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Fuzzy Translation in Fuzzy d-Ideals and Fuzzy d-Subalgebra

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Abstract

In this paper, we discuss about some properties such as fuzzy translation in fuzzy d-ideals and fuzzy d-subalgebra.

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1. Introduction

Fuzzy set theory was introduced by Zadeh in 1965 [6]. The study of fuzzy subsets and its applications to various mathematical contexts has given rise to what is now commonly called fuzzy mathematics. It forms a branch of mathematics including fuzzy set theory and fuzzy logic.

Fuzzy set theory was guided by the assumption that the classical sets were not natural appropriate or useful notions in describing the real-life problems because every object encountered in the real physical world carries some degree of fuzziness. Hence fuzzy set has become strong area of research in engineering, medical science, graph theory etc.

Algebraic structures play an important role in mathematics with wide range of application in many disciplines such as computer sciences, control engineering, theoretical physics, information systems and topological spaces. Since these ideas have been applied to other algebraic structures such as group, semigroup, ring, modules, vector spaces and topologies. It gives enthusiasm to the researchers to view various concepts and results from the area of abstract algebra in the broader frame work of fuzzy setting.

Fuzzy algebra is an important branch of fuzzy mathematics. In 1996, J. Negger and H.S. Kim [3] introduced the class of d-algebra which is a generalization of BCK-algebras and investigated relation between d-algebra and BCK-algebra. M. Akram and K.H. Dar [1] introduced the concepts fuzzy d-algebra, fuzzy subalgebra and fuzzy d-ideals of d-algebra.

2. Preliminaries

Definition:2.1 A d-algebra is a non-empty set X with a constant 0 and a binary operation $*$ satisfies the following axioms:

i. $x * x = 0$
ii. $0 * x = 0$
iii. $x * y = 0$ and $y * x = 0 \Rightarrow x = y, for\ all\ x, y \in X.$

Definition:2.2 A non-empty subset of a d-algebra X is called d-subalgebra of X if $x * y \in X, for\ all\ x, y \in X.$

Definition: 2.3 Let $(X,*,0)$ be a d-algebra and $a \in X$. Define $a * X = \{a * x / x \in X\}$. Then X is said to be edge if $a * X = \{0, a\}$ for all $a \in X$.

Definition: 2.4 Let X be a d-algebra and I be a subset of X, then I is called d-ideal of X if it satisfies the following conditions:

i. $0 \in I$

ii. $x * y \in I$ and $y \in I \Rightarrow x \in I$

iii. $x \in I$ and $y \in X \Rightarrow x * y \in I$.

Definition: 2.5 A fuzzy subset μ_A of X is called a fuzzy d-ideal of X if it satisfies the following condition:

i. $\mu_A(0) \geq \mu_A(x)$

ii. $\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\}$

iii. $\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}$ $\quad$ *for all* $x, y \in X$.

3. Fuzzy Translation in Fuzzy d - Ideals

Definition: 3.1 Let μ_A be a fuzzy subset of X and $\alpha \in [0, T]$. A mapping $(\mu_A)_\alpha^T: X \rightarrow [0,1]$ is said to be a fuzzy-α-translation of μ_A if it satisfies $(\mu_A)_\alpha^T(x) = \mu_A(x) + \alpha, for\ all\ x \in X$.

Theorem: 3.2 If μ_A is a fuzzy d-ideal of X, then the fuzzy-α-translation $(\mu_A)_\alpha^T$ of μ_A is a fuzzy d-ideal of X, for all $\alpha \in [0,1]$.

Proof:

Let μ_A be a fuzzy d-ideal of X and let $\alpha \in [0,1]$

Then $(\mu_A)_\alpha^T(0) = \mu_A(0) + \alpha$

$\geq \mu_A(x) + \alpha$

$= (\mu_A)_\alpha^T(x)$

$(\mu_A)_\alpha^T(0) \geq (\mu_A)_\alpha^T(x)$

$(\mu_A)_\alpha^T(x) = \mu_A(x) + \alpha$

$\geq min\{\mu_A(x * y), \mu_A(y)\} + \alpha$

$= min\{\mu_A(x * y) + \alpha, \quad \mu_A(y) + \alpha\}$

$= min\{(\mu_A)_\alpha^T(x * y), (\mu_A)_\alpha^T(y)\}$

$(\mu_A)_\alpha^T(x) \geq min\{(\mu_A)_\alpha^T(x * y), (\mu_A)_\alpha^T(y)\}$

$(\mu_A)_\alpha^T(x * y) = \mu_A(x * y) + \alpha$

$\geq min\{\mu_A(x), \mu_A(y)\} + \alpha$

$= min\{\mu_A(x) + \alpha, \quad \mu_A(y) + \alpha\}$

$= min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$

$(\mu_A)_\alpha^T(x * y) \geq min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$

Hence $(\mu_A)_\alpha^T$ of μ_A is a fuzzy d-ideal of X for all $\alpha \in [0,1]$.

Theorem: 3.3 Let μ_A be a fuzzy subset of X such that the fuzzy-α-translation $(\mu_A)_\alpha^T$ of μ_A is a fuzzy d-ideal of $X$for some $\alpha \in [0,1]$, then μ_A is a fuzzy d-ideal of X.

Proof:

Let $(\mu_A)_\alpha^T$ is a fuzzy d-ideal of X for some $\alpha \in [0,1]$

Let $x, y \in X$

$\mu_A(0) + \alpha = (\mu_A)_\alpha^T(0)$

$\geq (\mu_A)_\alpha^T(x)$

$= \mu_A(x) + \alpha$

$\mu_A(0) \geq \mu_A(x)$

$\mu_A(x) + \alpha = (\mu_A)_\alpha^T(x)$

$\geq min\{(\mu_A)_\alpha^T(x * y), (\mu_A)_\alpha^T(y)\}$

$= min\{\mu_A(x * y) + \alpha, \mu_A(y) + \alpha\}$

$= min\{\mu_A(x * y), \mu_A(y)\} + \alpha$

$\mu_A(x) \geq min\{\mu_A(x * y), \mu_A(y)\}$

$\mu_A(x * y) + \alpha = (\mu_A)_\alpha^T(x * y)$

$\geq min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$

$= min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\}$

$= min\{\mu_A(x), \mu_A(y)\} + \alpha$

$\mu_A(x * y) \geq min\{\mu_A(x), \mu_A(y)\}$

Hence μ_A is a fuzzy d-ideal of X.

4. Fuzzy Translation in Fuzzy d - Subalgebra

Definition: 4.1 A fuzzy set $\mu_A$in d-algebra X is called fuzzy d-subalgebra of X if it satisfies**,** $\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\} \quad for\ all\ x, y \in X$.

Theorem: 4.2 If μ_A is a fuzzy d-subalgebra of X, then the fuzzy-α-translation μ_α^T of μ_A is also a fuzzy d-subalgebra of X for all $\alpha \in [0,1]$.

Proof:

Let $x, y \in X$ and let $\alpha \in [0,1]$

Let μ_A be a fuzzy d-subalgebra of X .

Then $\quad \mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}$

$(\mu_A)_\alpha^T(xy) = \mu_A(xy) + \alpha$

$\geq min\{\mu_A(x), \mu_A(y)\} + \alpha$

$= min\{\mu_A(x) + \alpha, \quad \mu_A(y) + \alpha\}$

$= min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$

$(\mu_A)_\alpha^T(xy) \geq min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$

Hence $(\mu_A)_\alpha^T$ of μ_A is a fuzzy d-subalgebra of X for all $\alpha \in [0,1]$.

Theorem: 4.3 Let μ_A be a fuzzy subset of X such that the fuzzy-α-translation $(\mu_A)_\alpha^T$ of μ is a fuzzy d-subalgebra of X for some $\alpha \in [0,1]$, then μ_A is a fuzzy d-subalgebra of X.

Proof:

Let $(\mu_A)_\alpha^T$ is a fuzzy d-subalgebra of X for some $\alpha \in [0,1]$.

Let $x, y \in X$.

$\mu_A(xy) + \alpha = (\mu_A)_\alpha^T(xy)$

$\geq min\{(\mu_A)_\alpha^T(x), (\mu_A)_\alpha^T(y)\}$

$= min\{\mu_A(x) + \alpha, \mu_A(y) + \alpha\}$

$= min\{\mu_A(x), \mu_A(y)\} + \alpha$

$\mu_A(xy) \geq min\{\mu_A(x), \mu_A(y)\}$

Hence μ_A is a fuzzy d-subalgebra of X.

5. Conclusions

In this paper, we have given some ideas on fuzzy translation in fuzzy d-ideals and fuzzy d-subalgebras. Our further research will be focus on dot product and level set.

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