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History and new possible research directions of hyperstructures

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Abstract We present a summary of the origins and current developments of the theory of algebraic hyperstructures. We also sketch some possible lines of research .

Keywords. Hypergroupoid, hypergroups, fuzzy sets.

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1 The origins of the theory of hyperstructures

Hyperstructure theory was born in 1934, when Marty at the 8th Congress of Scandinavian Mathematiciens, gave the definition of hypergroup and illustrated some applications and showed its utility in the study of groups, algebraic functions and rational fractions. In the following years, around the 40's, several others worked on this subject: in France, the same Marty, Krasner, Kuntzman, Croisot, in USA Dresher and Ore, Prenowitz, Eaton, Griffith, Wall (who introduced a generalization of hypergroups, where the hyperproduct is a multiset, i.e. a set in which every element has a certain multiplicity); in Japan Utumi, in Spain San Juan, in Russia Vikhrov, in Uzbekistan Dietzman, in Italy Zappa.
In the 50's and 60's they worked on hyperstructures, in Romania Benado, in Czech Republic Drbohlav, in France Koskas, Sureau, In Greece Mittas, Stratigopoulos, in Italy Orsatti, Boccioni, in USA Prenowitz, Graetzer , Pickett, McAlister, in Japan Nakano, in Yugoslavia Dacic.

But it is above all since 70' that a more luxuriant flourishing of hyperstructures has been and is seen in Europe, Asia, America, Australia.

2. The most important definitions

Definition 1 Let H be a nonempty set and $P^*(H)$ the family of the nonempty subsets of H. A *multivalued operation* (said also *hyperoperation*) $< o >$ on H is a function which associates with every pair $(x, y) \in H^2$ a non empty subset of H denoted x o y.
An *algebraic hyperstructure* or simply a *hyperstructure* is a non empty set H, endowed with one or more hyperoperations.
A nonempty set H endowed with an hyperoperation $< o >$ is called *hypergroupoid* and is denoted $<H ; o >$. If $\{A, B\} \subseteq P^*(H)$, A o B denotes the set $\cup_{a \in A,\ b \in B}$ a o b.

Definition 2 A hypergroupoid $<H ; o>$ is called *semi-hypergroup* if

(I) $\forall (x, y, z) \in H^3$, $(x \circ y) \circ z = x \circ (y \circ z)$.

A hypergroupoid $<H ; o>$ is called *quasi-hypergroup* if

(II) $\forall (a, b) \in H^2$, $\exists (x, y) \in H^2$ such that $a \in b \circ x$, $a \in y \circ b$.

Definition 3 A hyperoperation is said *weak associative* if

(III) $\forall (x, y, z) \in H^3$, $(x \circ y) \circ z \cap x \circ (y \circ z) \neq \varnothing$ (see [141]).

Definition 4 A hypergroupoid $<H ; \circ>$ is called *hypergroup* if satisfies both (I) and (II).

Definition 5 A hyperoperation $< \circ>$ is said *commutative* if

(IV) $\forall (a, b) \in H^2$, $a \circ b = b \circ a$.

Definition 6 A hyperoperation is said *weak commutative* if

(V) $\forall (x, y) \in H^2$, $x \circ y \cap y \circ x \neq \varnothing$.

Definition 7 A H_v *– group* is a quasi-hypergroup $<H ; \circ>$ such that the hyperoperation $< \circ >$ is weak associative.

Let $<H ; \circ>$ be a commutative hypergroup. We denote with a / b the set $\{x \mid a \in x \circ b\}$.

Definition 8 A hypergroupoid $<H ; \circ>$ is called *join space* if it is a commutative hypergroup such that the following implication is satisfied

(VI) $\forall (a, b, c, d) \in H^4$, $a / b \cap c / d \neq \varnothing \Rightarrow a \circ d \cap b \circ c \neq \varnothing$.

3. The recent history of the theory

Currently one works successfully in hyperstructures, in several continents, I shall remember only some names of hyperstructure scientists since 1970.

Around the 70's and 80's, hyperstructures where cultivated especially:

- in Greece by Mittas and his school (Canonical Hypergroups and their applications, Vougiouklis and his school (H_v – groups);
- in Italy: by Corsini (Homomorphisms, Join Spaces, Quasicanonical Hypergroups, Complete Hypergroups, 1-Hypergroups, Cyclic Hypergroups etc.) and his school, Tallini G. (Hypergroups associated with Projective Planes), Rota, Procesi Ciampi (Hyperrings);
- in USA: by Prenowitz and Jantoshak (Join Spaces and Geometries, Homomorphisms), Roth (Character of hypergroups, Canonical hypergroups), Comer (Polygroups, Representations of hypergroups);
- in France by Krasner and Sureau, (Structure of Hypergroups), Koskas, (Semihypergroups associated with Groupoids). Deza;

- in Canada Rosenberg (Hypergroups associated with graphs, binary relations).

Around the 90's and more recently, many papers appeared, made in Europe, Asia, Asia , America and Australia.

Europe:

- In Greece
  - at Thessaloniki (Aristotle Univ.), Mittas, Konstantinidou, Serafimidis Kehagias, Ioulidis, Yatras, Synefaki,
  - at Alexandroupolis (Democritus Univ. of Thrace), T. Vougiouklis, Dramalidis, S.Vougiouklis,
  - at Patras (Patras Univ.), Stratigopoulos,
  - at Orestiada (Democritus Univ. of Thrace), Spartalis,
  - at Athens , Ch. Massouros, G. Massouros, G. Pinotsis;
- in Romania
  - at Iasi (Cuza Univ.), V. Leoreanu, Cristea, Tofan, Gontineac., Volf, L. Leoreanu,
  - at Cluj Napoca Babes Bolyai Univ.), Purdea, Pelea, Calugareanu,
  - at Constanta (Ovidius Univ.) Stefanescu;
- in Czech Republic
  - at Praha (Karlos Univ.) Kepka, Jezek, Drbohlav, (Agricultural Univ.) Nemec,
  - at Brno (Brno Univ. of Technology) J. Chvalina, (Military Academy of Brno) Hoskova, (Technical Univ. of Brno) L. Chvalinova, (Masaryk Univ.) Novotny, (University of Defence) Rackova, at Olomouc, (Palacky Univ.) Hort,
  - at Vyskov (Military Univ. of Ground Forces) Moucka;
- in Montenegro
  - at Podgorica (Univ. of Podgorica) Dasic, Rasovic;
- in Slovakia

- at Bratislava, (Comenius University), Kolibiar, (Slovak Techn. Univ.) Jasem,
- at Kosice, (Matematickz ustav SAV), Jakubik, (Safarik Univ.), Lihova, Repasky, Csontoova;

- in Italy
  - at Udine (Udine Univ.) Corsini,
  - at Messina (Messina Univ.) De Salvo, Migliorato, Lo Faro, Gentile,
  - at Rome (Universita' La Sapienza) G. Tallini, M. Scafati-Tallini, Rota, Procesi Ciampi, Peroni,
  - at Pescara (G. d'Annunzio Univ.) A. Maturo, S. Doria, B. Ferri,
  - at Teramo (Univ. di Teramo) Eugeni,
  - at L'Aquila (Univ. dell'Aquila) Innamorati, L. Berardi,
  - at Brescia (Universita' Cattolica del Sacro Cuore), Marchi,
  - at Lecce (Universita' di Lecce), Letizia, Lenzi,
  - at Palermo, (Univ. di Palermo), Falcone,
  - at Milano, (Politecnico), Mercanti, Cerritelli, Gelsomini;
- in France
  - at Clermont-Ferrand (Universite' des Math. Pures et Appl.) Sureau, M. Gutan, C. Gutan,
  - at Lyon, (Universite' Lyon 1), Bayon, Lygeros;
- in Spain
  - at Malaga, (Malaga Univ.) Martinez, Gutierrez, De Guzman, Cordero;
- in Finland
  - at Oulu, (Univ. of Oulu), Nieminen, Niemenmaa.

America

- In USA
  - at Charleston (The Citadel) Comer,
  - at New York (Brooklyn College, CUNY), Jantosciak,
  - at Cleveland, Ohio, (John Carroll Univ.), Olson, Ward,

- in Canada
    - at Montreal, (Universite' de Montreal), Rosenberg, Foldes,

Asia

- In Thailand
    - at Bangkok, (Chulalongkorn Univ), Kemprasit, Punkla , Chaopraknoi, Triphop, Tumsoun,
    - at Samutprakarn, (Hauchievw Chalermprakiet Univ.), Juntakharajorn,
    - at Phitsanulok, (Naresuan Univ.), C. Namnak.
- in Iran
    - at Babolsar (Mazandaran Univ.) Ameri, Razieh Mahjoob, Moghani, Hedayati, Alimohammadi,
    - at Yazd (Yazd Univ.) Davvaz, Koushky,
    - at Kerman, (Shahid Bahonar Univ.) Zahedi, Molaei, Torkzadeh, Khorashadi Zadeh, Hosseini, Mousavi, (Islamic Azad Univ.) Borumand Saeid
    - at Kashan (Univ. of Kashan) Ashrafi, Ali Hossein Zadeh,
    - at Tehran (Tehran Univ.) Darafsheh, Morteza Yavary, (Tarbiat Modarres Univ.) Iranmanesh, Iradmusa, Madanshekaf, (Iran Univ. of Sci. and Technology) Ghorbany, Alaeyan, (Shahid Beheshti Univ.) Mehdi Ebrahimi, Karimi, Mahmoudi,
    - at Zahedan (Sistan and Baluchestan Univ.) Borzooei, Hasankhani, Rezaei,
    - at Zanjan (Institute for Advanced Studies in Basic Sciences) Barghi
    - at Sari-Branch, (Islamic Azad Univ.), Roohi.
- in Korea
    - at Chinju (Gyeongsang National Univ.) Young Bae Jun, E. H. Roh,
    - at Taejon, (Chungnam National Univ.) Sang Cho Chung,

    - (Taejon Univ.) Byung-Mun Choi,
    - at Chungju (Chungju National Univ.) K.H. Kim.
- in India
    - at Kolkata, (Uni. of Calcutta), M.K. Sen, Dasgupta,Chowdhury,
    - at Tiruchendur, Tamilnadu (Adinatar College of Arts and Sciences), Asokkumar,Velrajan,
- in China
    - at Chongqing, (Chongqing three Gorges Univ.) Yuming Feng,
    - at Xi'an, (Northwest Univ.), Xiao Long Xin,
    - at Enshi, Hubei Province (Hubei Institute for Nationalities), Janming Zhan, Xueling Ma;
- in Japon
    - at Tokyo, (Hitotsubashi Univ. Kunitachi), Machida,
    - at Tagajo, Miyagi, (Tohoku Gakuin Univ.), Shoji Kyuno;
- in Jordan
    - at Karak, (Mu'tah Univ.) M.I. Al Ali;
- in Israel
    - at Ramat Gan, (Bar - Ilan Univ.), Feigelstock.

4. Join Spaces, Fuzzy Sets, Rough Sets

The *Join Spaces* were introduced by Prenowitz in the 40's and were utilized by him and later by him together with Jantosciak, to construct again several kinds of Geometry. Join spaces had already many other applications, as to Graphs, (Nieminen, Rosenberg, Bandelt, Mulder, Corsini), to Median Algebras (Bandelt-Hedlikova) to Hypergraphs (Corsini, Leoreanu), to Binary Relations (Chvalina, Rosenberg, Corsini, Corsini, Leoreanu, De Salvo-Lo Faro).

Fuzzy Sets were introduced in the 60's by an Iranian scientist who lives in USA, Zadeh [144]. He and others, in the following decades, found surprising applications to almost every field of science and

knowledge: from engineering to sociology, from agronomy to linguistic, from biology to computer science, from medicine to economy. From psychology to statistics and so on. They are now cultivated in all the world. Let's remember what is a Fuzzy Set. We know that a subset A of an universe H, can be represented as a function, the characteristic functions χ_A from H to the set $\{0,1\}$. The notion of fuzzy subset generalizes that one of characteristic function. One considers instead of the functions χ_A, functions μ_A from H to the closed real interval $[0,1]$. These functions, called "membership functions" express the degree of belonging of an element $x \in H$ to a subset A of H. To consider in a problem, a fuzzy subset instead of an usual (Cantor) subset, corresponds usually to think according to a multivalued logic instead of a bivalent logic. The reply to many questions in the science, and in the life, often is not possible in a dichotomic form, but it has a great variety of nuances. A Cantor subset A of the universe H, can be represented as the class of objects which satisfy a certain property p , so an element x does not belong to A if it does not satisfy p . But in the reality an object can satisfy p in a certain measure. Whence the advisability to size the satisfaction of p by a real number $\mu_A(x) \in [0,1]$.

Rough Sets, which have been proved to be a particular case of fuzzy sets (see [8]) are they also an important instrument for studying in depth some subjects of applied science. The first idea of rough set appears in the book by Shaefer [134] as pair of "inner and outer reductions" (see pages 117-119), in the context of Probability and Scientific Inference, but they became a well known subject of research in pure and applied mathematics, since Pawlak [121] considered them again and proved their utility in some topics of Artificial Intelligence as Decision Making, Data Analysis, Learning Machines, Switching Circuits.

Connections between fuzzy sets and algebraic hyperstructures were considered for the first time by Rosenfeld [130]. Many others worked in the same direction, that is studied algebraic structures endowed also with a fuzzy structure (see [1], [2], [3], [54],…, [59]).

Hyperstructures endowed with a fuzzy structure were considered by Ameri and Zahedi, Tofan, Davvaz, Borzooei, Hasankhani, Bolurian and others.

Definition 9 A *fuzzy subset* of a set H is a pair $(H; \mu_A)$ where μ_A is a function $\mu_A : H \to [0,1]$, A is the set $\{ x \in H \mid \mu_A(x) = 1 \}$.

Corsini proved in 1993, [17], that to every fuzzy subset of a set H one can associate a join space, where the hyperoperation is defined as follows:

(I) $\forall (x, y) \in H^2$, $x \bullet y = \{z \mid \min\{\mu(x), \mu(y)\} \leq \mu(z) \leq \max\{\mu(x), \mu(y)\}\}$.

Moreover in 2003 Corsini proved [29] that to every hypergroupoid <H, o> one can associate a fuzzy subset. See (II) :

(II) $\forall u \in H$, let $Q(u) = \{(x, y) \in H^2 \mid u \in x \circ y\}$, $q(u) = |Q(u)|$, $A(u) = \Sigma_{\{x,y\} \subseteq Q(u)} (1 / |x \circ y|)$, $\mu_A'(u) = A(u)/q(u)$.

If we have a hypergroupoid <H, ○”> and <○”> is a weak hyperoperation, we can associate with H the following fuzzy subset
(II”) Set $m_{x,y}(u)$ the multiplicity of the element u in the hyperproduct $x \circ y$. Set
$\mu_{x,y}(u) = m_{x,y}(u) / \Sigma (m_{x,y}(v) \mid v \in H, m_{x,y}(v) > 0)$
$Q(u) = \{(a,b) \in H^2 \mid m_{a,b}(u) > 0\}$, $q(u) = |Q(u)|$
$A(u) = \Sigma_{\{x,y\} \subset Q(u)} \mu_{x,y}(u)$, $\mu'(u) = A(u)/q(u)$.

Weak hyperstructures were introduced by Vougiouklis (1981) and were studied by many people especially by Vougiouklis and Spartalis.

So every fuzzy subset (and every hypergroupoid) determines a sequence of join spaces and of fuzzy subsets

Connections between hyperstructures and fuzzy sets have been considered by many people. In particular (I) and (II) opened research lines studied in deep by several scientists. In this context several

papers have been made in Italy, Romania, Greece, Iran, for example by Corsini, Leoreanu, Cristea, Serafimidis, Kehagias, Konstantinidou, Rosenberg. Hyperstructures endowed also with a fuzzy structure have been considered especially in Iran by Ameri, Zahedi, Davvaz and many others.

From (I) and (II) follows clearly that every hypergroupod (o fuzzy subset) determines a sequence of fuzzy subsets and hypergroupoids or of hypergroupoids and fuzzy subset) which is obtained applying consecutevely (II) e (I) (oppure (I) e (II))

The *fuzzy grade* (*minimum lengh* of such sequences) has been calculated for several classes of hyperstructures: Corsini-Cristea for i.p,s, hypergroups (a particular case of *canonical hypergroups*), and 1-hypergroups (hypergroups such that if ω is the heart of the hypergroup, $|\omega| = 1$). Corsini - Leoreanu for hypergroups associated with hypergraphs, Leoreanu for hypergroups associated with *rough sets*. Corsini and Cristea for *complete hypergroups*.

Let H be a set, R an equivalence relation in H and $\forall x \in H$, we denote the equivalence class of x by R(x).

It is known that with every binary relation R defined in a set H, a partial hypergroupoid corresponds defined

$$\forall (x,y) \in H^2 , x \cdot_R x = \{ u \mid xRu \} , x \cdot_R y = x \cdot_R x \cup y \cdot_R y$$

This structure that under certain conditions is a hypergroup was introduced by Rosenberg in 1996, see [102] and afterwards studied also by Corsini in Multiple Logic and Applications (1997) and by Corsini - Leoreanu in Algebra Universalis n. 43 (2000).
Hypergroupoids associated with multivalued functions, have been analyzed by Corsini and Razieh Majoob in 2010, see [40].

5. New lines of research

1) A research line could be to calculate the Fuzzy Grade of the hypergroupoid associated with a Binary relation
2) In Bull. Greek Math Society, Corsini has associated in different ways hypergropoids with an ordered set. It could be interesting to study the sequences of join spaces determined by these hypergroupoids.
3) Another research line could be to study the sequence of Join Spaces determined by a hypergroupoid endowed with a weak hyperoperation.
4) It would be interesting also to consider the sequence of fuzzy sets and join spaces determined by a *Chinese Hypergroupoid* (see [24])
5) In [25], [26] one has associated a hypergroupoid with a factor space, that is , given a function f from an universe U to a set of states X(f), and and a fuzzy subset of U, μ_f called the extension of f, one has considered the hyperoperation in U, $< \circ^f_{\mu f} >$. defined:
$x \circ^f_{\mu f} y = \{ w \mid \mu_f(w) \in [\sup. \{ \mu_f(z) \mid f(z) = f(x) \}, \sup. \{ \mu_f(v) \mid f(v) = f(y) \}] \}$
One proposes to determine the fuzzy grade of the hypergroupoid $<U; \circ^f_{\mu f} >$.
6) Set A a non empty set and $\mathcal{F}$ the set of functions f: $A \to P^*(A)$ such that $\cup_{x \in A} f(x) = A$. One considers the following hyperoperations in $\mathcal{F}(A)$, see [31]
(i) $f \circ_1 g = \{ h \in \mathcal{F} \mid \forall x \in A, h(x) \subset f(g(x)) \}$,
(ii) $f \circ_2 g = \{ h \in \mathcal{F} \mid \forall x \in A, h(x) \subset f(x) \cup g(x) \}$,
(iii) Let's suppose now $<A ; ò >$ to be a hypergroupoid. Then set for every $(f.g) \in \mathcal{F} \times \mathcal{F}$

$$f \circ_3 g = \{ h \in \mathcal{F} \mid \forall x \in A, h(x) \subset f(x) ò g(x) \},$$

Problems: Let's suppose $|A| = 3$
i* Determine the fuzzy grade of the hypergroupoid (i)
ii* Determine the fuzzy grade of the hypergroupoid (ii)
iii * Determine the fuzzy grade of the hypergroupoid (iii) , for some hypergroupoid $<A ; ò >$

(7) It is known that every hypergraph $<\Gamma ; \{A_i\} >$ determines a sequence of quasi-hypergroups $Q_0, Q_1, \ldots\ldots, Q_m$ (see [18], Th. 6). Set, for every k, m_k the membership function associated with Q_k and $J(Q_k)$ the corresponding join space. Set σ^* the sequence $< Q_0 , \mu_0 , J(Q_o) , \mu_1 , J(Q_1) , \ldots.., \mu_m , J(Q_m) >$, and set σ the sequence of membership functions and join spaces determined by the hypergroupoid Q_0 after (I) and (II)
It would be interesting to compare the two sequences σ and σ^* .

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