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On Intuitionistic Semi * Continuous Functions

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Abstract

In this paper we introduce intuitionistic semi * continuous and intuitionistic contra - semi * continuous functions via the concept of intuitionistic semi * open and intuitionistic semi * closed set respectively. Also, we investigate their properties and characterization.

Keywords: intuitionistic semi * open, intuitionistic semi * closed, intuitionistic semi * continuous, intuitionistic contra-semi * closed.

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1. Introduction

The concept of intuitionistic set introduce by D. Coker [2] in 1996 and also he [1] has introduced the concept of intuitionistic topological space. In [5], we introduced the concept of intuitionistic generalized closure operator and defined a new intuitionistic topology τ^* and studied their properties. Also, in [6] we introduced a new open set namely intuitionistic semi * open and studied their properties. In this paper we define intuitionistic semi * continuous and intuitionistic contra - semi * continuous functions via the concept of intuitionistic semi * open and intuitionistic semi * closed set respectively. Also, we investigate their properties and characterization.

2.Preliminaries

Definition 2.1 [1] Let X be a nonempty fixed set. An intuitionistic set (IS in short) $\tilde{A}$ is an object having the form $\tilde{A} =< X, A^1, A^2 >$ where A^1 and A^2 are subsets of X such that $A^1 \cap A^2 = \emptyset$. The set A^1 is called the set of member of $\tilde{A}$, while A^2 is called the set of non member of $\tilde{A}$.

Definition 2.2 [1] An intuitionistic topology (IT in short) by subsets of a nonempty set X is a family τ of IS's satisfying the following axioms.

(a) $\tilde{\emptyset}_I$, $\tilde{X}_I \in \tau$,

(b) $\tilde{G}_1 \cap \tilde{G}_2 \in \tau$ for every $\tilde{G}_1$, $\tilde{G}_2 \in \tau$, and

(c) $\cup \tilde{G}_i \in \tau$ for any arbitrary family $\{\tilde{G} : i \in J\} \subseteq \tau$.

The pair (X, τ) is called an intuitionistic topological space (ITS in short) and any IS $\tilde{A}$ in τ is called an intuitionistic open set (IOS). The complement of an IO set $\tilde{A}$ in is called an intuitionistic closed set (ICS).

Definition 2.3 [1] Let (X, τ) be an ITS and $\tilde{A} =< X, A^1, A^2 >$ be an IS in X. Then the interior and the closure of A are denoted by $Iint(\tilde{A})$ and $Icl(\tilde{A})$, and are defined as follows.

$Iint(\tilde{A}) = \cup \{\tilde{G} \mid \tilde{G}$ is an IOS and $\tilde{G} \subseteq \tilde{A}\}$ and

$Icl(\tilde{A}) = \cap \{\tilde{K} \mid \tilde{K}$ is an ICS and $\tilde{A} \subseteq \tilde{K}\}$.

Definition 2.4 [2] Let X be a nonempty set and $p \in X$ be a fixed element. Then the IS $\tilde{p}$ defined by $\tilde{p} =< X, \{p\}, \{p\}^c >$ is called an intuitionistic point (in short, IP).

Definition 2.5 [10] Let (X, τ) be an ITS and $\tilde{A} =< X, A^1, A^2 >$ be an IS in X, $\tilde{A}$ is said to be intuitionistic generalized closed set (briefly Ig – closed set) $Icl(\tilde{A}) \subseteq \tilde{U}$ whenever $\tilde{A} \subseteq \tilde{U}$ and $\tilde{U}$ is IOS in X.

Definition 2.6 [5] If $\tilde{A}$ is an IS of an ITS (X, τ), then the intuitionistic generalized closure of $\tilde{A}$ is defined as the intersection of all Ig – closed sets in X containing $\tilde{A}$ and is denoted by $Icl^*(\tilde{A})$.

Definition 2.7 [6] The IS $\tilde{A}$ of an ITS (X, τ) is called intuitionistic semi * open sets if there is an intuitionstic open set $\tilde{G}$ in X such that $\tilde{G} \subseteq \tilde{A} \subseteq Icl^*(\tilde{G})$.

Definition 2.8 [6] The intuitionistic semi * interior of $\tilde{A}$ is defined as the union of all intuitionistic semi * open sets of X contained in $\tilde{A}$. It is denoted by $IS^*int(\tilde{A})$.

Definition 2.9 An intuitionistic set $\tilde{A}$ of a ITS (X, τ) is called an intuitionistic semi * closed set if X - $\tilde{A}$ is intuitionistic semi * open.

Definition 2.10 The semi * closure of an IS $\tilde{A}$ is defined as the intersection of all intuitionistic semi * closed sets in X that containing $\tilde{A}$. It is denoted by $IS^*cl(\tilde{A})$.

Theorem 2.11 Let (X, τ) be an ITS and $\tilde{A}$ be an IS of X. Then

(i) $IS^*cl(X - \tilde{A}) = X - IS^*int(\tilde{A})$
(ii) $IS^*int(X - \tilde{A}) = X - IS^*cl(\tilde{A})$

Definition 2.12[9] A function $f : X \to Y$ is said to be intuitionistic semi continuous if $f^{-1}(\tilde{A})$ is ISO in X for every IOS $\tilde{A}$ in Y.

Theorem 2.13[6] Let (X, τ) be an ITS and $\tilde{A}$ be an IS of X. Then

(i) Every IOS is IS*O.
(ii) Every IS*O is ISO.
(iii) Every ICS is IS*C.
(iv) Every IS*C is ISC.

Theorem 2.14[6] Let (X, τ) be an ITS. Then

(i) If $\{\tilde{A}_\alpha\}$ is a collection of IS*O in X then $\cup \tilde{A}_\alpha$ is IS*O.
(ii) If $\tilde{A}$ is IS*O in X and $\tilde{B}$ is an IOS in X, then $\tilde{A} \cap \tilde{B}$ is IS*O in X.

Theorem 2.15[6] Let (X, τ) be an ITS and $\tilde{A}$ be an IS of X. Then

(i) $\tilde{A}$ is IS*O if and only if $IS^*int(\tilde{A}) = \tilde{A}$.
(ii) $\tilde{A}$ is IS*C if and only if $IS^*cl(\tilde{A}) = \tilde{A}$.

Theorem 2.16[6] Let (X, τ) be an ITS, $\tilde{A}$ be an IS of X and $\tilde{p} \in X$. Then $\tilde{p} \in IS{*}cl(\tilde{A})$ if and only if every IS*O in X containing $\tilde{p}$ intersects $\tilde{A}$.

Definition 2.17[7] Let (X, τ) be an ITS and $\tilde{A}$ be an IS of X. Then the intuitionistic semi * frontier of $\tilde{A}$ (denoted by $IS{*}Fr\,(\tilde{A})$) is defined by $IS{*}Fr\,(\tilde{A}) = IS{*}cl(\tilde{A}) - IS{*}int\,(\tilde{A})$.

Theorem 2.18[7] Let (X, τ) be an ITS and $\tilde{A}$ be an IS of X. Then

$$IS{*}Fr\,(\tilde{A}) = IS{*}cl(\tilde{A}) \cap IS{*}cl\,(\overline{\tilde{A}}).$$

3. Intuitionistic Semi * Continuous Functions

Definition 3.1 A function $f : X \to Y$ is said to be intuitionistic semi * continuous at $\tilde{p} \in X$ if for each intuitionistic open set $\tilde{U}$ of Y containing $f(\tilde{p})$, there is an intuitionistic semi open set $\tilde{V}$ in X such that $\tilde{p} \in \tilde{V}$ and $f(\tilde{V}) \subseteq \tilde{U}$.

Definition 3.2 A function $f : X \to Y$ is said to be intuitionistic semi * continuous if $f^{-1}(\tilde{U})$ is IS*O in X for every IOS $\tilde{U}$ in Y.

Theorem 3.3 Every intuitionistic continuous function is intuitionistic semi * continuous.

Proof: Let $f : X \to Y$ be intuitionistic continuous and $\tilde{U}$ be IO in Y. Then $f^{-1}(\tilde{U})$ is IO in X. Therefore by theorem 2.13(i), $f^{-1}(\tilde{U})$ is IS*O in X. Hence f is intuitionistic semi * continuous function.

Remark 3.4 The converse of the above theorem need not be true as seen from the succeeding example

Example 3.5 Let $X = \{i, j, k\} = Y$ and $\tau_1 = \{\tilde{X}_I, \tilde{\emptyset}_I, < X, \{i\},\{j, k\} >, < X, \{j\}, \{i, k\} >, < X, \{i, j\}, \{k\}>\}$, $\tau_2 = \{\tilde{X}_I, \tilde{\emptyset}_I, < X, \{j\},\{i, k\} >, < X, \{i\}, \{j\} >, < X, \{i, j\}, \emptyset>\}$. Let f: $(X, \tau_1) \to (Y, \tau_2)$ be defined by $f(i) = j$, $f(j) = i$, $f(k) = k$. Then f is intuitionistic semi * continuous. Let $\tilde{D} = < X, \{i, j\}, \emptyset>$. Then $f^{-1}(\tilde{D}) = < X, \{k, i\}, \emptyset>$ is not IOS in τ_1. Therefore f is not an intuitionistic continuous.

Corollary 3.6 Every constant function is intuitionistic semi * continuous function.

Proof: We know that every constant function is intuitionistic continuous function. Therefore by theorem 3.3 every constant function is intuitionistic semi * continuous function.

Theorem 3.7 Let ß be the intuitionistic basis of the intuitionistic topological space Y. Then the function f: $X \to Y$ is intuitionistic semi * continuous if and only if inverse image of every basic IOS in Y under the function f is IS*O in X.

Proof: Let f: $X \to Y$ be intuitionistic semi * continuous. Then the inverse image of every IOS in Y is IS*O in X. In particular, the inverse image of every basic IOS in Y is IS*O in X. Coversely, assume that $\tilde{V}$ be an IOS in Y. Then $\tilde{V} = \cup \tilde{B}_\alpha$ where $\tilde{B}_\alpha \in$ ß. Now $f^{-1}(\tilde{V}) = f^{-1}(\cup \tilde{B}_\alpha) = \cup f^{-1}(\tilde{B}_\alpha)$. Therefore by hypothesis, $f^{-1}(\tilde{B}_\alpha)$ is IS*O for each α. Then by theorem 2.14(i), $f^{-1}(\tilde{V})$ is IS*O. Hence the function f is intuitionistic semi * continuous.

Theorem 3.8 Every intuitionistic semi * continuous function is intuitionistic semi continuous.

Proof: Let f: $X \to Y$ be intuitionistic semi * continuous function and $\tilde{V}$ be an IOS in Y. Then $f^{-1}(\tilde{V})$ is IS*O in X. Therefore by theorem 2.13(ii), $f^{-1}(\tilde{V})$ is ISO in X. Hence f is intuitionistic semi continuous.

Remark 3.9 The following example shows that the converse of the above theorem need not be true.

Example 3.10 Let $X = \{i, j, k\} = Y$ and $\tau_1 = \{\widetilde{X}_I, \widetilde{\emptyset}_I$, < X, $\{j\}$,$\{i, k\}$ >, < X, $\{i\}$, $\{j\}$ >, < X, $\{i, j\}$, ∅>}, $\tau_2 = \{\widetilde{X}_I, \widetilde{\emptyset}_I$, < X, $\{i\}$,$\{j\}$ >, < X, $\{i, j\}$, ∅>}. Let f: $(X, \tau_1) \to (Y, \tau_2)$ be defined by $f(i) = j$, $f(j) = i$, $f(k) = k$. Then f is intuitionistic semi continuous. Let $\tilde{E}$ = < X, $\{j\}$, $\{i\}$>. Then $f^{-1}(\tilde{E})$ = < X, $\{j\}$, $\{i\}$> is not an IS*O in τ_1. Therefore f is not an intuitionistic semi * continuous.

Lemma 3.11 Let (X. τ) be an ITS and $\tilde{A}$ be an IS of X. Then

(i) $\tilde{A}$ is IS*O in X if and only if $Icl^*(Iint(\tilde{A})) = Icl^*(\tilde{A})$.

(ii) $\tilde{A}$ is IS*C in X if and only if $Iint^*(Icl(\tilde{A})) = Iint^*(\tilde{A})$.

Proof: (i) Let $\tilde{A}$ be an IS*O. Then by definition of IS*O we have $\tilde{A} \subseteq Icl^*(Iint(\tilde{A}))$. Hence $Icl^*(\tilde{A}) \subseteq Icl^*(Iint(\tilde{A}))$. Also we have $Iint(\tilde{A}) \subseteq \tilde{A}$, $Icl^*(Iint(\tilde{A})) \subseteq Icl^*(\tilde{A})$. Thus $Icl^*(Iint(\tilde{A})) = Icl^*(\tilde{A})$. On the other hand, let $Icl^*(Iint(\tilde{A})) = Icl^*(\tilde{A})$. Then by definition of IS*O, $\tilde{A}$ is IS*O.

(ii) $\tilde{A}$ is IS*C if and only if $X - \tilde{A}$ is *IS*O*. Then by (i) $\tilde{A}$ is IS*C if and only if $Icl^*(Iint(X - \tilde{A})) = Icl^*(X - \tilde{A})$. Hence $\tilde{A}$ is IS*C if and only if $Iint^*(Icl(\tilde{A})) = Iint^*(\tilde{A})$.

Theorem 3.12 Let *f*: $X \to Y$ be a function. Then the following are equivalent.

(i) *f* is intuitionistic semi * continuous.

(ii) *f* is intuitionistic semi * continuous at each IP of *X*.

(iii) $f^{-1}(\widetilde{U})$ is IS*C in *X* for every ICS $\widetilde{U}$ in *Y*.

(iv) $f(IS{*}cl(\tilde{B})) \subseteq Icl(f(\tilde{B}))$ for every IS $\tilde{B}$ of *X*.

(v) $IS{*}cl(f^{-1}(\tilde{A})) \subseteq f^{-1}(Icl(\tilde{A}))$ for every IS $\tilde{A}$ of *Y*.

(vi) $Iint{*}(Icl(f^{-1}(\tilde{E}))) = Iint{*}(f^{-1}(\tilde{E}))$ for every ICS $\tilde{E}$ in *Y*.

(vii) $Icl{*}(Iint(f^{-1}(\tilde{F}))) = Icl{*}(f^{-1}(\tilde{F}))$ for every IOS $\tilde{F}$ in *Y*.

(viii) $f^{-1}(Iint(\tilde{C}) \subseteq IS{*}int(f^{-1}(\tilde{F}))$ for every IS $\tilde{C}$ in *Y*.

Proof: (i) ⟹ (ii). Let *f*: $X \to Y$ be an intuitionistic semi * continuous. Let $\tilde{p} \in X$ and $\tilde{V}$ be an IOS in *Y* containing $f(\tilde{p})$. Then $\tilde{p} \in f^{-1}(\tilde{V})$. Since *f* is intuitionistic semi * continuous, $\widetilde{U} = f^{-1}(\tilde{V})$ is an IS*O in *X* containing $\tilde{p}$ such that $f(\widetilde{U}) \subseteq \tilde{V}$. Hence *f* is intuitionistic semi * continuous at each IP of *X*.

(ii)⟹ (iii). Let $\widetilde{U}$ be an ICS in *Y*. Then $\tilde{V} = Y - \widetilde{U}$ is an IOS in *Y*. Let $\tilde{p} \in f^{-1}(\tilde{V})$. Then $f(\tilde{p}) \in \tilde{V}$. By hypothesis, there is a IS*O set $\tilde{A}_p$ in *X* containing $\tilde{p}$ such that $f(\tilde{p}) \in f(\tilde{A}_p) \subseteq \tilde{V}$. Therefore $\tilde{A}_p \subseteq f^{-1}(\tilde{V})$. Hence $f^{-1}(\tilde{V}) = \cup \{\tilde{A}_p : \tilde{p} \in f^{-1}(\tilde{V})\}$. By theorem 2.14(*i*), $f^{-1}(\tilde{V})$ is IS*O in *X*. Thus $f^{-1}(\widetilde{U}) = f^{-1}(Y - \tilde{V}) = X - f^{-1}(\tilde{V})$ is IS*C in *X*. Hence $f^{-1}(\widetilde{U})$ is IS*C in *X* for every ICS $\widetilde{U}$ in *Y*.

(iii)⟹ (iv). Let $\tilde{B}$ be an IS of *X* and let $\widetilde{U}$ be an ICS containing $f(\tilde{B})$. Then by (iii), $f^{-1}(\widetilde{U})$ is *IS**C containing $\tilde{B}$. This implies that $IS{*}cl(\tilde{B}) \subseteq f^{-1}(\widetilde{U})$ and hence $f(IS{*}cl(\tilde{B})) \subseteq \widetilde{U}$. Thus $f(IS{*}cl(\tilde{B})) \subseteq Icl(f(\tilde{B}))$.

(iv)⟹ (v).Let $\tilde{A}$ be an IS of *Y*. Let $\tilde{B} = f^{-1}(\tilde{A})$. By assumption, $f(IS{*}cl(\tilde{B})) \subseteq Icl(f(\tilde{B}) \subseteq Icl(\tilde{A})$.This implies $(IS{*}cl(\tilde{B})) \subseteq f^{-1}(Icl(\tilde{A}))$. Hence $IS{*}cl(f^{-1}(\tilde{A})) \subseteq f^{-1}(Icl(\tilde{A}))$.

(v)⟹ (vi). Let $\tilde{E}$ be an ICS in *Y*. Then by (v), $IS{*}cl(f^{-1}(\tilde{E})) \subseteq f^{-1}(Icl(\tilde{E})) = f^{-1}(\tilde{E})$. Also we have $f^{-1}(\tilde{E}) \subseteq IS{*}cl(f^{-1}(\tilde{E}))$. Hence $IS{*}cl(f^{-1}(\tilde{E})) = f^{-1}(\tilde{E})$. Thus by theorem 2.15(*ii*), $f^{-1}(\tilde{E})$ is closed. Therefore by lemma 3.11 (ii) $Iint{*}(Icl(f^{-1}(\tilde{E}))) = Iint{*}(f^{-1}(\tilde{E}))$.

(vi)⟹ (vii). Let $\tilde{F}$ be an IOS in *Y*. Then $Y - \tilde{F}$ is ICS in *Y*. Therefore by assumption, $Iint{*}(Icl(f^{-1}(Y - \tilde{F}))) = Iint{*}(f^{-1}(Y - \tilde{F}))$.This implies that $Icl{*}(Iint(f^{-1}(\tilde{F}))) = Icl{*}(f^{-1}(\tilde{F}))$.

(vii)⟹ (i). Let $\widetilde{U}$ be an IOS in *Y*. Then by assumption, $Icl{*}(Iint(f^{-1}(\widetilde{U}))) = Icl{*}(f^{-1}(\widetilde{U}))$. Now by lemma 3.11 (i), $f^{-1}(\widetilde{U})$ is IS*O in *X*. Hence *f* is intuitionistic semi * continuous.

(i)⟹ (viii). Let $\tilde{C}$ be any IS of *Y*. Then $Iint(\tilde{C})$ is IOS in *Y*. By intuitionistic semi * continuity of *f*, $f^{-1}(Iint(\tilde{C}))$ is IS*O in *X* and it is contained in $f^{-1}(\tilde{C})$. Hence $f^{-1}(Iint(\tilde{C})) \subseteq IS{*}int(f^{-1}(\tilde{C}))$.

(viii)⟹ (i). Let $\widetilde{U}$ be an IOS in Y. Then $Iint(\widetilde{U}) = \widetilde{U}$. By (viii) $f^{-1}(\widetilde{U}) \subseteq IS{*}int(f^{-1}(\widetilde{U}))$ and hence $f^{-1}(\widetilde{U}) = IS{*}int(f^{-1}(\widetilde{U}))$. Therefore by theorem 2.15(*i*), $f^{-1}(\widetilde{U})$ is IS*O in X. Thus f is intuitionistic semi * continuous.

Theorem 3.13 The function f: $X \to Y$ is not an intuitionistic semi * continuous at an IP $\tilde{p}$ in X if and only if $\tilde{p}$ belongs to the intuitionistic semi * frontier of the inverse image of some IOS in Y containing $f(\tilde{p})$.

Proof: Let f be not an intuitionistic semi * continuous at an IP $\tilde{p}$. Then there is an IOS $\widetilde{U}$ in Y containing $f(\tilde{p})$ such that $f(\widetilde{V})$ is not an IS of $\widetilde{U}$ for every IS*O set $\widetilde{V}$ in X containing $\tilde{p}$. Hence $\widetilde{V} \cap (X - f^{-1}(\widetilde{U})) \neq \widetilde{\emptyset}_I$ for every IS*O set $\widetilde{V}$ containing $\tilde{p}$. By theorem 2.16, $\tilde{p} \in IS{*}cl(X - f^{-1}(\widetilde{U}))$. Also we have $\tilde{p} \in f^{-1}(\widetilde{U}) \subseteq IS{*}cl\,(f^{-1}(\widetilde{U}))$. Thus $\tilde{p} \in IS{*}cl\,(f^{-1}(\widetilde{U})) \cap IS{*}cl(X - f^{-1}(\widetilde{U}))$. Hence by theorem 2.18, $\tilde{p} \in IS{*}Fr(f^{-1}(\widetilde{U}))$. Conversely, let f be an intuitionistic semi * continuous at an IP $\tilde{p}$. Let $\widetilde{U}$ be any IOS in Y containing $f(\tilde{p})$. Then $f^{-1}(\widetilde{U})$ is an IS*O set in X containing $\tilde{p}$. Hence by theorem 2,15(*i*), $\tilde{p} \in IS{*}int(f^{-1}(\widetilde{U}))$. Thus $\tilde{p} \notin IS{*}Fr(f^{-1}(\widetilde{U}))$. This proves the theorem.

Theorem 3.14 Let f: $X \to \prod X_\alpha$ be an intuitionistic semi * continuous where $\prod X_\alpha$ is the intuitionistic product topology and $f(\tilde{p})= (f_\alpha(\tilde{p}))$. Then each coordinate function $f_\alpha : X \to X_\alpha$ is an intuitionistic semi * continuous.

Proof: Let $\widetilde{U}$ be an IOS in X_α. Then $f_\alpha^{-1}(\widetilde{U}) = (P_\alpha \circ f)^{-1}(\widetilde{U}) = f^{-1}(P_\alpha^{-1}(\widetilde{U}))$, where $P_\alpha : \prod X_\alpha \to X$, the projection map. Since P_α is intuitionistic continuous, $P_\alpha^{-1}(\widetilde{U})$ is IOS in $\prod X_\alpha$. Since f is intuitionistic semi * continuous, $f_\alpha^{-1}(\widetilde{U}) = f^{-1}(P_\alpha^{-1}(\widetilde{U}))$ is IS*O in X. Thus each f_α is intuitionistic semi * continuous.

Remark 3.15 The converse of the above theorem is not true in general. However the converse is true if IS*O(X) is ICS under finite intersection as seen in the following theorem.

Theorem 3.16 Let f: $X \to \prod X_\alpha$ be defined by $f(\tilde{p})= (f_\alpha(\tilde{p}))$ and $\prod X_\alpha$ be the intuitionistic product topology. Let IS*O(X) be ICS under finite intersection. If each coordinate function $f_\alpha : X \to X_\alpha$ is intuitionistic semi * continuous, then f is intuitionistic semi * continuous.

Proof: Let $\widetilde{U}$ be the basic IOS in $\prod X_\alpha$. Then $\widetilde{V} = \cap\ P_\alpha^{-1}(\widetilde{U})$ where each $\widetilde{U}$ is IOS in X_α, the intersection being taken over finitely many α's and where $P_\alpha : \prod X_\alpha \to X$ is the projection map. Now $f^{-1}(\widetilde{U}) = f^{-1}(\cap(P_\alpha^{-1}(\widetilde{U}_\alpha))) = \cap\ f^{-1}\ (P_\alpha^{-1}(\widetilde{U}_\alpha)) = \cap\ (P_\alpha \circ f)^{-1}(\widetilde{U}_\alpha) = \cap\ f_\alpha^{-1}(\widetilde{U}_\alpha)$ is IS*O, by hypothesis. Thus by theorem 3.7, *f* is intuitionistic semi * continuous.

Theorem 3.17 Let *f*: $X \to Y$ be intuitionistic continuous and g: $X \to Z$ be intuitionistic semi * continuous. Let *h*: $X \to Y \times Z$ be defined by $h(\tilde{p}) = (f(\tilde{p}), g(\tilde{p}))$ and $Y \times Z$ be the intuitionistic product topology. Then *h* is intuitionistic semi * continuous.

Proof: Using theorem 3.7, it is sufficient to prove that the inverse image under h of every basic IOS in $Y \times Z$ is IS*O in *X*. Let $\tilde{A} \times \tilde{B}$ be the basic IOS in $Y \times Z$. Then $h^{-1}(\tilde{A} \times \tilde{B}) = f^{-1}(\tilde{A}) \cap g^{-1}(\tilde{B})$. Now by intuitionistic continuity of *f*, $f^{-1}(\tilde{A})$ is IO in *X* and by intuitionistic semi * continuity of g, $g^{-1}(\tilde{B})$ is IS*O in *X*. Therefore by theorem 2.14(*ii*), $h^{-1}(\tilde{A} \times \tilde{B}) = f^{-1}(\tilde{A}) \cap g^{-1}(\tilde{B})$ is IS*O in *X*. Hence *h* is intuitionistic semi * continuous.

Theorem 3.18 Let *f*: $X \to Y$ be intuitionistic semi * continuous and *h*: $Y \to Z$ be an intuitionistic continuous. Then $h \circ f$: $X \to Z$ is intuitionistic semi * continuous.

Proof: Let $\widetilde{U}$ be an IOS in *Z*. Since h is intuitionistic continuous, $h^{-1}(\widetilde{U})$ is IOS in *Y*. Since f is intuitionistic semi * continuous, $f^{-1}(h^{-1}(\widetilde{U}))$ is IOS in *X*. Therefore $f^{-1}(h^{-1}(\widetilde{U})) = (h \circ f)^{-1})(\widetilde{U})$ is IS*O in *X*. Hence $h \circ f$ is intuitionistic semi * continuous.

Remark 3.19 From the above theorem it can be seen that the composition of two intuitionistic semi * continuous need not be intuitionistic semi * continuous.

Example 3.20 Let *X* = Y = Z ={*i*, *j*, *k*} and $\tau_1 = \{\widetilde{X}_I, \widetilde{\emptyset}_I$, < *X*, {*i*},{*j*, *k*} >, < *X*, {*k*}, {*i*, j} >, < *X*, {*i*, *k*}, {*b*}>}, $\tau_2 = \{\widetilde{X}_I, \widetilde{\emptyset}_I$, < *X*, {*j*},{*i*, *k*} >, < *X*, {*i*},{*j*} >, < *X*, {*i*, *j*}, ∅>} $\tau_3 = \{\widetilde{X}_I, \widetilde{\emptyset}_I$, < *X*, {*j*},{*i*} >, < *X*, {*i*, *j*}, ∅>}. Let *f*: $(X, \tau_1) \to (Y, \tau_2)$ be defined by *f*(*i*) = *i*, *f*(*j*) = *k*, *f*(*k*) = *j* and Let *g*: $(Y, \tau_2) \to (Z, \tau_3)$ be defined by *g*(*i*) = *j*, *g*(*j*) = *i*, *g*(*k*) = *k*. Then *f* and *g* are intuitionistic semi * continuous. Let $g \circ f : (X, \tau_1) \to (Z, \tau_3)$ and $\tilde{B}$ = < *X*, {*j*}, {*i*}>. Then $(g \circ f)^{-1}(\tilde{B})$ = *g*(*f*(< *X*, {*j*}, {*i*}>)) = *g*(<*X*, {*k*},{*i*}>) = <*X*, {*k*},{*i*}> is not an IS*O in (Z, τ_3). Therefore $g \circ f$ is not an intuitionistic semi * continuous.

Definition 3.21 A function *f*: $X \to Y$ is said to be intuitionistic contra semi * continuous if $f^{-1}(\widetilde{U})$ is IS*C in *X* for every IOS $\widetilde{U}$ in *Y*.

Remark 3.22 The concept of intuitionistic semi * continuity is free from intuitionistic contra semi * continuity.

Theorem 3.23 Let *f*: $X \to Y$ be the function. Then the following are equivalent

(i) f is intuitionistic contra semi * continuous.
(ii) For each $\tilde{p} \in X$ and each ISC $\tilde{E}$ in Y containing $f(\tilde{p})$, there exists a IS*O set $\tilde{V}$ in X containing $\tilde{p}$ such that $f(\tilde{V}) \subseteq \tilde{E}$.
(iii) The inverse image of each ISC in Y is IS*O in X.
(iv) $Icl^*(Iint(f^{-1}(\tilde{E}))) = Icl^*(f^{-1}(\tilde{E}))$ for every ICS $\tilde{E}$ in Y.
(v) $Iint^*(Icl(f^{-1}(\tilde{V}))) = Iint^*(f^{-1}(\tilde{V}))$ for every IOS $\tilde{V}$ in X.

Proof: (i) ⇒ (ii). Let $f: X \to Y$ be the intuitionistic contra semi * continuous function. Let $\tilde{p} \in X$ and $\tilde{E}$ be an ICS in Y containing $f(\tilde{p})$. Take $\tilde{U} = Y - \tilde{E}$. Then $\tilde{U}$ is an IOS in Y not containing $f(\tilde{p})$. Since f is intuitionistic contra semi * continuous, $f^{-1}(\tilde{U})$ is a intuitionistic semi * closed set in X not containing $\tilde{p}$. Therefore $f^{-1}(\tilde{U}) = X - f^{-1}(\tilde{E})$ is a IS*C in X not containing $\tilde{p}$. Thus $\tilde{V} = f^{-1}(\tilde{E})$ is a IS*O in X containing $\tilde{p}$ such that $f(\tilde{V}) \subseteq \tilde{E}$. Hence (i).
(ii) ⇒ (iii). Let $\tilde{E}$ be an ICS in Y and $\tilde{p} \in f^{-1}(\tilde{E})$. Then $f(\tilde{p}) \in \tilde{E}$. By assumption, there is an IS*O set $\tilde{V}_p$ in X containing $\tilde{p}$ such that $f(\tilde{p}) \in f(\tilde{V}_p) \subseteq \tilde{E}$. Therefore $\tilde{V}_p \subseteq f^{-1}(\tilde{E})$. Thus $f^{-1}(\tilde{E}) = \cup \{ \tilde{V}_p : \tilde{p} \in f^{-1}(\tilde{E})\}$. By theorem 2.14 (i) $f^{-1}(\tilde{E})$ is an IS*O in X. Hence (ii).
(iii) ⇒ (iv). Let $\tilde{E}$ be an ISC in Y. Then by hypothesis, $f^{-1}(\tilde{E})$ is IS*O in X. Hence from lemma 3.11(i), we have $Icl^*(Iint(f^{-1}(\tilde{E}))) = Icl^*(f^{-1}(\tilde{E}))$.
(iv) ⇒ (v). Let $\tilde{U}$ be an IOS in Y, Then $Y - \tilde{U}$ is an ICS in Y. By assumption, $Icl^*(Iint(f^{-1}(Y - \tilde{U}))) = Icl^*(f^{-1}(Y - \tilde{U}))$. Therefore $[Icl^*(Iint(f^{-1}(Y - \tilde{U})))]^c = [Icl^*(f^{-1}(Y - \tilde{U}))]^c$. Hence $Iint^*(Icl(f^{-1}(\tilde{U}))) = Iint^*(f^{-1}(\tilde{U}))$.
(v) ⇒ (i). Let $\tilde{U}$ be an IOS in Y. Then by assumption, $Iint^*(Icl(f^{-1}(\tilde{U}))) = Iint^*(f^{-1}(\tilde{U}))$. Therefore by lemma 3.11 (ii), $f^{-1}(\tilde{U})$ is IS*C in X. Thus f is an intuitionistic contra semi * continuous.

Theorem 3.24 Every intuitionistic contra continuous is intuitionistic contra semi * continuous.

Proof: Let $f: X \to Y$ be the intuitionistic contra continuous function and $\tilde{U}$ be an IOS in Y. Then $f^{-1}(\tilde{U})$ is an ICS in X. Hence f is intuitionistic contra semi * continuous.

Theorem 3.25 Every intuitionistic contra semi * continuous is intuitionistic contra semi continuous.

Proof: Let $f: X \to Y$ be the intuitionistic contra semi * continuous function and $\tilde{U}$ be an IOS in Y. Then $f^{-1}(\tilde{U})$ is an IS*C in X. Hence f is intuitionistic contra semi continuous.

Theorem 3.26 Let (X, τ_1) and (Y, τ_2) be an ITS.

(i) If f: $X \to Y$ is intuitionistic contra semi * continuous and g: $Y \to Z$ is intuitionistic contra continuous, then $g \circ f$: $X \to Z$ is intuitionistic semi * continuous.

(ii) If f: $X \to Y$ is intuitionistic semi * continuous and g: $Y \to Z$ is intuitionistic contra continuous, then $g \circ f$: $X \to Z$ is intuitionistic contra semi * continuous.

(iii) If f: $X \to Y$ is intuitionistic contra semi * continuous and g: $Y \to Z$ is intuitionistic continuous, then $g \circ f$: $X \to Z$ is intuitionistic contra semi * continuous.

Proof: Let $\widetilde{U}$ be an IOS in Z. Then g^{-1} ($\widetilde{U}$) is an ICS in Y. Since f is intuitionistic contra semi * continuous, $(g \circ f)^{-1}(\widetilde{U}) = f^{-1}$ (g^{-1} ($\widetilde{U}$)) is IS*C in X. Thus $g \circ f$ is intuitionistic semi * continuous.

(ii) and (iii) can be proved in a similar way.

4. Conclusions

In this paper, we dealt with intuitionistic semi * continuous and intuitionistic contra semi * continuous. In future we wish to do our research work in intuitionistic semi * separated, intuitionistic semi * connected, intuitionistic semi * compact, intuitionistic semi * irresolute continuous function and so on.

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