psi(d(Q_1Q_3...Q_{2n-1}x, P_0x)),\\
&\ \psi(d(Q_2Q_4...Q_{2n}y, P_1y)), \psi(d(Q_1Q_3...Q_{2n-1}x, P_1y)), \psi(d(Q_2Q_4...Q_{2n}y, P_0x)))\\
&\ + G(\psi(d(Q_1Q_3...Q_{2n-1}x, Q_2Q_4...Q_{2n}y)), \psi(d(Q_1Q_3...Q_{2n-1}x, P_0x)),\\
&\ \psi(d(Q_2Q_4...Q_{2n}y, P_1y)), \psi(d(Q_1Q_3...Q_{2n-1}x, P_1y), \psi(d(Q_2Q_4...Q_{2n}y, P_0x))) \leq 0,
\end{aligned}$$

for all $x, y \in \mathcal{X}$, some $F \in \mathcal{F}_D$, $G \in \mathcal{G}_D$ and ψ is an almost altering distance. Then Q_1, Q_2,...,Q_{2n}, P_0 and P_1 have a unique common fixed point in $\mathcal{X}$.

Proof. Let $\mathcal{Q}'_1 = Q_1Q_3...Q_{2n-1}$ and $\mathcal{Q}'_2 = Q_2Q_4...Q_{2n}$. Since $(P_0, \mathcal{Q}'_1)$ and $\mathcal{Q}'_2$ satisfy $(CLR)_{(P_0,\mathcal{Q}'_1)\mathcal{Q}'_2}$ property, there exists a sequence $\{u_n\}$ in $\mathcal{X}$ such that

$$\lim_{n\to\infty} P_0u_n = \lim_{n\to\infty} \mathcal{Q}'_1u_n = \lim_{n\to\infty} Q_1Q_3...Q_{2n-1}u_n = z,$$

where $z \in \mathcal{Q}'_1(\mathcal{X}) \cap \mathcal{Q}'_2(\mathcal{X}) = Q_1...Q_{2n-1}(\mathcal{X}) \cap Q_2...Q_{2n}(\mathcal{X})$.
Since $z \in Q_2Q_4...Q_{2n}(\mathcal{X})$, there exists $u \in \mathcal{X}$ such that $z = Q_2Q_4...Q_{2n}u$. Using

$(C3)$ with $x = u_n$ and $y = u$, we get

$$F(\psi(d(P_0u_n, P_1u)), \psi(d(Q_1Q_3...Q_{2n-1}u_n, Q_2Q_4...Q_{2n}u)), \psi(d(Q_1Q_3...Q_{2n-1}u_n, P_0u_n)),$$
$$\psi(d(Q_2Q_4...Q_{2n}u, P_1u)), \psi(d(Q_1Q_3...Q_{2n-1}u_n, P_1u)), \psi(d(Q_2Q_4...Q_{2n}u, P_0u_n)))$$
$$+ G(\psi(d(Q_1Q_3...Q_{2n-1}u_n, Q_2Q_4...Q_{2n}u)), \psi(d(Q_1Q_3...Q_{2n-1}u_n, P_0u_n)),$$
$$\psi(d(Q_2Q_4...Q_{2n}u, P_1u)), \psi(d(Q_1Q_3...Q_{2n-1}u_n, P_1u), \psi(d(Q_2Q_4...Q_{2n}u, P_0u_n))) \leq 0.$$

Taking limits as $n \to \infty$, we have

$$F(\psi(d(z, P_1u)), 0, 0, \psi(d(z, P_1u)), \psi(d(z, P_1u)), 0) + G(0, 0, \psi(d(z, P_1u)),$$
$$\psi(d(z, P_1u)), 0) \leq 0.$$

If $d(z, P_1u) > 0$, then

$$G(0, 0, \psi(d(z, P_1u)), \psi(z, P_1u), 0) > 0,$$

which implies that

$$F(\psi(d(z, P_1u)), 0, 0, \psi(d(z, P_1u)), \psi(d(z, P_1u), 0) < 0,$$

a contradiction of (F_{2D}). Hence $d(z, P_1u) = 0$ i.e., $z = P_1u = Q_2Q_4...Q_{2n}u$. Since $(P_1, Q_2Q_4...Q_{2n})$ is weakly compatible, we have

$$P_1z = P_1Q_2Q_4...Q_{2n}u = Q_2Q_4...Q_{2n}P_1u = Q_2Q_4...Q_{2n}z.$$

Since $z \in Q_1Q_3...Q_{2n-1}(\mathcal{X})$, which implies $z = Q_1Q_3...Q_{2n-1}v$ for some $v \in \mathcal{X}$. On putting $x = v$ and $y = u$ in $(C3)$, we have

$$F(\psi(d(P_0v, P_1u)), \psi(d(Q_1Q_3...Q_{2n-1}v, Q_2Q_4...Q_{2n}u)), \psi(d(Q_1Q_3...Q_{2n-1}v, P_0v)),$$
$$\psi(d(Q_2Q_4...Q_{2n}u, P_1u)), \psi(d(Q_1Q_3...Q_{2n-1}v, P_1u)), \psi(d(Q_2Q_4...Q_{2n}u, P_0v)))$$
$$+ G(\psi(d(Q_1Q_3...Q_{2n-1}v, Q_2Q_4...Q_{2n}u)), \psi(d(Q_1Q_3...Q_{2n-1}v, P_0v)),$$
$$\psi(d(Q_2Q_4...Q_{2n}u, P_1u)), \psi(d(Q_1Q_3...Q_{2n-1}v, P_1u), \psi(d(Q_2Q_4...Q_{2n}u, P_0v))) \leq 0.$$

On simplification, we get

$$F(\psi(d(P_0v, z)), 0, \psi(d(P_0v, z)), 0, 0, \psi(d(P_0v, z))) + G(0, \psi(d(P_0v, z)), 0, 0,$$
$$\psi(d(P_0v, z))) \leq 0.$$

If $d(P_0v, z) > 0$, then

$$G(0, \psi(d(P_0v, z)), 0, 0, \psi(d(P_0v, z))) > 0.$$

Therefore, we obtain

$$F(\psi(d(P_0v,z)),0,\psi(d(P_0v,z))0,0,\psi(d(P_0v,z))<0,$$

a contradiction of (F_{1D}). Hence $d(P_0v,z)=0$, which implies that $z=P_0v=Q_1Q_3...Q_{2n-1}v$. Since $(P_0,Q_1Q_3...Q_{2n-1})$ is weakly compatible, we get

$$P_0z=P_0Q_1Q_3...Q_{2n-1}v=Q_1Q_3...Q_{2n-1}P_0v=Q_1Q_3...Q_{2n-1}z.$$

Now, we prove that $z=P_1z$. On putting $x=v$ and $y=z$ in $(C3)$, we get

$$F(\psi(d(P_0v,P_1z)),\psi(d(Q_1Q_3...Q_{2n-1}v,Q_2Q_4...Q_{2n}z)),\psi(d(Q_1Q_3...Q_{2n-1}v,P_0v)),\\ \psi(d(Q_2Q_4...Q_{2n}z,P_1z)),\psi(d(Q_1Q_3...Q_{2n-1}v,P_1z)),\psi(d(Q_2Q_4...Q_{2n}z,P_0v)))\\ +G(\psi(d(Q_1Q_3...Q_{2n-1}v,Q_2Q_4...Q_{2n}z)),\psi(d(Q_1Q_3...Q_{2n-1}v,P_0v)),\\ \psi(d(Q_2Q_4...Q_{2n}z,P_1z)),\psi(d(Q_1Q_3...Q_{2n-1}v,P_1z),\psi(d(Q_2Q_4...Q_{2n}z,P_0v)))\leq 0,$$

which implies that

$$F(\psi(d(z,P_1z)),\psi(d(z,P_1z)),0,0,\psi(d(z,P_1z)),\psi(d(z,P_1z)))+G(\psi(d(z,P_1z)),0,0,\\ \psi(d(P_1z,z)),\psi(d(z,P_1z)))\leq 0.$$

If $d(z,P_1z)>0$, then

$$G(\psi(d(z,P_1z)),0,0,\psi(d(P_1z,z)),\psi(d(z,P_1z)))>0.$$

Thus from above, we get

$$F(\psi(d(z,P_1z)),\psi(d(z,P_1z)),0,0,\psi(d(z,P_1z)),\psi(d(z,P_1z)))<0,$$

a contradiction of (F_{3D}). Hence $d(z,P_1v)=0$ i.e., $P_1z=z$ and hence $P_1z=Q_2Q_4...Q_{2n}z=z$.
Further on putting $x=y=z$ in $(C3)$, we get

$$F(\psi(d(P_0z,P_1z)),\psi(d(Q_1Q_3...Q_{2n-1}z,Q_2Q_4...Q_{2n}z)),\psi(d(Q_1Q_3...Q_{2n-1}z,P_0z)),\\ \psi(d(Q_2Q_4...Q_{2n}z,P_1z)),\psi(d(Q_1Q_3...Q_{2n-1}z,P_1z)),\psi(d(Q_2Q_4...Q_{2n}z,P_0z)))\\ +G(\psi(d(Q_1Q_3...Q_{2n-1}z,Q_2Q_4...Q_{2n}z)),\psi(d(Q_1Q_3...Q_{2n-1}z,P_0z)),\\ \psi(d(Q_2Q_4...Q_{2n}z,P_1z)),\psi(d(Q_1Q_3...Q_{2n-1}z,P_1z),\psi(d(Q_2Q_4...Q_{2n}z,P_0z)))\leq 0.$$

On simplification, we have

$$F(\psi(d(P_0z,z)),\psi(d(P_0z,z)),0,0,\psi(d(P_0z,z)),\psi(d(P_0z,z)))+G(\psi(d(P_0z,z)),0,\\ 0,\psi(d(P_0z,z)),\psi(d(P_0z,z)))\leq 0.$$

If $d(P_0, z) > 0$, then

$$G(\psi(d(P_0z, z)), 0, , 0, \psi(d(P_0z, z)), \psi(d(P_0z, z))) > 0,$$

which implies that

$$F(\psi(d(P_0z, z)), \psi(d(P_0z, z)), 0, 0, \psi(d(P_0z, z)), \psi(d(P_0z, z))) < 0,$$

a contradiction of (F_{3D}). Hence $d(P_0z, z) = 0$ i.e., $P_0z = z$ and hence $P_0z = Q_1Q_3...Q_{2n-1}z = z$.
On putting $x = z$ and $y = Q_4...Q_{2n}z$ in $(C3)$ and using $(C1)$, $\mathcal{Q}_1' = Q_1Q_3...Q_{2n-1}$ and $\mathcal{Q}_2' = Q_2Q_4...Q_{2n}$, we get

$$\begin{aligned}&F(\psi(d(P_0z, P_1Q_4...Q_{2n}z)), \psi(d(\mathcal{Q}_1'z, \mathcal{Q}_2'Q_4...Q_{2n}z)), \psi(d(\mathcal{Q}_1'z, P_0z)),\\ &\quad\psi(d(\mathcal{Q}_2'Q_4...Q_{2n}z, P_1Q_4...Q_{2n}z)), \psi(d(\mathcal{Q}_1'z, P_1Q_4...Q_{2n}z)), \psi(d(\mathcal{Q}_2'Q_4...Q_{2n}z, P_0z)))\\ &\quad + G(\psi(d(\mathcal{Q}_1'z, \mathcal{Q}_2'Q_4...Q_{2n}z)), \psi(d(\mathcal{Q}_1'z, P_0z)), \psi(d(\mathcal{Q}_2'Q_4...Q_{2n}z, P_1Q_4...Q_{2n}z)),\\ &\quad\psi(d(\mathcal{Q}_1'z, P_1Q_4...Q_{2n}z)), \psi(d(\mathcal{Q}_2'Q_4...Q_{2n}z, P_0z))) \leq 0.\end{aligned}$$

From this we get

$$\begin{aligned}&F(\psi(d(z, Q_4...Q_{2n}z)), \psi(d(z, Q_4...Q_{2n}z)), 0, 0, \psi(d(z, Q_4...Q_{2n}z)), \psi(d(Q_4...Q_{2n}z, z)))\\ &\quad + G(\psi(d(z, Q_4...Q_{2n}z)), 0, 0, \psi(d(z, Q_4...Q_{2n}z)), \psi(d(Q_4...Q_{2n}z, z))) \leq 0.\end{aligned}$$

If $(d(z, Q_4...Q_{2n}z)) > 0$ then
$G(\psi(d(z, Q_4...Q_{2n}z)), 0, 0, \psi(d(z, Q_4...Q_{2n}z)), \psi(d(Q_4...Q_{2n}z, z))) > 0$.
Therefore, we have

$$\begin{aligned}&F(\psi(d(z, Q_4...Q_{2n}z)), \psi(d(z, Q_4...Q_{2n}z)), 0, 0, \psi(d(z, Q_4...Q_{2n}z)),\\ &\quad\psi(d(Q_4...Q_{2n}z, z))) < 0,\end{aligned}$$

a contradiction to (F_{3D}). Hence $d(z, Q_4...Q_{2n}z)) = 0$ i.e., $Q_4...Q_{2n}z = z$. Hence $Q_2Q_4...Q_{2n}z = Q_2z = z$. Continuing like this, we have

$$P_1z = Q_2z = Q_4z = ... = Q_{2n} = z. \tag{1}$$

On putting $x = Q_3...Q_{2n-1}z$ and $y = z$ in $(C3)$ and using $(C1)$, $\mathcal{Q}_1' = Q_1Q_3...Q_{2n-1}$ and $\mathcal{Q}_2' = Q_2Q_4...Q_{2n}$, we get

$$\begin{aligned}&F(\psi(d(P_0Q_3...Q_{2n-1}z, P_1z)), \psi(d(\mathcal{Q}_1'Q_3...Q_{2n-1}z, \mathcal{Q}_2'z)),\\ &\quad\psi(d(\mathcal{Q}_1'Q_3...Q_{2n-1}z, P_0Q_3...Q_{2n-1}z)), \psi(d(\mathcal{Q}_2'z, P_1z)),\\ &\quad\psi(d(\mathcal{Q}_1'Q_3...Q_{2n-1}z, P_1z)), \psi(d(\mathcal{Q}_2'z, P_0Q_3...Q_{2n-1}z)))\\ &\quad + G(\psi(d(\mathcal{Q}_1'Q_3...Q_{2n-1}z, \mathcal{Q}_2'z)), \psi(d(\mathcal{Q}_1'Q_3...Q_{2n-1}z, P_0Q_3...Q_{2n-1}z)),\\ &\quad\psi(d(\mathcal{Q}_2'z, P_1z)), \psi(d(\mathcal{Q}_1'Q_3...Q_{2n-1}z, P_1z)), \psi(d(\mathcal{Q}_2'z, P_0Q_3...Q_{2n-1}z))) \leq 0,\end{aligned}$$

which implies that

$$F(\psi(d(Q_3...Q_{2n-1}z,z)),\psi(d(Q_3...Q_{2n-1}z,z)),0,0,\psi(d(Q_3...Q_{2n-1}z,z)),\\ \psi(d(z,Q_3...Q_{2n-1}z)))+G(d(\psi(d(Q_3...Q_{2n-1}z,z)),0,0,\psi(d(Q_3...Q_{2n-1}z,z)),\\ \psi(d(z,Q_3...Q_{2n-1}z)))\leq 0.$$

If $d(z,Q_3...Q_{2n-1}z)>0$ then
$G(d(\psi(d(Q_3...Q_{2n-1}z,z)),0,0,\psi(d(Q_3...Q_{2n-1}z,z)),\psi(d(z,Q_3...Q_{2n-1}z)))>0$.
Thus from above, we obtain

$$F(\psi(d(Q_3...Q_{2n-1}z,z)),\psi(d(Q_3...Q_{2n-1}z,z)),0,0,\psi(d(Q_3...Q_{2n-1}z,z)),\\ \psi(d(z,Q_3...Q_{2n-1}z)))<0,$$

a contradiction to (F_{3D}). Hence $d(z,Q_3...Q_{2n-1}z)=0$ i.e., $Q_3...Q_{2n-1}z=z$. Hence $Q_1Q_3...Q_{2n-1}z=Q_1z=z$. Continuing like this, we have

$$P_0z=Q_1z=Q_3z=...=Q_{2n-1}=z. \tag{2}$$

Hence from (1) and (2), we have

$$P_0z=P_1=Q_1z=Q_2z=Q_3z=...=Q_{2n-1}=Q_{2n}z=z.$$

Therefore, z is a common fixed point of the given self mappings.
Uniqueness. Let w be another fixed point of the given mappings. Then $P_0w=P_1w=Q_1w=Q_2w=Q_3w=...=Q_{2n}w=w$. Suppose that $z\neq w$. Putting $x=z$ and $y=w$ in condition $(C3)$, we have

$$F(\psi(d(z,w)),\psi(d(z,w)),0,0,\psi(d(z,w)),\psi(d(w,z))\\ +G(\psi(d(z,w)),0,0,\psi(d(z,w)),\psi(d(w,z)))\leq 0.$$

If $d(z,w)>0$, then

$$G(\psi(d(z,w)),0,0,\psi(d(z,w)),\psi(d(w,z)))>0.$$

Therefore, we obtain

$$F(\psi(d(z,w)),\psi(d(z,w)),0,0,\psi(d(z,w)),\psi(d(w,z))<0,$$

a contradiction of (F_{3D}). Hence $z=w$. Therefore, z is a unique common fixed point of the given mappings. □

Now we prove a theorem for families of mappings.

Theorem 2.3. *Let $\{S_\alpha\}_{\alpha \in J}$ and $\{Q_i\}_{i=1}^{2p}$ be two families of self-mappings on a metric space $(\mathcal{X}, d)$. Suppose that there exists a fixed $\beta \in J$ such that:*
(C4) $Q_2(Q_4...Q_{2n}) = (Q_4...Q_{2n})Q_2$,
$Q_2Q_4(Q_6...Q_{2n}) = (Q_6...Q_{2n})Q_2Q_4$,
$\vdots$
$Q_2...Q_{2n-2}(Q_{2n}) = (Q_{2n})Q_2...Q_{2n-2}$,
$S_\beta(Q_4...S_{2n}) = (S_4...S_{2n})S_\beta$,
$S_\beta(Q_6...Q_{2n}) = (Q_6...Q_{2n})S_\beta$,
$\vdots$
$S_\beta Q_{2n} = Q_{2n}S_\beta$,
$Q_1(Q_3...Q_{2n-1}) = (Q_3...Q_{2n-1})Q_1$,
$Q_1Q_3(Q_5...Q_{2n-1}) = (Q_5...Q_{2n-1})Q_1Q_3$,
$\vdots$
$Q_1...Q_{2n-3}(Q_{2n-1}) = (Q_{2n-1})Q_1...Q_{2n-3}$,
$S_\alpha(Q_3...Q_{2n-1}) = (Q_3...Q_{2n-1})S_\alpha$,
$S_\alpha(Q_5...Q_{2n-1}) = (Q_5...Q_{2n-1})S_\alpha$,
$\vdots$
$S_\alpha S_{2n-1} = S_{2n-1}S_\alpha$,
(C5) the pairs $(S_\alpha, Q_1...Q_{2n-1})$ and $(S_\beta, Q_2...Q_{2n})$ are weakly compatible and $(S_\alpha, Q_1...Q_{2n-1})$ and $Q_2...Q_{2n}$ satisfy $(CLR)_{(S_\alpha, Q_1...Q_{2n-1})Q_2...Q_{2n}}$ property,
(C6)

$$\begin{aligned}&F(\psi(d(S_\alpha x, S_\beta y)), \psi(d(Q_1Q_3...Q_{2n-1}x, Q_2Q_4...Q_{2n}y)), \psi(d(Q_1Q_3...Q_{2n-1}x, S_\alpha x)),\\ &\quad \psi(d(Q_2Q_4...Q_{2n}y, S_\beta y)), \psi(d(Q_1Q_3...Q_{2n-1}x, S_\beta y)), \psi(d(Q_2Q_4...Q_{2n}y, S_\alpha x)))\\ &\quad + G(\psi(d(Q_1Q_3...Q_{2n-1}x, Q_2Q_4...Q_{2n}y)), \psi(d(Q_1Q_3...Q_{2n-1}x, S_\alpha x)),\\ &\quad \psi(d(Q_2Q_4...Q_{2n}y, S_\beta y)), \psi(d(Q_1Q_3...Q_{2n-1}x, S_\beta y)), \psi(d(Q_2Q_4...Q_{2n}y, S_\alpha x))) \le 0,\end{aligned}$$

for all $x, y \in \mathcal{X}$ and some $F \in \mathcal{F}_D$, $G \in \mathcal{G}_D$ and ψ is an almost altering distance. Then all S_α and Q_i have a unique common fixed point in $\mathcal{X}$.

Proof. Let $S_{\alpha 0}$ be a fixed element in $\{S_\alpha\}_{\alpha \in J}$. By Theorem 2.2 with $P_0 = S_\alpha$ and $P_1 = S_{\alpha 0}$ it follows that there exists some $u \in \mathcal{X}$ such that

$$S_\alpha u = S_{\alpha 0}u = Q_1Q_3...Q_{2n-1}u = Q_2Q_4...Q_{2n}u = u.$$

Let $\beta \in J$ be arbitrary. Then from $(C6)$, we get

$$\begin{aligned}&F(\psi(d(S_\alpha u, S_\beta u)), \psi(d(Q_1Q_3...Q_{2n-1}u, Q_2Q_4...Q_{2n}u)), \psi(d(Q_1Q_3...Q_{2n-1}u, S_\alpha u)),\\ &\quad \psi(d(Q_2Q_4...Q_{2n}u, S_\beta u)), \psi(d(Q_1Q_3...Q_{2n-1}u, S_\beta u)), \psi(d(Q_2Q_4...Q_{2n}u, S_\alpha u)))\\ &\quad + G(\psi(d(Q_1Q_3...Q_{2n-1}u, Q_2Q_4...Q_{2n}u)), \psi(d(Q_1Q_3...Q_{2n-1}u, S_\alpha u)),\\ &\quad \psi(d(Q_2Q_4...Q_{2n}u, S_\beta u)), \psi(d(Q_1Q_3...Q_{2n-1}u, S_\beta u)), \psi(d(Q_2Q_4...Q_{2n}u, S_\alpha u))) \le 0.\end{aligned}$$

Hence

$$F(\psi(d(u,S_\beta u)),\psi(d(u,u)),\psi(d(u,u)),\psi(d(u,S_\beta u)),\psi(d(u,S_\beta u)),\psi(d(u,u)))$$
$$+G(\psi(d(u,u)),\psi(d(u,u)),\psi(d(u,S_\beta u)),\psi(d(u,S_\beta u)),\psi(d(u,u)))\leq 0,$$

i.e.,

$$F(\psi(d(u,S_\beta u)),0,0,\psi(d(u,S_\beta u)),\psi(d(u,S_\beta u)),0)$$
$$+G(0,0,\psi(d(u,S_\beta u)),\psi(d(u,S_\beta u)),0)\leq 0.$$

If $d(u,S_\beta u)>0$, we get

$$G(0,0,\psi(d(u,S_\beta u)),\psi(d(u,S_\beta u)),0)>0,$$

which implies that

$$F(\psi(d(u,S_\beta u)),0,0,\psi(d(u,S_\beta u)),\psi(d(u,S_\beta u)),0)<0,$$

a contradiction by (F_{2D}) and hence $\psi(d(u,S_\beta u))=0$ i.e., $S_\beta u=u$ for each $\beta\in J$. Uniqueness follows easily. □

If we take $\psi(t)=t$ in Theorem 2.2, we get

Theorem 2.4. *Let $Q_1,Q_2,...,Q_{2n},P_0$ and P_1 be self mappings on a metric space $(\mathcal{X},d)$, satisfying conditions $(C1)$, $(C2)$ and the following condition:*
(C7)

$$F((d(P_0x,P_1y)),(d(Q_1Q_3...Q_{2n-1}x,Q_2Q_4...Q_{2n}y)),(d(Q_1Q_3...Q_{2n-1}x,P_0x)),$$
$$(d(Q_2Q_4...Q_{2n}y,P_1y)),(d(Q_1Q_3...Q_{2n-1}x,P_1y)),(d(Q_2Q_4...Q_{2n}y,P_0x)))$$
$$+G(d(Q_1Q_3...Q_{2n-1}x,Q_2Q_4...Q_{2n}y),d(Q_1Q_3...Q_{2n-1}x,P_0x),d(Q_2Q_4...Q_{2n}y,P_1y),$$
$$d(Q_1Q_3...Q_{2n-1}x,P_1y),d(Q_2Q_4...Q_{2n}y,P_0x))\leq 0,$$

for all $x,y\in\mathcal{X}$, some $F\in\mathcal{F}_D$, $G\in\mathcal{G}_D$ and ψ is an almost altering distance. Then Q_1, Q_2,...,Q_{2n}, P_0 and P_1 have a unique common fixed point in $\mathcal{X}$.

If we take $\psi(t)=t$ in Theorem 2.3, we get

Theorem 2.5. *Let $\{S_\alpha\}_{\alpha\in J}$ and $\{Q_i\}_{i=1}^{2p}$ be two families of self-mappings on a metric space $(\mathcal{X},d)$. Suppose that there exists a fixed $\beta\in J$ such that conditions $(C4)$ and $(C5)$ are satisfied. Moreover,*
(C8)

$$F(d(S_\alpha x,S_\beta y),d(Q_1Q_3...Q_{2n-1}x,Q_2Q_4...Q_{2n}y),d(Q_1Q_3...Q_{2n-1}x,S_\alpha x),$$
$$(d(Q_2Q_4...Q_{2n}y,S_\beta y),d(Q_1Q_3...Q_{2n-1}x,S_\beta y),d(Q_2Q_4...Q_{2n}y,S_\alpha x))$$
$$+G(d(Q_1Q_3...Q_{2n-1}x,Q_2Q_4...Q_{2n}y),d(Q_1Q_3...Q_{2n-1}x,S_\alpha x),$$
$$d(Q_2Q_4...Q_{2n}y,S_\beta y),d(Q_1Q_3...Q_{2n-1}x,S_\beta y),d(Q_2Q_4...Q_{2n}y,S_\alpha x))\leq 0,$$

for all $x,y\in\mathcal{X}$ and some $F\in\mathcal{F}_D$, $G\in\mathcal{G}_D$ and ψ is an almost altering distance. Then all S_α and Q_i have a unique common fixed point in $\mathcal{X}$.

Remark 2.1. *(i). Let ψ and ϕ be as in Theorem 1.1. Then*

$$F(u_1, ..., u_6) = \psi(u_1) - \psi(m(x, y))$$

and

$$G(v_1, ..., v_5) = \phi(m(x, y)).$$

Then $F(u, 0, u, 0, 0, u) = F(u, 0, 0, u, u, 0) = F(u, u, 0, 0, u, u) = 0$ and

$$G(v_1, ..., v_5) = \phi(max\{v_1, ..., v_5\}) > 0, \textit{ if one of } v_1, ..., v_5 > 0.$$

Hence $F \in \mathcal{F}_D$ and $G \in \mathcal{G}_D$. Then by Theorem 2.4, we get a generalization and extension of Theorem 1.1 for any even number of weakly compatible mappings. Similarly, Theorem 2.5 is a generalization and extension of Theorem 1.1 for families of weakly compatible mappings.
(ii). Theorems 2.2 and 2.3 are extension of Theorem 2.1 for any even number of weakly compatible mappings and families of weakly compatible mappings respectively.

Now we give an example in support of our theorems.

Example 2.1. *Let $\mathcal{X} = [0, 1]$ and d be usual metric on $\mathcal{X}$. Define*

$$S_\alpha(x) = \frac{x^4}{1 + x^4} \textit{ for each } \alpha \in J \textit{ and all } x \in \mathcal{X},$$

$$Q_i(x) = x^{\sqrt[n]{4}} \textit{ for each } i \in \{1, 2, ..., 2n\} \textit{ and all } x \in \mathcal{X}.$$

Then $Q_2Q_4...Q_{2n}x = x^4$, $Q_1Q_3...Q_{2n-1}x = x^4$.
The pairs $(S_\alpha, Q_1...Q_{2n-1})$ and $(S_\beta, Q_2...S_{2n})$ are weakly compatible..
Define implicit function F such that

$$\textit{Let } F(u_1, ..., u_6) = u_1 - \frac{9}{10} \max\{u_2, u_3, u_4, u_5, u_6\}.$$

and

$$G(t_1, ..., t_5) = \frac{1}{100(t_1 + t_2 + t_3 + t_4 + t_5)}.$$

Then $F \in \mathcal{F_D}$ and $G \in \mathcal{G_D}$. Thus all the conditions of Theorems 2.2 (for $\alpha = 0, 1$) and 2.3 are satisfied for $\psi(t) = t$ and 0 is the unique common fixed point of the mappings.

3 Application

In 2002, Branciari [4] obtained Banach contraction principle for mappings satisfying an integral type contraction condition. In the same way, we analyze Theorem 2.3 for mappings satisfying integral type contraction condition.

Lemma 3.1. *[19] Let $r : [0,\infty) \to [0,\infty)$ is a Lebesgue measurable mapping which is summable on each compact subset of $[0,\infty)$ such that $\int_0^\infty r(t)dt > 0$, for $\epsilon > 0$. Then $\psi(t) = \int_0^t r(x)dx$ is an almost altering distance.*

Theorem 3.1. *Let $\{S_\alpha\}_{\alpha\in J}$ and $\{Q_i\}_{i=1}^{2p}$ be two families of self-mappings on a metric space $(\mathcal{X}, d)$. Suppose that there exists a fixed $\beta \in J$ such that conditions $(C4)$ and $(C5)$ are satisfied. Moreover,*
(C12)

$$F\Bigg(\int_0^{d(S_\alpha x,S_\beta y)} r(t)dt, \int_0^{d(Q_1Q_3...Q_{2n-1}x,Q_2Q_4...Q_{2n}y)} r(t)dt, \int_0^{d(Q_1Q_3...Q_{2n-1}x,S_\alpha x)} r(t)dt,$$
$$\int_0^{d(Q_2Q_4...Q_{2n}y,S_\beta y)} r(t)dt, \int_0^{d(Q_1Q_3...Q_{2n-1}x,S_\beta y)} r(t)dt, \int_0^{d(Q_2Q_4...Q_{2n}y,S_\alpha x)} r(t)dt\Bigg)$$
$$+ G\Bigg(\int_0^{d(Q_1Q_3...Q_{2n-1}x,Q_2Q_4...Q_{2n}y)} r(t)dt, \int_0^{d(Q_1Q_3...Q_{2n-1}x,S_\alpha x)} r(t)dt,$$
$$\int_0^{d(Q_2Q_4...Q_{2n}y,S_\beta y)} r(t)dt, \int_0^{d(Q_1Q_3...Q_{2n-1}x,S_\beta y)} r(t)dt, \int_0^{d(Q_2Q_4...Q_{2n}y,S_\alpha x)} r(t)dt\Bigg) \leq 0,$$

for all $x, y \in \mathcal{X}$ and some $F \in \mathcal{F}_\mathcal{D}$ and $G \in \mathcal{G}_\mathcal{D}$. Then all S_α and Q_i have a unique common fixed point in $\mathcal{X}$.

Proof. Let $\psi(t)$ be as in Lemma 3.1. Then

$$\psi(d(S_\alpha x, S_\beta y)) = \int_0^{d(S_\alpha x,S_\beta y)} r(t)dt, \psi(d(Q_1Q_3...Q_{2n-1}x, S_\alpha x) = \int_0^{d(Q_1Q_3...Q_{2n-1}x,S_\alpha x)}$$
$$\psi(d(Q_1Q_3...Q_{2n-1}x, Q_2Q_4...Q_{2n}y)) = \int_0^{d(Q_1Q_3...Q_{2n-1}x,Q_2Q_4...Q_{2n}y)} r(t)dt,$$
$$\psi(d(Q_2Q_4...Q_{2n}y, S_\beta y)) = \int_0^{d(Q_2Q_4...Q_{2n}y,S_\beta y)} r(t)dt,$$
$$\psi(d(Q_1Q_3...Q_{2n-1}x, S_\beta y)) = \int_0^{d(Q_1Q_3...Q_{2n-1}x,S_\beta y)} r(t)dt,$$
$$\psi(d(Q_2Q_4...Q_{2n}y, S_\alpha x)) = \int_0^{d(Q_2Q_4...Q_{2n}y,S_\alpha x)} r(t)dt.$$

Hence the proof of Theorem 3.1 follows by Theorem 2.3. □

4 Conclusions

In this paper, we have established unique common fixed point theorems for families of weakly compatible mappings satisfying common limit range property and a mixed implicit relation. Our results generalize, extend and improve the results of Imdad [8] and Popa [17]. We provide an application for integral type contraction condition. In the end, we conclude that theory of fixed points can be extended in metric space for some applications as well and that the analogue of many known results can also be obtained in this literature.

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